Recitation Class 5

Q1 – A directed graph $G = (V, E)$ is semiconnected if, for all pairs of vertices $u, v \in V$, we have $u \rightsquigarrow v$ or $v \rightsquigarrow u$. Give an efficient algorithm to determine whether or not G is semiconnected. Prove that your algorithm is correct, and analyze its running time.

Answer:

First, solve this problem for special types of graphs which are DAGs. Consider the topological sort order of the vertices of a DAG. Show that a DAG is semiconnected if and only if there is a directed edge between every consecutive pair of vertices on the ordered vertices. For the general directed graph, first compute Strongly Connected Components of the graph. Since the strongly connected components of the graph is DAG, check that the above property (that there is a directed edge between every consecutive pair of vertices on the ordered vertices) holds or not. If it holds, the graph is semiconnected, otherwise it is not semiconnected.

The running time of the algorithm is $O(V + E)$.

Q2 – Give an efficient algorithm which takes as input a directed graph $G = (V, E)$, and determines whether or not, there is a vertex $s \in V$ from which all other vertices are reachable.

Answer:

Compute the strongly connected component of the graph call it G^{SCC}. Check how many vertices have in-degree 0, and if there is only one vertex with in-degree 0 in G^{SCC}, then there is some vertex $s \in V$ (in graph G) from which all other vertices are reachable, otherwise there is no such vertex $s \in V$ (in graph G) that all other vertices are reachable.

The running time of the algorithm is $O(V + E)$.

Q3 – In a data set, there are n data points. And there is a collection of m judgements. Each judgement is a label for a pair (i, j) of data. Each judgment is either “same”, which means data i and data j are of the same type, or either is “different”, which means data i and data j are of different types. There is a conjecture which data set consists of two types A and B.

Give an algorithm that determines that data is consistent with the conjecture or not, which means each data point is possible to be in exactly one category.

Answer:

We construct graph G, in the following way. For each data point in the data set, consider a vertex in the graph. For each judgement between a pair of points, consider an edge between the

associated vertices. For a moment, take the connected components of graph G, which forms by **only considering edges with label "same"**; If the data set is consistent, it is the case that all vertices in a component, should get the same category. Now probe each edge with label "different", if the two end points of the edge are inside a component, it is the case that there is a conflict and the data is not consistent with the conjecture. Otherwise, contract the components, such that each component is a single vertex. Now add edges with label "different". Call this graph G'. Check whether the graph G'is bipartite, if it is bipartite, then the data set is consistent with the conjecture, otherwise it is not.