$$P_n(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \cdots + \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n$$
$$= \sum_{k=0}^{n} \frac{f^{(k)}(x_0)}{k!}(x - x_0)^k$$

and

$$R_n(x) = \frac{f^{(n+1)}(\xi(x))}{(n+1)!}(x - x_0)^{n+1}.$$

error criterion conforming to two significant figures.

4.5 Use zero- through third-order Taylor series expansions to predict $f(3)$ for

$$f(x) = 25x^3 - 6x^2 + 7x - 88$$

using a base point at $x = 1$. Compute the true percent relative error ε_t for each approximation.

$f(3) = 554$

$f'(x) = 75x^2 - 12x + 7$

$f''(x) = 150x - 12$

$P_0(x) = f(x_0) \Rightarrow P_0(1) = 25 - 6 + 7 - 88 = -62$

$P_1(x) = f(x_0) + f'(x_0)(x - x_0) = f(1) + f'(1)(x-1)$

$= -62 + 70(x-1) = 70x - 132$

$\therefore P_1(3) = 78$

$P_2(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2$

$= f(1) + f'(1)(x-1) + \frac{f''(1)}{2}(x-1)^2$

$= -62 + 70(x-1) + 69(x-1)^2$

$P_2(3) = -62 + 140 + 276 = 354$

$P_3(x) = f(1) + \cdots + \frac{f'''(1)}{3!}(x-1)^3$

$P_3(3) = 354 + \frac{150}{3!}(8)$

$= 354 + 50 \cdot 4 = 554$

$\therefore P_0(3) = -62$

$P_1(3) = 78$

$P_2(3) = 354$

$P_3(3) = 554$

~> order로 error 추정.

$$v(t) = \frac{gm}{c}\left(1 - e^{-(c/m)t}\right)$$

$e^{-\frac{c}{m}t}$

$1 + \frac{t}{m} e^{-t/m\,t}$

4.12 Repeat Prob. 4.11 with $g = 9.81$, $t = 6$, $c = 12.5 \pm 1.5$, and $m = 50 \pm 2$.

$dv = \frac{g(tc+m)e^{-\frac{tc}{m}}}{c^2}\,dc$

$dv = \frac{g(m+ct)e^{-\frac{ct}{m}}}{cm}\,dm.$

$\therefore$