Modeling and forecasting of electricity prices and demand

Adam Spychała

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Abstract

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Keywords: electricity spot prices, day-ahead market, forecasting, power market

1 Market and data analysis

Danish power market has been transformed in the last several years drastically. Since 70s there was a lot of investments in renewable sources of energy, especially in the field of wind power, and much more since 2002 when first large scale offshore wind farm in the world has been finished - Horns Rev 1 (160 MW). For year 2019, total wind power generation capacity was 6128 MW[6].

National target for 2020 is over 50% of a energy consumption covered by wind power and it's likely to be achieved, as in 2019 they obtained 47% of coverage by domestic production[2]. Moreover they have finished construction of next large scale wind farm Horns Rev 3 in August 2019[2]. There are also defined next goals in last presented national energy strategy. For wind power consumption they aim for 70% in 2030[2]. Denmark is currently leader of wind power shares in the national production and its development.

The production of such significant part of energy from wind carries some risk. Wind speed is very fluctuant and variable even in a day cycle. There are no perfect methods of forecasting in the long term periods. It may occur in higher (or lower) demand in production from other, stable sources of energy to cover consumption. It's problematic to mark a common trend in wind power forecasts1. Thus it's helpful to correct prediction of electricity demand.

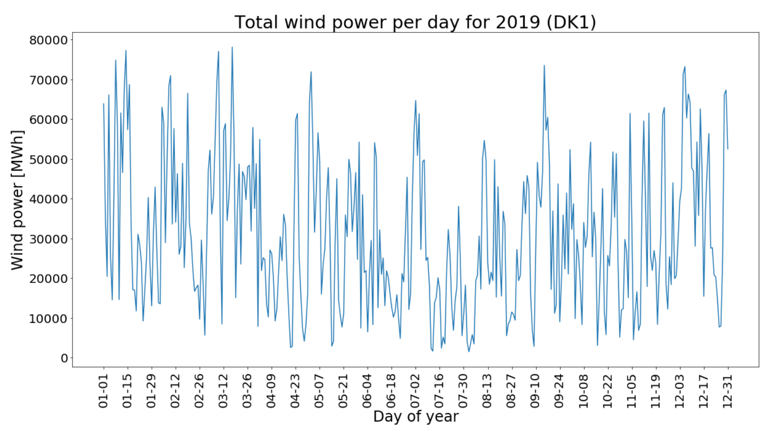

Figure 1: Wind power produced per day in 2019 (DK1)

Wind power generation can't be stopped quickly as production of conventional sources of energy (coal, gas). Because of that higher unexpected wind power production may lead to decreased prices on energy stocks and in some cases, prices can drop below zero. That was happening rarely in the era of conventional production and regulated electricity market, but it happens more often with more renewable power generators. Because energy network is quite connected with each other in the European Union, in a case of negative prices energy from Danish areas (DK1, DK2) is exported to the neighboring countries, mainly to the Germany, which Denmark has the biggest balance of energy export and import.

Negative prices obligate to use other approaches of price forecasting, than the old ones which were failing with unexpected domain of values. In my work I will present a few models to forecast price and consumption and point out the best approach to have optimal forecasts.

The electricity market in Denmark is divided into 2 areas (DK1 and DK2). First area (DK1) consists of regions: Nordjylland, Midtjylland and Syddanmark; second area (DK2) consists of regions Sjælland and Hovedstaden with the capital Copenhagen.

Figure 2: Danish electricity market

¡to remove¿ Denmark is located between two large energy producers: Germany and Sweden. Is connected directly with these countries and further with Norway and Netherlands. Energy link (Viking Link) with United Kingdom is under construction and connection with Poland is in the planning phase.

Denmark covers only small part of the its demand from the export, however their goals is to start exporting renewable energy from 2026[3]. ¡/to remove¿

2 Data analysis

I performed a few analysis for each dataset, although I don't include all of the charts and tables in this work. The rest is uploaded into the github repository.

2.1 Missing values

Data was very consistent and yet only single values were missing. These null values were replaced by average of the neighboring cells and in case of missing value in neighbor cell, value was fixed manually (with file fill_empty_cells.py). Half of day 2018-09-18 from Wind prognosis files was filled taking closest neighbors and counting average for whole vector (with file fill_empty_cells_wind_prognosis_DK.py). Number of missing values was reduced to zero.

Dataset	DK1	DK2
Consumption	5	5
Consumption prognosis	5	5
Price	21	12
Wind power	12	6
Wind power prognosis	18	19

Table 1: Missing values in files.

2.2 Consumption data

There was bigger consumption in area DK1 than DK2, compared for years 2016-2019 19.14, 19.41, 20.28, 20.37 TWh to 13.13, 13.03, 13.28, 13.16 TWh accordingly. We can see that consumption increased gradually in the area DK1, meanwhile in area DK2 was on the similar level.

We can spot three types of seasonal trends in the data: annual, weekly and daily. On the below chart showing consumption per day in 2019, we see that every weekend consumption value drops.

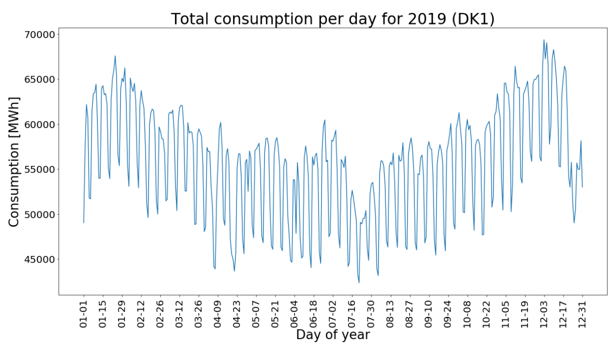

Figure 3: Consumption per day in 2019 (DK1)

Although it's not easy to spot, there is also annual trend with lower consumption during summer months and higher during winter. It's observable particularly in the DK2.

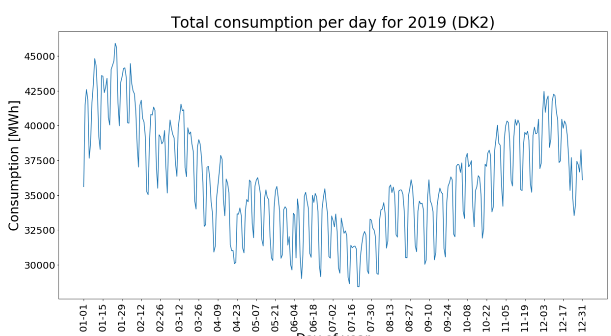

Figure 4: Consumption per day in 2019 (DK2)

Simple Moving Average with 14 days windows for each year shows clearly this trend. Although in the area DK1 it's not very sharp.

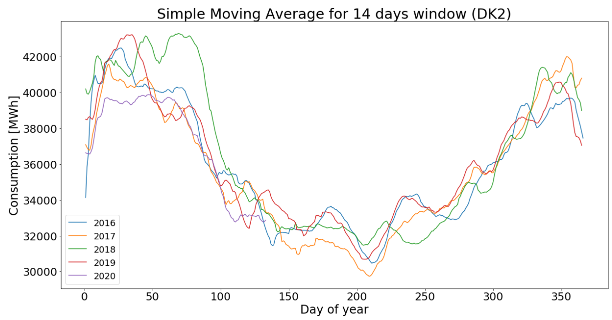

Figure 5: Simple Moving Average of consumption - 14 days window (DK2)

Last seasonal trend is daily which can be observed for each day of week, even holidays. There are two peaks of energy consumption each day, on the morning and evening. During weekend and holidays, morning peaks occurs later than during work days. There is also noticeable smaller consumption in the night.

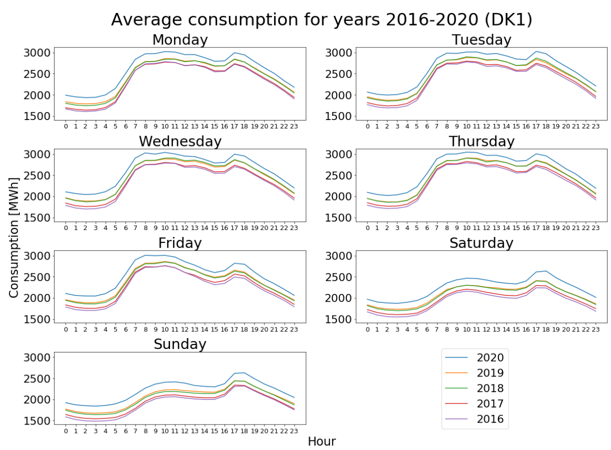

Figure 6: Average consumption per each hour of day for each day of week (DK1)

There is also one interesting thing observed in the 2020's data only in the area DK2. Evening daily peak is slightly shifted which can be caused epidemic COVID-19 or incomplete data of 2020 year.

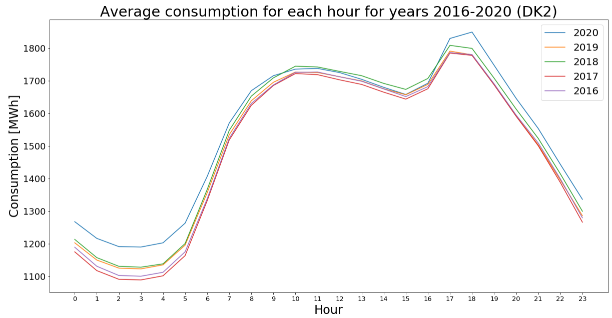

Figure 7: Average consumption per each hour of day for each year (DK2)

2.2.1 Nordpool's prognosis

The prognosis day-ahead given by Nordpool shows that those are less accurate over time, especially for area DK1 where consumption is larger than DK2. Level of accuracy for area DK2 is quite stable. Also there is no clear trend regarding the time of day. For DK1 best prediction are for night and for DK2 best prediction are performed for hours 7 and 15.

Area	Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	Total
DK1	2016	18.7	15.2	13.2	12.7	12.3	14.1	18.7	19.5	20.3	19.6	19.6	19.7	20.4	19.0	16.5	16.9	16.0	18.7	17.7	15.9	16.0	15.0	13.7	17.3	16.9
	2017	16.1	24.9	19.0	13.9	14.1	15.1	18.7	18.7	19.1	19.9	19.8	17.8	16.3	15.4	15.1	16.1	21.1	22.1	20.7	17.2	18.0	17.5	19.4	22.5	18.3
	2018	18.8	26.7	17.7	15.9	14.1	15.9	20.6	24.8	21.0	23.6	21.3	20.1	22.5	21.0	20.9	19.8	19.2	21.7	18.3	19.9	24.9	22.5	23.1	25.8	20.8
	2019	22.0	29.3	21.8	17.9	17.2	18.1	23.0	24.9	22.0	27.3	24.7	22.7	23.0	20.6	19.2	19.2	20.2	21.5	22.6	22.4	25.2	24.0	25.1	30.3	22.7
	2020	21.7	26.9	23.4	20.5	16.8	20.5	34.8	34.2	26.4	32.8	32.9	22.6	22.2	27.4	24.6	22.2	22.3	36.1	32.4	23.6	26.4	24.5	27.7	32.4	26.5
	Total	19.1	24.3	18.4	15.5	14.6	16.2	21.4	23.0	21.1	23.5	22.3	20.3	20.7	19.7	18.5	18.3	19.4	22.2	20.9	19.2	21.5	20.1	20.9	24.7	20.2

Figure 8: MAE for consumption for each hour and year with total figures (DK1)

Area	Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	Total
DK2	2016	23.8	19.6	19.0	20.3	22.4	22.9	20.9	17.1	23.6	27.6	24.3	21.4	24.1	22.0	20.0	19.2	23.3	26.4	24.5	22.9	24.5	24.6	20.7	15.9	22.1
	2017	19.6	21.9	21.7	20.2	18.8	18.5	22.7	16.8	20.2	21.8	21.6	20.8	20.2	20.5	20.6	16.7	18.0	24.4	23.3	21.2	20.8	22.8	22.8	27.4	21.0
	2018	18.1	25.2	24.7	20.8	19.5	20.2	27.5	18.8	21.2	24.8	26.8	26.7	27.0	26.1	22.4	17.5	20.7	24.7	22.5	22.4	26.8	29.1	25.9	23.2	23.4
	2019	16.1	23.9	22.8	17.5	16.3	15.9	21.2	14.3	16.9	17.9	21.2	21.6	18.6	17.5	17.1	15.3	18.7	24.5	22.1	18.8	22.2	22.6	21.1	22.7	19.5
	2020	17.4	18.2	15.8	12.4	15.0	17.6	25.5	22.1	24.4	31.0	36.4	27.6	23.5	26.0	21.7	13.9	15.9	28.1	27.5	18.3	17.2	17.1	14.6	18.9	21.1
	Total	19.2	22.3	21.5	19.1	18.9	19.2	23.3	17.2	20.8	23.7	24.5	23.0	22.6	21.9	20.1	16.9	19.8	25.3	23.5	21.1	23.1	24.1	22.0	22.0	21.5

Figure 9: MAE for consumption for each hour and year with total figures (DK2)

2.3 Wind power data

Wind power production increased significantly within last 4 years. In the area DK1 from 9.41 to 11.26 TWh and for area DK2 from 2.37 to 3.22 TWh, so about 35% more.

So far in the consumption data we could spot any trend whereas in the wind power data there is no distinct trend.

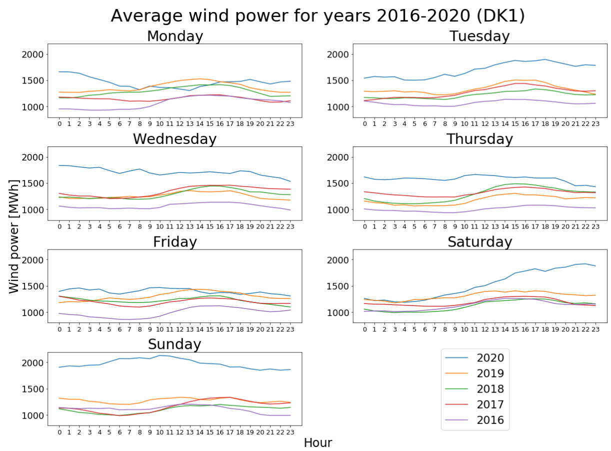

Figure 10: Wind power production per hour of a day for each day of week (DK1)

Simple Moving Average also doesn't show anything recurrent, thus we can assume any annual trend.

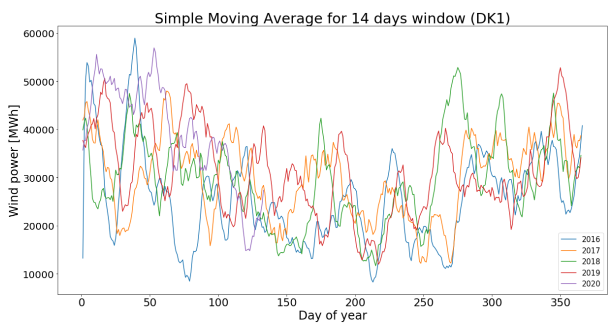

Figure 11: Simple moving average of wind power production per each hour for each day of week (DK1)

Only chart of average wind power for each hour suggest there may be a daily trend, however data from area DK2 doesn't confirm this assumption.

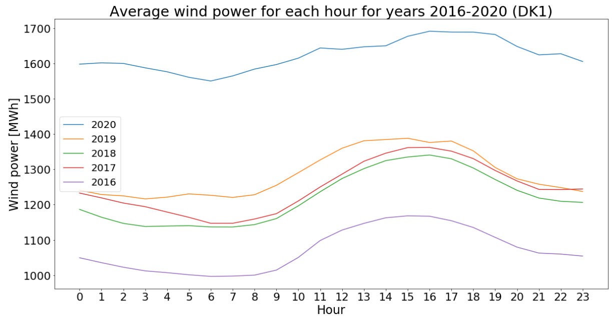

Figure 12: Average wind power production per hour of a day for each year (DK1)

2.3.1 Nordpool's prognosis

I calculated Mean Absolute Error for prognosis performed by Nordpool for each hour and year. The results for area DK1 are rounded to the integer. We can spot that the newer data is the bigger error for prognosis with over 80% growth in 2020 (until 12th May) comparing to the previous year. Another thing we can notice that the error in the night hours is lower than others.

Area	Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	Total
DK1	2016	125	131	133	134	141	145	144	152	155	157	164	163	163	180	191	194	189	189	187	189	185	186	182	180	165
	2017	152	156	161	158	162	166	167	170	169	168	183	196	197	198	199	202	204	202	197	194	194	195	199	200	183
	2018	154	149	150	156	155	156	162	162	174	175	184	186	192	201	209	217	225	215	207	211	208	207	210	227	187
	2019	216	216	208	213	217	219	221	217	218	217	226	229	238	249	253	255	261	255	256	241	232	240	246	258	233
	2020	410	379	381	389	401	400	425	411	381	382	386	404	431	429	451	447	484	470	432	424	437	455	449	462	422
	Total	182	181	181	184	188	191	194	195	196	196	205	211	217	225	233	236	242	236	230	227	224	227	229	237	211

Figure 13: MAE for wind power for each hour and year with total figures (DK1)

The results for area DK2 are similar, however numbers are smaller due to the lower capacity of wind farms in this part of Denmark.

Area	Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	Total
DK2	2016	45.7	45.7	45.9	45.0	47.4	48.7	50.3	51.5	50.6	49.1	49.6	52.2	49.6	49.9	51.5	51.3	53.2	58.0	60.8	59.4	59.4	60.7	65.5	66.1	52.8
	2017	55.5	57.4	57.1	56.5	53.7	54.7	56.0	56.3	59.7	60.0	62.2	65.0	67.9	70.0	67.1	68.9	68.2	68.8	70.1	72.6	72.2	70.4	68.3	69.3	63.7
	2018	52.5	54.2	53.9	54.1	55.2	56.4	56.2	59.5	60.7	60.1	60.1	62.5	60.9	59.2	59.8	65.1	67.3	73.6	75.4	70.7	70.9	68.5	66.6	69.6	62.2
	2019	58.4	60.0	56.5	58.1	55.2	55.1	56.7	60.3	64.2	62.9	66.3	67.7	65.9	65.1	63.5	64.9	67.6	69.6	71.0	68.6	67.9	72.4	73.6	73.4	64.4
	2020	68.6	66.1	65.7	67.2	66.7	66.6	64.3	63.7	61.1	72.8	73.0	70.1	74.4	79.9	78.6	76.9	87.7	84.3	78.0	76.6	82.5	87.8	89.8	96.0	74.9
	Total	54.3	55.3	54.4	54.6	54.0	54.8	55.6	57.5	59.0	59.3	60.7	62.5	62.2	62.6	62.0	63.7	66.0	68.9	70.0	68.5	68.8	69.7	70.3	71.8	61.9

Figure 14: MAE for wind power for each hour and year with total figures (DK2)

2.4 Price data

In the 2020 price of energy decreased compared to the previous years. However we don't know what impact had epidemic on this data so it's hard to retrieve any conclusion.

Year	DK1	DK2
2016	184.435	206.825
2017	224.005	227.575
2018	329.235	339.345
2019	289.270	295.390
2020	142.520	149.865

Table 2: Average price of energy per each year (DKK/MWh)

As in the consumption data we can also notice seasonal trends in this category, but this time only two kinds: daily and weekly. There is no annual trend in the price data.

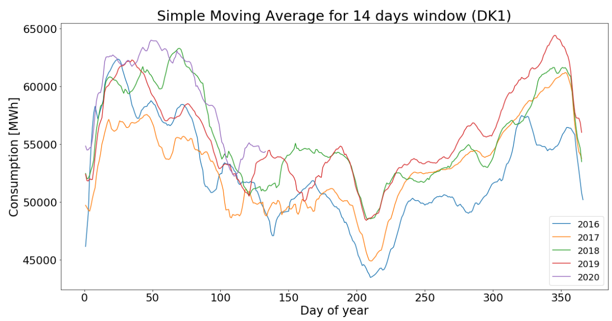

Figure 15: Simple moving average of price for each year - 14 days window (DK1)

Electricity prices are lower during weekends due to the lower consumption and there are also 2 daily peaks each day, either weekend (including holidays) and work days.

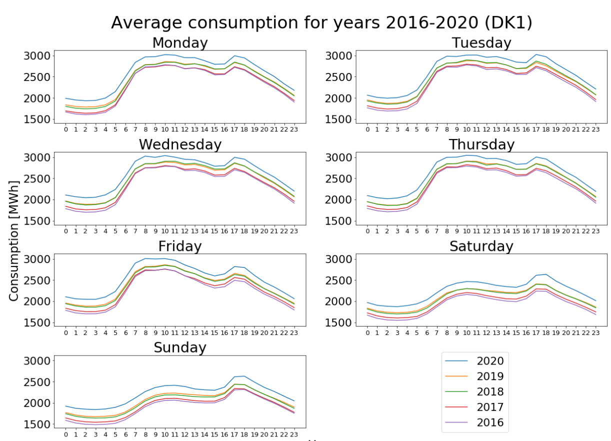

Figure 16: Average price per hour of a day for each day of week (DK1)

2.4.1 Negative prices

There were 417 negative prices in the area DK1 in the years 2016-2020 and 288 in the DK2. The dataset for 2020 ends on 12th May, however there are already 84 negative prices compared to the 133 in the full year 2019. Negative prices occur more often during nights than the peaks of consumption.

year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	Total
2016	3	7	8	8	6	4	5	2	1	1	0	0	1	2	3	2	2	1	0	0	1	2	0	4	63
2017	6	6	8	7	6	5	6	4	1	2	2	2	2	4	4	5	4	0	0	0	1	3	3	4	85
2018	4	5	5	6	7	6	6	3	0	0	1	0	1	2	2	1	0	0	0	0	0	1	0	2	52
2019	9	9	12	14	11	5	6	4	3	3	4	3	6	9	11	8	4	2	1	1	1	1	1	5	133
2020	5	3	6	7	7	7	4	3	2	2	4	3	3	4	6	7	6	1	1	0	0	1	1	1	84
Total	27	30	39	42	37	27	27	16	7	8	11	8	13	21	26	23	16	4	2	1	3	8	5	16	417

Figure 17: Number of negative prices for each hour, year and totally (DK1)

2.4.2 Correlation between price and wind power

We can suspect that with bigger values of wind power price is lower. Pearson's correlation shows that mostly it's weak correlation, only for 2020 year there are moderate values.

Correlation is a little bit higher for night hours than rest of the day.

Area	Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
	2016	-0.41	-0.42	-0.41	-0.41	-0.40	-0.36	-0.36	-0.40	-0.44	-0.44	-0.42	-0.41	-0.40	-0.39	-0.38	-0.37	-0.35	-0.35	-0.41	-0.43	-0.38	-0.37	-0.37	-0.37
	2017	-0.62	-0.62	-0.61	-0.58	-0.54	-0.52	-0.46	-0.46	-0.51	-0.53	-0.53	-0.55	-0.56	-0.54	-0.53	-0.51	-0.50	-0.51	-0.60	-0.63	-0.62	-0.60	-0.62	-0.61
DK1	2018	-0.45	-0.46	-0.49	-0.50	-0.48	-0.44	-0.36	-0.38	-0.42	-0.44	-0.46	-0.45	-0.43	-0.41	-0.40	-0.39	-0.42	-0.41	-0.45	-0.46	-0.44	-0.46	-0.47	-0.46
	2019	-0.50	-0.50	-0.52	-0.53	-0.51	-0.46	-0.38	-0.37	-0.38	-0.39	-0.41	-0.42	-0.41	-0.40	-0.38	-0.35	-0.33	-0.31	-0.40	-0.51	-0.56	-0.56	-0.56	-0.56
	2020	-0.53	-0.63	-0.81	-0.84	-0.83	-0.83	-0.72	-0.68	-0.69	-0.69	-0.68	-0.64	-0.68	-0.69	-0.72	-0.74	-0.75	-0.73	-0.72	-0.72	-0.69	-0.65	-0.69	-0.73
	2016	-0.36	-0.40	-0.41	-0.40	-0.34	-0.29	-0.27	-0.23	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.23	-0.18	-0.19	-0.24	-0.28	-0.31	-0.31	-0.28	-0.32
	2017	-0.61	-0.61	-0.61	-0.61	-0.57	-0.50	-0.45	-0.42	-0.42	-0.43	-0.44	-0.46	-0.44	-0.43	-0.42	-0.38	-0.37	-0.37	-0.42	-0.47	-0.53	-0.58	-0.62	-0.63
DK2	2018	-0.52	-0.53	-0.53	-0.53	-0.51	-0.48	-0.39	-0.31	-0.30	-0.33	-0.35	-0.38	-0.36	-0.35	-0.34	-0.33	-0.33	-0.31	-0.34	-0.38	-0.40	-0.43	-0.47	-0.50
	2019	-0.50	-0.52	-0.52	-0.52	-0.49	-0.43	-0.31	-0.32	-0.31	-0.28	-0.26	-0.25	-0.25	-0.25	-0.23	-0.23	-0.24	-0.26	-0.37	-0.44	-0.48	-0.46	-0.49	-0.50
	2020	-0.50	-0.46	-0.43	-0.35	-0.39	-0.51	-0.58	-0.38	-0.39	-0.40	-0.35	-0.40	-0.38	-0.34	-0.31	-0.29	-0.32	-0.38	-0.36	-0.43	-0.53	-0.64	-0.75	-0.75

Figure 18: Correlation between price and wind power for each hour and year (DK1 and DK2)

2.5 Holidays

Usage of the electricity decreases during weekends and public holidays and this has a significant effect on a prediction especially during Christmas or Easter. Due to that fact, each day was aligned with variable holiday with following value based of occurrence of day of week or public holiday:

- 1 - National Holidays (e.g. Easter Monday)[7]
- 1 - Sundays
- 0 - Not a public holidays (e.g. New Year's Eve)
- 0 - Other days

Each holiday is treated in weekday dummies as a Sunday.

3 Methodology

I decided to choose Danish market for forecasting and so on I want to apply methods and models which are most fitting to the characteristics of this market. I performed forecasting for 3 categories of values: wind power, consumption and price. In each category I make prognosis for few models which are described in Forecasting section. My forecasting framework for all of them is following:

1. Data preparation

2. Preliminary data analysis
3. Data forecasting (cycle for each model and day)

    Data normalization

    Day-ahead forecasting
4. Verifying models' performance

All calculations were conducted with Python3.7 language and its libraries, i.e. numpy, pandas, scipy. All the scripts made are accessible in public GIT repository https://github.com/ajescode/energyForecast.

Data I used to perform forecasts has been downloaded from the official webpage of the Nordpool power exchange[5]. Datasets are divided into the year files and periods (hours, weeks etc.). I managed to download following datasets (valid for the day 14.05.2020):

- Consumption - hourly
- Consumption prognosis - hourly
- Wind power - hourly
- Wind power prognosis - hourly
- Elspot prices (as Price) - hourly

All of the datasets were available for years 2013-2020, except for Consumption prognosis (2015-2020). So I decided to focus on analysis only on the period 2015-2020 (2015.01.01-2020.05.12), because 4 years time frame is still sufficient for calculations.

Units of downloaded data are following:

- Consumption and Wind Power- MWh
- Price - DKK/MWh

The files were downloaded, merged, split for regions DK1 and DK2, pivoted in order to have separated hours as parameters for each day and merged for all years. Example for consumption DK1 is presented below.

	date	holiday	0	1	2	...	22	23
0	2016-01-01	1	1818.0	1741.0	1660.0	...	1858.0	1713.0
1	2016-01-02	0	1615.0	1510.0	1461.0	...	2027.0	1822.0
2	2016-01-03	1	1724.0	1665.0	1671.0	...	2127.0	1998.0
3	2016-01-04	0	1844.0	1803.0	1789.0	...	2293.0	2079.0
4	2016-01-05	0	1940.0	1891.0	1952.0	...	2372.0	2193.0

Table 3: First 5 rows of merged file Consumption DK1.

Data forecasting is performed for day ahead because given Wind power prognosis and Consumption prognosis is supplying only one day in advance. For the computations is considered number of previous days to have insights of possible value and its called as calibration window. It may take values of week multiples (7 days) because of the seasonal trend of energy production. Because chosen data has limit of 1583 days, the maximum calibration window I consider is 728 days, around 2 years.

I consider few models for each category of forecasts (consumption, price and wind power) which I explain in the end of this section. General equation for each model (except of benchmark models for wind power and consumption) can be presented as:

$$\hat{Y}_{d,h} = \sum_{i=1}^{n} \beta_{d,h,i} X^i_{d,h} \tag{1}$$

, where $Y_{d,h}$ is prediction and $X^i_{d,h}$ is variable for given day (d) and hour (h). We can predict values for next whole day only, but basing on the variables from whole calibration window. Coefficients $\beta_{h,i}$ have been approximated by Ordinary least squares (OLS).

$$\hat{\vec{\beta}} = (X^T X)^{-1} X^T Y \tag{2}$$

,where X is matrix of independent variables, and Y is vector of dependant variables of size of calibration window (multiple of 7 days).

Because prices of energy due to the wind dependence can have negative values I need to normalize values. I used function $asinh$ which was empirically confirmed to have best results for danish market among 4 different normalization functions.[1].

$$Y_{d,h} = asinh(y_{d,h}) \equiv \log(y_{d,h} + \sqrt{y^2_{d,h} + 1}) \tag{3}$$

, where $Y_{d,h}$ is transformed value used for forecasts either as independent variable or dependent variable. Independent values in the model must be also transformed. $y_{d,h}$ is normalized value in calibration window by:

$$y_{d,h} = \frac{1}{b_{d,h}}(y^*_{d,h} - a_{d,h}) \tag{4}$$

, where a is median and b is median absolute deviation (MAD) for given day and hour in the calibration window. I used both simple normalization (4), and one with $asinh$ function (3) for each category of forecasting.

Inverse function for transformation and normalization is following:

$$y^*_{d,h} = b_{d,h} sinh(Y_{d,h}) + a_{d,h} \tag{5}$$

For each forecast there is error analysis with Mean Average Error (MAE), Mean Root Square Error (MRSE). All forecasts in the appropriate categories are compared to each other in order to get optimal result.

$$MAE = \frac{1}{24T}\sum_{d=1}^{T}\sum_{h=1}^{24} |\hat{\varepsilon}_{d,h}| \equiv \frac{1}{24T}\sum_{d=1}^{T}\sum_{h=1}^{24} |y_{d,h} - \hat{y}_{d,h}| \tag{6}$$

$$RMSE = \sqrt{\frac{1}{24D}\sum_{d=1}^{T}\sum_{h=1}^{24} \hat{\varepsilon}^2_{d,h}} \equiv \sqrt{\frac{1}{24D}\sum_{d=1}^{T}\sum_{h=1}^{24} (y_{d,h} - \hat{y}_{d,h})^2} \tag{7}$$

, where T is size of calibration window. Also I performed error analysis for each hour and day of forecasts and compared average values of them.

$$MAE_h = \frac{1}{T}\sum_{d=1}^{T} |\hat{\varepsilon}_{d,h}| \equiv \frac{1}{T}\sum_{d=1}^{T} |y_{d,h} - \hat{y}_{d,h}| \tag{8}$$

$$MAE_d = \frac{1}{24}\sum_{h=1}^{24} |\hat{\varepsilon}_{d,h}| \equiv \frac{1}{24}\sum_{h=1}^{24} |y_{d,h} - \hat{y}_{d,h}| \tag{9}$$

$$RMSE_h = \sqrt{\frac{1}{T}\sum_{d=1}^{T} \hat{\varepsilon}^2_{d,h}} \equiv \sqrt{\frac{1}{T}\sum_{d=1}^{T} (y_{d,h} - \hat{y}_{d,h})^2} \tag{10}$$

$$RMSE_d = \sqrt{\frac{1}{24}\sum_{h=1}^{24}\hat{\varepsilon}_{d,h}^2} \equiv \sqrt{\frac{1}{24}\sum_{h=1}^{24}(y_{d,h} - \hat{y}_{d,h})^2} \tag{11}$$

4 Demand forecasting

First considering model is a benchmark model containing consumption forecast provided by Nordpool's database for day ahead ($FL_{t,h}$). Its performance is analyzed in the section Demand data prognosis.

$$C_{d,h} = FC_{d,h} \tag{C1}$$

Second model is extension of benchmark model (C1), where Di, d is a vector of the values 0,1 for the corresponding day of a week. For day (d) which is Monday $D_{1,d} = 1$ and $D_{2,d} = D_{3,d} = ... = D_{7,d} = 0$, for a day which is Tuesday $D_{2,d} = 1$ and $D_{1,d} = D_{3,d} = ... = D_{7,d} = 0$ etc.

$$C_{d,h} = \sum_{i=1}^{7}\beta_{0,i}D_{i,d} + \beta_1 FC_{d,h} + \varepsilon_{d,h} \tag{C2}$$

Third model is extension of second model (C2), where are added 3 parameters according to a similar-day technique: consumption of previous day ($C_{d-1,h}$), 2 days ago ($C_{d-2,h}$) and week ago ($C_{d-7,h}$).

$$C_{d,h} = \sum_{i=1}^{7}\beta_{0,i}D_{i,d} + \beta_1 FC_{d,h} + \beta_1 C_{d-1,h} + \beta_2 C_{d-2,h} + \beta_3 C_{d-7,h} + \varepsilon_{d,h} \tag{C3}$$

Fourth model is extension of third model (C3) where is also considered forecast of the wind power download from Nordpool's database, denoted as $FW_{d,h}$.

$$C_{d,h} = \sum_{i=1}^{7}\beta_{0,i}D_{i,d} + \beta_1 FC_{d,h} + \beta_2 C_{d-1,h} + \beta_3 C_{d-2,h} + \beta_4 C_{d-7,h} + \beta_5 FW_{d,h} + \varepsilon_{d,h} \tag{C4}$$

4.1 Results

The forecasts were performed with several different settings for each area (DK1 and DK2):

- Calibration window: 182, 364, 728
- Normalization function: standard, asinh
- Dates: 2019.01.01-2019.12.31 (2019 year),
  2019.05.13-2020.05.12 (last year),
  2020.01.01-2020.05.12,
  2019.01.01-2020.05.12

For both areas results with *asinh* were significantly worse than with standard normalization function. Especially for calibration window 182. The forecast with standard normalization was better for Models 2-4 than Nordpool's prognosis, however better accuracy was for longer calibration windows. Because drops of consumption level in 2020 results including this period were less accurate.

Area	Dates	Window	Model 1	Model 2 (asinh)	Model 3 (asinh)	Model 4 (asinh)	Model 2	Model 3	Model 4
DK1	2019.01.01-2019.12.31	182	22.704395	40.377925	40.765326	40.879039	21.026883	21.364330	21.460788
		364	22.704395	30.833751	30.931238	31.006436	20.573811	20.604376	20.635657
		728	22.704395	30.258628	30.174136	30.211663	20.481759	20.412800	20.376542
	2019.01.01-2020.05.12	182	23.707915	41.317094	41.652774	41.752153	21.997153	22.340290	22.405802
		364	23.707915	32.807189	32.730434	32.785073	21.632464	21.703457	21.699693
		728	23.707915	32.845550	32.563926	32.601543	21.529247	21.492532	21.440592
	2019.05.13-2020.05.12	182	24.522143	41.839699	42.221620	42.324837	22.830232	23.161873	23.225991
		364	24.522143	34.482139	34.269649	34.319270	22.445105	22.511653	22.468517
		728	24.522143	34.322466	33.969337	33.987965	22.326490	22.299777	22.222493
	2020.01.01-2020.05.12	182	26.461936	43.894514	44.088254	44.148294	24.659924	25.018678	24.999263
		364	26.461936	38.223013	37.668078	37.666292	24.537791	24.719730	24.619793
		728	26.461936	39.944996	39.122371	39.160236	24.403931	24.455705	24.360732

Figure 19: MAE on the consumption forecasts (DK1)

Area	Dates	Window	Model 1	Model 2 (asinh)	Model 3 (asinh)	Model 4 (asinh)	Model 2	Model 3	Model 4
DK2	2019.01.01-2019.12.31	182	19.473916	33.230658	33.424981	33.556035	18.507033	18.161387	18.286632
		364	19.473916	23.304240	23.062514	23.127045	17.811062	17.613376	17.785547
		728	19.473916	23.699155	23.504598	23.489713	17.651220	17.388470	17.547174
	2019.01.01-2020.05.12	182	19.887383	33.692910	33.947067	34.068954	18.586461	18.283000	18.378010
		364	19.887383	23.376331	23.024670	23.070509	18.201139	18.048743	18.162581
		728	19.887383	24.116272	23.740377	23.709035	18.264652	18.062187	18.184073
	2019.05.13-2020.05.12	182	18.999658	31.378130	31.573865	31.642235	17.583754	17.199495	17.268510
		364	18.999658	21.217481	20.912567	20.957331	17.432061	17.209056	17.214983
		728	18.999658	22.225234	21.948706	21.915605	17.415524	17.236912	17.306714
	2020.01.01-2020.05.12	182	21.022086	34.961497	35.379858	35.476589	18.804438	18.616750	18.628786
		364	21.022086	23.574174	22.920814	22.915354	19.271651	19.243547	19.197300
		728	21.022086	25.260992	24.387441	24.310934	19.948130	19.911108	19.931954

Figure 20: MAE on the consumption forecasts (DK2)

Area	Dates	Window	Model 1	Model 2 (asinh)	Model 3 (asinh)	Model 4 (asinh)	Model 2	Model 3	Model 4
DK2	2019.01.01-2019.12.31	182	28.341912	57.056903	57.506133	57.759529	26.721173	26.069790	26.258225
		364	28.341912	32.889380	31.791148	31.857514	26.022295	25.295621	25.533021
		728	28.341912	33.664516	32.294136	32.271689	25.987201	25.183820	25.309949
	2019.01.01-2020.05.12	182	29.430146	58.385491	59.151430	59.339332	27.523442	26.866673	27.006186
		364	29.430146	33.337281	32.137641	32.190566	27.170554	26.395746	26.538476
		728	29.430146	34.103514	32.625531	32.595495	27.328482	26.428567	26.534730
	2019.05.13-2020.05.12	182	28.078188	53.902529	54.809712	54.748425	26.054504	25.396351	25.481365
		364	28.078188	29.966140	29.232460	29.281532	25.934630	25.242161	25.236591
		728	28.078188	30.569884	29.802082	29.768587	25.981558	25.227106	25.285332
	2020.01.01-2020.05.12	182	32.228407	61.885241	63.447886	63.473178	29.613698	28.941090	28.959774
		364	32.228407	34.536652	33.069892	33.087360	30.097546	29.202719	29.119978
		728	32.228407	35.280228	33.518167	33.468041	30.709911	29.576661	29.637043

Figure 21: RMSE on the consumption forecasts (DK1)

Area	Dates	Window	Model 1	Model 2 (asinh)	Model 3 (asinh)	Model 4 (asinh)	Model 2	Model 3	Model 4
DK1	2019.01.01-2019.12.31	182	36.902095	69.147136	69.699399	69.826043	35.185087	35.451488	35.453872
		364	36.902095	49.596527	49.710845	49.839186	34.817941	34.784771	34.754983
		728	36.902095	48.507415	48.334706	48.417899	34.743648	34.675315	34.613163
	2019.01.01-2020.05.12	182	36.780154	71.261926	71.419734	71.554337	34.936314	35.254252	35.263161
		364	36.780154	59.391927	56.455358	56.721813	34.637305	34.680831	34.637497
		728	36.780154	66.047064	60.961407	61.090178	34.560341	34.537831	34.469738
	2019.05.13-2020.05.12	182	39.326498	72.316495	72.337533	72.505304	37.479944	37.799352	37.796761
		364	39.326498	65.372788	61.574367	61.908548	37.224406	37.262406	37.190576
		728	39.326498	73.436774	67.217973	67.374424	37.138652	37.121964	37.038202
	2020.01.01-2020.05.12	182	36.443407	76.766925	75.940983	76.095987	34.244308	34.707205	34.734402
		364	36.443407	80.356959	71.779368	72.319654	34.136663	34.393967	34.313009
		728	36.443407	99.379622	86.623593	86.835512	34.052210	34.157681	34.073025

Figure 22: RMSE on the consumption forecasts (DK2)

5 Wind power forecasting

First model is presented and analyzed in the section *Wind power data prognosis*. This is a benchmark model for further forecasts, where $FW_{t,h} + \varepsilon_t$ is a day ahead forecast of wind power taken from Nordpool's data.

$$W_{t,h} = FW_{t,h} \tag{W1}$$

Second model includes additionally day of week vector as the second demand model (C2).

$$W_{t,h} = \sum_{i=1}^{7} \beta_{0,i} D_{i,d} + \beta FW_{t,h} + \varepsilon_{d,h} \tag{W2}$$

Third model is extension of second model (W2) and contains also wind power from previous day ($W_{t-1,h}$) and 2 days ago ($W_{t-1,h}$) from the corresponding hour.

$$W_{t,h} = \sum_{i=1}^{7} \beta_{0,i} D_{i,d} + \beta_1 FW_{t,h} + \beta_2 W_{t-1,h} + \beta_3 W_{t-2,h} + \varepsilon_{d,h} \tag{W3}$$

Fourth model is extension of third one (W3) and contains additionally parameter of consumption forecast from previous day denoted as $FL_{t,h}$.

$$W_{t,h} = \sum_{i=1}^{7} \beta_{0,i} D_{i,d} + \beta_1 FW_{t,h} + \beta_2 W_{t-1,h} + \beta_3 W_{t-2,h} + \beta_4 FC_{t,h} + \varepsilon_{d,h} \tag{W4}$$

5.1 Results

The retrieved results from wind power forecasting with standard normalization were significantly better for DK1, and very slightly better for DK2 comparing to the Nordpool's forecast. *Asinh* function as normalization was useful only for part of cases with short calibration window, so I don't consider it as succesful results. MAE for Model 4 with 728 days window and for full 2019 year is 6,6% lower, while MAE for last period (2020.01.01-2020.05-12) is 22,19% lower.

Area	Dates	Window	Model 1	Model 2 (asinh)	Model 3 (asinh)	Model 4 (asinh)	Model 2	Model 3	Model 4
DK1	2019.01.01-2019.12.31	182	234.505308	257.618128	259.217127	259.929772	217.478916	219.693092	220.001034
		364	234.505308	258.494888	259.708202	259.503366	217.177879	218.212622	218.529144
		728	234.505308	258.537776	258.675203	258.022547	218.872158	219.069252	219.064942
	2019.01.01-2020.05.12	182	283.895457	275.953249	278.209823	279.151491	235.274013	237.847026	238.300715
		364	283.895457	295.505857	296.964391	296.016090	239.146582	240.612433	240.579080
		728	283.895457	315.423075	315.350516	312.618201	248.212884	248.436243	247.801243
	2019.05.13-2020.05.12	182	304.049977	277.287942	279.245624	281.134477	236.693538	239.211003	240.226255
		364	304.049977	295.273433	296.312244	295.538486	243.185350	244.585107	244.556479
		728	304.049977	326.742085	326.318958	323.256497	256.066520	256.193458	255.405858
	2020.01.01-2020.05.12	182	419.439850	326.271437	330.332636	331.902824	284.110181	287.667971	288.521645
		364	419.439850	397.077314	399.208821	396.220181	299.436631	302.085601	301.092064
		728	419.439850	471.536867	470.888032	462.448378	328.734424	329.029865	326.664025

Figure 23: MAE on the wind power forecasts (DK1)

Area	Dates	Window	Model 1	Model 2 (asinh)	Model 3 (asinh)	Model 4 (asinh)	Model 2	Model 3	Model 4
DK2	2019.01.01-2019.12.31	182	64.516667	80.646820	81.464406	81.117840	66.312175	66.869509	67.068310
		364	64.516667	78.611877	78.642126	78.084783	65.032888	65.272429	65.175668
		728	64.516667	76.183505	76.255634	76.059755	65.407998	65.534689	65.536840
	2019.01.01-2020.05.12	182	67.261881	82.837225	83.904873	83.621384	69.068615	69.888059	70.093752
		364	67.261881	82.274283	82.540406	82.179327	67.589801	68.034580	68.040300
		728	67.261881	81.739830	81.959521	81.866628	67.493742	67.711453	67.768841
	2019.05.13-2020.05.12	182	66.814094	83.174766	84.490223	84.151104	69.954498	70.736871	70.795956
		364	66.814094	78.516309	78.865089	78.761522	68.159593	68.515858	68.578528
		728	66.814094	80.800997	81.179748	81.046728	67.891692	68.032725	68.047130
	2020.01.01-2020.05.12	182	74.795739	88.848488	90.602396	90.492013	76.633279	78.172049	78.396656
		364	74.795739	92.325247	93.238693	93.416235	74.606895	75.614920	75.901884
		728	74.795739	96.988392	97.613044	97.802785	73.217778	73.685281	73.894259

Figure 24: MAE on the wind power forecasts (DK2)

Area	Dates	Window	Model 1	Model 2 (asinh)	Model 3 (asinh)	Model 4 (asinh)	Model 2	Model 3	Model 4
DK1	2019.01.01-2019.12.31	182	318.384899	367.489059	369.193070	368.878292	290.354599	292.139811	292.894480
		364	318.384899	366.263058	367.687652	367.200224	291.047309	291.751225	292.323555
		728	318.384899	363.631430	364.043984	363.075531	292.788017	293.021460	293.042919
	2019.01.01-2020.05.12	182	406.869604	398.507940	401.168220	401.122645	321.310501	323.896706	324.118359
		364	406.869604	441.763189	442.604058	441.151312	330.377883	331.423292	331.739072
		728	406.869604	488.802754	488.103420	483.676908	346.422368	346.399049	345.970348
	2019.05.13-2020.05.12	182	436.840443	406.496547	408.846682	408.653622	327.239721	329.807483	329.796350
		364	436.840443	446.751304	446.944226	445.319835	340.344860	341.254752	341.364980
		728	436.840443	516.323439	514.813697	510.169295	362.431378	362.043568	361.528382
	2020.01.01-2020.05.12	182	584.516298	473.302871	478.054583	478.577964	393.959310	398.243803	397.396892
		364	584.516298	602.144032	602.073439	598.889130	419.791015	421.531960	421.373847
		728	584.516298	729.213484	726.890877	717.074711	462.704143	462.232981	460.991772

Figure 25: RMSE on the wind power forecasts (DK1)

Area	Dates	Window	Model 1	Model 2 (asinh)	Model 3 (asinh)	Model 4 (asinh)	Model 2	Model 3	Model 4
DK2	2019.01.01-2019.12.31	182	89.692353	116.154447	117.190195	116.498073	90.576995	91.036874	91.390983
		364	89.692353	110.471759	110.326201	109.651575	89.196772	89.297236	89.237428
		728	89.692353	105.140561	105.256967	104.918033	89.525286	89.519276	89.464245
	2019.01.01-2020.05.12	182	93.437162	117.509184	118.914050	118.290468	93.841940	94.697121	95.107356
		364	93.437162	114.542670	114.884931	114.473742	92.486348	92.898494	92.978135
		728	93.437162	113.682709	114.033447	113.881175	92.547781	92.708086	92.759200
	2019.05.13-2020.05.12	182	93.684466	119.121235	120.798422	120.100026	94.989410	95.765894	96.047434
		364	93.684466	108.018185	108.491043	108.303648	93.450492	93.775196	93.870937
		728	93.684466	113.029049	113.398263	113.215839	93.766138	93.844232	93.853261
	2020.01.01-2020.05.12	182	103.017205	121.149259	123.521383	123.075404	102.267973	104.082899	104.630389
		364	103.017205	125.035577	126.554807	126.768528	100.964946	102.131280	102.545264
		728	103.017205	134.363592	135.223585	135.465737	100.376066	100.943080	101.252321

Figure 26: RMSE on the wind power forecasts (DK2)

6 Price forecasting

First forecasting model of price consists of the same daf of week vector ($D_{i,d}$) as the second demand model (C2).

$$P_{d,h} = \sum_{i=1}^{7} \beta_{0,i} D_{i,d} + \varepsilon_{d,h} \tag{P1}$$

Second model is an extension of first model (P1), where were added parameters of price for previous days: 1 day ago, 2 days ago and 1 week ago.

$$P_{d,h} = \sum_{i=1}^{7} \beta_{0,i} D_{i,d} + \beta_1 p_{d-1,h} + \beta_2 p_{d-2,h} + \beta_3 p_{d-7,h} + \varepsilon_{d,h} \tag{P2}$$

Third model is an extension of second model (P2) which consists additionally parameters of minimum ($p_{d-1,min}$) and maximum value ($p_{d-1,max}$) of previous day and value of price for last hour of previous day ($p_{d-1,24}$).

$$P_{d,h} = \sum_{i=1}^{7} \beta_{0,i} D_{i,d} + \beta_1 p_{d-1,h} + \beta_2 p_{d-2,h} + \beta_3 p_{d-7,h} + \beta_4 p_{d-1,min} + \beta_5 p_{d-1,max} + \beta_6 p_{d-1,24} + \varepsilon_{d,h} \tag{12}$$

Fourth model is an extension of third model (12) which is an expert model ARX2[4] with two exogenous variables: wind power prognosis ($FW_{d,h}$) and consumption prognosis ($FC_{d,h}$).

$$P_{d,h} = \sum_{i=1}^{7} \beta_{0,i} D_{i,d} + \beta_1 p_{d-1,h} + \beta_2 p_{d-2,h} + \beta_3 p_{d-7,h} + \beta_4 p_{d-1,min} + \beta_5 p_{d-1,max} + \beta_6 p_{d-1,24} + \beta_7 FW_{d,h} + \beta_8 FC_{d,h} + \varepsilon_{d,h} \tag{P4}$$

In the last fifth model, Nordpool's prognosis for wind and consumption are replaced with the forecast of Model 4 (W4) for wind and model C4 for consumption. However only values for predicted period were replaced by better prognosis, not full rolling window, that's why it was very inefficient to

$$P_{d,h} = \sum_{i=1}^{7} \beta_{0,i} D_{i,d} + \beta_1 p_{d-1,h} + \beta_2 p_{d-2,h} + \beta_3 p_{d-7,h} + \beta_4 p_{d-1,min} + \beta_5 p_{d-1,max} + \beta_6 p_{d-1,24} + \beta_7 FW_{d,h} + \beta_8 FC_{d,h} + \varepsilon_{d,h} \tag{P5}$$

6.1 Results

With more complex models results are much better, however *asinh* function surprisingly doesn't improve predictions in the every case. Disproportion is especially big in the last period (2020.01.01-2020.05-12) and Model 4.

I performed Model 5 hoping that with better predictions of wind power and consumption than original ones, price will be more accurate. Because these predictions replaced only values for prediction period, not rolling window, coefficients could be overfitted to the worse values trying to predict using improved ones.

Area	Dates	Window	P1	P1 (asinh)	P1 (asinh-hp)	P2	P2 (asinh)	P2 (asinh-hp)	P3	P3 (asinh)	P3 (asinh-hp)	P4	P4 (asinh)	P4 (asinh-hp)	P5	P5 (asinh)
DK1	2019.01.01-2019.12.31	182	62.419447	61.248905	61.248905	50.970812	47.862838	47.738136	45.531671	42.763101	42.691759	41.832735	38.053032	37.917382	41.789153	38.164474
		364	66.905983	67.163867	67.163867	49.816323	47.516155	47.173496	43.599339	41.380780	41.245874	41.600593	38.068337	37.814441	41.895198	38.249817
		728	59.616463	59.869545	59.869545	48.271333	46.798962	48.272424	42.218329	41.285560	42.293601	38.539624	36.953780	38.032519	38.773604	37.099335
	2019.01.01-2020.05.12	182	74.037510	73.752601	73.752601	54.304729	52.422105	52.272366	47.144198	45.256767	45.152057	42.638636	40.310793	40.175984	42.790111	40.480601
		364	81.583109	83.185105	83.185105	53.909704	53.190293	52.804557	45.803505	44.940818	44.797602	42.979752	41.426176	41.180891	43.460681	41.715465
		728	86.491722	87.441830	87.441830	52.921611	52.983252	53.732966	44.599572	44.803385	45.378040	40.125764	40.397561	41.097301	40.339211	40.469730
	2019.05.13-2020.05.12	182	71.894074	72.552328	72.552328	51.030218	50.348744	50.251831	44.979770	43.838396	43.789638	39.770383	38.890162	38.840113	39.980648	39.042576
		364	83.140187	85.593875	85.593875	51.939930	52.136530	51.694149	44.333355	44.243325	44.046129	42.318592	41.587801	41.127362	43.343499	42.180723
		728	90.324035	91.364589	91.364589	50.441822	51.137697	50.892249	42.781262	43.371479	43.193533	38.666817	39.451380	39.398347	39.421250	39.850178
	2020.01.01-2020.05.12	182	105.921667	108.067257	108.067257	63.454202	64.934380	64.715930	51.569554	52.100288	51.904001	44.850317	46.506904	46.374403	46.425426	47.449435
		364	121.862442	127.153167	127.153167	65.143420	68.762175	68.258221	51.852532	54.710845	54.544825	46.764663	50.641299	50.419645	48.747856	51.967810
		728	160.247132	163.110131	163.110131	65.683651	69.955175	68.718664	51.134560	54.457565	53.842851	44.478704	49.848538	49.508171	46.424097	51.241317

Figure 27: MAE on the price forecasts (DK1)

Area	Dates	Window	P1	P1 (asinh)	P1 (asinh-hp)	P2	P2 (asinh)	P2 (asinh-hp)	P3	P3 (asinh)	P3 (asinh-hp)	P4	P4 (asinh)	P4 (asinh-hp)	P5	P5 (asinh)
DK2	2019.01.01-2019.12.31	182	60.685783	59.265083	59.265083	47.103923	43.958484	43.868928	42.800522	40.041798	40.019143	40.867716	37.631112	37.592842	40.917539	37.656573
		364	66.643572	65.894549	65.894549	46.536799	43.736972	43.409012	41.864222	39.129699	38.977039	41.773766	37.899312	37.706766	41.737298	37.872167
		728	57.641463	56.997347	56.997347	44.836256	42.947585	44.126417	40.620861	38.820265	39.730950	38.663786	36.541061	37.185542	38.646771	36.530471
	2019.01.01-2020.05.12	182	72.088336	71.634624	71.634624	50.790176	49.063672	48.960707	44.593005	42.871472	42.829081	42.240303	40.706097	40.665848	42.268304	40.722874
		364	80.282137	81.045797	81.045797	50.643427	49.600087	49.256643	43.848764	42.678249	42.549245	43.034634	41.318684	41.161845	43.023788	41.311466
		728	84.403612	84.125969	84.125969	49.712208	49.083361	49.654152	42.993153	42.220931	42.751613	40.725420	40.009271	40.376973	40.703890	39.996084
	2019.05.13-2020.05.12	182	68.703284	69.280873	69.280873	47.331708	47.005937	46.951134	42.340374	41.453055	41.455745	38.983829	38.603889	38.608243	38.997465	38.620568
		364	80.654765	82.458458	82.458458	48.135879	48.324643	47.966416	41.888804	41.748732	41.650334	41.183152	40.570452	40.354765	41.105611	40.537650
		728	88.502231	88.186925	88.186925	47.364384	47.302565	47.090488	41.124731	40.735721	40.701233	38.987806	38.711242	38.577699	38.941618	38.684554
	2020.01.01-2020.05.12	182	103.381058	105.581108	105.581108	60.906583	63.074152	62.934386	49.512224	50.637121	50.540565	46.007178	49.144966	49.099286	45.955950	49.141889
		364	117.711280	122.626291	122.626291	61.913499	65.690589	65.304652	49.295062	52.416750	52.352668	46.494911	50.702674	50.643831	46.430872	50.649240
		728	157.848607	158.576699	158.576699	63.093580	65.922146	64.824254	49.503579	51.553585	51.041401	46.383287	49.527291	49.135411	46.274236	49.455233

Figure 28: MAE on the price forecasts (DK2)

Area	Dates	Window	P1	P1 (asinh)	P1 (asinh-hp)	P2	P2 (asinh)	P2 (asinh-hp)	P3	P3 (asinh)	P3 (asinh-hp)	P4	P4 (asinh)	P4 (asinh-hp)	P5	P5 (asinh)
DK1	2019.01.01-2019.12.31	182	88.552095	89.377633	89.377633	75.854601	74.598665	74.391426	68.473893	66.034200	65.874428	59.011419	55.578274	55.358938	59.203002	55.901061
		364	93.087824	94.352975	94.352975	75.095306	73.924743	73.360790	66.647750	64.475040	64.032671	59.280699	56.405672	55.988147	59.773234	56.644245
		728	85.255977	86.419926	86.419926	73.416049	72.257950	72.577855	65.340032	64.153815	64.052560	56.731674	56.140598	56.348611	57.210817	56.408465
	2019.01.01-2020.05.12	182	100.419523	101.646696	101.646696	76.764606	76.322214	76.089317	68.376690	66.772849	66.585285	58.859893	57.016755	56.812191	59.237037	57.384865
		364	108.382915	110.865235	110.865235	76.532520	76.967591	76.365457	66.867873	66.328687	65.917485	59.490655	58.841660	58.443448	60.213819	59.297428
		728	117.686433	119.273737	119.273737	75.655026	76.389407	76.153396	65.855836	66.199160	65.827934	57.012862	58.590169	58.521558	57.489391	58.782734
	2019.05.13-2020.05.12	182	95.465230	96.983670	96.983670	68.212685	68.563190	68.403369	62.299381	61.418993	61.309870	53.964120	54.091201	53.982518	54.301694	54.277113
		364	107.685682	110.712737	110.712737	69.306409	70.734229	70.164132	61.345989	61.916248	61.600884	56.388890	56.859771	56.380539	57.812438	57.691593
		728	121.312715	122.971495	122.971495	68.434178	70.469465	69.744520	60.344236	61.666861	61.048242	52.893555	55.460732	55.039993	53.987241	55.944617
	2020.01.01-2020.05.12	182	127.430779	129.475793	129.475793	79.208289	80.863726	80.565305	68.109219	68.759214	68.498242	58.442031	60.789827	60.621708	60.666018	62.211354
		364	142.139512	146.937698	146.937698	80.344770	84.758849	84.061342	67.468281	71.168134	70.832876	60.063080	65.059942	64.704741	62.703120	66.998609
		728	178.639717	181.030724	181.030724	81.484104	86.722178	85.198452	67.251065	71.512389	70.470768	57.777514	64.838924	64.107598	60.331381	66.571225

Figure 29: RMSE on the price forecasts (DK1)

Area	Dates	Window	P1	P1 (asinh)	P1 (asinh-hp)	P2	P2 (asinh)	P2 (asinh-hp)	P3	P3 (asinh)	P3 (asinh-hp)	P4	P4 (asinh)	P4 (asinh-hp)	P5	P5 (asinh)
DK2	2019.01.01-2019.12.31	182	85.807352	85.896984	85.896984	71.018341	69.555000	69.379977	64.046663	62.180679	62.084748	57.322139	55.413446	55.348356	57.369391	55.425448
		364	90.943848	90.895794	90.895794	70.897828	69.132619	68.657715	63.118444	60.741802	60.320563	58.595830	55.930580	55.589629	58.551300	55.904560
		728	81.697935	81.828635	81.828635	69.490904	67.728746	68.029236	62.115683	60.256858	60.166623	56.427961	55.223516	55.394403	56.432042	55.227725
	2019.01.01-2020.05.12	182	97.529695	98.309403	98.309403	72.873760	72.475196	72.296759	64.566985	63.442992	63.341161	58.309209	57.725638	57.671154	58.334621	57.728349
		364	105.054135	106.585367	106.585367	72.893222	72.939020	72.466732	63.805347	63.127536	62.784119	59.408044	58.730318	58.470746	59.389992	58.725930
		728	114.468197	114.616814	114.616814	72.423808	72.149589	71.992683	63.374743	62.639554	62.341294	57.839339	58.055362	57.989223	57.827832	58.044470
	2019.05.13-2020.05.12	182	91.720525	93.090487	93.090487	64.406940	65.130937	65.019584	58.413321	57.804075	57.766398	53.052848	53.549639	53.547140	53.075160	53.578512
		364	103.403839	105.674446	105.674446	65.255122	66.661286	66.234275	57.587360	58.073291	57.893616	54.955283	55.073300	54.847794	54.872352	55.052359
		728	118.232453	118.064362	118.064362	65.196015	66.062437	65.507953	57.358018	57.480783	57.091545	52.927365	53.710299	53.434019	52.874757	53.679855
	2020.01.01-2020.05.12	182	124.137247	126.252027	126.252027	77.738504	79.943070	79.755259	65.973857	66.784762	66.667665	60.935991	63.640964	63.611442	60.889485	63.634190
		364	136.477624	140.938122	140.938122	78.107829	82.487579	82.016132	65.653539	69.253864	69.095171	61.582040	65.804607	65.731905	61.502552	65.732906
		728	175.342124	175.538274	175.538274	79.921663	83.082018	81.890066	66.708073	68.755588	67.952733	61.546601	65.198037	64.576840	61.404119	65.103546

Figure 30: RMSE on the price forecasts (DK2)

7 Conclusions

Almost all of the assumptions came true, forecasts for wind power and consumption were performed better than original prepared by Nordpool. Even models for wind power performed well however wind power predicting is not typical time-series problem, because more important factors influencing such as atmospheric models were not known and not used in this work.

Main finding is that period of forecast matters in using the most efficient model. For some periods more accurate were shorter calibration windows, because anomalies in the 2020 caused by COVID-19 changed seasonal numbers like consumption and thus price which was significantly lower in 2020 than 2019.

One assumption I found invalid was that *asinh* normalization function improves price predictions. It completely fails for wind power and consumption forecasting, but should be useful for price, but yet it works worse for periods including 2020 and especially for period which includes only 2020 year. The forecast Model 4 with asinh normalization performs 7.34% (DK1) and 16.34% (DK2) worse comparing MAE of the results.

Experiment of replacing original forecasts of Nordpool with better, forecasted results unfortunately failed but I still see potential with bigger calibration window, which can be used to trained better forecasts for whole considering period of price prediction. But for this purpose it's necessary to have more data.

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