College of Computer & Information Science — CS5800
Northeastern University — Algorithms
Fall 2017 — 4 December 2017

Instructions:

- This quiz is closed book and closed notes. Please use both sides of the page.
- Please write clearly and legibly. Grading will be based on both clarity and correctness.

Name: ________________________________

Sample Solution to Quiz 4

Network flow (2 + 1 + 2 = 5 points)

In the attached sheet, you are given a flow network. The capacity of each edge appears as a label next to the edge, and the numbers in the boxes give the amount of flow on each edge. Edges without boxed numbers have no flow being sent on them.

(a) What is the current value of the total flow from s to t? Draw the residual network (with residual capacities) for the given flow.

Answer: Current value of total flow is 15.

Let G be the given network. We obtain the residual network G' as follows. G' has the same vertices as G. For each edge (i, j) in G with capacity $c(i, j)$ and flow $f(i, j)$, include edge (i, j) in G' with capacity $c(i, j) - f(i, j)$ and include edge (j, i) in G' with capacity $f(i, j)$.

(b) List an augmenting path from s to t in the residual network (by listing the vertices in order along the path). What is the maximum amount of flow you can send on this path? What is the new value of the total flow from s to t, after using the augmenting path?

Answer: $s \rightarrow a \rightarrow c \rightarrow b \rightarrow d \rightarrow t$ with a flow of 2 across the path.

After using the augmenting path, the total flow is 17.

(c) Let G be a flow network with source s and sink t. Let C denote any minimum capacity cut (also referred to as a min-cut) separating s from t. Let e be an arbitrary edge crossing the cut C (from the side containing s to the side containing t). Let v denote the value of maximum flow from s to t in G.

Indicate whether the following statement is true: if we decrease the capacity of e by 1, then the value of maximum flow from s to t will become $v - 1$.

If you claim that the statement is true, give a brief proof; otherwise, give a counterexample.

Answer: True. Since C is a minimum cut, its capacity equals the maximum flow v. After we decrease the capacity of e by 1, the capacity of C decreases by 1 to $v - 1$. For every other cut C', the capacity either stays the same (which is at least v) or decreases by 1 (so at least $v - 1$). Therefore, the capacity of the minimum cut is exactly $v - 1$. Hence the new maximum flow is exactly $v - 1$.