Sample Solution to Quiz 2

1. (4 points) Depth-first-search

Prove or disprove the following statement. For a directed graph G, and vertices u and v of G, if there is a path from u to v in G and if $d[u] < d[v]$ in a depth-first search of G, then v is a descendant of u in the depth-first forest produced by that depth-first search. (Recall that $d[x]$ denotes the discovery time of x in a given depth-first search.)

Answer: The statement is false. Consider the graph with three vertices s, u and v, and three edges $(s, u), (s, v), (u, s)$. Note that there is a path $u \to s \to v$. If we start the DFS from s, then we can have the following traversal order.

discover s, discover u, finish u, discover v, finish v, finish s

So $d[u] < d[v]$. But v is not a descendant of u; they are both children of s.

2. (6 points) Directed Acyclic Graphs

Give an algorithm that takes as input a directed acyclic graph $G = (V, E)$ and returns a path that visits every vertex of the graph; if no such path exists, then your algorithm should indicate so.

State the worst-case running time of your algorithm, in terms of the number of vertices and edges of G.

Your grade for this question will be determined on the basis of the correctness of your algorithm and its efficiency, given by its worst-case running time. Partial credit may be given for non-optimal algorithms provided they are correct and well explained.

Answer:

1. Find topological ordering of G. Let this order be $v_1, v_2, \ldots, v_n$.
2. Check if there is an edge (v_i, v_{i+1}) for each $1 \leq i < n$. If this is true for all i, return the path $v_1 \to v_2 \to \cdots \to v_{n-1} \to v_n$. If this is not true, then indicate that no desired path exists.

The first step takes $\Theta(n + m)$ time where n is the number of vertices and m the number of edges. The second step takes $\Theta(n)$ time. The total time is $\Theta(n + m)$.