Northeastern University

DS5020 - Introduction to Linear Algebra and Probability for Data Science

Assignment 06 - Fall 2018

1) **{30 pts}** Consider the system of differential equations

$$\frac{dx_1(t)}{dt} = x_1(t) - x_2(t) - x_3(t)$$
$$\frac{dx_2(t)}{dt} = -x_1(t) + 2x_2(t) - x_3(t)$$
$$\frac{dx_1(t)}{dt} = -x_1(t) - x_2(t) + 2x_3(t)$$

   a. Express the system in a matrix form.
   b. Find the general solution of the system.
   c. Find the solution of the system with the initial value $x_1 = 0, x_2 = 1, x_3 = 5$.

2) **{30 pts}** Find SVD of the matrix $A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$.

   a. Find eigenvalues and eigenvectors of $A^T A$.
   b. Form the diagonal matrix Σ using singular values.
   c. Find the eigenvectors of AA^T.
   d. Form the orthonormal vectors U and V using a, b and c above. If needed complete the matrices to squares by adding orthonormal vectors from null spaces A^T and A.
   e. Verify $A = U\Sigma V^T$.

3) **{20 pts}**

   a. Prove that the diagonal elements of a triangular matrix are its eigenvalues.
   b. Construct a matrix whose eigenvalues and singular values are the same.

c. Construct a matrix whose largest singular value is twice its largest eigenvalue.
d. Prove that the diagonal elements of a triangular matrix are its eigenvalues.

4) **{20 pts}** Use any programming package you like to compute the best rank one approximation of the below matrix.

$$\begin{matrix} 3 & 7 & 3 & 6 & 4 & 3 & 10 & 2 & 4 \\ 9 & 5 & 8 & 8 & 2 & 7 & 2 & 10 & 3 \\ 3 & 4 & 8 & 10 & 8 & 7 & 9 & 1 & 9 \\ 10 & 9 & 4 & 2 & 4 & 8 & 6 & 8 & 5 \\ 4 & 6 & 6 & 6 & 6 & 5 & 10 & 9 & 10 \\ 2 & 6 & 1 & 5 & 2 & 1 & 1 & 9 & 2 \\ 3 & 10 & 1 & 1 & 7 & 3 & 5 & 1 & 3 \end{matrix}$$