College of Computer & Information Science | CS5800
Northeastern University | Algorithms
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Sample Solution to Quiz 1

Problem 1. (5 points) Stable Matching

Decide whether you think the following is true or false. If it is true, give a short explanation. If it is false, give a counterexample.

Consider an instance of the stable matching problem with $n \geq 2$ men and n women in which there exists a man m and a woman w such that m ranks last on the preference list of w and w ranks last on the preference list of m. **True or False?** There is no stable matching containing the pair (m, w).

Answer: False. Consider the following instance with 2 men M_1, M_2 and 2 women W_1, W_2. Suppose the preferences are the following.

$M_1 : W_1, W_2$
$M_2 : W_1, W_2$
$W_1 : M_1, M_2$
$W_2 : M_1, M_2$

The only stable matching is (M_1, W_1), (M_2, W_2) since otherwise both M_1 and W_1 would want to swap. In this stable matching the pair (M_2, W_2) is such that M_2 ranks last on W_2's list and W_2 ranks last on M_2's list.

Problem 2. (5 points) Asymptotic Notation

Let $f(n)$, $g(n)$, and $h(n)$ be positive monotonically increasing functions such that $f(n) = \Omega(g(n))$ and $g(n) = \Omega(h(n))$. Prove or disprove: $f(n) \cdot g(n) = \Omega(h(n)^2)$.

Answer: True.

Since $f(n) = O(g(n))$, there exists constants $c_1, n_1 > 0$ such that $f(n) \geq c_1 g(n)$ for all $n \geq n_1$.

Since $g(n) = O(h(n))$, there exists constants $c_2, n_2 > 0$ such that $f(n) \geq c_2 g(n)$ for all $n \geq n_2$.

Therefore, if $n_3 = \max\{n_1, n_2\}$, then for $n \geq n_3$, we have:

$$\begin{aligned} f(n) \cdot g(n) &\geq c_1 g(n) \cdot c_2 h(n) \\ &\geq c_1 c_2^2 h(n)^2. \end{aligned}$$

So, for $c_3 = c_1 c_2^2$, we obtain that there exists constants $c_3, n_3 > 0$ such that $f(n) \cdot g(n) \geq c_3 h(n)^2$ for all $n \geq n_3$.