College of Computer & Information Science
Northeastern University
Spring 2019

CS5800
Algorithms
5 April 2019

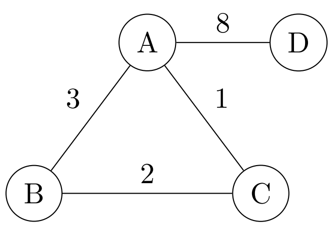
Sample Solution to Quiz 3

1. (4 points) Minimum Spanning Tree

Determine if the following statement is True or False. If you believe it is true provide a proof, and if you consider it false, provide a counterexample.

True or False? $G = (V, E)$ is a connected, undirected graph with positive costs c_e for each $e \in E$. If graph G has more than $|V| - 1$ edges, and there is a unique heaviest edge e^*, then this edge e^* cannot be part of a minimum spanning tree.

Answer: *False* .
It is possible that the heaviest edge is the only edge across a cut, which implies it must be part of any MST for the graph. Here is an example.



Here, (A, D) is the heaviest cost edge, yet part of the MST.

2. (6 points) Cell Tower Locations

Consider a long, quiet country road with houses scattered very sparsely along it. You can picture the road as a long line segment, with an eastern endpoint and a western endpoint. The locations of the houses are given as $L_i > 0 : 1 \leq i \leq n$. Further, suppose that despite the bucolic setting, the residents of all these houses are avid cell phone users. You want to place cell phone base stations at certain points $B_j : 1 \leq j \leq m$ along the road, so that every house is within four miles of one of the base stations.

Our objective is to minimize m while meeting the requirement of proximity of base stations to the houses. Give an efficient algorithm that achieves this goal. State the running time of your algorithm in terms of n. Your grade will depend on the correctness and the efficiency of your algorithm. No proof of correctness or analysis of running time is required.

Answer: Sort the locations of the houses L_i in increasing order as measured in miles from the eastern endpoint. The problem can be solved using a greedy strategy where the first cell tower is placed 4 miles west of the first location yet to be covered. Skip past all of the locations L_i that are within 4 miles of the base station in either direction. Place the next base station 4 miles west of the next location to be covered, etc. until all locations are covered.

PLACECELLTOWERS($L[1 \dots n]$)
$i \leftarrow 1; j \leftarrow 0$
while $i \leq n$ **do**
    $j \leftarrow j + 1$
    Comment: Place a cell tower 4 miles west of L[i]
    $B[j] \leftarrow L[i] + 4$
    while $L[i] \leq B[j] + 4$ **do**
        $i \leftarrow i + 1$
    end
end
$m \leftarrow j$
Comment: $B[1], \dots B[m]$ hold the optimal bell tower locations.
return ($B[1 \dots m]$)

As the algorithm requires a linear scan of the house locations, its worst-case time complexity is $O(n)$.