College of Computer & Information Science
Northeastern University
Spring 2019

CS5800
Algorithms
22 April 2019

Sample Solutions to Quiz 4

1. (3 points) Shortest Paths

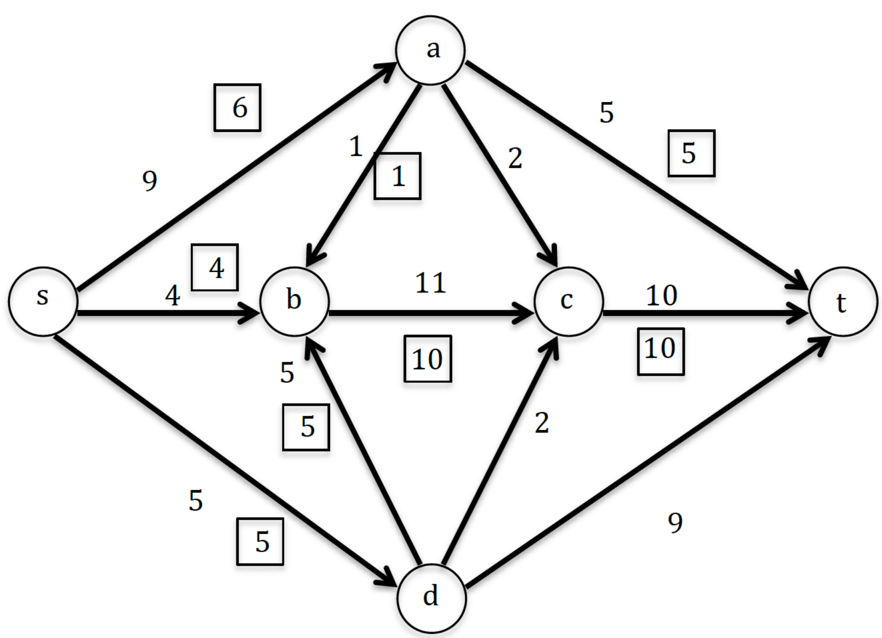
Indicate whether the following claim is true or false. Justify your answer.

- Let G be a directed graph with arbitrary weights (nonnegative or negative) on edges, and s and t be two vertices in G. Let p denote a shortest path from s to t. If we multiply the weight of every edge in the graph by 2, then p remains a shortest path from s to t.

Answer: True.
Multiplying every edge weight by 2 doubles the weight of every path. Hence, for any two paths P_1 and P_2, if weight of P_1 was at most that of P_2 earlier, then the revised weight of P_1 will be at most the revised weight of P_2. So p remains a shortest path.

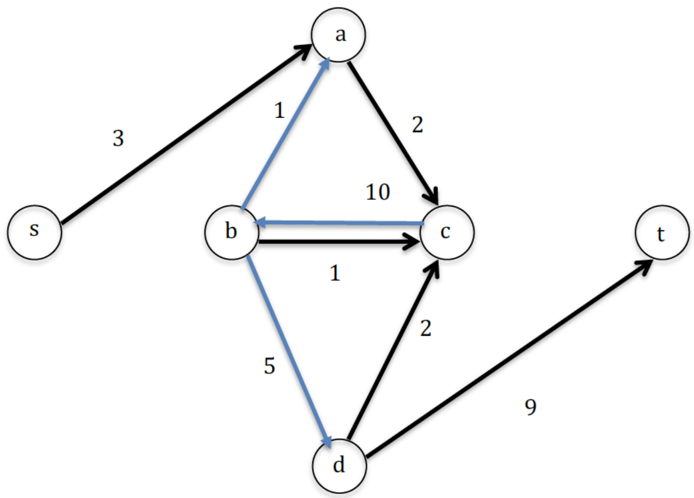
2. (3 + 2 + 2 = 7 points) Network Flows

In the flow network given below, the capacity of each edge appears as a label next to the edge, and the numbers in the boxes give the amount of flow on each edge. Edges without boxed numbers have no flow being sent on them.



(a) What is the current value of the total flow from s to t? Draw the residual graph for the given flow.

Answer: Current value of total flow is 15. The residual network is below. Edges into s and out of t are not drawn.



(b) List an augmenting path from s to t in the residual graph (by listing the vertices in order along the path). What is the maximum amount of flow you can send on this path? What is the new value of the total flow from s to t after using the augmenting path?

Answer: $s \to a \to c \to b \to d \to t$. Maximum flow we can send on this path is 2. New value of flow = 17.

(c) What is the maximum flow from s to t? Find the corresponding minimum cut.

Answer: The maximum flow that can be achieved is 17. The corresponding min-cut is given the vertices $\{s, a\}$ and $\{b, c, d, t\}$. The edges across the cut are: $(s, b), (s, d), (a, b), (a, c), (a, t)$ with capacities of 4, 5, 1, 2, 5 which add up 17.