Deep Learning - IIT Ropar

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Outline

Rank of a matrix

Definition: The number of independent rows or the number of independent columns in a matrix is called rank of a matrix

Q Find the Rank of given Matrix: $\begin{bmatrix} 1 & 2 & 3 \\ 1 & -1 & 0 \\ 2 & 1 & 3 \end{bmatrix}$ $R_3 = R_1 + R_2$

(A) 1

(B) 2

(C) 3

(D) 0

Rank of a matrix

Definition: The Highest order of Non-zero minor of matrix is called rank of a matrix.

$\rho(A)$: Rank of A

- det $|A_{n\times n}| \neq 0 \implies \rho(A) = order = n$
- If $|A_{n\times n}| = 0 \implies \rho(A) < order = n$
  - if $\exists$ a non zero minor of order (n-1), $\therefore \rho(A) = (n-1)$
  - If all the minor of order (n-1) are zeros then $\rho(A) < (n-1)$
  - if $\exists$ a non zero zero minor of order (n-2), $\therefore \rho(A) = (n-2)$. so on ...

Q Find the Rank of given Matrix: $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$

$|A| = 1(36-36) - 2(18-18) + 3(18-18) = 0$

$|A| = 0$, $n = 3$

$\rho(A) < n$

det(minors of A of order n-1) = 0 $\rho(A) < 2$

det(minors of A of order (n-2)=1) $\neq 0$

(A) 1

(B) 2

(C) 3

(D) 0

Rank of a matrix

Definition: The number of non zero rows in echelon form of a matrix.

Q Find the Rank of given Matrix: $A \begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$

(A) 1

(B) 2

(C) 3

(D) 4

$R_2 \to 2R_2 + R_1, \quad R_3 \to 2R_3 + R_1$

$$\begin{bmatrix} -2 & -1 & -3 & -1 \\ 0 & 3 & 3 & -3 \\ 0 & -1 & -1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$R_3 \to 3R_3 + R_2, \quad R_4 \to 3R_4 - R_2$

Row Echelon form of given matrix

$$\begin{bmatrix} -2 & -1 & -3 & -1 \\ 0 & 3 & 3 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \to 2 \text{ non-zero rows}$$

Eigen Values

Solving $\left|A - \lambda I\right| = 0$, i.e, the characteristics equation of A.

We get $\lambda_1, \lambda_2, ..., \lambda_n$ are the eigen values of A.

$|A - \lambda I| = 0$

$I \Rightarrow$ Identity matrix

Q: Find the Eigen value of A$= \begin{bmatrix} 1 & -1 & 5 \\ 0 & 5 & 6 \\ 0 & -6 & 5 \end{bmatrix}$

Soln

Solve $|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & -1 & 5 \\ 0 & 5-\lambda & 6 \\ 0 & -6 & 5-\lambda \end{vmatrix} = 0 \Rightarrow (1-\lambda)\left[(5-\lambda)^2 + 36\right] + 1(0-0) + 5(0-0)$$

$$(5-\lambda)^2 + 36 = 0$$

$$(5-\lambda)^2 = -36$$

$$5-\lambda = \pm 6i$$

$$\lambda = 5 \mp 6i$$

$$\Rightarrow (1-\lambda)(\lambda^2 - 10\lambda + 25 + 36) = 0$$

$$(1-\lambda)(\lambda^2 - 10\lambda + 61) = 0$$

$$\lambda = 1; \quad \lambda = 5 \mp 6i$$

Eigen values $\lambda_1 = 1, \lambda_2 = 5-6i, \lambda_3 = 5+6i$

~~Rank of a matrix~~ Probability

Definition: The probability of an event "E" is defined as number of outcomes in E divided by the total number of possible outcomes.

$$\boxed{P = \frac{N(E)}{N(S)}}$$

$N(E) \rightarrow$ no. of event in which E has occured

$N(S) \rightarrow$ Sample space or Total no. of events

Q: There are 5 red and 7 green balls in a bag. Three balls are drawn from it. Find the probability that 1 ball is red 2 balls are green.

(A) $\frac{1}{4}$

(B) $\frac{2}{35}$

(C) $\frac{3}{7}$

(D) $\frac{21}{44}$ ✓

$$P(E) = \frac{N(E)}{N(S)}$$

$\boxed{\begin{array}{l}\text{5 Red}\\ \text{7 Green}\end{array}}$

N(S) = No. of way in which 3 balls can be picked = ${}^{12}C_3$

N(E) = 1 balls is red & 2 balls are green can be choosen

$\downarrow$ ${}^{5}C_1$ $\quad$ $\downarrow$ ${}^{7}C_2$

Probability = $\dfrac{{}^{5}C_1 \times {}^{7}C_2}{{}^{12}C_3}$

$$= \frac{\frac{5!}{4! \times 1!} \times \frac{7!}{5! \times 2!}}{\frac{12!}{9! \times 3!}}$$

$$= \frac{\cancel{5}^{1} \times 21 \times \cancel{3} \times \cancel{2}}{\cancel{12} \times 11 \times \cancel{10}} = \frac{21}{44}$$

$\cancel{4}$ 2 $\qquad$ 2

Conditional Probability

Definition: If A and B are sequential Events, IF P(B) is given then Probability of event A is called conditional probability of A.

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

Probability of event A such that event B has occured = P(A/B)

Q A fair dice is rolled twice and obtain two numbers $X1=$ result of the first roll and $X2=$ result of the second roll. Given that $X1 + X2 = 7$, what is the probability that $X1 = 4$ or $X2 = 4$?

(A) $\frac{1}{3}$ ✓

(B) $\frac{1}{2}$

(C) $\frac{1}{18}$

(D) $\frac{1}{9}$

B : → $x_1 + x_2 = 7$: B: (1,6), (2,5), (3,4), (4,3), (5,2), (6,1)

$$B \left\{ \begin{array}{cc} x_1 & x_2 \\ 1, & 6 \\ 2, & 5 \\ 3, & 4 \\ 4, & 3 \\ 2, & 5 \\ 6, & 1 \end{array} \right\}$$

$P(B) = \frac{6}{36} = 1/6$

A: (3,4), (4,3)

NPTEL

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{2/36}{6/36} = \frac{2}{6} = 1/3$$

THANK YOU