Modeling and forecasting of electricity prices and demand

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Abstract

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Keywords: electricity spot prices, day-ahead market, forecasting, power market

1 Introduction

2 Market analysis

Danish power market has been transformed in the last several years drastically. Since 70s there was a lot of investments in renewable sources of energy, especially in the field of wind power, and much more since 2002 when first large scale offshore wind farm in the world has been finished - Horns Rev 1 (160 MW). For year 2019, total wind power generation capacity was 6128 MW[5].

National target for 2020 is over 50% of a energy consumption covered by wind power and it's likely to be achieved, as in 2019 they obtained 47% of coverage by domestic production[2]. Moreover they have finished construction of next large scale wind farm Horns Rev 3 in August 2019[2]. There are also defined next goals in last presented national energy strategy. For wind power consumption they aim for 70% in 2030[2]. Denmark is currently leader of wind power shares in the national production and its development.

The production of such significant part of energy from wind carries some risk. Wind speed is very fluctuant and variable even in a day cycle. There are no perfect methods of forecasting in the long term periods. It may occur in higher (or lower) demand in production from other, stable sources of energy to cover consumption. It's problematic to mark a common trend in wind power forecasts1. Thus it's helpful to correct prediction of electricity demand.

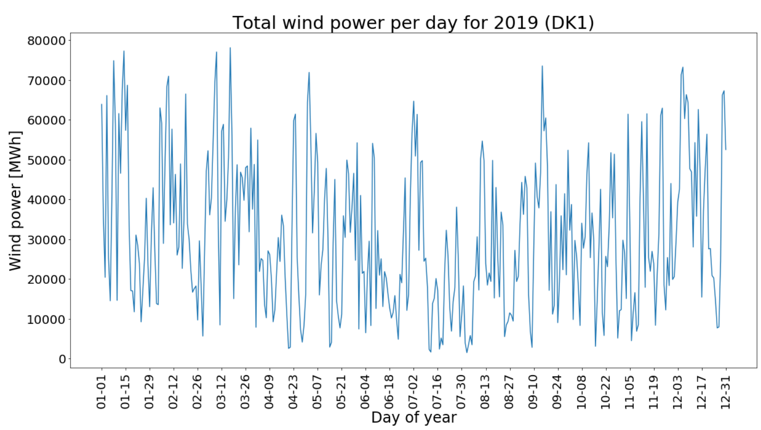

Figure 1: Wind power produced per day in 2019 (DK1)

Wind power generation can't be stopped quickly as production of conventional sources of energy (coal, gas). Because of that higher unexpected wind power production may lead to decreased prices on energy stocks and in some cases, prices can drop below zero. That was happening rarely in the era of conventional production and regulated electricity market, but it happens more often with more renewable power generators. Because energy network is quite connected with each other in the European Union, in a case of negative prices energy from Danish areas (DK1, DK2) is exported to the neighboring countries, mainly to the Germany, which Denmark has the biggest balance of energy export and import.

Negative prices obligate to use other approaches of price forecasting, than the old ones which were failing with unexpected domain of values. In my work I will present a few models to forecast price and consumption and point out the best approach to have optimal forecasts.

The electricity market in Denmark is divided into 2 areas (DK1 and DK2). First area (DK1) consists of regions: Nordjylland, Midtjylland and Syddanmark; second area (DK2) consists of regions Sjælland and Hovedstaden with the capital Copenhagen.

Figure 2: Danish electricity market

3 Data analysis

Data I used to perform forecasts has been downloaded from the official webpage of the Nordpool power exchange[4]. Datasets are divided into the year files and periods (hours, weeks etc.). I managed to download following datasets (valid for the day 14.05.2020):

- Consumption - hourly
- Consumption prognosis - hourly
- Wind power - hourly
- Wind power prognosis - hourly
- Elspot prices (as Price) - hourly

All of the datasets were available for years 2013-2020, except for Consumption prognosis (2015-2020). So I decided to focus on analysis only on the period 2015-2020 (2015.01.01-2020.05.12), because 4 years time frame is still sufficient for calculations.

Units of downloaded data are following:

- Consumption and Wind Power- MWh
- Price - DKK/MWh

The files were downloaded, merged, split for regions DK1 and DK2, pivoted in order to have separated hours as parameters for each day and merged for all years. Example for consumption DK1 is presented below.

	date	holiday	0	1	2	...	22	23
0	2016-01-01	1	1818.0	1741.0	1660.0	...	1858.0	1713.0
1	2016-01-02	0	1615.0	1510.0	1461.0	...	2027.0	1822.0
2	2016-01-03	1	1724.0	1665.0	1671.0	...	2127.0	1998.0
3	2016-01-04	0	1844.0	1803.0	1789.0	...	2293.0	2079.0
4	2016-01-05	0	1940.0	1891.0	1952.0	...	2372.0	2193.0

Table 1: First 5 rows of merged file Consumption DK1.

I performed a few analysis for each dataset, although I didn't include all of the charts and the tables in this work. The rest is uploaded into the github repository.

3.1 Missing values

Data was very consistent and yet only single values were missing. These null values were replaced by average of the neighboring cells and in case of missing value in neighbor cell, value was fixed manually (with file fill_empty_cells.py). Half of day 2018-09-18 from Wind prognosis files was filled taking closest neighbors and counting average for whole vector (with file fill_empty_cells_wind_prognosis_DK.py). Number of missing values was reduced to zero.

Dataset	DK1	DK2
Consumption	5	5
Consumption prognosis	5	5
Price	21	12
Wind power	12	6
Wind power prognosis	18	19

Table 2: Missing values in files.

3.2 Consumption data

Elictricity consumption was higher in the area DK1 than DK2, compared for years 2016-2019 19.14, 19.41, 20.28, 20.37 TWh to 13.13, 13.03, 13.28, 13.16 TWh accordingly. We can see that consumption increased gradually in the area DK1, meanwhile in area DK2 was on the similar level.

We can spot three types of seasonal trends in the data: annual, weekly and daily. On the below chart showing consumption per day in 2019, we see that every weekend consumption value drops.

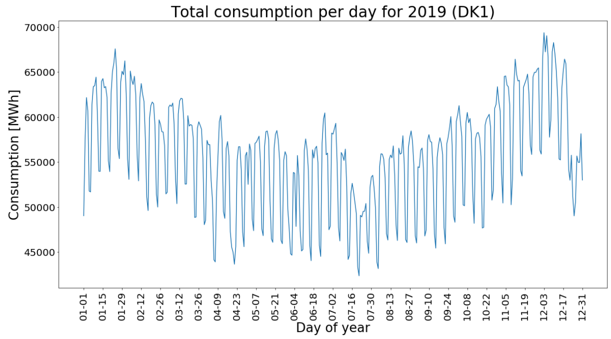

Figure 3: Consumption per day in 2019 (DK1)

Although it's not easy to spot in the DK1, there is annual trend with lower consumption during summer months and higher during winter. It's observable particularly in the DK2.

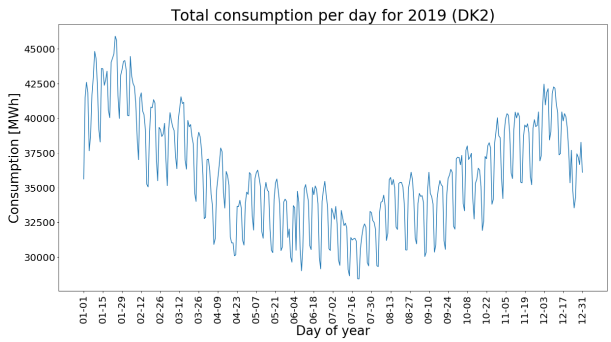

Figure 4: Consumption per day in 2019 (DK2)

Simple moving average with 14 days windows for each year shows clearly this trend. Although in the area DK1 it's not very sharp.

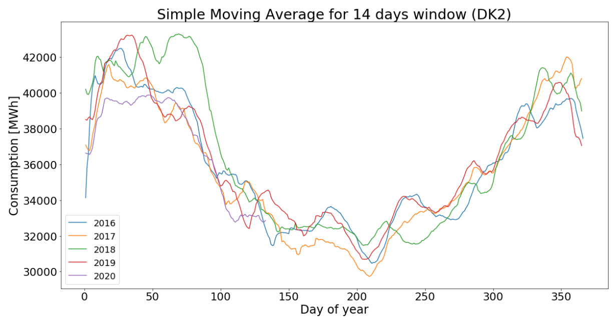

Figure 5: Simple moving average of consumption - 14 days window (DK2)

Last seasonal trend is daily which can be observed for each day of a week, even holidays. There are two peaks of energy consumption each day, in the morning and evening. During weekends and holidays, morning peaks are slightly shifted than during work days. There is also noticeable smaller consumption in the night.

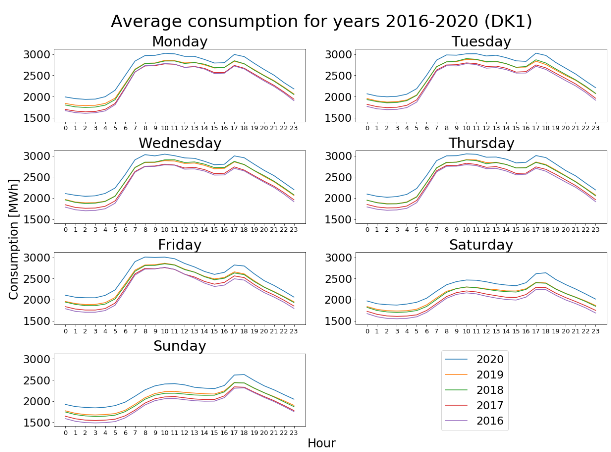

Figure 6: Average consumption per each hour of day for each day of week (DK1)

There is also one interesting thing observed in the 2020's data only in the area DK2. Evening daily peak is slightly shifted which can be caused epidemic COVID-19 or incomplete data of 2020 year.

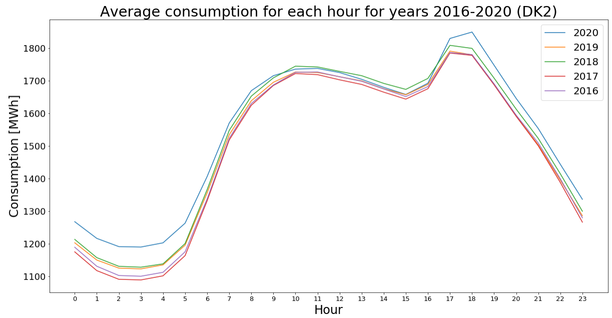

Figure 7: Average consumption per each hour of day for each year (DK2)

3.2.1 Nordpool's prognosis

The prognosis day-ahead given by Nordpool shows that those are less accurate over time, especially for area DK1 where consumption is larger than DK2. Level of accuracy for area DK2 is quite stable. Also there is no clear trend regarding the time of day. For DK1 best prediction are for night and for DK2 best prediction are performed for hours 7 and 15.

Area	Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	Total
DK1	2016	18.7	15.2	13.2	12.7	12.3	14.1	18.7	19.5	20.3	19.6	19.6	19.7	20.4	19.0	16.5	16.9	16.0	18.7	17.7	15.9	16.0	15.0	13.7	17.3	16.9
	2017	16.1	24.9	19.0	13.9	14.1	15.1	18.7	18.7	19.1	19.9	19.8	17.8	16.3	15.4	15.1	16.1	21.1	22.1	20.7	17.2	18.0	17.5	19.4	22.5	18.3
	2018	18.8	26.7	17.7	15.9	14.1	15.9	20.6	24.8	21.0	23.6	21.3	20.1	22.5	21.0	20.9	19.8	19.2	21.7	18.3	19.9	24.9	22.5	23.1	25.8	20.8
	2019	22.0	29.3	21.8	17.9	17.2	18.1	23.0	24.9	22.0	27.3	24.7	22.7	23.0	20.6	19.2	19.2	20.2	21.5	22.6	22.4	25.2	24.0	25.1	30.3	22.7
	2020	21.7	26.9	23.4	20.5	16.8	20.5	34.8	34.2	26.4	32.8	32.9	22.6	22.2	27.4	24.6	22.2	22.3	36.1	32.4	23.6	26.4	24.5	27.7	32.4	26.5
	Total	19.1	24.3	18.4	15.5	14.6	16.2	21.4	23.0	21.1	23.5	22.3	20.3	20.7	19.7	18.5	18.3	19.4	22.2	20.9	19.2	21.5	20.1	20.9	24.7	20.2

Figure 8: MAE for consumption for each hour and year with total figures (DK1)

Area	Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	Total
DK2	2016	23.8	19.6	19.0	20.3	22.4	22.9	20.9	17.1	23.6	27.6	24.3	21.4	24.1	22.0	20.0	19.2	23.3	26.4	24.5	22.9	24.5	24.6	20.7	15.9	22.1
	2017	19.6	21.9	21.7	20.2	18.8	18.5	22.7	16.8	20.2	21.8	21.6	20.8	20.2	20.5	20.6	16.7	18.0	24.4	23.3	21.2	20.8	22.8	22.8	27.4	21.0
	2018	18.1	25.2	24.7	20.8	19.5	20.2	27.5	18.8	21.2	24.8	26.8	26.7	27.0	26.1	22.4	17.5	20.7	24.7	22.5	22.4	26.8	29.1	25.9	23.2	23.4
	2019	16.1	23.9	22.8	17.5	16.3	15.9	21.2	14.3	16.9	17.9	21.2	21.6	18.6	17.5	17.1	15.3	18.7	24.5	22.1	18.8	22.2	22.6	21.1	22.7	19.5
	2020	17.4	18.2	15.8	12.4	15.0	17.6	25.5	22.1	24.4	31.0	36.4	27.6	23.5	26.0	21.7	13.9	15.9	28.1	27.5	18.3	17.2	17.1	14.6	18.9	21.1
	Total	19.2	22.3	21.5	19.1	18.9	19.2	23.3	17.2	20.8	23.7	24.5	23.0	22.6	21.9	20.1	16.9	19.8	25.3	23.5	21.1	23.1	24.1	22.0	22.0	21.5

Figure 9: MAE for consumption for each hour and year with total figures (DK2)

3.3 Wind power data

Wind power production increased significantly within last 4 years. In the area DK1 from 9.41 to 11.26 TWh and for area DK2 from 2.37 to 3.22 TWh, so about 35% more.

So far in the consumption data we could spot trends whereas in the wind power data there is no distinct trend.

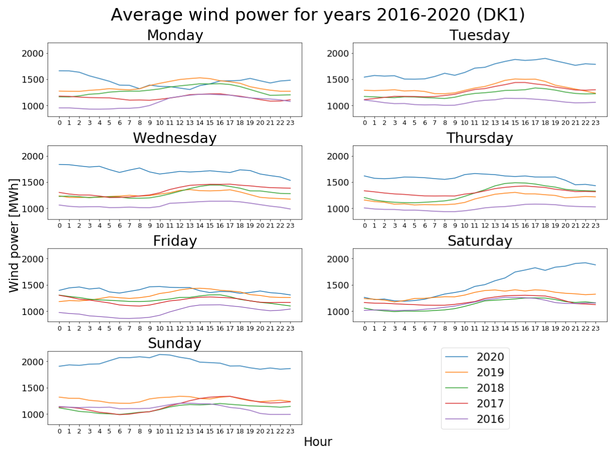

Figure 10: Wind power production per hour of a day for each day of week (DK1)

Simple moving average also doesn't show anything recurrent, thus we can't assume any annual trend.

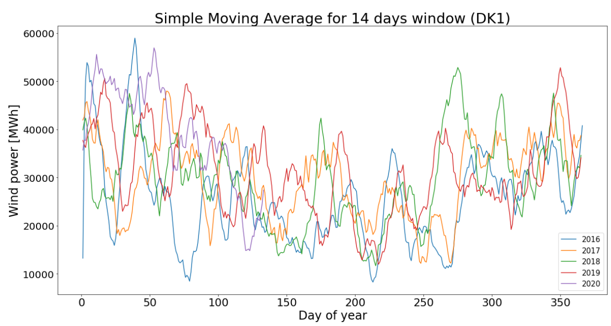

Figure 11: Simple moving average of wind power production per each hour for each day of week (DK1)

Only chart of average wind power for each hour suggest there may be a daily trend, however data from area DK2 doesn't confirm this assumption.

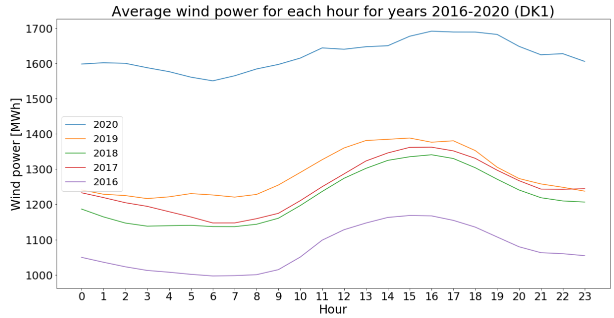

Figure 12: Average wind power production per hour of a day for each year (DK1)

3.3.1 Nordpool's prognosis

I calculated mean absolute error for prognosis performed by Nordpool for each hour and year. The results for area DK1 are rounded to the whole number. We can spot that error for prognosis is bigger for newer data with over 80% growth in 2020 (until 12th May) comparing to the previous year. Another thing we can notice that the error in the night hours is lower than others.

Area	Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	Total
DK1	2016	125	131	133	134	141	145	144	152	155	157	164	163	163	180	191	194	189	189	187	189	185	186	182	180	165
	2017	152	156	161	158	162	166	167	170	169	168	183	196	197	198	199	202	204	202	197	194	194	195	199	200	183
	2018	154	149	150	156	155	156	162	162	174	175	184	186	192	201	209	217	225	215	207	211	208	207	210	227	187
	2019	216	216	208	213	217	219	221	217	218	217	226	229	238	249	253	255	261	255	256	241	232	240	246	258	233
	2020	410	379	381	389	401	400	425	411	381	382	386	404	431	429	451	447	484	470	432	424	437	455	449	462	422
	Total	182	181	181	184	188	191	194	195	196	196	205	211	217	225	233	236	242	236	230	227	224	227	229	237	211

Figure 13: MAE for wind power and Nordpool prognosis for each hour and year (DK1)

The results for area DK2 are similar, however numbers are smaller due to the lower capacity of wind farms in this part of Denmark.

Area	Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	Total
DK2	2016	45.7	45.7	45.9	45.0	47.4	48.7	50.3	51.5	50.6	49.1	49.6	52.2	49.6	49.9	51.5	51.3	53.2	58.0	60.8	59.4	59.4	60.7	65.5	66.1	52.8
	2017	55.5	57.4	57.1	56.5	53.7	54.7	56.0	56.3	59.7	60.0	62.2	65.0	67.9	70.0	67.1	68.9	68.2	68.8	70.1	72.6	72.2	70.4	68.3	69.3	63.7
	2018	52.5	54.2	53.9	54.1	55.2	56.4	56.2	59.5	60.7	60.1	60.1	62.5	60.9	59.2	59.8	65.1	67.3	73.6	75.4	70.7	70.9	68.5	66.6	69.6	62.2
	2019	58.4	60.0	56.5	58.1	55.2	55.1	56.7	60.3	64.2	62.9	66.3	67.7	65.9	65.1	63.5	64.9	67.6	69.6	71.0	68.6	67.9	72.4	73.6	73.4	64.4
	2020	68.6	66.1	65.7	67.2	66.7	66.6	64.3	63.7	61.1	72.8	73.0	70.1	74.4	79.9	78.6	76.9	87.7	84.3	78.0	76.6	82.5	87.8	89.8	96.0	74.9
	Total	54.3	55.3	54.4	54.6	54.0	54.8	55.6	57.5	59.0	59.3	60.7	62.5	62.2	62.6	62.0	63.7	66.0	68.9	70.0	68.5	68.8	69.7	70.3	71.8	61.9

Figure 14: MAE for wind power and Nordpool prognosis for each hour and year (DK2)

3.4 Price data

In the 2020 price of energy decreased compared to the previous years. However we don't know what kind of the impact had epidemic on this data so it's hard to come up with any conclusion.

Year	DK1	DK2
2016	184.435	206.825
2017	224.005	227.575
2018	329.235	339.345
2019	289.270	295.390
2020	142.520	149.865

Table 3: Average price of energy per each year (DKK/MWh)

As in the consumption data we can also notice seasonal trends in this category, but this time only two kinds: daily and weekly. There is no annual trend in the price data.

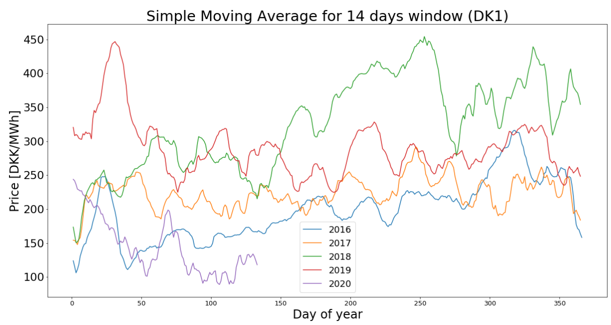

Figure 15: Simple moving average of price for each year - 14 days window (DK1)

Electricity prices are lower during weekends due to the lower consumption and there are also 2 daily peaks each day, either weekend (including holidays) and work days.

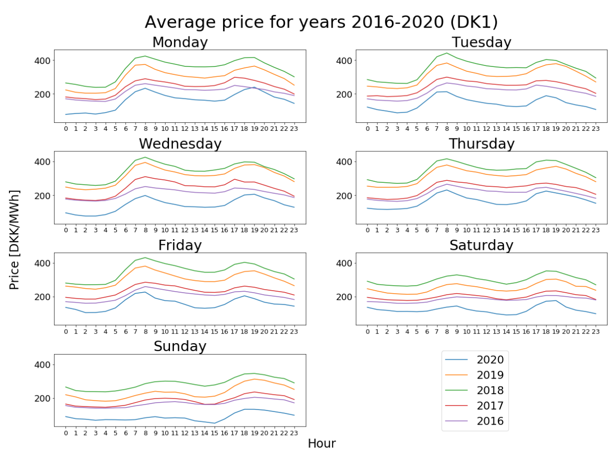

Figure 16: Average price per hour of a day for each day of week (DK1)

3.4.1 Negative prices

There were 417 negative prices in the area DK1 in the years 2016-2020 and 288 in the DK2. The dataset for 2020 ends on 12th May, however there are already 84 negative prices compared to the 133 in the full year 2019. Negative prices occur more often during nights than the peaks of consumption.

year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	Total
2016	3	7	8	8	6	4	5	2	1	1	0	0	1	2	3	2	2	1	0	0	1	2	0	4	63
2017	6	6	8	7	6	5	6	4	1	2	2	2	2	4	4	5	4	0	0	0	1	3	3	4	85
2018	4	5	5	6	7	6	6	3	0	0	1	0	1	2	2	1	0	0	0	0	0	1	0	2	52
2019	9	9	12	14	11	5	6	4	3	3	4	3	6	9	11	8	4	2	1	1	1	1	1	5	133
2020	5	3	6	7	7	7	4	3	2	2	4	3	3	4	6	7	6	1	1	0	0	1	1	1	84
Total	27	30	39	42	37	27	27	16	7	8	11	8	13	21	26	23	16	4	2	1	3	8	5	16	417

Figure 17: Number of negative prices for each hour, year and totally (DK1)

3.4.2 Correlation between price and wind power

We can suspect that with bigger values of wind power price is lower. Pearson's correlation shows that mostly it's weak correlation, only for 2020 year there are moderate values.

Correlation is a little bit higher for night hours than rest of the day.

Area	Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
DK1	2016	-0.41	-0.42	-0.41	-0.41	-0.40	-0.36	-0.36	-0.40	-0.44	-0.44	-0.42	-0.41	-0.40	-0.39	-0.38	-0.37	-0.35	-0.35	-0.41	-0.43	-0.38	-0.37	-0.37	-0.37
	2017	-0.62	-0.62	-0.61	-0.58	-0.54	-0.52	-0.46	-0.46	-0.51	-0.53	-0.53	-0.55	-0.56	-0.54	-0.53	-0.51	-0.50	-0.51	-0.60	-0.63	-0.62	-0.60	-0.62	-0.61
	2018	-0.45	-0.46	-0.49	-0.50	-0.48	-0.44	-0.36	-0.38	-0.42	-0.44	-0.46	-0.45	-0.43	-0.41	-0.40	-0.39	-0.42	-0.41	-0.45	-0.46	-0.44	-0.46	-0.47	-0.46
	2019	-0.50	-0.50	-0.52	-0.53	-0.51	-0.46	-0.38	-0.37	-0.38	-0.39	-0.41	-0.42	-0.41	-0.40	-0.38	-0.35	-0.33	-0.31	-0.40	-0.51	-0.56	-0.56	-0.56	-0.56
	2020	-0.53	-0.63	-0.81	-0.84	-0.83	-0.83	-0.72	-0.68	-0.69	-0.69	-0.68	-0.64	-0.68	-0.69	-0.72	-0.74	-0.75	-0.73	-0.72	-0.72	-0.69	-0.65	-0.69	-0.73
DK2	2016	-0.36	-0.40	-0.41	-0.40	-0.34	-0.29	-0.27	-0.23	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.23	-0.18	-0.19	-0.24	-0.28	-0.31	-0.31	-0.28	-0.32
	2017	-0.61	-0.61	-0.61	-0.61	-0.57	-0.50	-0.45	-0.42	-0.42	-0.43	-0.44	-0.46	-0.44	-0.43	-0.42	-0.38	-0.37	-0.37	-0.42	-0.47	-0.53	-0.58	-0.62	-0.63
	2018	-0.52	-0.53	-0.53	-0.53	-0.51	-0.48	-0.39	-0.31	-0.30	-0.33	-0.35	-0.38	-0.36	-0.35	-0.34	-0.33	-0.33	-0.31	-0.34	-0.38	-0.40	-0.43	-0.47	-0.50
	2019	-0.50	-0.52	-0.52	-0.52	-0.49	-0.43	-0.31	-0.32	-0.31	-0.28	-0.26	-0.25	-0.25	-0.25	-0.23	-0.23	-0.24	-0.26	-0.37	-0.44	-0.48	-0.46	-0.49	-0.50
	2020	-0.50	-0.46	-0.43	-0.35	-0.39	-0.51	-0.58	-0.38	-0.39	-0.40	-0.35	-0.40	-0.38	-0.34	-0.31	-0.29	-0.32	-0.38	-0.36	-0.43	-0.53	-0.64	-0.75	-0.75

Figure 18: Correlation between price and wind power for each hour and year (DK1 and DK2)

3.5 Holidays

Usage of the electricity decreases during weekends and public holidays and this has a significant effect on a prediction especially during Christmas or Easter. Due to that fact, each day was aligned with variable holiday with following value based of occurrence of day of week or public holiday:

- 1 - National Holidays (e.g. Easter Monday)[6]
- 1 - Sundays
- 0 - Not a public holidays (e.g. New Year's Eve)
- 0 - Other days

Each holiday is treated in weekday dummies as a Sunday.

4 Methodology

I decided to choose Danish market for forecasting and so on I want to apply methods and models most fitting to the characteristics of this market. I performed forecasting for 3 categories of values: wind power, consumption and price. In each category I made prognosis for few models which are described in Forecasting section. My forecasting framework for all of them is following:

1. Data preparation

2. Preliminary data analysis
3. Data forecasting (cycle for each model and day)

    Optional deseasonalization

    Data normalization

    Day-ahead forecasting

4. Verifying models' performance

All calculations were conducted with Python 3.7 language and its libraries, i.e. numpy, pandas, scipy. All the scripts and results made are accessible in public GIT repository https://github.com/ajescode/energyForecast.

Wind power prognosis and Consumption prognosis is provided only one day in advance so data forecasting is also performed day ahead. I used 3 calibration windows to compute predictions. They may take values of week multiples (7 days) because of the seasonal trend of energy production. Because chosen data has a limit of 1583 days, the maximum calibration window I considered is 728 days, around 2 years.

I consider few models for each category of forecasts (consumption, price and wind power) which I explain later in this section. General equation for each model (except of benchmark models for wind power and consumption) can be presented as:

$$\hat{Y}_{d,h} = \sum_{i=1}^{n} \beta_{d,h,i} X^i_{d,h}, \tag{1}$$

where $Y_{d,h}$ is prediction and $X^i_{d,h}$ is variable for given day (d) and hour (h). We can predict values for next whole day only, but basing on the variables from whole calibration window. Coefficients $\beta_{h,i}$ have been approximated by ordinary least squares (OLS).

$$\vec{\hat{\beta}} = (X^T X)^{-1} X^T Y, \tag{2}$$

where X is matrix of independent variables, and Y is vector of dependant variables of size of calibration window (multiple of 7 days).

Because prices of energy due to the wind dependence can have negative values I needed to normalize values. I used function $asinh$ which was empirically confirmed to have best results for danish market among 4 different normalization functions.[1].

$$X_{d,h} = asinh(x_{d,h}) \equiv \log(x_{d,h} + \sqrt{x^2_{d,h} + 1}), \tag{asinh}$$

where $Y_{d,h}$ is transformed value used for forecasts either as independent variable or dependent variable. Independent values in the model must be also transformed. $y_{d,h}$ is normalized in a calibration window by:

$$x_{d,h} = \frac{1}{b_{d,h}}(x^*_{d,h} - a_{d,h}), \tag{std}$$

where a is median and b is median absolute deviation (MAD) for given day and hour in the calibration window. $y^*_{d,h}$ is original value of a parameter without any transformation yet. I used both simple normalization (std), and one with $asinh$ function (asinh) for each category of forecasting.

Inverse function for transformation and normalization is following:

$$x^*_{d,h} = b_{d,h} sinh(X_{d,h}) + a_{d,h}, \tag{3}$$

To eliminate seasonal component from the data I applied Hodrick-Prescott filter[7] before standard normalization with smoothing parameter derived from Ravn and Uhlig (2002) [8] and adjusted to the daily data as $1600^4 * 6.25 = 110930628906.25$.

Types of transformations used:

1. Standard normalization (std)
2. Standard normalization (std) + asinh (asinh)
3. Standard normalization (std) + asinh (asinh) + HP filter

4.1 Demand forecasting

First considering model is a benchmark model containing consumption forecast provided by Nordpool's database for day ahead ($FL_{t,h}$). Its performance is analyzed in the section Demand data prognosis.

$$C_{d,h} = FC_{d,h} \tag{C1}$$

Second model is extension of benchmark model (C1), where Di, d is a vector of the values 0,1 for the corresponding day of a week. For day (d) which is Monday $D_{1,d} = 1$ and $D_{2,d} = D_{3,d} = ... = D_{7,d} = 0$, for a day which is Tuesday $D_{2,d} = 1$ and $D_{1,d} = D_{3,d} = ... = D_{7,d} = 0$ etc.

$$C_{d,h} = \sum_{i=1}^{7} \beta_{0,i} D_{i,d} + \beta_1 FC_{d,h} + \varepsilon_{d,h} \tag{C2}$$

Third model is extension of second model (C2), where are added 3 parameters according to a similar-day technique: consumption of previous day ($C_{d-1,h}$), 2 days ago ($C_{d-2,h}$) and week ago ($C_{d-7,h}$).

$$C_{d,h} = \sum_{i=1}^{7} \beta_{0,i} D_{i,d} + \beta_1 FC_{d,h} + \beta_1 C_{d-1,h} + \beta_2 C_{d-2,h} + \beta_3 C_{d-7,h} + \varepsilon_{d,h} \tag{C3}$$

Fourth model is extension of third model (C3) where is also considered forecast of the wind power download from Nordpool's database, denoted as $FW_{d,h}$.

$$C_{d,h} = \sum_{i=1}^{7} \beta_{0,i} D_{i,d} + \beta_1 FC_{d,h} + \beta_2 C_{d-1,h} + \beta_3 C_{d-2,h} + \beta_4 C_{d-7,h} + \beta_5 FW_{d,h} + \varepsilon_{d,h} \tag{C4}$$

4.2 Wind power forecasting

First model is presented and analyzed in the section *Wind power data prognosis*. This is a benchmark model for further forecasts, where $FW_{t,h} + \varepsilon_t$ is a day ahead forecast of wind power taken from Nordpool's data.

$$W_{t,h} = FW_{t,h} \tag{W1}$$

Second model includes additionally day of week vector as the second demand model (C2).

$$W_{t,h} = \sum_{i=1}^{7} \beta_{0,i} D_{i,d} + \beta FW_{t,h} + \varepsilon_{d,h} \tag{W2}$$

Third model is extension of second model (W2) and contains also wind power from previous day ($W_{t-1,h}$) and 2 days ago ($W_{t-1,h}$) from the corresponding hour.

$$W_{t,h} = \sum_{i=1}^{7} \beta_{0,i} D_{i,d} + \beta_1 FW_{t,h} + \beta_2 W_{t-1,h} + \beta_3 W_{t-2,h} + \varepsilon_{d,h} \tag{W3}$$

Fourth model is extension of third one (W3) and contains additionally parameter of consumption forecast from previous day denoted as $FL_{t,h}$.

$$W_{t,h} = \sum_{i=1}^{7} \beta_{0,i} D_{i,d} + \beta_1 FW_{t,h} + \beta_2 W_{t-1,h} + \beta_3 W_{t-2,h} + \beta_4 FC_{t,h} + \varepsilon_{d,h} \tag{W4}$$

4.3 Price forecasting

First forecasting model of price consists of the same daf of week vector ($D_{i,d}$) as the second demand model (C2).

$$P_{d,h} = \sum_{i=1}^{7} \beta_{0,i} D_{i,d} + \varepsilon_{d,h} \tag{P1}$$

Second model is an extension of first model (P1), where were added parameters of price for previous days: 1 day ago, 2 days ago and 1 week ago.

$$P_{d,h} = \sum_{i=1}^{7} \beta_{0,i} D_{i,d} + \beta_1 p_{d-1,h} + \beta_2 p_{d-2,h} + \beta_3 p_{d-7,h} + \varepsilon_{d,h} \tag{P2}$$

Third model is an extension of second model (P2) which consists additionally parameters of minimum ($p_{d-1,min}$) and maximum value ($p_{d-1,max}$) of previous day and value of price for last hour of previous day ($p_{d-1,24}$).

$$P_{d,h} = \sum_{i=1}^{7} \beta_{0,i} D_{i,d} + \beta_1 p_{d-1,h} + \beta_2 p_{d-2,h} + \beta_3 p_{d-7,h} + \beta_4 p_{d-1,min} + \beta_5 p_{d-1,max} + \beta_6 p_{d-1,24} + \varepsilon_{d,h} \tag{4}$$

Fourth model is an extension of third model (4) which is an expert model ARX2[3] with two exogenous variables: wind power prognosis ($FW_{d,h}$) and consumption prognosis ($FC_{d,h}$).

$$\begin{aligned} P_{d,h} = \sum_{i=1}^{7} \beta_{0,i} D_{i,d} + \beta_1 p_{d-1,h} + \beta_2 p_{d-2,h} + \beta_3 p_{d-7,h} + \beta_4 p_{d-1,min} + \beta_5 p_{d-1,max} \\ + \beta_6 p_{d-1,24} + \beta_7 FW_{d,h} + \beta_8 FC_{d,h} + \varepsilon_{d,h} \end{aligned} \tag{P4}$$

In the last fifth model, Nordpool's prognosis for wind and consumption are replaced with the best forecasts acquired from the wind and consumption prognosis. However only values in predicted period were replaced by better prognosis, not full rolling window, that's why eventually it wasn't very efficient.

Models W2 and C2 were used for area DK1 and for are DK2 best forecasts were achieved in W1 and C3.

$$\begin{aligned} P_{d,h} = \sum_{i=1}^{7} \beta_{0,i} D_{i,d} + \beta_1 p_{d-1,h} + \beta_2 p_{d-2,h} + \beta_3 p_{d-7,h} + \beta_4 p_{d-1,min} + \beta_5 p_{d-1,max} \\ + \beta_6 p_{d-1,24} + \beta_7 FW^*_{d,h} + \beta_8 FC^*_{d,h} + \varepsilon_{d,h} \end{aligned} \tag{P5}$$

5 Results

For each forecast there is error analysis with Mean Average Error (MAE), Mean Root Square Error (MRSE). All forecasts in the appropriate categories are compared to each other in order to get optimal result.

$$MAE = \frac{1}{24T} \sum_{d=1}^{T} \sum_{h=1}^{24} |\hat{\varepsilon}_{d,h}| \equiv \frac{1}{24T} \sum_{d=1}^{T} \sum_{h=1}^{24} |y_{d,h} - \hat{y}_{d,h}| \tag{5}$$

$$RMSE = \sqrt{\frac{1}{24D} \sum_{d=1}^{T} \sum_{h=1}^{24} \hat{\varepsilon}^2_{d,h}} \equiv \sqrt{\frac{1}{24D} \sum_{d=1}^{T} \sum_{h=1}^{24} (y_{d,h} - \hat{y}_{d,h})^2} \tag{6}$$

, where T is size of calibration window. Also I performed error analysis for each hour and day of forecasts and compared average values of them.

$$MAE_h = \frac{1}{T}\sum_{d=1}^{T}|\hat{\varepsilon}_{d,h}| \equiv \frac{1}{T}\sum_{d=1}^{T}|y_{d,h} - \hat{y}_{d,h}| \tag{7}$$

$$MAE_d = \frac{1}{24}\sum_{h=1}^{24}|\hat{\varepsilon}_{d,h}| \equiv \frac{1}{24}\sum_{h=1}^{24}|y_{d,h} - \hat{y}_{d,h}| \tag{8}$$

$$RMSE_h = \sqrt{\frac{1}{T}\sum_{d=1}^{T}\hat{\varepsilon}_{d,h}^2} \equiv \sqrt{\frac{1}{T}\sum_{d=1}^{T}(y_{d,h} - \hat{y}_{d,h})^2} \tag{9}$$

$$RMSE_d = \sqrt{\frac{1}{24}\sum_{h=1}^{24}\hat{\varepsilon}_{d,h}^2} \equiv \sqrt{\frac{1}{24}\sum_{h=1}^{24}(y_{d,h} - \hat{y}_{d,h})^2} \tag{10}$$

5.1 Results

The forecasts were performed with several different settings for each area (DK1 and DK2):

- Calibration window: 182, 364, 728
- Normalization function: standard, asinh
- Dates: 2019.01.01-2019.12.31 (2019 year),
  2019.05.13-2020.05.12 (last year),
  2020.01.01-2020.05.12,
  2019.01.01-2020.05.12

For both areas results with *asinh* were significantly worse than with standard normalization function. Especially for calibration window 182. The forecast with standard normalization was better for Models 2-4 than Nordpool's prognosis, however better accuracy was for longer calibration windows. Because drops of consumption level in 2020 results including this period were less accurate.

Area	Dates	Window	C1	C2	C2 (asinh)	C2 (asinh-hp)	C3	C3 (asinh)	C3 (asinh-hp)	C4	C4 (asinh)	C4 (asinh-hp)
DK1	2019.01.01-2019.12.31	182	22.704395	21.026883	21.278522	24.146593	21.364330	21.673405	24.363970	21.460788	21.809971	24.525773
		364	22.704395	20.573811	20.645187	36.088857	20.604376	20.709898	36.207233	20.635657	20.754207	36.237594
		728	22.704395	20.481759	20.555259	59.594126	20.412800	20.500747	59.367897	20.376542	20.478937	59.430217
	2019.01.01-2020.05.12	182	23.707915	21.997153	22.308181	24.704135	22.340290	22.671243	24.961505	22.405802	22.767146	25.066242
		364	23.707915	21.632464	21.657050	35.899516	21.703457	21.771440	35.888645	21.699693	21.789942	35.840132
		728	23.707915	21.529247	21.564290	60.553174	21.492532	21.552894	60.401634	21.440592	21.530552	60.139280
	2019.05.13-2020.05.12	182	24.522143	22.830232	23.090781	25.466853	23.161873	23.435499	25.681428	23.225991	23.518861	25.772308
		364	24.522143	22.445105	22.460981	36.512977	22.511653	22.562559	36.503980	22.468517	22.548466	36.525488
		728	24.522143	22.326490	22.336533	61.610408	22.299777	22.328969	61.434464	22.222493	22.296693	61.013695
	2020.01.01-2020.05.12	182	26.461936	24.659924	25.133939	26.234230	25.018678	25.409673	26.601357	24.999263	25.393981	26.549485
		364	26.461936	24.537791	24.433966	35.379896	24.719730	24.684696	35.014327	24.619793	24.632375	34.749354
		728	26.461936	24.403931	24.333436	63.185148	24.455705	24.440366	63.238582	24.360732	24.416565	62.085207

Figure 19: MAE on the consumption forecasts (DK1)

Area	Dates	Window	C1	C2	C2 (asinh)	C2 (asinh-hp)	C3	C3 (asinh)	C3 (asinh-hp)	C4	C4 (asinh)	C4 (asinh-hp)
DK2	2019.01.01-2019.12.31	182	19.473916	18.507033	19.146465	19.095119	18.161387	19.027368	18.997484	18.286632	19.223036	19.203095
		364	19.473916	17.811062	17.745990	17.753539	17.613376	17.914266	17.981829	17.785547	18.122210	18.202078
		728	19.473916	17.651220	17.609941	18.066932	17.388470	17.538987	18.094095	17.547174	17.729982	18.257902
	2019.01.01-2020.05.12	182	19.887383	18.586461	19.264721	19.206957	18.283000	19.105458	19.065401	18.378010	19.263226	19.226775
		364	19.887383	18.201139	18.174379	18.146137	18.048743	18.323249	18.331821	18.162581	18.469107	18.490195
		728	19.887383	18.264652	18.218040	18.507439	18.062187	18.193035	18.579753	18.184073	18.350364	18.725199
	2019.05.13-2020.05.12	182	18.999658	17.583754	18.232112	18.174849	17.199495	17.826469	17.795239	17.268510	17.952312	17.922835
		364	18.999658	17.432061	17.423749	17.304601	17.209056	17.335372	17.309250	17.214983	17.360200	17.347680
		728	18.999658	17.415524	17.376481	17.504240	17.236912	17.320106	17.646085	17.306714	17.427983	17.743832
	2020.01.01-2020.05.12	182	21.022086	18.804438	19.589257	19.513881	18.616750	19.319767	19.251787	18.628786	19.373522	19.291762
		364	21.022086	19.271651	19.350033	19.223566	19.243547	19.445644	19.292325	19.197300	19.421117	19.280890
		728	21.022086	19.948130	19.886882	19.716346	19.911108	19.987976	19.912574	19.931954	20.052917	20.007632

Figure 20: MAE on the consumption forecasts (DK2)

Area	Dates	Window	C1	C2	C2 (asinh)	C2 (asinh-hp)	C3	C3 (asinh)	C3 (asinh-hp)	C4	C4 (asinh)	C4 (asinh-hp)
DK1	2019.01.01-2019.12.31	182	36.902095	35.185087	35.344016	37.602830	35.451488	35.622875	37.730507	35.453872	35.631172	37.774140
		364	36.902095	34.817941	34.850912	47.367852	34.784771	34.823640	47.469686	34.754983	34.809466	47.501731
		728	36.902095	34.743648	34.782387	68.035598	34.675315	34.710660	67.890235	34.613163	34.670277	67.922423
	2019.01.01-2020.05.12	182	36.780154	34.936314	35.136187	37.079822	35.254252	35.439543	37.317340	35.263161	35.468348	37.373501
		364	36.780154	34.637305	34.613302	46.577215	34.680831	34.672863	46.569575	34.637497	34.667538	46.542720
		728	36.780154	34.560341	34.560053	68.624267	34.537831	34.536093	68.539502	34.469738	34.515632	68.304825
	2019.05.13-2020.05.12	182	39.326498	37.479944	37.558227	39.360457	37.799352	37.876119	39.573333	37.796761	37.889856	39.605372
		364	39.326498	37.224406	37.185267	48.520139	37.262406	37.240220	48.511530	37.190576	37.216973	48.510441
		728	39.326498	37.138652	37.116591	70.515760	37.121964	37.090541	70.442033	37.038202	37.070943	70.060431
	2020.01.01-2020.05.12	182	36.443407	34.244308	34.559411	35.605049	34.707205	34.931470	36.159214	34.734402	35.017611	36.251262
		364	36.443407	34.136663	33.952674	44.335037	34.393967	34.255666	44.004851	34.313009	34.275016	43.803104
		728	36.443407	34.052210	33.942406	70.214434	34.157681	34.052419	70.290518	34.073025	34.087626	69.343442

Figure 21: RMSE on the consumption forecasts (DK1)

Area	Dates	Window	C1	C2	C2 (asinh)	C2 (asinh-hp)	C3	C3 (asinh)	C3 (asinh-hp)	C4	C4 (asinh)	C4 (asinh-hp)
DK2	2019.01.01-2019.12.31	182	28.341912	26.721173	27.365739	27.380132	26.069790	27.250059	27.259121	26.258225	27.592408	27.607307
		364	28.341912	26.022295	25.941888	26.102319	25.295621	25.607190	25.788850	25.533021	25.891212	26.070721
		728	28.341912	25.987201	25.943130	26.573661	25.183820	25.311004	25.999175	25.309949	25.471407	26.150760
	2019.01.01-2020.05.12	182	29.430146	27.523442	28.195916	28.190292	26.866673	27.885702	27.880920	27.006186	28.156564	28.155279
		364	29.430146	27.170554	27.129984	27.223927	26.395746	26.626181	26.739204	26.538476	26.812680	26.925465
		728	29.430146	27.328482	27.266396	27.688507	26.428567	26.451328	26.946304	26.534730	26.595967	27.093507
	2019.05.13-2020.05.12	182	28.078188	26.054504	26.730579	26.712049	25.396351	26.136359	26.127461	25.481365	26.285462	26.280740
		364	28.078188	25.934630	25.927904	25.951411	25.242161	25.328041	25.399280	25.236591	25.339533	25.412132
		728	28.078188	25.981558	25.936765	26.218604	25.227106	25.186989	25.605089	25.285332	25.282989	25.696835
	2020.01.01-2020.05.12	182	32.228407	29.613698	30.357800	30.302578	28.941090	29.559993	29.520144	28.959774	29.649709	29.607049
		364	32.228407	30.097546	30.150959	30.087965	29.202719	29.240778	29.188769	29.119978	29.192458	29.142670
		728	32.228407	30.709911	30.605356	30.539657	29.576661	29.354062	29.389156	29.637043	29.462445	29.526490

Figure 22: RMSE on the consumption forecasts (DK2)

5.2 Results

The retrieved results from wind power forecasting with standard normalization were significantly better for DK1, and very slightly better for DK2 comparing to the Nordpool's forecast. *Asinh* function as normalization was useful only for part of cases with short calibration window, so I don't consider it as succesful results. MAE for Model 4 with 728 days window and for full 2019 year is 6,6% lower, while MAE for last period (2020.01.01-2020.05-12) is 22,19% lower.

Area	Dates	Window	W1	W2	W2 (asinh)	W2 (asinh-hp)	W3	W3 (asinh)	W3 (asinh-hp)	W4	W4 (asinh)	W4 (asinh-hp)
DK1	2019.01.01-2019.12.31	182	234.505308	217.478916	215.894285	214.734093	219.693092	218.731698	217.737519	220.001034	220.022164	218.051361
		364	234.505308	217.177879	214.967982	210.802901	218.212622	216.089489	212.260496	218.529144	216.377558	213.243288
		728	234.505308	218.872158	217.126530	225.231826	219.069252	217.362971	226.196376	219.064942	217.330238	227.035245
	2019.01.01-2020.05.12	182	283.895457	235.274013	233.854067	232.446844	237.847026	237.060609	235.862607	238.300715	238.356255	237.177084
		364	283.895457	239.146582	236.620141	231.800738	240.612433	238.204647	233.671114	240.579080	238.089518	234.534196
		728	283.895457	248.212884	245.156871	246.454817	248.436243	245.377615	247.586395	247.801243	244.526811	248.140149
	2019.05.13-2020.05.12	182	304.049977	236.693538	235.344291	232.420561	239.211003	238.645877	236.146228	240.226255	240.646629	238.419249
		364	304.049977	243.185350	240.488133	231.351282	244.585107	242.083375	233.337450	244.556479	241.979656	234.401496
		728	304.049977	256.066520	252.439740	242.497874	256.193458	252.528921	243.402873	255.405858	251.475802	244.158334
	2020.01.01-2020.05.12	182	419.439850	284.110181	283.142189	281.057025	287.667971	287.361758	285.604391	288.521645	288.671619	289.664972
		364	419.439850	299.436631	296.041480	289.426382	302.085601	298.896623	292.429577	301.092064	297.674971	292.964132
		728	419.439850	328.734424	322.082241	304.698365	329.029865	322.259910	306.288327	326.664025	319.164021	306.059622

Figure 23: MAE on the wind power forecasts (DK1)

Area	Dates	Window	W1	W2	W2 (asinh)	W2 (asinh-hp)	W3	W3 (asinh)	W3 (asinh-hp)	W4	W4 (asinh)	W4 (asinh-hp)
DK2	2019.01.01-2019.12.31	182	64.516667	66.312175	67.041271	67.430750	66.869509	67.751983	68.146756	67.068310	67.955680	68.213032
		364	64.516667	65.032888	65.481516	67.865333	65.272429	65.812336	68.170554	65.175668	65.856513	68.517516
		728	64.516667	65.407998	66.025683	74.755683	65.534689	66.271785	75.014849	65.536840	66.257624	74.773940
	2019.01.01-2020.05.12	182	67.261881	69.068615	69.601714	69.801059	69.888059	70.549723	70.766295	70.093752	70.666007	70.899635
		364	67.261881	67.589801	67.930268	69.831930	68.034580	68.495361	70.353998	68.040300	68.552511	70.617038
		728	67.261881	67.493742	67.898529	76.177646	67.711453	68.247345	76.362209	67.768841	68.255633	76.294865
	2019.05.13-2020.05.12	182	66.814094	69.954498	70.563876	70.938860	70.736871	71.411201	71.787756	70.795956	71.347557	71.738147
		364	66.814094	68.159593	68.434445	70.686016	68.515858	68.841288	71.038799	68.578528	68.863972	71.171989
		728	66.814094	67.891692	68.020489	77.013305	68.032725	68.214296	77.052352	68.047130	68.208949	77.347609
	2020.01.01-2020.05.12	182	74.795739	76.633279	76.628492	76.306042	78.172049	78.227734	77.955258	78.396656	78.104122	78.272644
		364	74.795739	74.606895	74.650530	75.228980	75.614920	75.858551	76.346155	75.901884	75.951300	76.378885
		728	74.795739	73.217778	73.038292	80.080026	73.685281	73.668994	80.059851	73.894259	73.738890	80.468834

Figure 24: MAE on the wind power forecasts (DK2)

Area	Dates	Window	W1	W2	W2 (asinh)	W2 (asinh-hp)	W3	W3 (asinh)	W3 (asinh-hp)	W4	W4 (asinh)	W4 (asinh-hp)
DK1	2019.01.01-2019.12.31	182	318.384899	290.354599	289.350292	287.506028	292.139811	292.189688	290.268436	292.894480	294.049368	291.467793
		364	318.384899	291.047309	288.841140	282.372527	291.751225	289.493793	283.042036	292.323555	290.126353	284.527390
		728	318.384899	292.788017	291.134008	294.664987	293.021460	291.211083	294.975405	293.042919	291.190151	296.566314
	2019.01.01-2020.05.12	182	406.869604	321.310501	319.353973	317.260953	323.896706	323.210446	321.181606	324.118359	324.393927	322.290028
		364	406.869604	330.377883	326.508866	319.133670	331.423292	327.757590	320.640958	331.739072	328.055338	321.895844
		728	406.869604	346.422368	341.366710	333.418528	346.399049	341.339365	334.226772	345.970348	340.225888	335.177616
	2019.05.13-2020.05.12	182	436.840443	327.239721	325.396891	321.700473	329.807483	329.491191	326.005186	329.796350	330.636509	327.049059
		364	436.840443	340.344860	336.064696	324.715410	341.254752	337.330041	326.191809	341.364980	337.397003	327.469087
		728	436.840443	362.431378	356.537514	335.715455	362.043568	356.166111	336.085349	361.528382	354.800850	337.455252
	2020.01.01-2020.05.12	182	584.516298	393.959310	390.010557	387.348329	398.243803	396.049659	393.739949	397.396892	395.895395	394.698623
		364	584.516298	419.791015	412.576209	403.149800	421.531960	415.020963	406.327037	421.373847	414.689457	407.194263
		728	584.516298	462.704143	451.359800	421.861258	462.232981	451.145863	423.656866	460.991772	448.022791	423.421463

Figure 25: RMSE on the wind power forecasts (DK1)

Area	Dates	Window	W1	W2	W2 (asinh)	W2 (asinh-hp)	W3	W3 (asinh)	W3 (asinh-hp)	W4	W4 (asinh)	W4 (asinh-hp)
DK2	2019.01.01-2019.12.31	182	89.692353	90.576995	91.334043	91.834672	91.036874	92.039990	92.501746	91.390983	92.396428	92.807545
		364	89.692353	89.196772	89.561408	91.981586	89.297236	89.758430	92.171465	89.237428	89.861052	92.319077
		728	89.692353	89.525286	90.005933	98.795012	89.519276	90.081383	98.981029	89.464245	90.029751	98.660941
	2019.01.01-2020.05.12	182	93.437162	93.841940	94.340480	94.740480	94.697121	95.458095	95.829849	95.107356	95.807643	96.235375
		364	93.437162	92.486348	92.686460	94.756095	92.898494	93.271515	95.365956	92.978135	93.407041	95.534596
		728	93.437162	92.547781	92.751826	100.833367	92.708086	93.037354	101.132533	92.759200	93.046806	101.061235
	2019.05.13-2020.05.12	182	93.684466	94.989410	95.648314	96.316917	95.765894	96.666991	97.315014	96.047434	96.905310	97.562547
		364	93.684466	93.450492	93.627295	96.193874	93.775196	94.087528	96.688866	93.870937	94.134153	96.749909
		728	93.684466	93.766138	93.821730	102.537750	93.844232	93.973813	102.713099	93.853261	93.941780	102.908014
	2020.01.01-2020.05.12	182	103.017205	102.267973	102.137556	102.291831	104.082899	104.264453	104.419775	104.630389	104.599218	105.069560
		364	103.017205	100.964946	100.766002	101.983261	102.131280	102.294669	103.628061	102.545264	102.510215	103.848843
		728	103.017205	100.376066	99.900514	106.226577	100.943080	100.704789	106.814605	101.252321	100.864072	107.373171

Figure 26: RMSE on the wind power forecasts (DK2)

5.3 Results

With more complex models results are much better, however *asinh* function surprisingly doesn't improve predictions in the every case. Disproportion is especially big in the last period (2020.01.01-2020.05-12) and Model 4.

I performed Model 5 hoping that with better predictions of wind power and consumption than original ones, price will be more accurate. Because these predictions replaced only values for prediction period, not rolling window, coefficients could be overfitted to the worse values trying to predict using improved ones.

Area	Dates	Window	P1	P1 (asinh)	P1 (asinh-hp)	P2	P2 (asinh)	P2 (asinh-hp)	P3	P3 (asinh)	P3 (asinh-hp)	P4	P4 (asinh)	P4 (asinh-hp)	P5	P5 (asinh)
DK1	2019.01.01-2019.12.31	182	62.419447	61.248905	61.248905	50.970812	47.862838	47.738136	45.531671	42.763101	42.691759	41.832735	38.053032	37.917382	41.789153	38.164474
		364	66.905983	67.163867	67.163867	49.816323	47.516155	47.173496	43.599339	41.380780	41.245874	41.600593	38.068337	37.814441	41.895198	38.249817
		728	59.616463	59.869545	59.869545	48.271333	46.798962	48.272424	42.218329	41.285560	42.293601	38.539624	36.953780	38.032519	38.773604	37.099335
	2019.01.01-2020.05.12	182	74.037510	73.752601	73.752601	54.304729	52.422105	52.272366	47.144198	45.256767	45.152057	42.638636	40.310793	40.175984	42.790111	40.480601
		364	81.583109	83.185105	83.185105	53.909704	53.190293	52.804557	45.803505	44.940818	44.797602	42.979752	41.426176	41.180891	43.460681	41.715465
		728	86.491722	87.441830	87.441830	52.921611	52.983252	53.732966	44.599572	44.803385	45.378040	40.125764	40.397561	41.097301	40.339211	40.469730
	2019.05.13-2020.05.12	182	71.894074	72.552328	72.552328	51.030218	50.348744	50.251831	44.979770	43.838396	43.789638	39.770383	38.890162	38.840113	39.980648	39.042576
		364	83.140187	85.593875	85.593875	51.939930	52.136530	51.694149	44.333355	44.243325	44.046129	42.318592	41.587801	41.127362	43.343499	42.180723
		728	90.324035	91.364589	91.364589	50.441822	51.137697	50.892249	42.781262	43.371479	43.193533	38.666817	39.451380	39.398347	39.421250	39.850178
	2020.01.01-2020.05.12	182	105.921667	108.067257	108.067257	63.454202	64.934380	64.715930	51.569554	52.100288	51.904001	44.850317	46.506904	46.374403	46.425426	47.449435
		364	121.862442	127.153167	127.153167	65.143420	68.762175	68.258221	51.852532	54.710845	54.544825	46.764663	50.641299	50.419645	48.747856	51.967810
		728	160.247132	163.110131	163.110131	65.683651	69.955175	68.718664	51.134560	54.457565	53.842851	44.478704	49.848538	49.508171	46.424097	51.241317

Figure 27: MAE on the price forecasts (DK1)

Area	Dates	Window	P1	P1 (asinh)	P1 (asinh-hp)	P2	P2 (asinh)	P2 (asinh-hp)	P3	P3 (asinh)	P3 (asinh-hp)	P4	P4 (asinh)	P4 (asinh-hp)	P5	P5 (asinh)
DK2	2019.01.01-2019.12.31	182	60.685783	59.265083	59.265083	47.103923	43.958484	43.868928	42.800522	40.041798	40.019143	40.867716	37.631112	37.592842	40.917539	37.656573
		364	66.643572	65.894549	65.894549	46.536799	43.736972	43.409012	41.864222	39.129699	38.977039	41.773766	37.899312	37.706766	41.737298	37.872167
		728	57.641463	56.997347	56.997347	44.836256	42.947585	44.126417	40.620861	38.820265	39.730950	38.663786	36.541061	37.185542	38.646771	36.530471
	2019.01.01-2020.05.12	182	72.088336	71.634624	71.634624	50.790176	49.063672	48.960707	44.593005	42.871472	42.829081	42.240303	40.706097	40.665848	42.268304	40.722874
		364	80.282137	81.045797	81.045797	50.643427	49.600087	49.256643	43.848764	42.678249	42.549245	43.034634	41.318684	41.161845	43.023788	41.311466
		728	84.403612	84.125969	84.125969	49.712208	49.083361	49.654152	42.993153	42.220931	42.751613	40.725420	40.009271	40.376973	40.703890	39.996084
	2019.05.13-2020.05.12	182	68.703284	69.280873	69.280873	47.331708	47.005937	46.951134	42.340374	41.453055	41.455745	38.983829	38.603889	38.608243	38.997465	38.620568
		364	80.654765	82.458458	82.458458	48.135879	48.324643	47.966416	41.888804	41.748732	41.650334	41.183152	40.570452	40.354765	41.105611	40.537650
		728	88.502231	88.186925	88.186925	47.364384	47.302565	47.090488	41.124731	40.735721	40.701233	38.987806	38.711242	38.577699	38.941618	38.684554
	2020.01.01-2020.05.12	182	103.381058	105.581108	105.581108	60.906583	63.074152	62.934386	49.512224	50.637121	50.540565	46.007178	49.144966	49.099286	45.955950	49.141889
		364	117.711280	122.626291	122.626291	61.913499	65.690589	65.304652	49.295062	52.416750	52.352668	46.494911	50.702674	50.643831	46.430872	50.649240
		728	157.848607	158.576699	158.576699	63.093580	65.922146	64.824254	49.503579	51.553585	51.041401	46.383287	49.527291	49.135411	46.274236	49.455233

Figure 28: MAE on the price forecasts (DK2)

Area	Dates	Window	P1	P1 (asinh)	P1 (asinh-hp)	P2	P2 (asinh)	P2 (asinh-hp)	P3	P3 (asinh)	P3 (asinh-hp)	P4	P4 (asinh)	P4 (asinh-hp)	P5	P5 (asinh)
DK1	2019.01.01-2019.12.31	182	88.552095	89.377633	89.377633	75.854601	74.598665	74.391426	68.473893	66.034200	65.874428	59.011419	55.578274	55.358938	59.203002	55.901061
		364	93.087824	94.352975	94.352975	75.095306	73.924743	73.360790	66.647750	64.475040	64.032671	59.280699	56.405672	55.988147	59.773234	56.644245
		728	85.255977	86.419926	86.419926	73.416049	72.257950	72.577855	65.340032	64.153815	64.052560	56.731674	56.140598	56.348611	57.210817	56.408465
	2019.01.01-2020.05.12	182	100.419523	101.646696	101.646696	76.764606	76.322214	76.089317	68.376690	66.772849	66.585285	58.859893	57.016755	56.812191	59.237037	57.384865
		364	108.382915	110.865235	110.865235	76.532520	76.967591	76.365457	66.867873	66.328687	65.917485	59.490655	58.841660	58.443448	60.213819	59.297428
		728	117.686433	119.273737	119.273737	75.655026	76.389407	76.153396	65.855836	66.199160	65.827934	57.012862	58.590169	58.521558	57.489391	58.782734
	2019.05.13-2020.05.12	182	95.465230	96.983670	96.983670	68.212685	68.563190	68.403369	62.299381	61.418993	61.309870	53.964120	54.091201	53.982518	54.301694	54.277113
		364	107.685682	110.712737	110.712737	69.306409	70.734229	70.164132	61.345989	61.916248	61.600884	56.388890	56.859771	56.380539	57.812438	57.691593
		728	121.312715	122.971495	122.971495	68.434178	70.469465	69.744520	60.344236	61.666861	61.048242	52.893555	55.460732	55.039993	53.987241	55.944617
	2020.01.01-2020.05.12	182	127.430779	129.475793	129.475793	79.208289	80.863726	80.565305	68.109219	68.759214	68.498242	58.442031	60.789827	60.621708	60.666018	62.211354
		364	142.139512	146.937698	146.937698	80.344770	84.758849	84.061342	67.468281	71.168134	70.832876	60.063080	65.059942	64.704741	62.703120	66.998609
		728	178.639717	181.030724	181.030724	81.484104	86.722178	85.198452	67.251065	71.512389	70.470768	57.777514	64.838924	64.107598	60.331381	66.571225

Figure 29: RMSE on the price forecasts (DK1)

Area	Dates	Window	P1	P1 (asinh)	P1 (asinh-hp)	P2	P2 (asinh)	P2 (asinh-hp)	P3	P3 (asinh)	P3 (asinh-hp)	P4	P4 (asinh)	P4 (asinh-hp)	P5	P5 (asinh)
DK2	2019.01.01-2019.12.31	182	85.807352	85.896984	85.896984	71.018341	69.555000	69.379977	64.046663	62.180679	62.084748	57.322139	55.413446	55.348356	57.369391	55.425448
		364	90.943848	90.895794	90.895794	70.897828	69.132619	68.657715	63.118444	60.741802	60.320563	58.595830	55.930580	55.589629	58.551300	55.904560
		728	81.697935	81.828635	81.828635	69.490904	67.728746	68.029236	62.115683	60.256858	60.166623	56.427961	55.223516	55.394403	56.432042	55.227725
	2019.01.01-2020.05.12	182	97.529695	98.309403	98.309403	72.873760	72.475196	72.296759	64.566985	63.442992	63.341161	58.309209	57.725638	57.671154	58.334621	57.728349
		364	105.054135	106.585367	106.585367	72.893222	72.939020	72.466732	63.805347	63.127536	62.784119	59.408044	58.730318	58.470746	59.389992	58.725930
		728	114.468197	114.616814	114.616814	72.423808	72.149589	71.992683	63.374743	62.639554	62.341294	57.839339	58.055362	57.989223	57.827832	58.044470
	2019.05.13-2020.05.12	182	91.720525	93.090487	93.090487	64.406940	65.130937	65.019584	58.413321	57.804075	57.766398	53.052848	53.549639	53.547140	53.075160	53.578512
		364	103.403839	105.674446	105.674446	65.255122	66.661286	66.234275	57.587360	58.073291	57.893616	54.955283	55.073300	54.847794	54.872352	55.052359
		728	118.232453	118.064362	118.064362	65.196015	66.062437	65.507953	57.358018	57.480783	57.091545	52.927365	53.710299	53.434019	52.874757	53.679855
	2020.01.01-2020.05.12	182	124.137247	126.252027	126.252027	77.738504	79.943070	79.755259	65.973857	66.784762	66.667665	60.935991	63.640964	63.611442	60.889485	63.634190
		364	136.477624	140.938122	140.938122	78.107829	82.487579	82.016132	65.653539	69.253864	69.095171	61.582040	65.804607	65.731905	61.502552	65.732906
		728	175.342124	175.538274	175.538274	79.921663	83.082018	81.890066	66.708073	68.755588	67.952733	61.546601	65.198037	64.576840	61.404119	65.103546

Figure 30: RMSE on the price forecasts (DK2)

6 Conclusions

Almost all of the assumptions came true, forecasts for wind power and consumption were performed better than original prepared by Nordpool. Even models for wind power performed well however wind power predicting is not typical time-series problem, because more important factors influencing such as atmospheric models were not known and not used in this work.

Main finding is that period of forecast matters in using the most efficient model. For some periods more accurate were shorter calibration windows, because anomalies in the 2020 caused by COVID-19 changed seasonal numbers like consumption and thus price which was significantly lower in 2020 than 2019.

One assumption I found invalid was that *asinh* normalization function improves price predictions. It completely fails for wind power and consumption forecasting, but should be useful for price, but yet it works worse for periods including 2020 and especially for period which includes only 2020 year. The forecast Model 4 with asinh normalization performs 7.34% (DK1) and 16.34% (DK2) worse comparing MAE of the results.

Experiment of replacing original forecasts of Nordpool with better, forecasted results unfortunately failed but I still see potential with bigger calibration window, which can be used to trained better forecasts for whole considering period of price prediction. But for this purpose it's necessary to have more data.

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