Capacity Analysis of a Restaurant with Variable Family Sizes

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1 Introduction

This report presents an analysis of a restaurant's operations using a queueing model to identify optimal resource allocation, specifically focusing on chef staffing levels. Customers arrive at the restaurant in groups ranging from 1 to 7 individuals, with arrivals occurring at a consistent rate throughout operating hours. The service times, however, vary depending on group size, with larger groups generally requiring longer service durations. This variability creates unique challenges in managing customer flow and maintaining a high standard of service.

In Figure 1, the plot in row 1, column 1 illustrates the distribution of customer arrivals. Observing this distribution reveals an approximately exponential pattern, though not strictly so, as group size 2 arrives more frequently than other groups, followed by groups 1, 3, 4, 5, 6, and 7 in decreasing order.

2 Problem Description

The restaurant's customer arrivals follow an Exponential Distribution with an average rate of $\lambda = 7$ customers per hour. The service rate, μ, depends on the size of the arriving group, with larger groups requiring more time. This analysis aims to determine the impact of varying the number of chefs on the restaurant's service capacity and efficiency.

3 Assumptions

- Family sizes range from 1 to 7 people.
- Constant arrival rate $\lambda = 7$ customers/hour.
- Service rate μ varies by family size.
- Queue length is limited to 1000 people.
- Simulation period is 6000 hours.

4 Methodology

The methodology for this analysis applies an M/G/K/1000 queueing model, a framework designed for systems with exponential arrival patterns and generalized service times, implemented through a discrete-event simulation. Customer arrivals follow a Poisson process with an average rate of $\lambda = 7$ customers per hour, simulating steady demand over time. The queue is set to a maximum capacity of 1000, creating an environment that can accommodate high customer volumes without frequently turning customers away due to space limitations.

Customer groups are assumed to follow an approximate exponential distribution with sizes ranging from 1 to 7. The service rate, μ, depends on group size, calculated as $\mu = \frac{10}{\text{group size}}$, meaning larger groups take more time to serve. To simulate variability in chef availability, an Erlang distribution is applied, which effectively captures the impact of different staffing levels on service performance. The Erlang distribution, commonly used in queueing theory, represents the sum of multiple exponential random variables and allows this model to simulate realistic variations in service demand.

The simulation tracks three primary events—customer arrivals, departures, and system state measurements—updating event times continuously through an event-driven loop. Upon arrival, customer groups enter the queue unless the system is at full capacity, in which case they are recorded as "left without service." Departures occur based on the variable service rate, and each departure frees up space in the queue, allowing more arrivals when capacity permits.

To assess the impact of chef staffing, simulations are conducted across different chef counts (e.g., 10, 20, etc.). Key performance metrics include queue length, sojourn time (total time spent in the system), and the frequency of groups leaving without service. The simulation results are presented with histograms of group sizes, queue lengths, and sojourn times, showing the probability distributions of each. By analyzing these distributions, this model provides insights into optimal chef staffing configurations that balance customer satisfaction and resource efficiency, helping to maintain low wait times and effective resource utilization.

5 Simulation Results

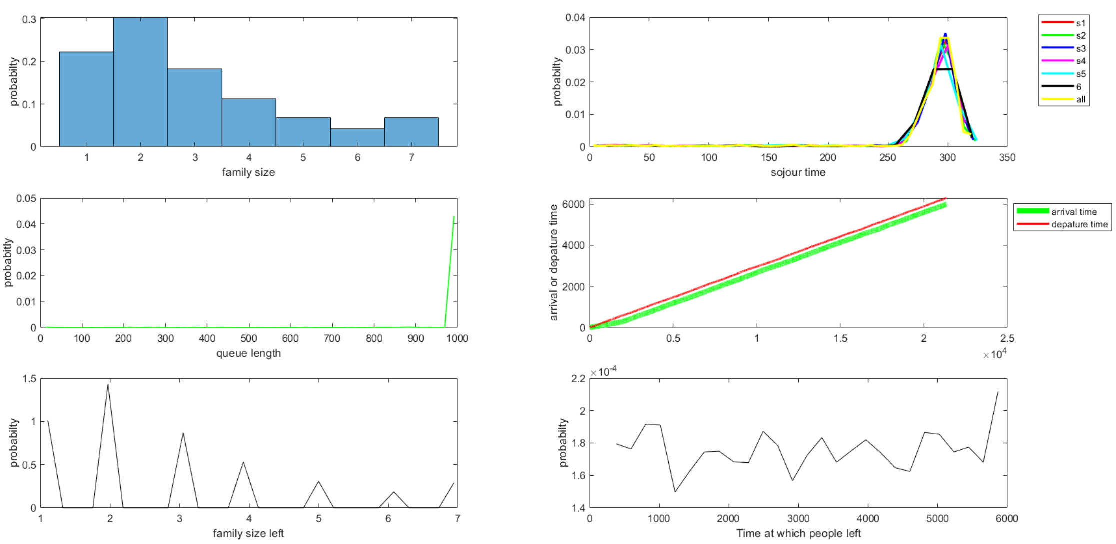

Figure 1: Simulation results for numbers of chefs 1

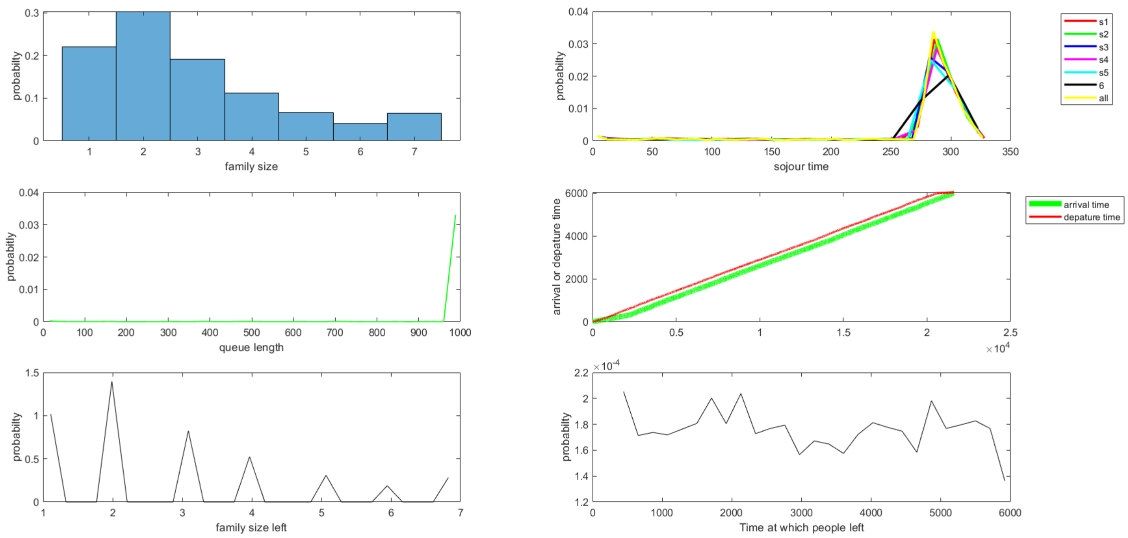

Figure 2: Simulation results for numbers of chefs 10

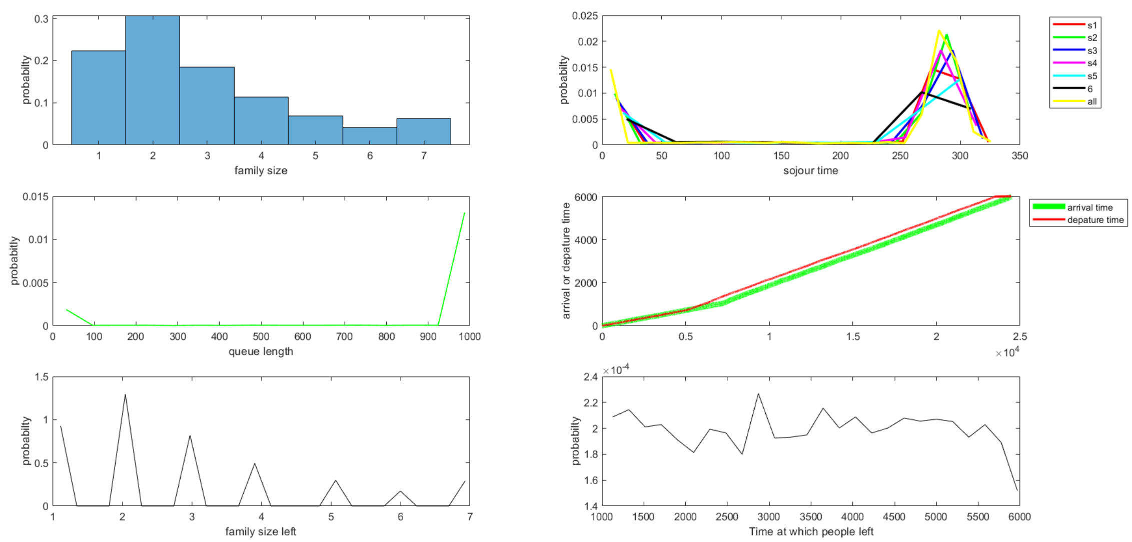

Figure 3: Simulation results for numbers of chefs 20

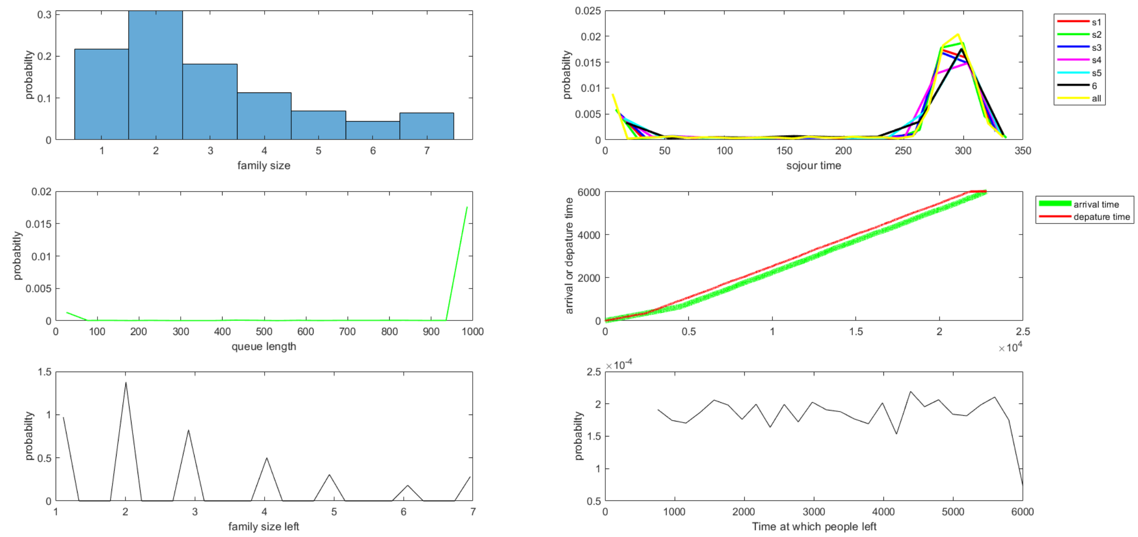

Figure 4: Simulation results for numbers of chefs 30

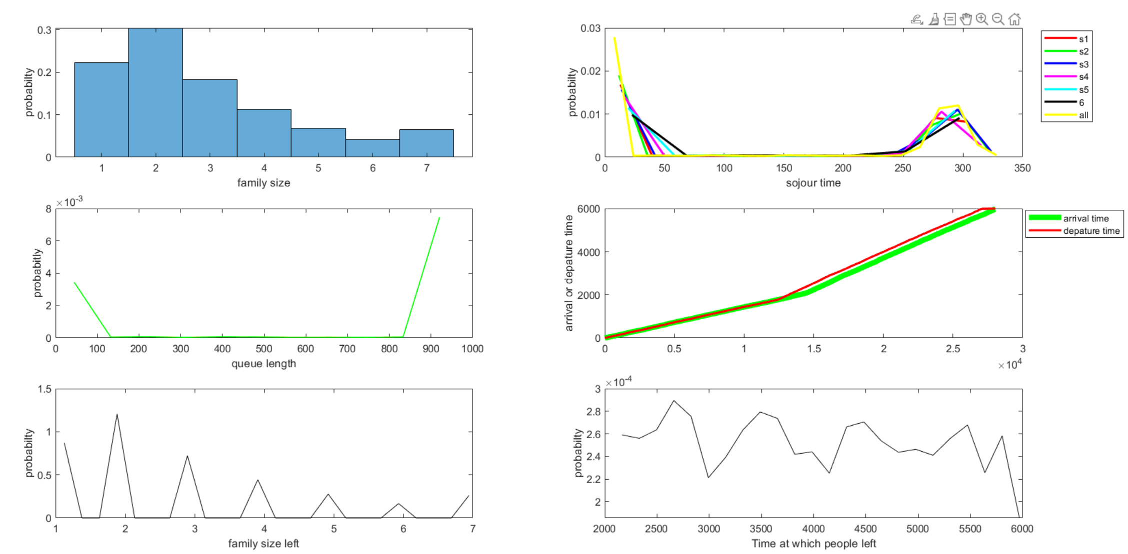

Figure 5: Simulation results for numbers of chefs 31

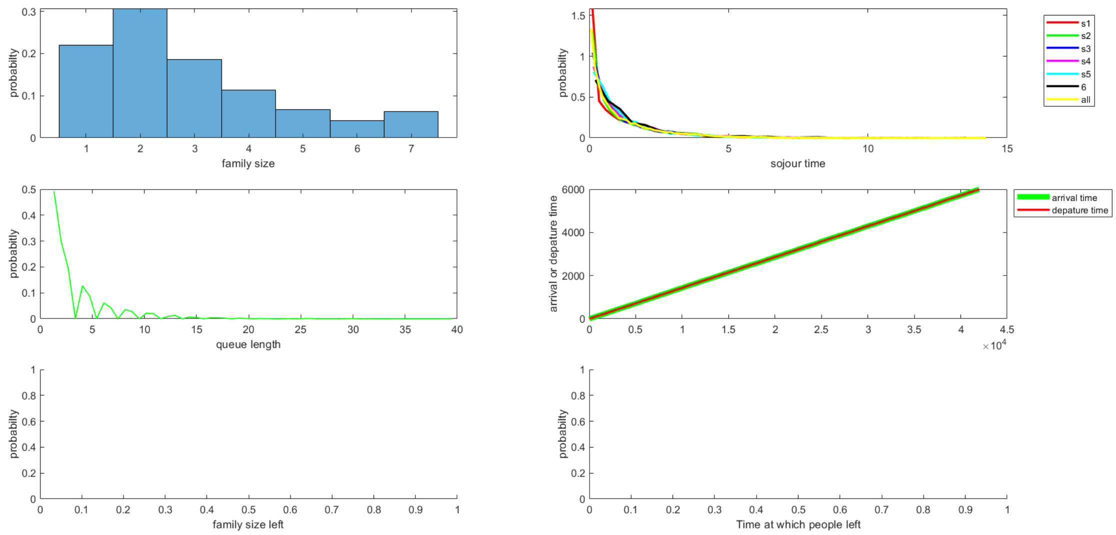

Figure 6: Simulation results for numbers of chefs 41

The graphs for different chef counts, shown in Figures 1, 2, 3, 4, 5, and 6, illustrate how increasing the number of chefs impacts key metrics: sojourn time (row 1, column 2), queue length (row 2, column 1), departure and arrival times (row 2, column 2), distribution of customers who left without service (row 3, column 1), and the histogram of times at which customers left (row 3, column 2). These graphs demonstrate how key metrics evolve as chef staffing levels change.

With higher chef counts, the sojourn time distribution appears to transition from a Gaussian shape to an exponential form. For example, with one chef, the sojourn time initially shows a triangular shape, but extending the simulation time to 10,000 hours reveals a more Gaussian distribution. The queue length frequently reaches its limit of 1000 for lower chef counts, indicating that the queue is always full with fewer chefs. Additionally, the histogram of customers leaving without service mirrors the group arrival distribution (row 1, column 1), reflecting a similar pattern in both metrics.

6 Conclusions

As seen in Figure 6, no customers leave the restaurant without receiving service, and the queue length does not reach its maximum limit. The sojourn time follows an exponential distribution with minimal time spent in the system. Therefore, the optimal number of chefs required for the restaurant is determined to be 41.