College of Computer & Information Science
Northeastern University
Fall 2018

CS5800
MS Algorithms
7 November 2018

Problem Set 6 (due Wednesday, November 14, 11:59 PM)

Instructions:

- The assignment is due at the time and date specified. Late assignments will be accepted, up until Thursday, November 15, 11:59 PM. *Note, however, that you can use at most 3 late days for your problem set and programming assignment submissions throughout the course of the term.*
- We encourage you to attempt and work out all of the problems on your own. You are permitted to study with friends and discuss the problems; however, *you must write up you own solutions, in your own words.*
- Please refrain from searching online or asking your peers or other students for solutions. The best way to learn the material is to attempt the problem yourself, and if you are stuck, identify where and why you are stuck and seek help to overcome the associated hurdles.
- If you do collaborate with any of the other students on any problem, please *list all your collaborators in your submission for each problem.*
- We require that all homework submissions be neat, organized, and *typeset*. You may use plain text or a word processor like Microsoft Word or LaTeX for your submissions. If you need to draw any diagrams, however, you may draw them with your hand.

Problem 1. (7 points) Project management

Suppose you are a high-level manager in a software firm and you are managing n software projects. You are asked to assign m of the programmers in your firm among these n projects. Assume that all of the programmers are equally competent.

After some careful thought, you have figured out how much benefit i programmers will bring to project j. View this benefit as a number. Formally put, for each project j, you have computed an array $A_j[0..m]$ where $A_j[i]$ is the benefit obtained by assigning i programmers to project j. Assume that $A_j[i]$ is nondecreasing with increasing i.

Design a dynamic programming algorithm to determine how many programmers you will assign to each project such that the total benefit obtained over all projects is maximized. Analyze its running time.

Problem 2. (7 points) Organizing congressional districts

The midterm elections are just over, and it was a really close election. As the neutral Election Commissioner for the state of Dynamica, you are worried that the two parties Maroon and Gray will indulge in reorganizing the precincts so as to make favorable maps.

There are n precincts $P_1, P_2, \ldots, P_n$, each containing m registered voters. You also find out the number m_i of registered Maroon voters and g_i of registered Gray voters in precinct P_i. We say that the precincts are *vulnerable* if there is a way to split the n precincts into two districts, each containing $n/2$ precincts in such a way that the same party holds a majority in both districts (assume n is even).

Give a dynamic programming algorithm to determine whether the given precincts are vulnerable. Your algorithm must take time polynomial in the total number of voters in the n precincts.

Problem 3. (6 points) Planning a company party

You are consulting for a corporation that is planning a company party. The company has a hierarchical structure that can be captured as a tree rooted at the president. Thus, every node of the tree represents an employee and the parent $\pi(v)$ of any node v (except the root) represents the immediate supervisor of the person. The personnel office has ranked each employee v with a personality rating $\sigma(v)$, which is a nonnegative integer. In order to make the party fun for all attendees, the president does not want both an employee and his or her immediate supervisor to attend.

Given the tree structure of the company hierarchy and the personality ratings of each person, describe a dynamic programming algorithm to make up a guest list that maximizes the sum of the personality ratings of the guests in the list. Analyze the time complexity of your algorithm in terms of the number of employees.