Modeling and forecasting of electricity prices and demand

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June 2020

Abstract

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Keywords: electricity spot prices, day-ahead market, forecasting, power market

1 Introduction

We have experienced big growth of wind power industry in the recent years and massive number of new installments of the wind turbines. Some countries have significantly increasing shares of renewable energy in the overall production. I will focus especially on a wind energy, which is becoming the most important source in some regions.

Better prognosis of the renewable energy production, consumption may have positive impact on the final predicted price of energy. Smaller errors may lead to economical profits for companies operating and buying energy on the power exchanges.

In this work I made a simulations in order to achieve the best forecasts of consumption and price for Danish market using day-ahead data. There are plenty of possible approaches using, i.e. regression models[10], neural networks[6][11], probabilistic methods[8]. I decided to use regression model with best fitting expert models from [2] implementing modifications of HP filter and testing different variations of expert models with different lengths of calibration window.

In the Chapter 2 I reviewed analysis of the Danish market. In the Chapter 3 I presented characteristics of available data provided by Nordpool power exchange. In the Chapter 4 I presented used methods to perform forecasts. In the Chapter 5 I presented empirical results of the all forecasts and in the last Chapter 6 I came up with conclusions.

2 Market analysis

Danish power market has been transformed in the last several years drastically. Since 70s there was a lot of investments in renewable sources of energy, especially in the field of wind power, and much more since 2002 when first large scale offshore wind farm in the world has been finished - Horns Rev 1 (160 MW). For year 2019, total wind power generation capacity was 6128 MW[7].

National target for 2020 is over 50% of a energy consumption covered by wind power and it's likely to be achieved, as in 2019 they obtained 47% of coverage by domestic production[3]. Moreover they have finished construction of next large scale wind farm Horns Rev 3 in August 2019[3]. There are also defined next goals in last presented national energy strategy. For wind power consumption they aim for 70% in 2030[3]. Denmark is currently leader of wind power shares in the national production and its development.

The production of such significant part of energy from wind carries some risk. Wind speed is very fluctuant and variable even in a day cycle. There are no perfect methods of forecasting in the long term periods. It may occur in higher (or lower) demand in production from other, stable sources of

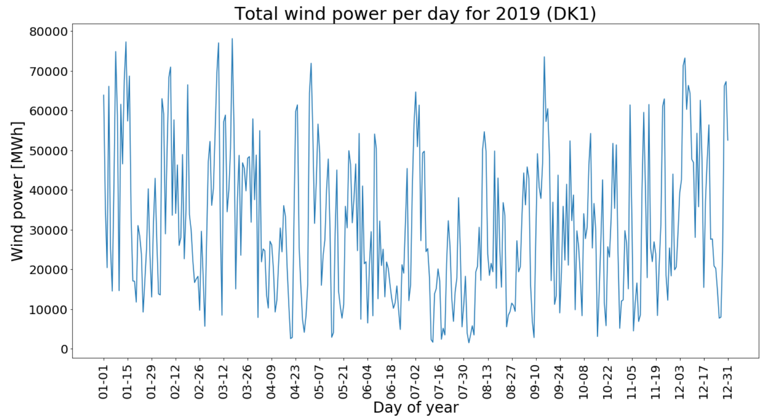

Figure 1: Wind power produced per day in 2019 (DK1)

energy to cover consumption. It's problematic to mark a common trend in wind power forecasts1. Thus it's helpful to correct prediction of electricity demand.

Wind power generation can't be stopped quickly as production of conventional sources of energy (coal, gas). Because of that higher unexpected wind power production may lead to decreased prices on energy stocks and in some cases, prices can drop below zero. That was happening rarely in the era of conventional production and regulated electricity market, but it happens more often with more renewable power generators. Because energy network is quite connected with each other in the European Union, in a case of negative prices energy from Danish areas (DK1, DK2) is exported to the neighboring countries, mainly to the Germany, which Denmark has the biggest balance of energy export and import.

Negative prices obligate to use other approaches of price forecasting, than the old ones which were failing with unexpected domain of values. In my work I will present a few models to forecast price and consumption and point out the best approach to have optimal forecasts.

The electricity market in Denmark is divided into 2 areas (DK1 and DK2). First area (DK1) consists of regions: Nordjylland, Midtjylland and Syddanmark; second area (DK2) consists of regions Sjælland and Hovedstaden with the capital Copenhagen.

Figure 2: Danish electricity market

3 Data analysis

Data I used to perform forecasts has been downloaded from the official webpage of the Nordpool power exchange[5]. Datasets are divided into the year files and periods (hours, weeks etc.). I managed to download following datasets (valid for the day 14.05.2020):

- Consumption - hourly
- Consumption prognosis - hourly
- Wind power - hourly
- Wind power prognosis - hourly
- Elspot prices (as Price) - hourly

All of the datasets were available for years 2013-2020, except for Consumption prognosis (2015-2020). So I decided to focus on analysis only on the period 2015-2020 (2015.01.01-2020.05.12), because 4 years time frame is still sufficient for calculations.

Units of downloaded data are following:

- Consumption and Wind Power- MWh
- Price - DKK/MWh

The files were downloaded, merged, split for regions DK1 and DK2, pivoted in order to have separated hours as parameters for each day and merged for all years. Example for consumption DK1 is presented below.

I performed a few analysis for each dataset, although I didn't include all of the charts and the tables in this work. The rest is uploaded into the github repository.

	date	holiday	0	1	2	...	22	23
0	2016-01-01	1	1818.0	1741.0	1660.0	...	1858.0	1713.0
1	2016-01-02	0	1615.0	1510.0	1461.0	...	2027.0	1822.0
2	2016-01-03	1	1724.0	1665.0	1671.0	...	2127.0	1998.0
3	2016-01-04	0	1844.0	1803.0	1789.0	...	2293.0	2079.0
4	2016-01-05	0	1940.0	1891.0	1952.0	...	2372.0	2193.0

Table 1: First 5 rows of merged file Consumption DK1.

3.1 Missing values

Data was very consistent and yet only single values were missing. These null values were replaced by average of the neighboring cells and in case of missing value in neighbor cell, value was fixed manually (with file fill_empty_cells.py). Half of day 2018-09-18 from Wind prognosis files was filled taking closest neighbors and counting average for whole vector (with file fill_empty_cells_wind_prognosis_DK.py). Number of missing values was reduced to zero.

Dataset	DK1	DK2
Consumption	5	5
Consumption prognosis	5	5
Price	21	12
Wind power	12	6
Wind power prognosis	18	19

Table 2: Missing values in files.

3.2 Consumption data

Elictricity consumption was higher in the area DK1 than DK2, compared for years 2016-2019 19.14, 19.41, 20.28, 20.37 TWh to 13.13, 13.03, 13.28, 13.16 TWh accordingly. We can see that consumption increased gradually in the area DK1, meanwhile in area DK2 was on the similar level.

We can spot three types of seasonal trends in the data: annual, weekly and daily. On the below chart showing consumption per day in 2019, we see that every weekend consumption value drops.

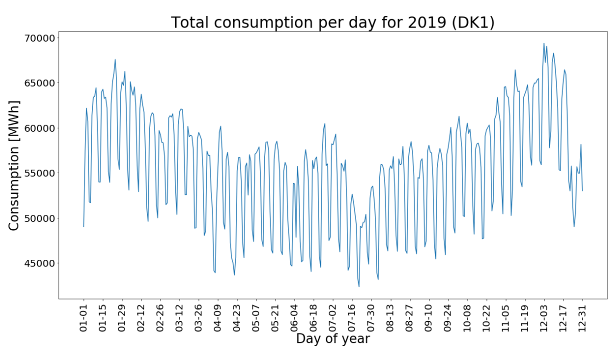

Figure 3: Consumption per day in 2019 (DK1)

Although it's not easy to spot in the DK1, there is annual trend with lower consumption during summer months and higher during winter. It's observable particularly in the DK2.

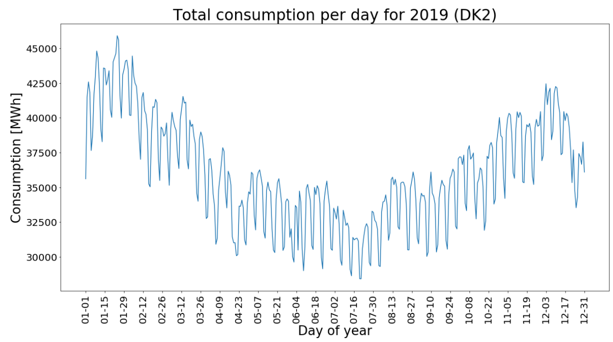

Figure 4: Consumption per day in 2019 (DK2)

Simple moving average with 14 days windows for each year shows clearly this trend. Although in the area DK1 it's not very sharp.

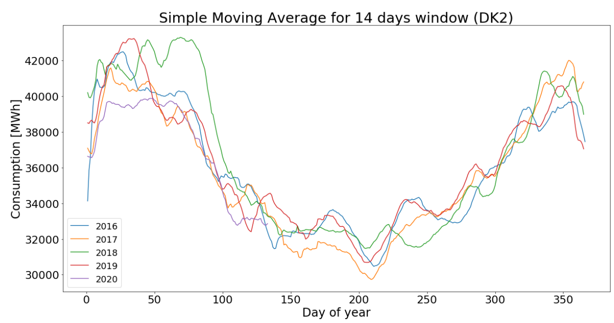

Figure 5: Simple moving average of consumption - 14 days window (DK2)

Last seasonal trend is daily which can be observed for each day of a week, even holidays. There are two peaks of energy consumption each day, in the morning and evening. During weekends and holidays, morning peaks are slightly shifted than during work days. There is also noticeable smaller consumption in the night.

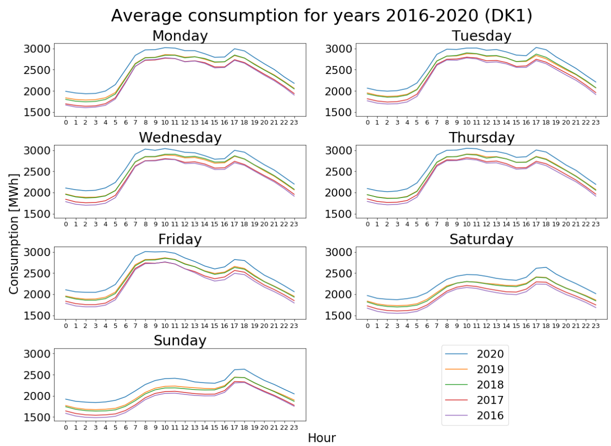

Figure 6: Average consumption per each hour of day for each day of week (DK1)

There is also one interesting thing observed in the 2020's data only in the area DK2. Evening daily peak is slightly shifted which can be caused epidemic COVID-19 or incomplete data of 2020 year.

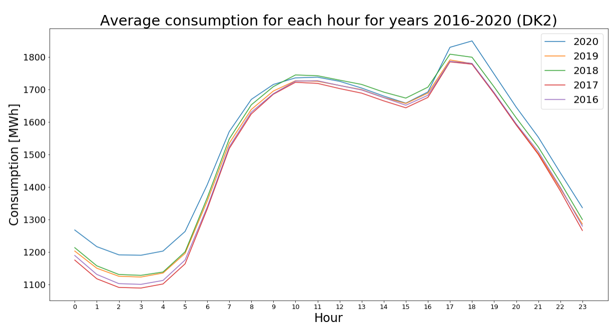

Figure 7: Average consumption per each hour of day for each year (DK2)

3.2.1 Nordpool's prognosis

The prognosis day-ahead given by Nordpool shows that those are less accurate over time, especially for area DK1 where consumption is larger than DK2. Level of accuracy for area DK2 is quite stable. Also there is no clear trend regarding the time of day. For DK1 best prediction are for night and for DK2 best prediction are performed for hours 7 and 15.

Area	Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	Total
DK1	2016	18.7	15.2	13.2	12.7	12.3	14.1	18.7	19.5	20.3	19.6	19.6	19.7	20.4	19.0	16.5	16.9	16.0	18.7	17.7	15.9	16.0	15.0	13.7	17.3	16.9
	2017	16.1	24.9	19.0	13.9	14.1	15.1	18.7	18.7	19.1	19.9	19.8	17.8	16.3	15.4	15.1	16.1	21.1	22.1	20.7	17.2	18.0	17.5	19.4	22.5	18.3
	2018	18.8	26.7	17.7	15.9	14.1	15.9	20.6	24.8	21.0	23.6	21.3	20.1	22.5	21.0	20.9	19.8	19.2	21.7	18.3	19.9	24.9	22.5	23.1	25.8	20.8
	2019	22.0	29.3	21.8	17.9	17.2	18.1	23.0	24.9	22.0	27.3	24.7	22.7	23.0	20.6	19.2	19.2	20.2	21.5	22.6	22.4	25.2	24.0	25.1	30.3	22.7
	2020	21.7	26.9	23.4	20.5	16.8	20.5	34.8	34.2	26.4	32.8	32.9	22.6	22.2	27.4	24.6	22.2	22.3	36.1	32.4	23.6	26.4	24.5	27.7	32.4	26.5
	Total	19.1	24.3	18.4	15.5	14.6	16.2	21.4	23.0	21.1	23.5	22.3	20.3	20.7	19.7	18.5	18.3	19.4	22.2	20.9	19.2	21.5	20.1	20.9	24.7	20.2

Figure 8: MAE for consumption for each hour and year with total figures (DK1)

Area	Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	Total
DK2	2016	23.8	19.6	19.0	20.3	22.4	22.9	20.9	17.1	23.6	27.6	24.3	21.4	24.1	22.0	20.0	19.2	23.3	26.4	24.5	22.9	24.5	24.6	20.7	15.9	22.1
	2017	19.6	21.9	21.7	20.2	18.8	18.5	22.7	16.8	20.2	21.8	21.6	20.8	20.2	20.5	20.6	16.7	18.0	24.4	23.3	21.2	20.8	22.8	22.8	27.4	21.0
	2018	18.1	25.2	24.7	20.8	19.5	20.2	27.5	18.8	21.2	24.8	26.8	26.7	27.0	26.1	22.4	17.5	20.7	24.7	22.5	22.4	26.8	29.1	25.9	23.2	23.4
	2019	16.1	23.9	22.8	17.5	16.3	15.9	21.2	14.3	16.9	17.9	21.2	21.6	18.6	17.5	17.1	15.3	18.7	24.5	22.1	18.8	22.2	22.6	21.1	22.7	19.5
	2020	17.4	18.2	15.8	12.4	15.0	17.6	25.5	22.1	24.4	31.0	36.4	27.6	23.5	26.0	21.7	13.9	15.9	28.1	27.5	18.3	17.2	17.1	14.6	18.9	21.1
	Total	19.2	22.3	21.5	19.1	18.9	19.2	23.3	17.2	20.8	23.7	24.5	23.0	22.6	21.9	20.1	16.9	19.8	25.3	23.5	21.1	23.1	24.1	22.0	22.0	21.5

Figure 9: MAE for consumption for each hour and year with total figures (DK2)

3.3 Wind power data

Wind power production increased significantly within last 4 years. In the area DK1 from 9.41 to 11.26 TWh and for area DK2 from 2.37 to 3.22 TWh, so about 35% more.

So far in the consumption data we could spot trends whereas in the wind power data there is no distinct trend.

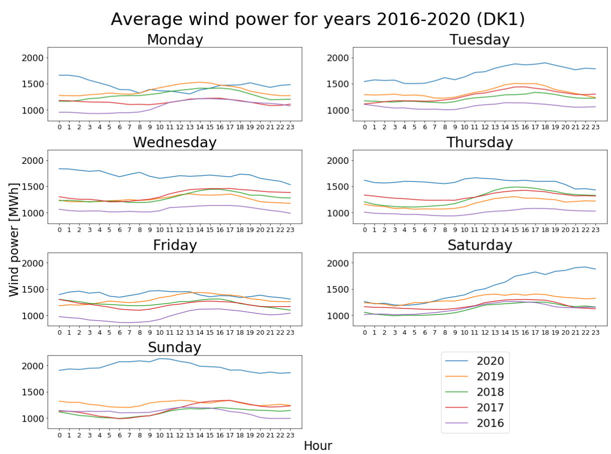

Figure 10: Wind power production per hour of a day for each day of week (DK1)

Simple moving average also doesn't show anything recurrent, thus we can't assume any annual trend.

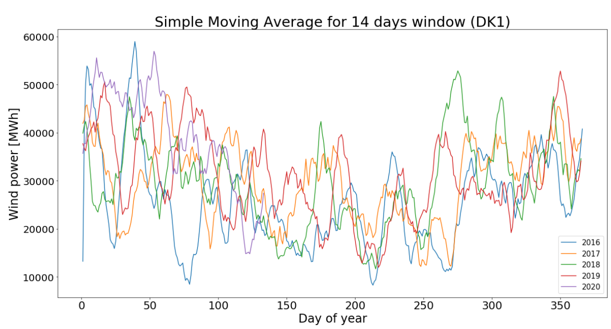

Figure 11: Simple moving average of wind power production per each hour for each day of week (DK1)

Only chart of average wind power for each hour suggest there may be a daily trend, however data from area DK2 doesn't confirm this assumption.

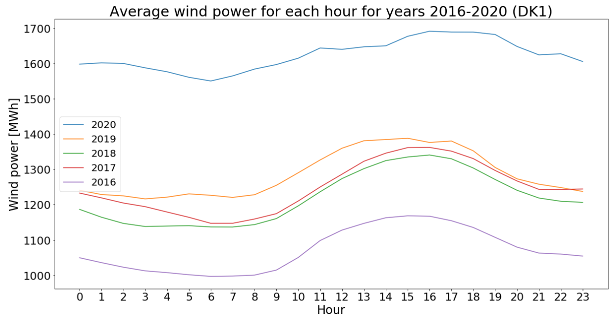

Figure 12: Average wind power production per hour of a day for each year (DK1)

3.3.1 Nordpool's prognosis

I calculated mean absolute error for prognosis performed by Nordpool for each hour and year. The results for area DK1 are rounded to the whole number. We can spot that error for prognosis is bigger for newer data with over 80% growth in 2020 (until 12th May) comparing to the previous year. Another thing we can notice that the error in the night hours is lower than others.

Area	Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	Total
DK1	2016	125	131	133	134	141	145	144	152	155	157	164	163	163	180	191	194	189	189	187	189	185	186	182	180	165
	2017	152	156	161	158	162	166	167	170	169	168	183	196	197	198	199	202	204	202	197	194	194	195	199	200	183
	2018	154	149	150	156	155	156	162	162	174	175	184	186	192	201	209	217	225	215	207	211	208	207	210	227	187
	2019	216	216	208	213	217	219	221	217	218	217	226	229	238	249	253	255	261	255	256	241	232	240	246	258	233
	2020	410	379	381	389	401	400	425	411	381	382	386	404	431	429	451	447	484	470	432	424	437	455	449	462	422
	Total	182	181	181	184	188	191	194	195	196	196	205	211	217	225	233	236	242	236	230	227	224	227	229	237	211

Figure 13: MAE for wind power and Nordpool prognosis for each hour and year (DK1)

The results for area DK2 are similar, however numbers are smaller due to the lower capacity of wind farms in this part of Denmark.

Area	Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	Total
DK2	2016	45.7	45.7	45.9	45.0	47.4	48.7	50.3	51.5	50.6	49.1	49.6	52.2	49.6	49.9	51.5	51.3	53.2	58.0	60.8	59.4	59.4	60.7	65.5	66.1	52.8
	2017	55.5	57.4	57.1	56.5	53.7	54.7	56.0	56.3	59.7	60.0	62.2	65.0	67.9	70.0	67.1	68.9	68.2	68.8	70.1	72.6	72.2	70.4	68.3	69.3	63.7
	2018	52.5	54.2	53.9	54.1	55.2	56.4	56.2	59.5	60.7	60.1	60.1	62.5	60.9	59.2	59.8	65.1	67.3	73.6	75.4	70.7	70.9	68.5	66.6	69.6	62.2
	2019	58.4	60.0	56.5	58.1	55.2	55.1	56.7	60.3	64.2	62.9	66.3	67.7	65.9	65.1	63.5	64.9	67.6	69.6	71.0	68.6	67.9	72.4	73.6	73.4	64.4
	2020	68.6	66.1	65.7	67.2	66.7	66.6	64.3	63.7	61.1	72.8	73.0	70.1	74.4	79.9	78.6	76.9	87.7	84.3	78.0	76.6	82.5	87.8	89.8	96.0	74.9
	Total	54.3	55.3	54.4	54.6	54.0	54.8	55.6	57.5	59.0	59.3	60.7	62.5	62.2	62.6	62.0	63.7	66.0	68.9	70.0	68.5	68.8	69.7	70.3	71.8	61.9

Figure 14: MAE for wind power and Nordpool prognosis for each hour and year (DK2)

3.4 Price data

In the 2020 price of energy decreased compared to the previous years. However we don't know what kind of the impact had epidemic on this data so it's hard to come up with any conclusion.

Year	DK1	DK2
2016	184.435	206.825
2017	224.005	227.575
2018	329.235	339.345
2019	289.270	295.390
2020	142.520	149.865

Table 3: Average price of energy per each year (DKK/MWh)

As in the consumption data we can also notice seasonal trends in this category, but this time only two kinds: daily and weekly. There is no annual trend in the price data.

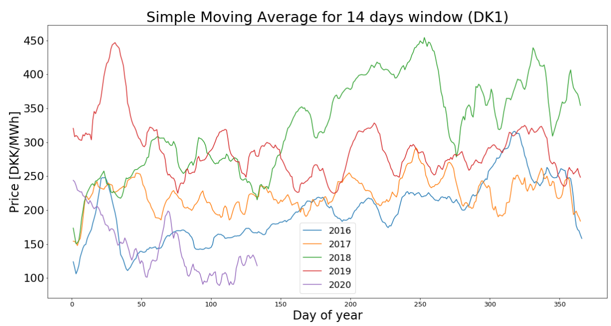

Figure 15: Simple moving average of price for each year - 14 days window (DK1)

Electricity prices are lower during weekends due to the lower consumption and there are also 2 daily peaks each day, either weekend (including holidays) and work days.

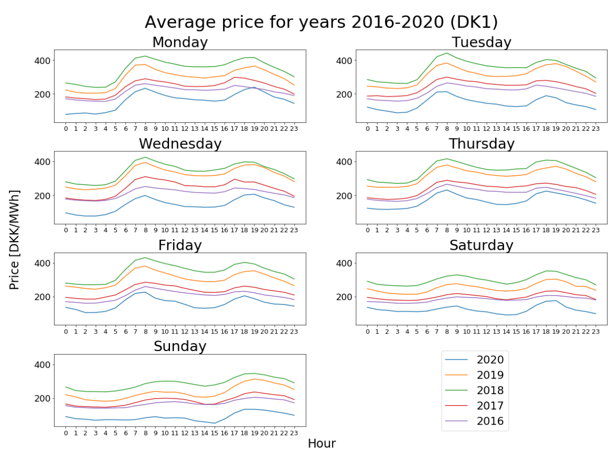

Figure 16: Average price per hour of a day for each day of week (DK1)

3.4.1 Negative prices

There were 417 negative prices in the area DK1 in the years 2016-2020 and 288 in the DK2. The dataset for 2020 ends on 12th May, however there are already 84 negative prices compared to the 133 in the full year 2019. Negative prices occur more often during nights than the peaks of consumption.

year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	Total
2016	3	7	8	8	6	4	5	2	1	1	0	0	1	2	3	2	2	1	0	0	1	2	0	4	63
2017	6	6	8	7	6	5	6	4	1	2	2	2	2	4	4	5	4	0	0	0	1	3	3	4	85
2018	4	5	5	6	7	6	6	3	0	0	1	0	1	2	2	1	0	0	0	0	0	1	0	2	52
2019	9	9	12	14	11	5	6	4	3	3	4	3	6	9	11	8	4	2	1	1	1	1	1	5	133
2020	5	3	6	7	7	7	4	3	2	2	4	3	3	4	6	7	6	1	1	0	0	1	1	1	84
Total	27	30	39	42	37	27	27	16	7	8	11	8	13	21	26	23	16	4	2	1	3	8	5	16	417

Figure 17: Number of negative prices for each hour, year and totally (DK1)

3.4.2 Correlation between price and wind power

We can suspect that with bigger values of wind power price is lower. Pearson's correlation shows that mostly it's weak correlation, only for 2020 year there are moderate values.

Correlation is a little bit higher for night hours than rest of the day.

Area	Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
DK1	2016	-0.41	-0.42	-0.41	-0.41	-0.40	-0.36	-0.36	-0.40	-0.44	-0.44	-0.42	-0.41	-0.40	-0.39	-0.38	-0.37	-0.35	-0.35	-0.41	-0.43	-0.38	-0.37	-0.37	-0.37
	2017	-0.62	-0.62	-0.61	-0.58	-0.54	-0.52	-0.46	-0.46	-0.51	-0.53	-0.53	-0.55	-0.56	-0.54	-0.53	-0.51	-0.50	-0.51	-0.60	-0.63	-0.62	-0.60	-0.62	-0.61
	2018	-0.45	-0.46	-0.49	-0.50	-0.48	-0.44	-0.36	-0.38	-0.42	-0.44	-0.46	-0.45	-0.43	-0.41	-0.40	-0.39	-0.42	-0.41	-0.45	-0.46	-0.44	-0.46	-0.47	-0.46
	2019	-0.50	-0.50	-0.52	-0.53	-0.51	-0.46	-0.38	-0.37	-0.38	-0.39	-0.41	-0.42	-0.41	-0.40	-0.38	-0.35	-0.33	-0.31	-0.40	-0.51	-0.56	-0.56	-0.56	-0.56
	2020	-0.53	-0.63	-0.81	-0.84	-0.83	-0.83	-0.72	-0.68	-0.69	-0.69	-0.68	-0.64	-0.68	-0.69	-0.72	-0.74	-0.75	-0.73	-0.72	-0.72	-0.69	-0.65	-0.69	-0.73
DK2	2016	-0.36	-0.40	-0.41	-0.40	-0.34	-0.29	-0.27	-0.23	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.23	-0.18	-0.19	-0.24	-0.28	-0.31	-0.31	-0.28	-0.32
	2017	-0.61	-0.61	-0.61	-0.61	-0.57	-0.50	-0.45	-0.42	-0.42	-0.43	-0.44	-0.46	-0.44	-0.43	-0.42	-0.38	-0.37	-0.37	-0.42	-0.47	-0.53	-0.58	-0.62	-0.63
	2018	-0.52	-0.53	-0.53	-0.53	-0.51	-0.48	-0.39	-0.31	-0.30	-0.33	-0.35	-0.38	-0.36	-0.35	-0.34	-0.33	-0.33	-0.31	-0.34	-0.38	-0.40	-0.43	-0.47	-0.50
	2019	-0.50	-0.52	-0.52	-0.52	-0.49	-0.43	-0.31	-0.32	-0.31	-0.28	-0.26	-0.25	-0.25	-0.25	-0.23	-0.23	-0.24	-0.26	-0.37	-0.44	-0.48	-0.46	-0.49	-0.50
	2020	-0.50	-0.46	-0.43	-0.35	-0.39	-0.51	-0.58	-0.38	-0.39	-0.40	-0.35	-0.40	-0.38	-0.34	-0.31	-0.29	-0.32	-0.38	-0.36	-0.43	-0.53	-0.64	-0.75	-0.75

Figure 18: Correlation between price and wind power for each hour and year (DK1 and DK2)

3.5 Holidays

Usage of the electricity decreases during weekends and public holidays and this has a significant effect on a prediction especially during Christmas or Easter. Due to that fact, each day was aligned with variable holiday with following value based of occurrence of day of week or public holiday:

- 1 - National Holidays (e.g. Easter Monday)[9]
- 1 - Sundays
- 0 - Not a public holidays (e.g. New Year's Eve)
- 0 - Other days

Each holiday is treated in weekday dummies as a Sunday.

4 Methodology

I decided to choose Danish market for forecasting and so on I want to apply methods and models most fitting to the characteristics of this market. I performed forecasting for 3 categories of values: wind power, consumption and price. In each category I made prognosis for few models which are described in Forecasting section. My forecasting framework for all of them is following:

1. Data preparation

2. Preliminary data analysis
3. Data forecasting (cycle for each model and day)

    Optional deseasonalization

    Data normalization

    Day-ahead forecasting
4. Verifying models' performance

All calculations were conducted with Python 3.7 language and its libraries, i.e. numpy, pandas, scipy. All the scripts and results made are accessible in public GIT repository https://github.com/ajescode/energyForecast.

Wind power prognosis and Consumption prognosis is provided only one day in advance so data forecasting is also performed day ahead. I used 3 calibration windows to compute predictions. They may take values of week multiples (7 days) because of the seasonal trend of energy production. Because chosen data has a limit of 1583 days, the maximum calibration window I considered is 728 days, around 2 years.

I consider few models for each category of forecasts (consumption, price and wind power) which I explain later in this section. General equation for each model (except of benchmark models for wind power and consumption) can be presented as:

$$\hat{Y}_{d,h} = \sum_{i=1}^{n} \beta_{d,h,i} X^i_{d,h}, \tag{1}$$

where $Y_{d,h}$ is prediction and $X^i_{d,h}$ is variable for given day (d) and hour (h). We can predict values for next whole day only, but basing on the variables from whole calibration window. Coefficients $\beta_{h,i}$ have been approximated by ordinary least squares (OLS).

$$\vec{\hat{\beta}} = (X^T X)^{-1} X^T Y, \tag{2}$$

where X is matrix of independent variables, and Y is vector of dependant variables of size of calibration window (multiple of 7 days).

Because prices of energy due to the wind dependence can have negative values I needed to normalize values. I used function $asinh$ which was empirically confirmed to have best results for danish market among 4 different normalization functions.[1].

$$X_{d,h} = asinh(x_{d,h}) \equiv \log(x_{d,h} + \sqrt{x^2_{d,h} + 1}), \tag{asinh}$$

where $Y_{d,h}$ is transformed value used for forecasts either as independent variable or dependent variable. Independent values in the model must be also transformed. $y_{d,h}$ is normalized in a calibration window by:

$$x_{d,h} = \frac{1}{b_{d,h}}(x^*_{d,h} - a_{d,h}), \tag{std}$$

where a is median and b is median absolute deviation (MAD) for given day and hour in the calibration window. $y^*_{d,h}$ is original value of a parameter without any transformation yet. I used both simple normalization (std), and one with $asinh$ function (asinh) for each category of forecasting.

Inverse function for transformation and normalization is following:

$$x^*_{d,h} = b_{d,h} sinh(X_{d,h}) + a_{d,h}, \tag{3}$$

To eliminate seasonal component from the data I applied Hodrick-Prescott filter[12] before standard normalization with smoothing parameter derived from Ravn and Uhlig (2002) [13] and adjusted to the daily data as $1600^4 * 6.25 = 110930628906.25$.

Types of transformations used:

1. Standard normalization (std)
2. Standard normalization (std) + asinh (asinh)
3. Standard normalization (std) + asinh (asinh) + HP filter

Despite of used transformation, the forecasts were performed with several different settings for each area (DK1 and DK2):

- Calibration window: 182, 364, 728
- Predicted dates: 2019.01.01-2019.12.31 (2019 year),
  2019.05.13-2020.05.12 (last year),
  2020.01.01-2020.05.12,
  2019.01.01-2020.05.12

4.1 Demand forecasting

First considering model is a benchmark model containing consumption forecast provided by Nordpool's database for day ahead ($FL_{t,h}$). Its performance is analyzed in the section Demand data prognosis.

$$C_{d,h} = FC_{d,h} \tag{C1}$$

Second model is extension of benchmark model (C1), where Di, d is a vector of the values 0,1 for the corresponding day of a week. For day (d) which is Monday $D_{1,d} = 1$ and $D_{2,d} = D_{3,d} = ... = D_{7,d} = 0$, for a day which is Tuesday $D_{2,d} = 1$ and $D_{1,d} = D_{3,d} = ... = D_{7,d} = 0$ etc.

$$C_{d,h} = \sum_{i=1}^{7} \beta_{0,i} D_{i,d} + \beta_1 FC_{d,h} + \varepsilon_{d,h} \tag{C2}$$

Third model is extension of second model (C2), where are added 3 parameters according to a similar-day technique: consumption of previous day ($C_{d-1,h}$), 2 days ago ($C_{d-2,h}$) and week ago ($C_{d-7,h}$).

$$C_{d,h} = \sum_{i=1}^{7} \beta_{0,i} D_{i,d} + \beta_1 FC_{d,h} + \beta_1 C_{d-1,h} + \beta_2 C_{d-2,h} + \beta_3 C_{d-7,h} + \varepsilon_{d,h} \tag{C3}$$

Fourth model is extension of third model (C3) where is also considered forecast of the wind power download from Nordpool's database, denoted as $FW_{d,h}$.

$$C_{d,h} = \sum_{i=1}^{7} \beta_{0,i} D_{i,d} + \beta_1 FC_{d,h} + \beta_2 C_{d-1,h} + \beta_3 C_{d-2,h} + \beta_4 C_{d-7,h} + \beta_5 FW_{d,h} + \varepsilon_{d,h} \tag{C4}$$

4.2 Wind power forecasting

First model is presented and analyzed in the section *Wind power data prognosis.* This is a benchmark model for further forecasts, where $FW_{t,h} + \varepsilon_t$ is a day ahead forecast of wind power taken from Nordpool's data.

$$W_{t,h} = FW_{t,h} \tag{W1}$$

Second model includes additionally day of week vector as the second demand model (C2).

$$W_{t,h} = \sum_{i=1}^{7} \beta_{0,i} D_{i,d} + \beta FW_{t,h} + \varepsilon_{d,h} \tag{W2}$$

Third model is extension of second model (W2) and contains also wind power from previous day ($W_{t-1,h}$) and 2 days ago ($W_{t-1,h}$) from the corresponding hour.

$$W_{t,h} = \sum_{i=1}^{7} \beta_{0,i} D_{i,d} + \beta_1 FW_{t,h} + \beta_2 W_{t-1,h} + \beta_3 W_{t-2,h} + \varepsilon_{d,h} \tag{W3}$$

Fourth model is extension of third one (W3) and contains additionally parameter of consumption forecast from previous day denoted as $FL_{t,h}$.

$$W_{t,h} = \sum_{i=1}^{7} \beta_{0,i} D_{i,d} + \beta_1 FW_{t,h} + \beta_2 W_{t-1,h} + \beta_3 W_{t-2,h} + \beta_4 FC_{t,h} + \varepsilon_{d,h} \tag{W4}$$

4.3 Price forecasting

First forecasting model of price consists of the same daf of week vector ($D_{i,d}$) as the second demand model (C2).

$$P_{d,h} = \sum_{i=1}^{7} \beta_{0,i} D_{i,d} + \varepsilon_{d,h} \tag{P1}$$

Second model is an extension of first model (P1), where were added parameters of price for previous days: 1 day ago, 2 days ago and 1 week ago.

$$P_{d,h} = \sum_{i=1}^{7} \beta_{0,i} D_{i,d} + \beta_1 p_{d-1,h} + \beta_2 p_{d-2,h} + \beta_3 p_{d-7,h} + \varepsilon_{d,h} \tag{P2}$$

Third model is an extension of second model (P2) which consists additionally parameters of minimum ($p_{d-1,min}$) and maximum value ($p_{d-1,max}$) of previous day and value of price for last hour of previous day ($p_{d-1,24}$).

$$P_{d,h} = \sum_{i=1}^{7} \beta_{0,i} D_{i,d} + \beta_1 p_{d-1,h} + \beta_2 p_{d-2,h} + \beta_3 p_{d-7,h} + \beta_4 p_{d-1,min} + \beta_5 p_{d-1,max} + \beta_6 p_{d-1,24} + \varepsilon_{d,h} \tag{4}$$

Fourth model is an extension of third model (4) which is an expert model ARX2[4] with two exogenous variables: wind power prognosis ($FW_{d,h}$) and consumption prognosis ($FC_{d,h}$).

$$\begin{aligned} P_{d,h} = \sum_{i=1}^{7} \beta_{0,i} D_{i,d} + \beta_1 p_{d-1,h} + \beta_2 p_{d-2,h} + \beta_3 p_{d-7,h} + \beta_4 p_{d-1,min} + \beta_5 p_{d-1,max} \\ + \beta_6 p_{d-1,24} + \beta_7 FW_{d,h} + \beta_8 FC_{d,h} + \varepsilon_{d,h} \end{aligned} \tag{P4}$$

In the last fifth model, Nordpool's prognosis for wind and consumption are replaced with the best forecasts acquired from the wind and consumption prognosis. However only values in predicted period were replaced by better prognosis, not full rolling window, that's why eventually it wasn't very efficient.

Different models for different periods and areas were used. Summary of best models used were presented in next chapter.

$$\begin{aligned} P_{d,h} = \sum_{i=1}^{7} \beta_{0,i} D_{i,d} + \beta_1 p_{d-1,h} + \beta_2 p_{d-2,h} + \beta_3 p_{d-7,h} + \beta_4 p_{d-1,min} + \beta_5 p_{d-1,max} \\ + \beta_6 p_{d-1,24} + \beta_7 FW^*_{d,h} + \beta_8 FC^*_{d,h} + \varepsilon_{d,h} \end{aligned} \tag{P5}$$

5 Empirical results

For each forecast there is error analysis with Mean average error (MAE), Mean root square error (MRSE). All forecasts in the appropriate categories are compared to each other in order to get optimal result.

$$MAE = \frac{1}{24T}\sum_{d=1}^{T}\sum_{h=1}^{24}|\hat{\varepsilon}_{d,h}| \equiv \frac{1}{24T}\sum_{d=1}^{T}\sum_{h=1}^{24}|y_{d,h} - \hat{y}_{d,h}|, \tag{5}$$

$$RMSE = \sqrt{\frac{1}{24T}\sum_{d=1}^{T}\sum_{h=1}^{24}\hat{\varepsilon}_{d,h}^2} \equiv \sqrt{\frac{1}{24T}\sum_{d=1}^{T}\sum_{h=1}^{24}(y_{d,h} - \hat{y}_{d,h})^2}, \tag{6}$$

where T is size of calibration window.

From each consumption, wind and price prognosis were chosen the best results for each period and area and performed Diebold-Mariano test based on MAD criterion.

5.1 Demand forecasts

For both areas results with *asinh* were significantly worse than with standard normalization function. Especially for calibration window 182. The forecast with standard normalization was better for Models 2-4 than Nordpool's prognosis, however better accuracy was for longer calibration windows. Because drops of consumption level in 2020 results including this period were less accurate.

Area	Dates	Window	C1	C2	C2 (asinh)	C2 (asinh-hp)	C2 (hp)	C3	C3 (asinh)	C3 (asinh-hp)	C3 (hp)	C4	C4 (asinh)	C4 (asinh-hp)	C4 (hp)
DK1	2019.01.01-2019.12.31	182	22.7044	21.0269	21.2785	24.1466	23.8343	21.3643	21.6734	24.3640	24.0067	21.4608	21.8100	24.5258	24.1166
		364	22.7044	20.5738	20.6452	36.0889	36.0494	20.6044	20.7099	36.2072	36.1524	20.6357	20.7542	36.2376	36.1422
		728	22.7044	20.4818	20.5553	59.5941	59.8155	20.4128	20.5007	59.3679	59.5639	20.3765	20.4789	59.4302	59.6149
	2019.01.01-2020.05.12	182	23.7079	21.9972	22.3082	24.7041	24.3825	22.3403	22.6712	24.9615	24.5930	22.4058	22.7671	25.0662	24.6507
		364	23.7079	21.6325	21.6570	35.8995	35.7843	21.7035	21.7714	35.8886	35.8010	21.6997	21.7899	35.8401	35.7073
		728	23.7079	21.5292	21.5643	60.5532	60.5844	21.4925	21.5529	60.4016	60.4206	21.4406	21.5306	60.1393	60.0978
	2019.05.13-2020.05.12	182	24.5221	22.8302	23.0908	25.4669	25.1809	23.1619	23.4355	25.6814	25.3691	23.2260	23.5189	25.7723	25.4160
		364	24.5221	22.4451	22.4610	36.5130	36.3553	22.5117	22.5626	36.5040	36.3466	22.4685	22.5485	36.5255	36.3298
		728	24.5221	22.3265	22.3365	61.6104	61.6929	22.2998	22.3290	61.4345	61.4787	22.2225	22.2967	61.0137	60.9890
	2020.01.01-2020.05.12	182	26.4619	24.6599	25.1339	26.2342	25.8871	25.0187	25.4097	26.6014	26.2021	24.9993	25.3940	26.5495	26.1163
		364	26.4619	24.5378	24.4340	35.3799	35.0568	24.7197	24.6847	35.0143	34.8365	24.6198	24.6324	34.7494	34.5140
		728	26.4619	24.4039	24.3334	63.1851	62.6947	24.4557	24.4404	63.2386	62.7718	24.3607	24.4166	62.0852	61.4231

Figure 19: MAE on the consumption forecasts (DK1)

Area	Dates	Window	C1	C2	C2 (asinh)	C2 (asinh-hp)	C2 (hp)	C3	C3 (asinh)	C3 (asinh-hp)	C3 (hp)	C4	C4 (asinh)	C4 (asinh-hp)	C4 (hp)
DK2	2019.01.01-2019.12.31	182	19.4739	18.5070	19.1465	19.0951	18.4566	18.1614	19.0274	18.9975	18.1329	18.2866	19.2230	19.2031	18.2637
		364	19.4739	17.8111	17.7460	17.7535	17.7854	17.6134	17.9143	17.9818	17.6679	17.7855	18.1222	18.2021	17.8651
		728	19.4739	17.6512	17.6099	18.0669	18.0412	17.3885	17.5390	18.0941	17.8905	17.5472	17.7300	18.2579	18.0342
	2019.01.01-2020.05.12	182	19.8874	18.5865	19.2647	19.2070	18.5321	18.2830	19.1055	19.0654	18.2456	18.3780	19.2632	19.2268	18.3462
		364	19.8874	18.2011	18.1744	18.1461	18.1316	18.0487	18.3232	18.3318	18.0357	18.1626	18.4691	18.4902	18.1706
		728	19.8874	18.2647	18.2180	18.5074	18.4816	18.0622	18.1930	18.5798	18.3977	18.1841	18.3504	18.7252	18.5168
	2019.05.13-2020.05.12	182	18.9997	17.5838	18.2321	18.1748	17.5352	17.1995	17.8265	17.7952	17.1671	17.2685	17.9523	17.9228	17.2436
		364	18.9997	17.4321	17.4237	17.3046	17.2795	17.2091	17.3354	17.3093	17.1650	17.2150	17.3602	17.3477	17.1868
		728	18.9997	17.4155	17.3765	17.5042	17.4841	17.2369	17.3201	17.6461	17.5203	17.3067	17.4280	17.7438	17.5863
	2020.01.01-2020.05.12	182	21.0221	18.8044	19.5893	19.5139	18.7391	18.6167	19.3198	19.2518	18.5548	18.6288	19.3735	19.2918	18.5725
		364	21.0221	19.2717	19.3500	19.2236	19.0815	19.2435	19.4456	19.2923	19.0451	19.1973	19.4211	19.2809	19.0090
		728	21.0221	19.9481	19.8869	19.7163	19.6904	19.9111	19.9880	19.9126	19.7895	19.9320	20.0529	20.0076	19.8412

Figure 20: MAE on the consumption forecasts (DK2)

Area	Dates	Window	C1	C2	C2 (asinh)	C2 (asinh-hp)	C2 (hp)	C3	C3 (asinh)	C3 (asinh-hp)	C3 (hp)	C4	C4 (asinh)	C4 (asinh-hp)	C4 (hp)
DK1	2019.01.01-2019.12.31	182	36.9021	35.1851	35.3440	37.6028	37.3393	35.4515	35.6229	37.7305	37.4808	35.4539	35.6312	37.7741	37.5051
		364	36.9021	34.8179	34.8509	47.3679	47.2368	34.7848	34.8236	47.4697	47.3370	34.7550	34.8095	47.5017	47.3267
		728	36.9021	34.7436	34.7824	68.0356	68.1554	34.6753	34.7107	67.8902	67.9854	34.6132	34.6703	67.9224	68.0114
	2019.01.01-2020.05.12	182	36.7802	34.9363	35.1362	37.0798	36.8179	35.2543	35.4395	37.3173	37.0468	35.2632	35.4683	37.3735	37.0368
		364	36.7802	34.6373	34.6133	46.5772	46.4119	34.6808	34.6729	46.5696	46.4417	34.6375	34.6675	46.5427	46.3598
		728	36.7802	34.5603	34.5601	68.6243	68.5919	34.5378	34.5361	68.5395	68.4949	34.4697	34.5156	68.3048	68.2087
	2019.05.13-2020.05.12	182	39.3265	37.4799	37.5582	39.3605	39.2253	37.7994	37.8761	39.5733	39.4245	37.7968	37.8899	39.6054	39.3842
		364	39.3265	37.2244	37.1853	48.5201	48.3321	37.2624	37.2402	48.5115	48.3304	37.1906	37.2170	48.5104	48.2850
		728	39.3265	37.1387	37.1166	70.5158	70.5085	37.1220	37.0905	70.4420	70.3902	37.0382	37.0709	70.0604	69.9530
	2020.01.01-2020.05.12	182	36.4434	34.2443	34.5594	35.6050	35.3475	34.7072	34.9315	36.1592	35.8289	34.7344	35.0176	36.2513	35.7201
		364	36.4434	34.1367	33.9527	44.3350	44.0687	34.3940	34.2557	44.0049	43.8910	34.3130	34.2750	43.8031	43.5964
		728	36.4434	34.0522	33.9424	70.2144	69.7759	34.1577	34.0524	70.2905	69.8741	34.0730	34.0876	69.3434	68.7471

Figure 21: RMSE on the consumption forecasts (DK1)

Area	Dates	Window	C1	C2	C2 (asinh)	C2 (asinh-hp)	C2 (hp)	C3	C3 (asinh)	C3 (asinh-hp)	C3 (hp)	C4	C4 (asinh)	C4 (asinh-hp)	C4 (hp)
DK2	2019.01.01-2019.12.31	182	28.3419	26.7212	27.3657	27.3801	26.7393	26.0698	27.2501	27.2591	26.0859	26.2582	27.5924	27.6073	26.2755
		364	28.3419	26.0223	25.9419	26.1023	26.1619	25.2956	25.6072	25.7888	25.4555	25.5330	25.8912	26.0707	25.6926
		728	28.3419	25.9872	25.9431	26.5737	26.5666	25.1838	25.3110	25.9992	25.8112	25.3099	25.4714	26.1508	25.9294
	2019.01.01-2020.05.12	182	29.4301	27.5234	28.1959	28.1903	27.5225	26.8667	27.8857	27.8809	26.8643	27.0062	28.1566	28.1553	27.0045
		364	29.4301	27.1706	27.1300	27.2239	27.2374	26.3957	26.6262	26.7392	26.4785	26.5385	26.8127	26.9255	26.6226
		728	29.4301	27.3285	27.2664	27.6885	27.6941	26.4286	26.4513	26.9463	26.8598	26.5347	26.5960	27.0935	26.9706
	2019.05.13-2020.05.12	182	28.0782	26.0545	26.7306	26.7120	26.0410	25.3964	26.1364	26.1275	25.3848	25.4814	26.2855	26.2807	25.4729
		364	28.0782	25.9346	25.9279	25.9514	25.9351	25.2422	25.3280	25.3993	25.2849	25.2366	25.3395	25.4121	25.2846
		728	28.0782	25.9816	25.9368	26.2186	26.2190	25.2271	25.1870	25.6051	25.5894	25.2853	25.2830	25.6968	25.6474
	2020.01.01-2020.05.12	182	32.2284	29.6137	30.3578	30.3026	29.5657	28.9411	29.5600	29.5201	28.8928	28.9598	29.6497	29.6070	28.9111
		364	32.2284	30.0975	30.1510	30.0880	29.9914	29.2027	29.2408	29.1888	29.1018	29.1200	29.1925	29.1427	29.0221
		728	32.2284	30.7099	30.6054	30.5397	30.5754	29.5767	29.3541	29.3892	29.5469	29.6370	29.4624	29.5265	29.6409

Figure 22: RMSE on the consumption forecasts (DK2)

5.2 Wind power forecasts

The retrieved results from wind power forecasting with standard normalization were significantly better for DK1, and very slightly better for DK2 comparing to the Nordpool's forecast. *Asinh* function as normalization was useful only for part of cases with short calibration window, so I don't consider it as succesful results. MAE for Model 4 with 728 days window and for full 2019 year is 6,6% lower, while MAE for last period (2020.01.01-2020.05-12) is 22,19% lower.

Area	Dates	Window	W1	W2	W2 (asinh)	W2 (asinh-hp)	W2 (hp)	W3	W3 (asinh)	W3 (asinh-hp)	W3 (hp)	W4	W4 (asinh)	W4 (asinh-hp)	W4 (hp)
DK1	2019.01.01-2019.12.31	182	234.5053	217.4789	215.8943	214.7341	215.6551	219.6931	218.7317	217.7375	218.1050	220.0010	220.0222	218.0514	217.8571
		364	234.5053	217.1779	214.9680	210.8029	211.5826	218.2126	216.0895	212.2605	213.0257	218.5291	216.3776	213.2433	213.6965
		728	234.5053	218.8722	217.1265	225.2318	224.0590	219.0693	217.3630	226.1964	224.8970	219.0649	217.3302	227.0352	225.1335
	2019.01.01-2020.05.12	182	283.8955	235.2740	233.8541	232.4468	233.3719	237.8470	237.0606	235.8626	236.1541	238.3007	238.3563	237.1771	236.9041
		364	283.8955	239.1466	236.6201	231.8007	232.9267	240.6124	238.2046	233.6711	234.7300	240.5791	238.0895	234.5342	235.1725
		728	283.8955	248.2129	245.1569	246.4548	246.1251	248.4362	245.3776	247.5864	247.0758	247.8012	244.5268	248.1401	247.2035
	2019.05.13-2020.05.12	182	304.0500	236.6935	235.3443	232.4206	233.2747	239.2110	238.6459	236.1462	236.2941	240.2263	240.6466	238.4192	237.8396
		364	304.0500	243.1853	240.4881	231.3513	232.7017	244.5851	242.0834	233.3375	234.4189	244.5565	241.9797	234.4015	235.0060
		728	304.0500	256.0665	252.4397	242.4979	242.1728	256.1935	252.5289	243.4029	242.7863	255.4059	251.4758	244.1583	242.9778
	2020.01.01-2020.05.12	182	419.4398	284.1102	283.1422	281.0570	281.9934	287.6680	287.3618	285.6044	285.6874	288.5216	288.6716	289.6650	289.1759
		364	419.4398	299.4366	296.0415	289.4264	291.5025	302.0856	298.8966	292.4296	294.2944	301.0921	297.6750	292.9641	294.1102
		728	419.4398	328.7344	322.0822	304.6984	306.6825	329.0299	322.2599	306.2883	307.9427	326.6640	319.1640	306.0596	307.7717

Figure 23: MAE on the wind power forecasts (DK1)

Area	Dates	Window	W1	W2	W2 (asinh)	W2 (asinh-hp)	W2 (hp)	W3	W3 (asinh)	W3 (asinh-hp)	W3 (hp)	W4	W4 (asinh)	W4 (asinh-hp)	W4 (hp)
DK2	2019.01.01-2019.12.31	182	64.5167	66.3122	67.0413	67.4307	66.6758	66.8695	67.7520	68.1468	67.1943	67.0683	67.9557	68.2130	67.2834
		364	64.5167	65.0329	65.4815	67.8653	67.2296	65.2724	65.8123	68.1706	67.3852	65.1757	65.8565	68.5175	67.5938
		728	64.5167	65.4080	66.0257	74.7557	73.8559	65.5347	66.2718	75.0148	74.0370	65.5368	66.2576	74.7739	73.8716
	2019.01.01-2020.05.12	182	67.2619	69.0686	69.6017	69.8011	69.2103	69.8881	70.5497	70.7663	70.0300	70.0938	70.6660	70.8996	70.2510
		364	67.2619	67.5898	67.9303	69.8319	69.2777	68.0346	68.4954	70.3540	69.6380	68.0403	68.5525	70.6170	69.8619
		728	67.2619	67.4937	67.8985	76.1776	75.3422	67.7115	68.2473	76.3622	75.4676	67.7688	68.2556	76.2949	75.4954
	2019.05.13-2020.05.12	182	66.8141	69.9545	70.5639	70.9389	70.2817	70.7369	71.4112	71.7878	71.0638	70.7960	71.3476	71.7381	71.0793
		364	66.8141	68.1596	68.4344	70.6860	70.2773	68.5159	68.8413	71.0388	70.5430	68.5785	68.8640	71.1720	70.7202
		728	66.8141	67.8917	68.0205	77.0133	76.4850	68.0327	68.2143	77.0524	76.4654	68.0471	68.2089	77.3476	76.7213
	2020.01.01-2020.05.12	182	74.7957	76.6333	76.6285	76.3060	76.1660	78.1720	78.2277	77.9553	77.8123	78.3967	78.1041	78.2726	78.3955
		364	74.7957	74.6069	74.6505	75.2290	74.8985	75.6149	75.8586	76.3462	75.8204	75.9019	75.9513	76.3789	76.0864
		728	74.7957	73.2178	73.0383	80.0800	79.4211	73.6853	73.6690	80.0599	79.3938	73.8943	73.7389	80.4688	79.9520

Figure 24: MAE on the wind power forecasts (DK2)

Area	Dates	Window	W1	W2	W2 (asinh)	W2 (asinh-hp)	W2 (hp)	W3	W3 (asinh)	W3 (asinh-hp)	W3 (hp)	W4	W4 (asinh)	W4 (asinh-hp)	W4 (hp)
DK1	2019.01.01-2019.12.31	182	318.3849	290.3546	289.3503	287.5060	288.0211	292.1398	292.1897	290.2684	289.8196	292.8945	294.0494	291.4678	290.2309
		364	318.3849	291.0473	288.8411	282.3725	283.6819	291.7512	289.4938	283.0420	284.2917	292.3236	290.1264	284.5274	285.1055
		728	318.3849	292.7880	291.1340	294.6650	294.8072	293.0215	291.2111	294.9754	295.0011	293.0429	291.1902	296.5663	295.5779
	2019.01.01-2020.05.12	182	406.8696	321.3105	319.3540	317.2610	318.7936	323.8967	323.2104	321.1816	321.4703	324.1184	324.3939	322.2900	321.7824
		364	406.8696	330.3779	326.5089	319.1337	321.9901	331.4233	327.7576	320.6410	323.1410	331.7391	328.0553	321.8958	323.6097
		728	406.8696	346.4224	341.3667	333.4185	336.1887	346.3990	341.3394	334.2268	336.6006	345.9703	340.2259	335.1776	337.0265
	2019.05.13-2020.05.12	182	436.8404	327.2397	325.3969	321.7005	323.0743	329.8075	329.4912	326.0052	325.8949	329.7964	330.6365	327.0491	326.1218
		364	436.8404	340.3449	336.0647	324.7154	327.8384	341.2548	337.3300	326.1918	328.7467	341.3650	337.3970	327.4691	329.1208
		728	436.8404	362.4314	356.5375	335.7155	339.1086	362.0436	356.1661	336.0853	338.9054	361.5284	354.8008	337.4553	339.5970
	2020.01.01-2020.05.12	182	584.5163	393.9593	390.0106	387.3483	390.9928	398.2438	396.0497	393.7399	395.5255	397.3969	395.8954	394.6986	395.6478
		364	584.5163	419.7910	412.5762	403.1498	409.0879	421.5320	415.0210	406.3270	411.3181	421.3738	414.6895	407.1943	411.1519
		728	584.5163	462.7041	451.3598	421.8613	429.7469	462.2330	451.1459	423.6569	430.5881	460.9918	448.0228	423.4215	430.7499

Figure 25: RMSE on the wind power forecasts (DK1)

Area	Dates	Window	W1	W2	W2 (asinh)	W2 (asinh-hp)	W2 (hp)	W3	W3 (asinh)	W3 (asinh-hp)	W3 (hp)	W4	W4 (asinh)	W4 (asinh-hp)	W4 (hp)
DK2	2019.01.01-2019.12.31	182	89.6924	90.5770	91.3340	91.8347	90.9437	91.0369	92.0400	92.5017	91.3492	91.3910	92.3964	92.8075	91.6583
		364	89.6924	89.1968	89.5614	91.9816	91.2586	89.2972	89.7584	92.1715	91.2894	89.2374	89.8611	92.3191	91.3162
		728	89.6924	89.5253	90.0059	98.7950	97.7557	89.5193	90.0814	98.9810	97.8200	89.4642	90.0298	98.6609	97.6008
	2019.01.01-2020.05.12	182	93.4372	93.8419	94.3405	94.7405	94.1276	94.6971	95.4581	95.8298	94.9402	95.1074	95.8076	96.2354	95.3743
		364	93.4372	92.4863	92.6865	94.7561	94.1766	92.8985	93.2715	95.3660	94.5679	92.9781	93.4070	95.5346	94.7055
		728	93.4372	92.5478	92.7518	100.8334	99.9302	92.7081	93.0374	101.1325	100.1006	92.7592	93.0468	101.0612	100.1567
	2019.05.13-2020.05.12	182	93.6845	94.9894	95.6483	96.3169	95.5403	95.7659	96.6670	97.3150	96.2831	96.0474	96.9053	97.5625	96.5129
		364	93.6845	93.4505	93.6273	96.1939	95.6844	93.7752	94.0875	96.6889	96.0140	93.8709	94.1342	96.7499	96.1234
		728	93.6845	93.7661	93.8217	102.5378	101.8433	93.8442	93.9738	102.7131	101.9201	93.8533	93.9418	102.9080	102.1248
	2020.01.01-2020.05.12	182	103.0172	102.2680	102.1376	102.2918	102.3577	104.0829	104.2645	104.4198	104.1612	104.6304	104.5992	105.0696	104.8981
		364	103.0172	100.9649	100.7660	101.9833	101.7556	102.1313	102.2947	103.6281	103.0306	102.5453	102.5102	103.8488	103.4377
		728	103.0172	100.3761	99.9005	106.2266	105.6682	100.9431	100.7048	106.8146	106.1078	101.2523	100.8641	107.3732	106.8575

Figure 26: RMSE on the wind power forecasts (DK2)

5.3 Price forecasts

With more complex models results are much better, however *asinh* function surprisingly doesn't improve predictions in the every case. Disproportion is especially big in the last period (2020.01.01-2020.05-12) and Model 4.

I performed Model 5 hoping that with better predictions of wind power and consumption than original ones, price will be more accurate. Because these predictions replaced only values for prediction period, not rolling window, coefficients could be overfitted to the worse values trying to predict using improved ones.

Area	Dates	Window	P1	P1 (asinh)	P1 (asinh-hp)	P2	P2 (asinh)	P2 (asinh-hp)	P3	P3 (asinh)	P3 (asinh-hp)	P4	P4 (asinh)	P4 (asinh-hp)	P5	P5 (asinh)
DK1	2019.01.01-2019.12.31	182	62.419447	61.248905	61.248905	50.970812	47.862838	47.738136	45.531671	42.763101	42.691759	41.832735	38.053032	37.917382	41.789153	38.164474
		364	66.905983	67.163867	67.163867	49.816323	47.516155	47.173496	43.599339	41.380780	41.245874	41.600593	38.068337	37.814441	41.895198	38.249817
		728	59.616463	59.869545	59.869545	48.271333	46.798962	48.272424	42.218329	41.285560	42.293601	38.539624	36.953780	38.032519	38.773604	37.099335
	2019.01.01-2020.05.12	182	74.037510	73.752601	73.752601	54.304729	52.422105	52.272366	47.144198	45.256767	45.152057	42.638636	40.310793	40.175984	42.790111	40.480601
		364	81.583109	83.185105	83.185105	53.909704	53.190293	52.804557	45.803505	44.940818	44.797602	42.979752	41.426176	41.180891	43.460681	41.715465
		728	86.491722	87.441830	87.441830	52.921611	52.983252	53.732966	44.599572	44.803385	45.378040	40.125764	40.397561	41.097301	40.339211	40.469730
	2019.05.13-2020.05.12	182	71.894074	72.552328	72.552328	51.030218	50.348744	50.251831	44.979770	43.838396	43.789638	39.770383	38.890162	38.840113	39.980648	39.042576
		364	83.140187	85.593875	85.593875	51.939930	52.136530	51.694149	44.333355	44.243325	44.046129	42.318592	41.587801	41.127362	43.343499	42.180723
		728	90.324035	91.364589	91.364589	50.441822	51.137697	50.892249	42.781262	43.371479	43.193533	38.666817	39.451380	39.398347	39.421250	39.850178
	2020.01.01-2020.05.12	182	105.921667	108.067257	108.067257	63.454202	64.934380	64.715930	51.569554	52.100288	51.904001	44.850317	46.506904	46.374403	46.425426	47.449435
		364	121.862442	127.153167	127.153167	65.143420	68.762175	68.258221	51.852532	54.710845	54.544825	46.764663	50.641299	50.419645	48.747856	51.967810
		728	160.247132	163.110131	163.110131	65.683651	69.955175	68.718664	51.134560	54.457565	53.842851	44.478704	49.848538	49.508171	46.424097	51.241317

Figure 27: MAE on the price forecasts (DK1)

Area	Dates	Window	P1	P1 (asinh)	P1 (asinh-hp)	P2	P2 (asinh)	P2 (asinh-hp)	P3	P3 (asinh)	P3 (asinh-hp)	P4	P4 (asinh)	P4 (asinh-hp)	P5	P5 (asinh)
DK2	2019.01.01-2019.12.31	182	60.685783	59.265083	59.265083	47.103923	43.958484	43.868928	42.800522	40.041798	40.019143	40.867716	37.631112	37.592842	40.917539	37.656573
		364	66.643572	65.894549	65.894549	46.536799	43.736972	43.409012	41.864222	39.129699	38.977039	41.773766	37.899312	37.706766	41.737298	37.872167
		728	57.641463	56.997347	56.997347	44.836256	42.947585	44.126417	40.620861	38.820265	39.730950	38.663786	36.541061	37.185542	38.646771	36.530471
	2019.01.01-2020.05.12	182	72.088336	71.634624	71.634624	50.790176	49.063672	48.960707	44.593005	42.871472	42.829081	42.240303	40.706097	40.665848	42.268304	40.722874
		364	80.282137	81.045797	81.045797	50.643427	49.600087	49.256643	43.848764	42.678249	42.549245	43.034634	41.318684	41.161845	43.023788	41.311466
		728	84.403612	84.125969	84.125969	49.712208	49.083361	49.654152	42.993153	42.220931	42.751613	40.725420	40.009271	40.376973	40.703890	39.996084
	2019.05.13-2020.05.12	182	68.703284	69.280873	69.280873	47.331708	47.005937	46.951134	42.340374	41.453055	41.455745	38.983829	38.603889	38.608243	38.997465	38.620568
		364	80.654765	82.458458	82.458458	48.135879	48.324643	47.966416	41.888804	41.748732	41.650334	41.183152	40.570452	40.354765	41.105611	40.537650
		728	88.502231	88.186925	88.186925	47.364384	47.302565	47.090488	41.124731	40.735721	40.701233	38.987806	38.711242	38.577699	38.941618	38.684554
	2020.01.01-2020.05.12	182	103.381058	105.581108	105.581108	60.906583	63.074152	62.934386	49.512224	50.637121	50.540565	46.007178	49.144966	49.099286	45.955950	49.141889
		364	117.711280	122.626291	122.626291	61.913499	65.690589	65.304652	49.295062	52.416750	52.352668	46.494911	50.702674	50.643831	46.430872	50.649240
		728	157.848607	158.576699	158.576699	63.093580	65.922146	64.824254	49.503579	51.553585	51.041401	46.383287	49.527291	49.135411	46.274236	49.455233

Figure 28: MAE on the price forecasts (DK2)

Area	Dates	Window	P1	P1 (asinh)	P1 (asinh-hp)	P2	P2 (asinh)	P2 (asinh-hp)	P3	P3 (asinh)	P3 (asinh-hp)	P4	P4 (asinh)	P4 (asinh-hp)	P5	P5 (asinh)
DK1	2019.01.01-2019.12.31	182	88.552095	89.377633	89.377633	75.854601	74.598665	74.391426	68.473893	66.034200	65.874428	59.011419	55.578274	55.358938	59.203002	55.901061
		364	93.087824	94.352975	94.352975	75.095306	73.924743	73.360790	66.647750	64.475040	64.032671	59.280699	56.405672	55.988147	59.773234	56.644245
		728	85.255977	86.419926	86.419926	73.416049	72.257950	72.577855	65.340032	64.153815	64.052560	56.731674	56.140598	56.348611	57.210817	56.408465
	2019.01.01-2020.05.12	182	100.419523	101.646696	101.646696	76.764606	76.322214	76.089317	68.376690	66.772849	66.585285	58.859893	57.016755	56.812191	59.237037	57.384865
		364	108.382915	110.865235	110.865235	76.532520	76.967591	76.365457	66.867873	66.328687	65.917485	59.490655	58.841660	58.443448	60.213819	59.297428
		728	117.686433	119.273737	119.273737	75.655026	76.389407	76.153396	65.855836	66.199160	65.827934	57.012862	58.590169	58.521558	57.489391	58.782734
	2019.05.13-2020.05.12	182	95.465230	96.983670	96.983670	68.212685	68.563190	68.403369	62.299381	61.418993	61.309870	53.964120	54.091201	53.982518	54.301694	54.277113
		364	107.685682	110.712737	110.712737	69.306409	70.734229	70.164132	61.345989	61.916248	61.600884	56.388890	56.859771	56.380539	57.812438	57.691593
		728	121.312715	122.971495	122.971495	68.434178	70.469465	69.744520	60.344236	61.666861	61.048242	52.893555	55.460732	55.039993	53.987241	55.944617
	2020.01.01-2020.05.12	182	127.430779	129.475793	129.475793	79.208289	80.863726	80.565305	68.109219	68.759214	68.498242	58.442031	60.789827	60.621708	60.666018	62.211354
		364	142.139512	146.937698	146.937698	80.344770	84.758849	84.061342	67.468281	71.168134	70.832876	60.063080	65.059942	64.704741	62.703120	66.998609
		728	178.639717	181.030724	181.030724	81.484104	86.722178	85.198452	67.251065	71.512389	70.470768	57.777514	64.838924	64.107598	60.331381	66.571225

Figure 29: RMSE on the price forecasts (DK1)

Area	Dates	Window	P1	P1 (asinh)	P1 (asinh-hp)	P2	P2 (asinh)	P2 (asinh-hp)	P3	P3 (asinh)	P3 (asinh-hp)	P4	P4 (asinh)	P4 (asinh-hp)	P5	P5 (asinh)
DK2	2019.01.01-2019.12.31	182	85.807352	85.896984	85.896984	71.018341	69.555000	69.379977	64.046663	62.180679	62.084748	57.322139	55.413446	55.348356	57.369391	55.425448
		364	90.943848	90.895794	90.895794	70.897828	69.132619	68.657715	63.118444	60.741802	60.320563	58.595830	55.930580	55.589629	58.551300	55.904560
		728	81.697935	81.828635	81.828635	69.490904	67.728746	68.029236	62.115683	60.256858	60.166623	56.427961	55.223516	55.394403	56.432042	55.227725
	2019.01.01-2020.05.12	182	97.529695	98.309403	98.309403	72.873760	72.475196	72.296759	64.566985	63.442992	63.341161	58.309209	57.725638	57.671154	58.334621	57.728349
		364	105.054135	106.585367	106.585367	72.893222	72.939020	72.466732	63.805347	63.127536	62.784119	59.408044	58.730318	58.470746	59.389992	58.725930
		728	114.468197	114.616814	114.616814	72.423808	72.149589	71.992683	63.374743	62.639554	62.341294	57.839339	58.055362	57.989223	57.827832	58.044470
	2019.05.13-2020.05.12	182	91.720525	93.090487	93.090487	64.406940	65.130937	65.019584	58.413321	57.804075	57.766398	53.052848	53.549639	53.547140	53.075160	53.578512
		364	103.403839	105.674446	105.674446	65.255122	66.661286	66.234275	57.587360	58.073291	57.893616	54.955283	55.073300	54.847794	54.872352	55.052359
		728	118.232453	118.064362	118.064362	65.196015	66.062437	65.507953	57.358018	57.480783	57.091545	52.927365	53.710299	53.434019	52.874757	53.679855
	2020.01.01-2020.05.12	182	124.137247	126.252027	126.252027	77.738504	79.943070	79.755259	65.973857	66.784762	66.667665	60.935991	63.640964	63.611442	60.889485	63.634190
		364	136.477624	140.938122	140.938122	78.107829	82.487579	82.016132	65.653539	69.253864	69.095171	61.582040	65.804607	65.731905	61.502552	65.732906
		728	175.342124	175.538274	175.538274	79.921663	83.082018	81.890066	66.708073	68.755588	67.952733	61.546601	65.198037	64.576840	61.404119	65.103546

Figure 30: RMSE on the price forecasts (DK2)

6 Conclusions

Almost all of the assumptions came true, forecasts for wind power and consumption were performed better than original prepared by Nordpool. Even models for wind power performed well however wind power predicting is not typical time-series problem, because more important factors influencing such as atmospheric models were not known and not used in this work.

Main finding is that period of forecast matters in using the most efficient model. For some periods more accurate were shorter calibration windows, because anomalies in the 2020 caused by COVID-19 changed seasonal numbers like consumption and thus price which was significantly lower in 2020 than 2019.

One assumption I found invalid was that *asinh* normalization function improves price predictions. It completely fails for wind power and consumption forecasting, but should be useful for price, but yet it works worse for periods including 2020 and especially for period which includes only 2020 year. The forecast Model 4 with asinh normalization performs 7.34% (DK1) and 16.34% (DK2) worse comparing MAE of the results.

Experiment of replacing original forecasts of Nordpool with better, forecasted results unfortunately failed but I still see potential with bigger calibration window, which can be used to trained better forecasts for whole considering period of price prediction. But for this purpose it's necessary to have more data.

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