Control System Engineering

[BASIC CONCEPTS IN CONTROL SYSTEM]

[Basic Concepts In Control System, Block Diagram Algebra]

Dt: 8/8/11

Basic Concept

→ **System:** A combination of different physical components that are connected or related together to form an entire unit to achieve a certain objective is called a system.

→ **Control:** The meaning of control is to regulate or direct or command a system so that a desired objective is reached.

→ **Control System:** It is a combination of different physical elements linked or connected in such a manner as to regulate or command itself to obtain a certain o/p.

Control System consist of:

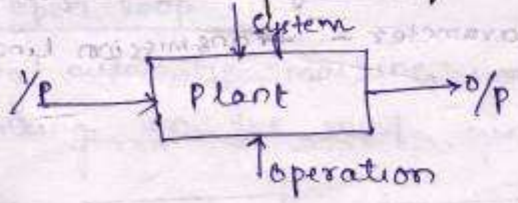
→ I/p (cause)

→ O/p (effect)

→ Way to achieve I/p & O/p.

→ Control action

Systematic Diagram of control system:



Plant: It is defined as a portion of the system which is to be controlled or regulated. It is also known as process

Controller :-

It is an element within the system itself or external to the system and it controls the plant

Disturbance :

External
Internal

Classification of control system :

- → Time Invariant – R, L, C
- → Time variant – AC, traffic light (change with time)

- → Linear → obeys ohm law
- → Non linear

- → Continuous
- → ~~Discontinuous~~ Discrete.

- → Deterministic
- → Stochastic

- → Lumped parameter – any electrical n/w.
- → Distributed parameter – transmission Line

- → SISO
- → MISO

- → Static
- → Dynamic

Deterministic : The response to I/P and to external disturbances of a control system is predictable and repeatable while viceversa is **stochastic system**.

Lumped parameter :-

It is a control system which can be represented by ordinary differential eqn.

Distributed parameter:

It is control system which can be represented by partial differential eqn.

Static:

O/P at any time depends on I/P at that time.

Dynamic:

present O/P depends on past I/P

Based on Control Action:

(a) Open loop control system

(b) closed loop control system.

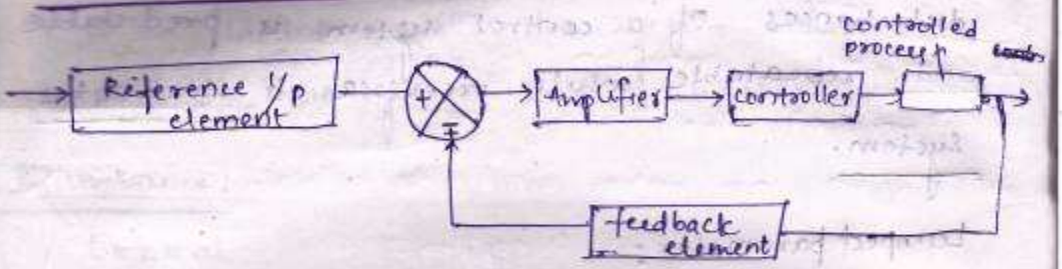
open loop C.S

Eg: automatic machine, immersion heater, a man walking on the road with his eyes closed.

I/P → [Controller] → [controlled problem] → O/P

Here O/P is not compared to I/P, simple system and no maintanance reqn and controlled O/p.

Closed loop C.S (feedback C.S)



Here O/p is compared to I/p, complex system.

open loop C.S	closed loop C.S
→ Not reliable	Reliable
→ Easier to build	Difficult to build
→ optimization is not possible	optimization is possible.

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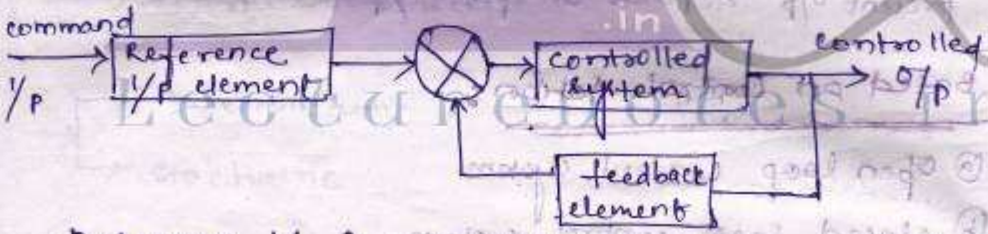
Components of closed loop system:

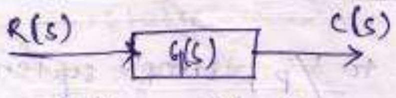


Reference I/p is proportional to command I/p

Error detector:

gives a error signal from a measured o/p signal by comparing with reference I/p element.

Transfer function:



$$G(s) = \left.\frac{C(s)}{R(s)}\right|_{I.C = 0.}$$

It is the ratio of laplace transform of o/p function and laplace transform of i/p function considering the initial condition is zero.

controlled o/p.

$$v_c(0^-) = v_c(0^+)$$

$$i_L(0^-) = i_L(0^+).$$

Laplace Transform:-

Let us consider a function

$s = \sigma + j\omega$

σ → neper frequency, $j\omega$ → radian frequency

s = frequency function.

$$L\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt = F(s)$$

$$L\{t^n\} = \frac{n!}{s^{n+1}}$$

Initial value Theorem:

$$\lim_{t \to 0} f(t) = \lim_{s \to \infty} sF(s)$$

Final value Theorem:

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$$

Derivative function:

$$L\{y'\} = sL\{y\} - y(0^-)$$

$$L\{y''\} = s^2 L\{y\} - sy(0^-) - y'(0^-)$$

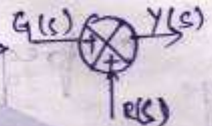
$$L\{y'''\} = s^3 L\{y\} - s^2 y(0^-) - sy'(0^-) - y''(0^-).$$

Block Diagram Algebra

It is the graphical representation of I/P & O/P behaviour of a system.

Important points

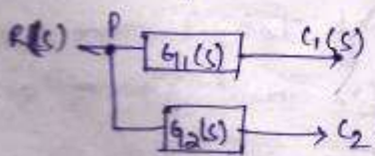
1) functional block
2) Summing point
3) Take off point
4) forward and feedback path

Summing point →  $Y(s) = G(s) + R(s)$.

Take off point:

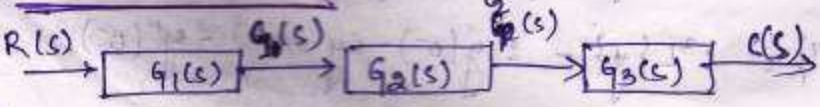
Take off point on a branch is a physical point in the system where the desired signal is tapped off to utilise elsewhere.

P can be called as take off point where application of one I/P source to two or more systems is represented by take off point.



Block diagram rearrangement (Reduction):

(i) Block in cascade:



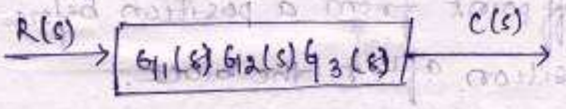
$$G_1(s) = \frac{C_1(s)}{R(s)}$$

$$G_2(s) = \frac{C_2(s)}{C_1(s)}$$

$$G_3(s) = \frac{C(s)}{C_2(s)}.$$

$$G_1(s)\,G_2(s)\,G_3(s) = \frac{C(s)}{R(s)}$$

Single block representation:

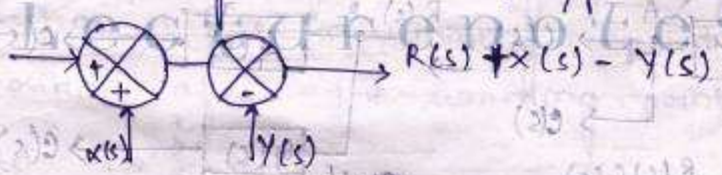


(ii) Summing point:

① 

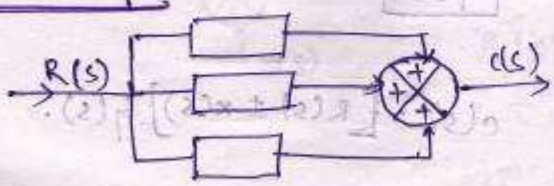
$C(s) = R(s) + X(s) - Y(s).$

② The consecutive summing point can be interchange so as the o/p remains constant.



③ Interchanging the summing point.

(III) Block in parallel:-



$$C(s) = R(s)\,G_1(s) + R(s)\,G_2(s) + R(s)\,G_3(s)$$

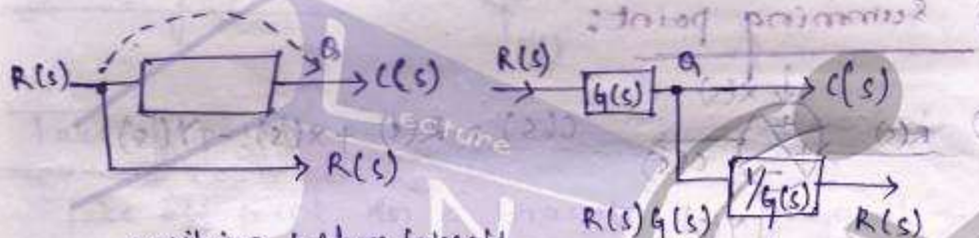
$$C(s) = R(s)\left[G_1(s) + G_2(s) + G_3(s)\right]$$

$$\frac{C(s)}{R(s)} = G_1(s) + G_2(s) + G_3(s).$$

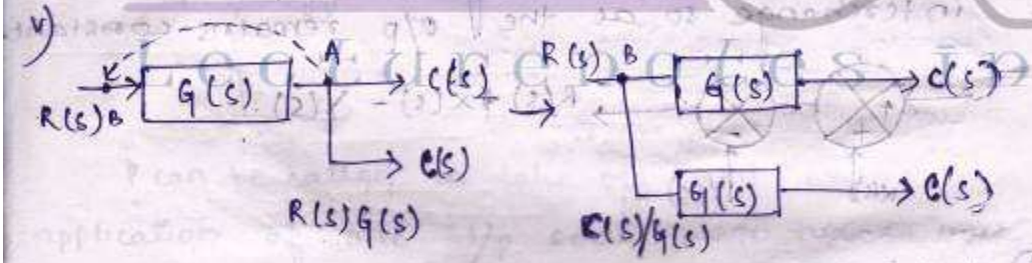
Single block representation:

$$R(s) \rightarrow \boxed{G_1(s)+G_2(s)+G_3(s)} \rightarrow C(s)$$

iv) shifting a takeoff point from a position before the block to a position after the block.



position before takeoff   position after takeoff



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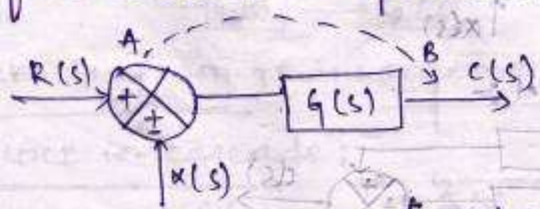
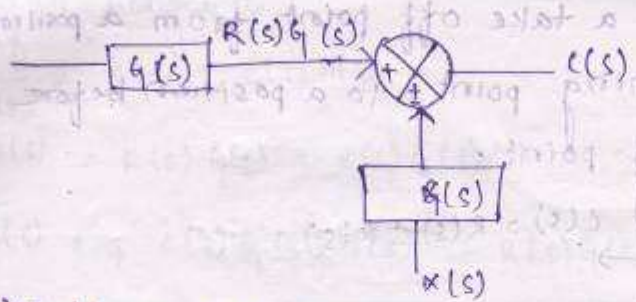
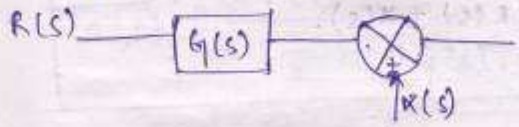
(vi) shifting of a summing point from a position before a block to position after block.



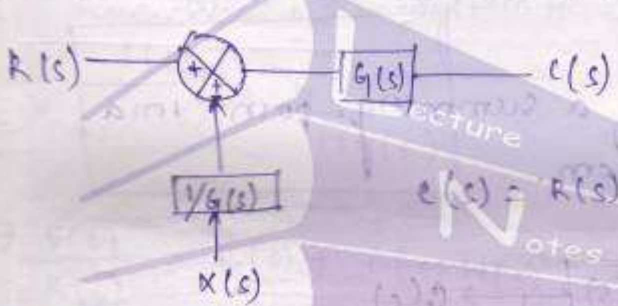
$$C(s) = \left[R(s) \pm X(s)\right] G(s).$$



(VII) Shifting of a summing point from a position after the block to position before the block.

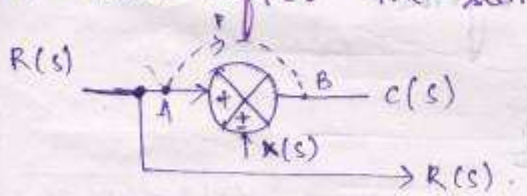


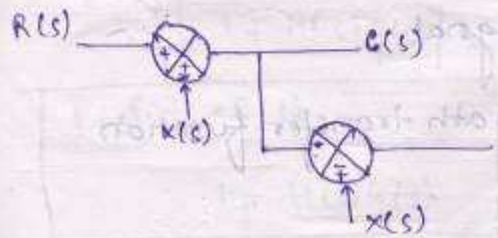
$C(s) = R(s)G(s) \pm X(s)$



$C(s) = R(s)G(s) \pm X(s)$.

(VIII) Shifting of a take off point from a position before a summing point to a position after the summing point.

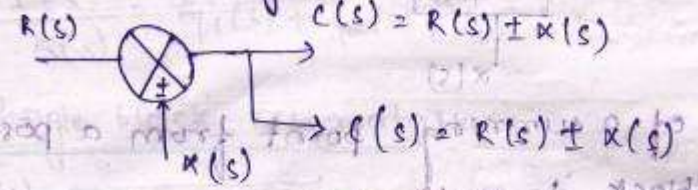


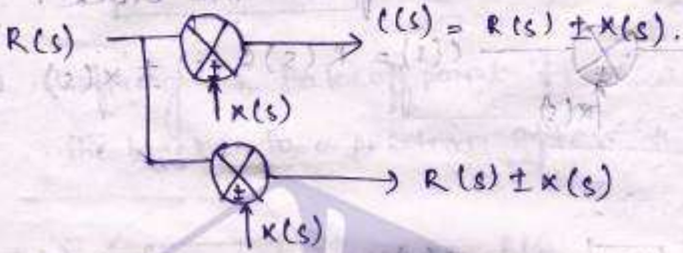


$R(s) \pm X(s) \mp X(s)$

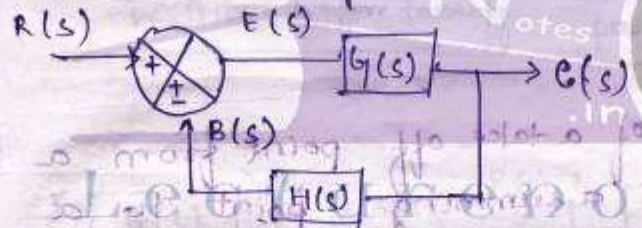
$= R(s)$.

ix) Shifting of a take off point from a position after the summing point to a position before the summing point.





x) Elimination of a summing point in a closed loop system.



$R(s)$ = Reference i/p signal

$C(s)$ = o/p signal

$E(s)$ = error signal

$G(s)$ = forward path transfer function.

$B(s)$ = feedback signal

$H(s)$ = feedback path transfer function

$B(s) = H(s)\,C(s)$

$E(s) = R(s) \pm B(s)$

$C(s) = E(s)\,G(s)$.

Eqⁿ ① changes to

$$\frac{C(s)}{G(s)} = R(s) \pm C(s)H(s)$$

$$\Rightarrow C(s) = R(s)G(s) \pm C(s)G(s)H(s)$$

$$\Rightarrow C(s) \mp C(s)G(s)H(s) = R(s)G(s)$$

$$\Rightarrow C(s)\left[1 \mp G(s)H(s)\right] = R(s)G(s)$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1 \mp H(s)G(s)}}$$

when $H(s) = 1$, the system is unity feed back system

$$\boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1 \mp G(s)}}$$

$$\rightarrow \frac{E(s)}{R(s)} = \frac{R(s) \pm C(s)H(s)}{R(s)} = 1 \pm \frac{H(s)C(s)}{R(s)}$$

$$= 1 \pm \frac{H(s)E(s)G(s)}{R(s)}$$

$$= \frac{R(s) \pm H(s)E(s)G(s)}{R(s)}$$

$$\Rightarrow E(s) = R(s) \pm H(s)E(s)G(s)$$

$$E(s)\left[1 \mp H(s)G(s)\right] = R(s)$$

$$\boxed{E(s) = \frac{R(s)}{1 \mp H(s)G(s)}}$$ Error ratio.

$$B(s) = C(s)\,H(s)$$
$$= E(s)\,G(s)\,H(s)$$

$$\boxed{\frac{B(s)}{E(s)} = G(s)\,H(s)} \longrightarrow \text{open loop transfer function}$$

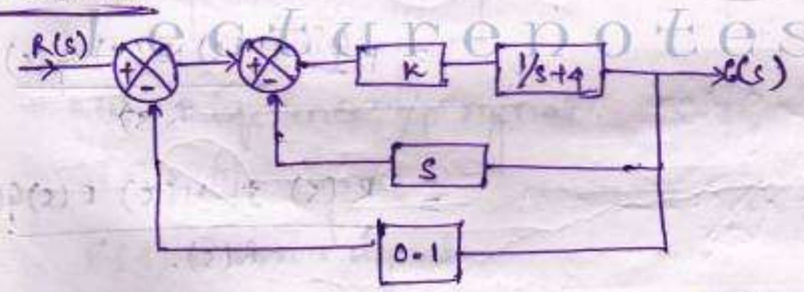
$\rightarrow$ $B(s) = G(s)\,H(s)$

$$= E(s)\,G(s)\,H(s)$$
$$= [R(s) \pm B(s)]\,G(s)\,H(s)$$
$$= R(s)\,G(s)\,H(s) \pm B(s)\,G(s)\,H(s)$$

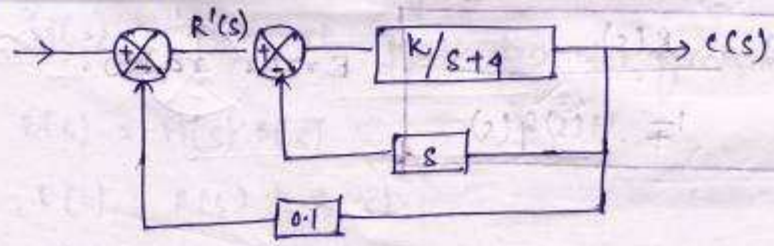
$$B(s)[1 \mp G(s)\,H(s)] = R(s)\,G(s)\,H(s)$$

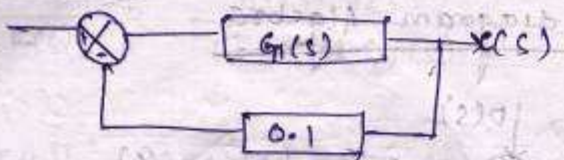
$$\boxed{\frac{B(s)}{R(s)} = \frac{G(s)\,H(s)}{1 \mp G(s)\,H(s)}} \longrightarrow \text{primary feedback ratio.}$$

Problem 1

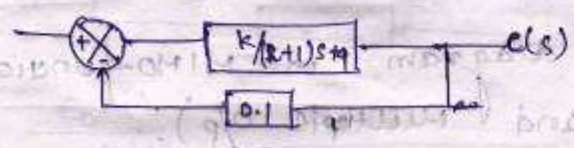


Reduce the block diagram and find out the T.F of total system.





$$\frac{C_1(s)}{R'(s)} = \frac{k/s+4}{1+\frac{sk}{s+4}} = \frac{k}{s+4+sk}$$

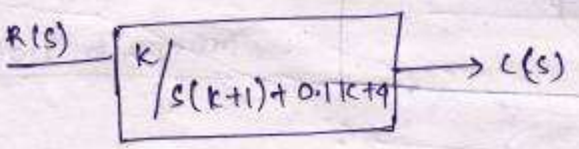


$$G(s) = \frac{k}{s(k+1)+4} \qquad H(s) = 0.1$$

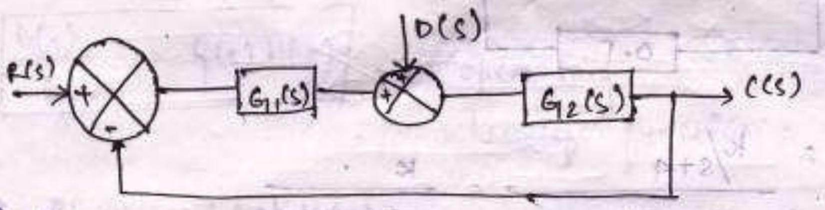
$$\frac{C(s)}{R(s)} = \frac{\frac{k}{s(k+1)+4}}{1+\frac{0.1k}{s(k+1)+4}}$$

$$= \frac{k}{s(k+1)+0.1k+4}.$$

So



Problem 3 — Block diagram Algebra — 16/8/11



Reduce the block diagram and find out the Transfer function.

The above block diagram is MIMO-function (Multiple i/p and Multiple o/p).

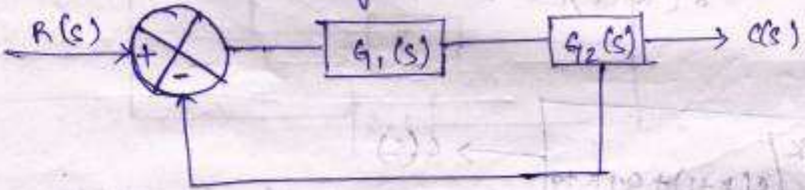
Rule 11

When two or more input act on a system Then total output is obtained by adding the effect of each individual input separately.

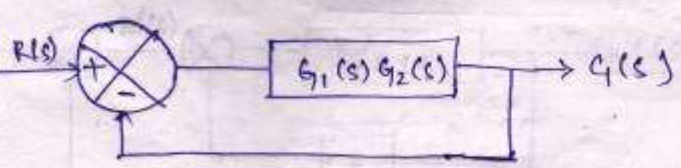
$C(s) = C_1(s) + C_2(s)$

$= R_1(s)G(s) + R_2(s)G(s)$.

Step 1 Considering R(s)

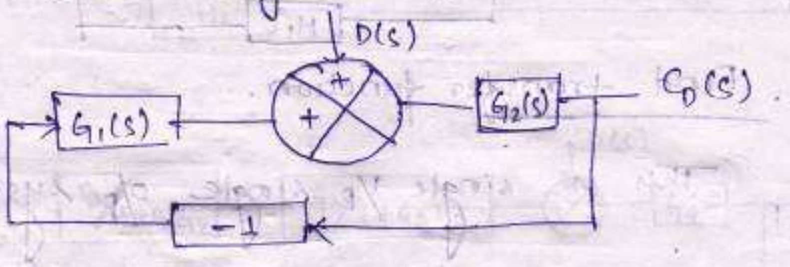


~~Step 2~~



$$\frac{G(s)}{R(s)} = \frac{G_1(s)\,G_2(s)}{1+G_1(s)\,G_2(s)\cdot 1} = \frac{G_1(s)\,G_2(s)}{1+G_1(s)G_2(s)}.$$

Step II Considering D(s)



$$\frac{C_D(s)}{D(s)} = \frac{G_2(s)}{1-G_2(s)G_1(s)(-1)}$$

$$= \frac{G_2(s)}{1+G_2(s)G_1(s)}.$$

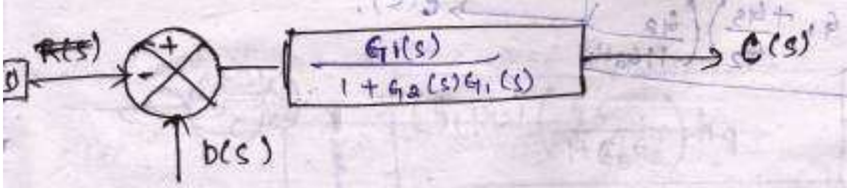
Step III

Total o/p $C(s) = C_R(s)\,\cancel{R(s)} + C_D(s)\,\cancel{D(s)}$

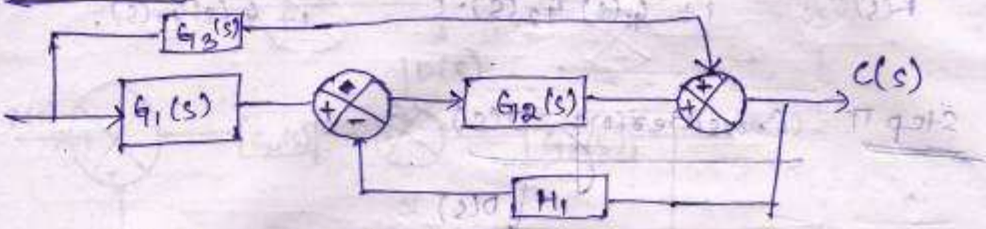
$$= \frac{G_2(s)G_1(s)\,R(s)}{1+G_2(s)G_1(s)} + \frac{G_2(s)\,D(s)}{1+G_2(s)G_1(s)}$$

$$= \frac{G_2(s)\left[G_1(s)R(s)+D(s)\right]}{(1+G_2(s)G_1(s))}$$

Block diagram



~~Block~~ Problem 4

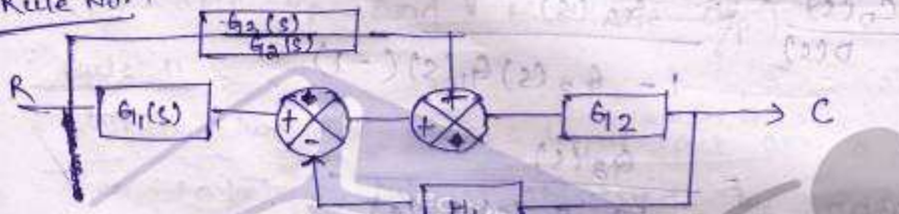


Find transfer function.

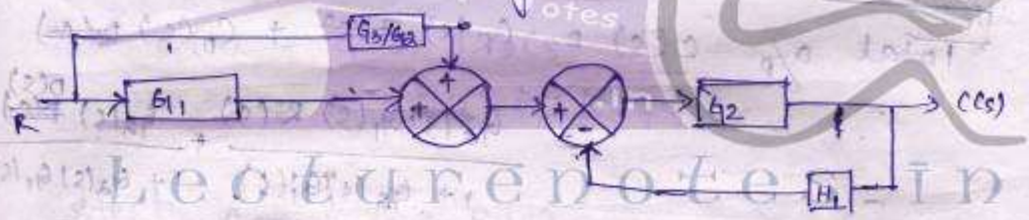
Sol^n

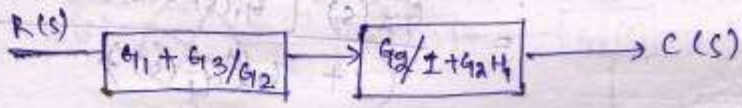
Step 1 This is single i/p single o/p system.

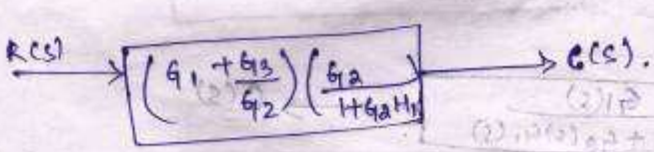
Rule No. 4



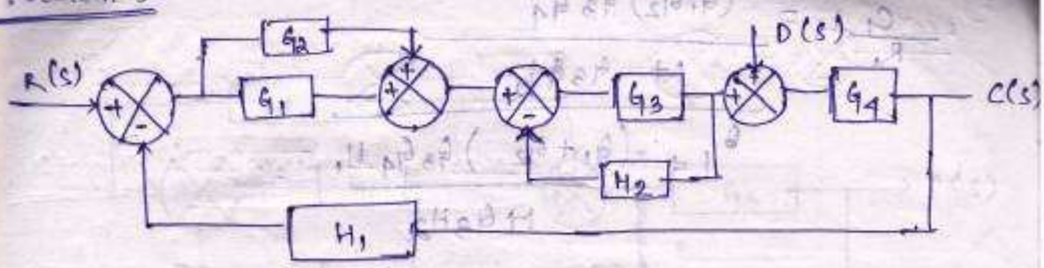
→ Consecutive summing point can be interchanged

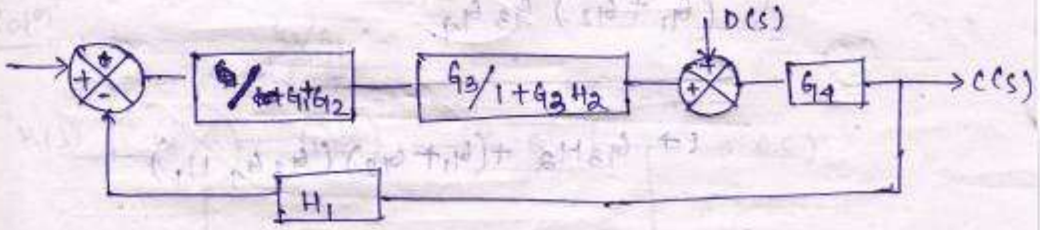


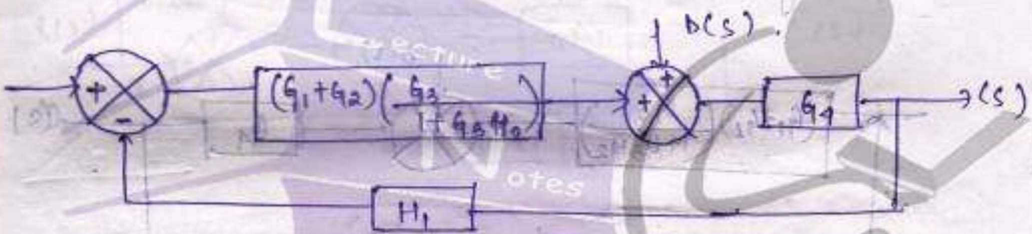




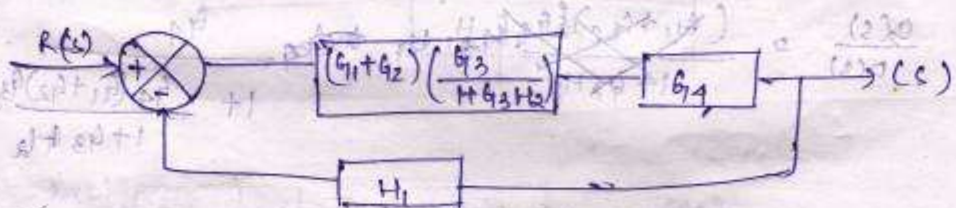
Problem 5

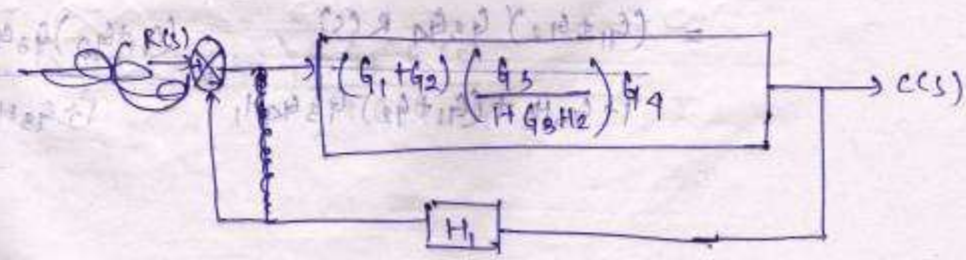






Considering R(s)



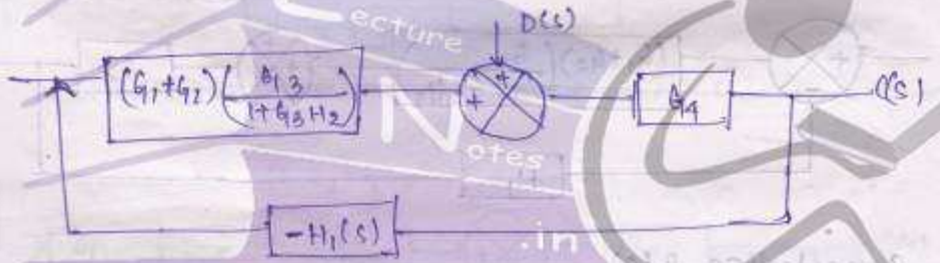


$$R(s) \longrightarrow \boxed{\frac{(G_1+G_2)G_3G_4}{1+G_3H_2} \Big/ 1+\frac{(G_1+G_2)G_3G_4H_1}{1+G_3H_2}} \longrightarrow C(s)$$

$$\frac{C_1}{R_1} = \frac{\dfrac{(G_1+G_2)G_3G_4}{1+G_3H_2}}{1+\dfrac{(G_1+G_2)G_3G_4H_1}{1+G_3H_2}}$$

$$= \frac{(G_1+G_2)G_3G_4}{1+G_3H_2+(G_1+G_2)(G_3G_4H_1)}$$

Considering D(s)

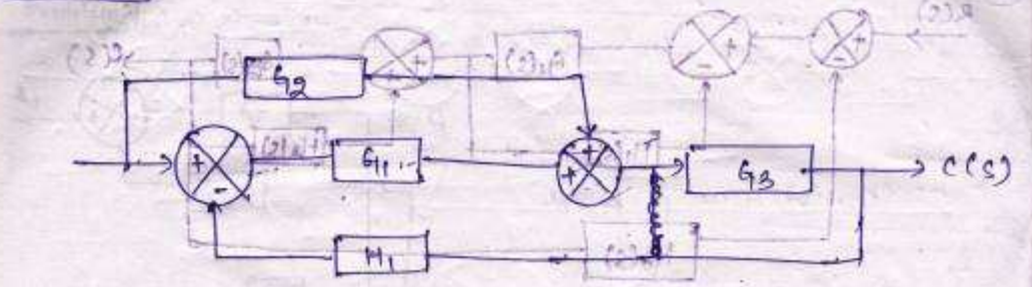


$$\frac{C_2(s)}{D(s)} = \cancel{\frac{(G_1+G_2)G_3G_4H_1}{1+G_3H_2}} = \frac{G_4}{1+\dfrac{G_4(G_1+G_2)G_3\times(-H_1)}{1+G_3H_2}}$$

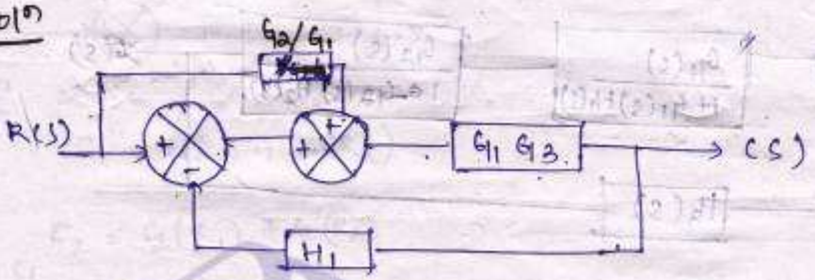
So, $C(s) = C_1(s) + C_2(s)$

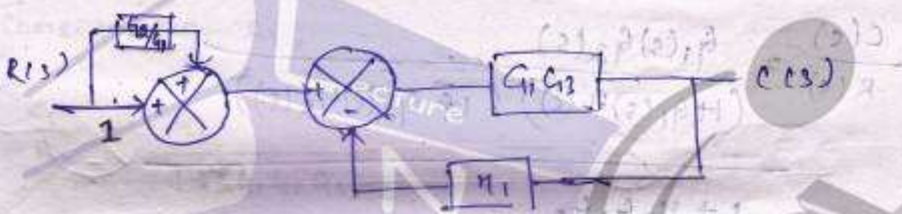
$$= \frac{(G_1+G_2)G_3G_4R(s)}{1+G_3H_2+(G_1+G_2)G_3G_4H_1} + \frac{(G_1+G_2)G_3G_4H_1}{1+G_3H_2}$$

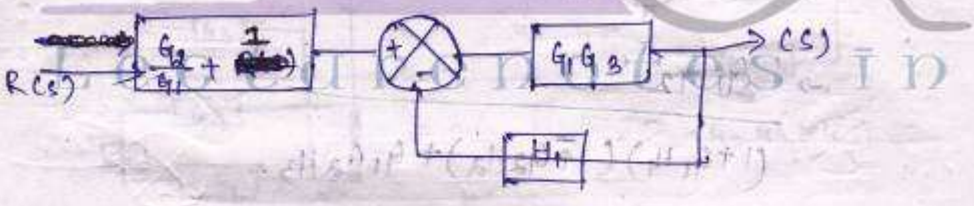
Problem 6



Soln

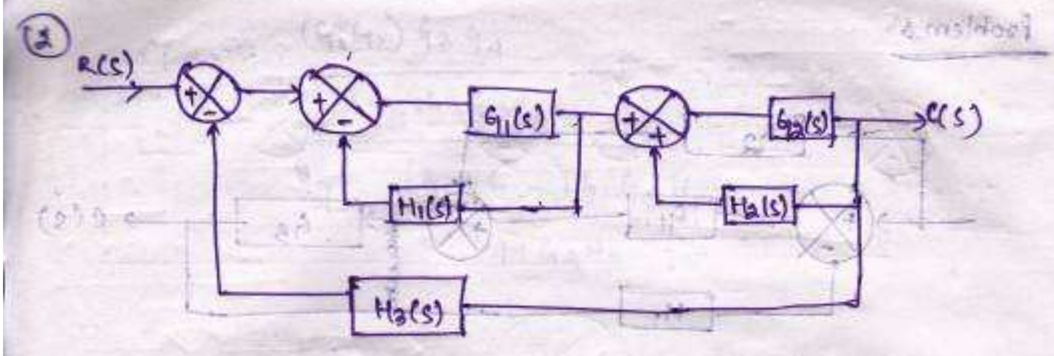


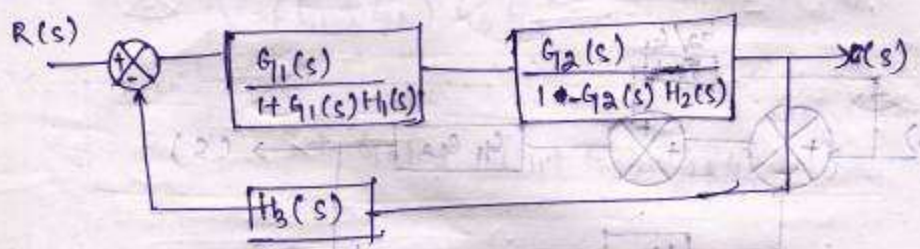




$$C(s) = \left(\frac{G_2}{G_1} + 1\right) \frac{G_1 G_3 \, R(s)}{1 + G_1 G_3 H_1}$$

(2)

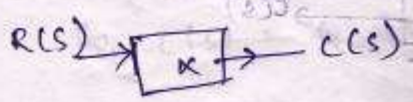




$$\frac{C(s)}{R(s)} = \frac{\dfrac{G_1(s)G_2(s)}{(1+G_1(s)H_1(s))(1\mp G_2(s)H_2(s))}}{1+\dfrac{H_3G_1G_2}{(1+G_1H_1)(1\mp G_2H_2)}}$$

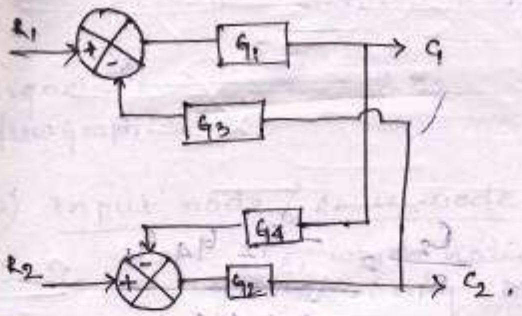
$$= \frac{G_1G_2}{(1+G_1H_1)(1\mp G_2H_2)+G_1G_2H_3}.$$

$= K$



Dt: 19/08/2011

Problem 4



Find C_1 and C_2.

Soln

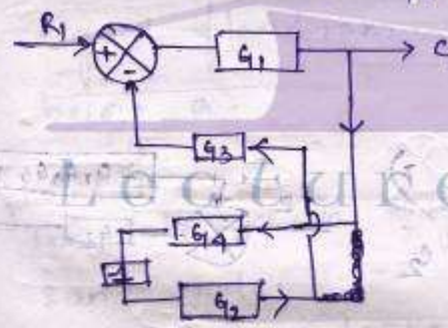
$$C_1 = C_1(R_1) + C_1(R_2)$$

$$C_2 = C_2(R_1) + C_2(R_2)$$

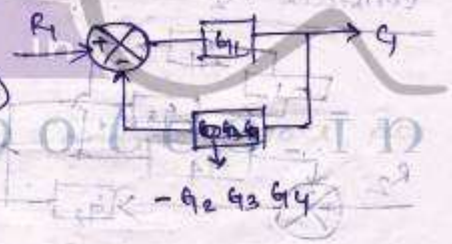
for C_1

Considering R_1,

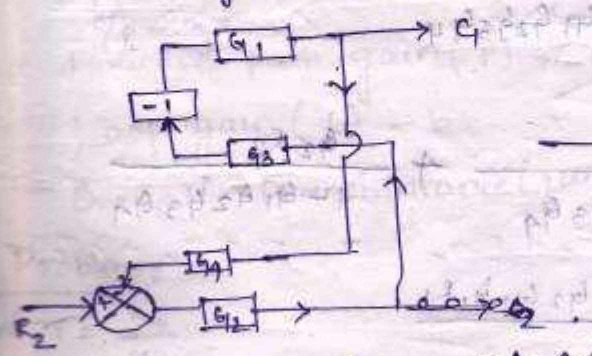
$$\frac{C_1}{R_1} = \frac{G_1}{1 - G_1 G_2 G_3 G_4}$$

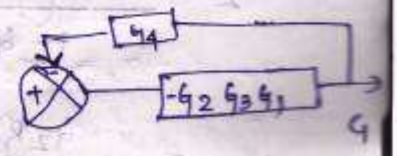


2)



Considering R_2



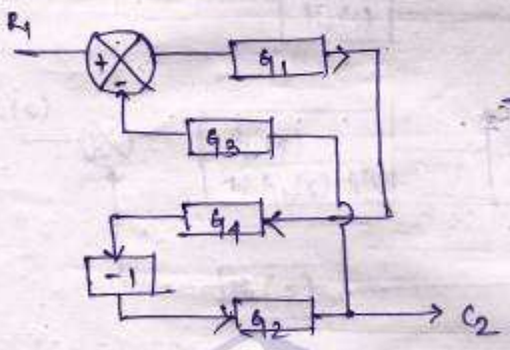


$$\text{So, } \frac{C_1}{R_2} = \frac{-G_2 G_3 G_1}{1 - G_2 G_3 G_1 G_4}$$

$$C = \frac{G_1 R_1}{1 - G_1 G_2 G_3 G_4} + \frac{-G_2 G_3 G_1 R_2}{1 - G_1 G_2 G_3 G_4}$$

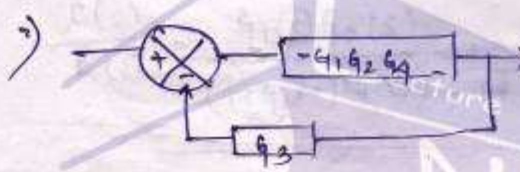
For C_2

consider R_1

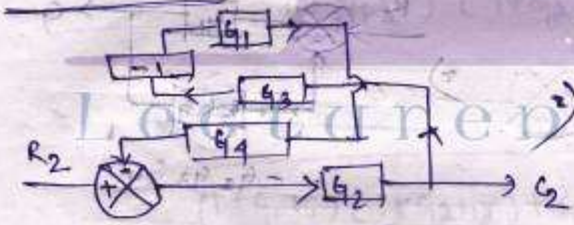


$$\frac{C_2}{R_1} = \frac{-G_2 G_4 G_1}{1 - G_1 G_2 G_3 G_4}.$$

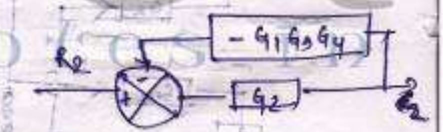
i)



consider R_2



ii)



$$\frac{C_2}{R_2} = \frac{G_2}{1 - G_1 G_2 G_3 G_4}$$

$$C_2 = \frac{-G_2 G_4 G_1 R_1}{1 - G_1 G_2 G_3 G_4} + \frac{G_2 R_2}{1 - G_1 G_2 G_3 G_4}$$

$$= \frac{G_2 R_2 - G_2 G_4 G_1 R_1}{1 - G_1 G_2 G_3 G_4}$$