



$$T = \frac{1}{2} M V_1^2 + \frac{1}{2} m |\vec{v}_2|^2$$

$$x_2 = x_1 - l \cdot \sin \theta$$

$$y_2 = l \cdot \cos \theta$$

$$\vec{v}_2 = \frac{d}{dt}(x_2, y_2) = (\dot{x}_1 - l \cos \theta \dot{\theta}, -l \sin \theta \dot{\theta})$$

$$|\vec{v}_2|^2 = (\dot{x}_1 - l \cos \theta \dot{\theta})^2 + (l \sin \theta \dot{\theta})^2$$

$$= \dot{x}_1^2 - 2l \dot{x}_1 \cos \theta \dot{\theta} + l^2 \cos^2 \theta \dot{\theta}^2 + l^2 \sin^2 \theta \dot{\theta}^2$$

$$= \dot{x}_1^2 - 2l \dot{x}_1 \dot{\theta} \cos \theta + l^2 \dot{\theta}^2$$

$$T = \frac{1}{2} M \dot{x}^2 + \frac{1}{2} m (\dot{x}^2 - 2l \dot{x} \dot{\theta} \cos \theta + l^2 \dot{\theta}^2) = \frac{1}{2} (\tilde{M} + m) \dot{x}^2 - m l \dot{x} \dot{\theta} \cos \theta + \frac{1}{2} m l^2 \dot{\theta}^2$$

$$U = -\sin \theta l g m$$

$$\mathcal{L} = T - U$$

$$\text{states: } x, \dot{x}, \theta, \dot{\theta}$$

$$\frac{d}{dt} \left( \frac{\partial \mathcal{L}}{\partial \dot{x}} \right) - \frac{\partial \mathcal{L}}{\partial x} = Q_x$$

$$\mathcal{L} = \frac{1}{2} \tilde{M} \dot{x}^2 - m l \dot{x} \dot{\theta} \cos \theta + \frac{1}{2} m l^2 \dot{\theta}^2 + \sin \theta l g m$$

$$x: \frac{\partial \mathcal{L}}{\partial \dot{x}} = \tilde{M} \dot{x} - m l \dot{\theta} \cos \theta \xrightarrow{\frac{d}{dt}} \tilde{M} \ddot{x} - m l \ddot{\theta} \cos \theta + m l \dot{\theta}^2 \sin \theta$$

$$\frac{\partial \mathcal{L}}{\partial x} = 0 \Rightarrow \tilde{M} \ddot{x} - m l \ddot{\theta} \cos \theta + m l \dot{\theta}^2 \sin \theta = F$$

$$\theta: \frac{\partial \mathcal{L}}{\partial \dot{\theta}} = -m l \dot{x} \cos \theta + m l^2 \dot{\theta} \xrightarrow{\frac{d}{dt}} -m l \ddot{x} \cos \theta + m l \dot{x} \sin \theta \dot{\theta} + m l^2 \ddot{\theta}$$

$$\frac{\partial \mathcal{L}}{\partial \theta} = m l \dot{x} \sin \theta + \cos \theta l g m \Rightarrow -m l \ddot{x} \cos \theta + m l \dot{x} \sin \theta \dot{\theta} + m l^2 \ddot{\theta} - m l \dot{x} \sin \theta - \sin \theta l g m = 0$$

$$x: (\tilde{M} + m) \ddot{x} - m l \ddot{\theta} \cos \theta + m l \dot{\theta}^2 \sin \theta = F$$

$$\theta: l \ddot{\theta} - \sin \theta g = \ddot{x} \cos \theta$$