Gram-Schmidt Process: Worked out solution for exercise in class

Example: $a=\begin{bmatrix}-1\\-1\\0\end{bmatrix}$ $b=\begin{bmatrix}1\\0\\1\end{bmatrix}$ $c=\begin{bmatrix}1\\1\\1\end{bmatrix}$ (exercise)

$$q_1 = \frac{a}{||a||} = \frac{1}{\sqrt{2}}\begin{bmatrix}-1\\-1\\0\end{bmatrix}$$

$$e = b - (q_1^T b)q_1 = \begin{bmatrix}1\\0\\1\end{bmatrix} - \left(\frac{1}{\sqrt{2}}[-1 \quad -1 \quad 0]\begin{bmatrix}1\\0\\1\end{bmatrix}\right)\frac{1}{\sqrt{2}}\begin{bmatrix}-1\\-1\\0\end{bmatrix} = \begin{bmatrix}1\\0\\1\end{bmatrix} - \frac{1}{2}\begin{bmatrix}1\\1\\0\end{bmatrix} = \begin{bmatrix}1/2\\-1/2\\1\end{bmatrix}$$

$$q_2 = \frac{e}{||e||} = \frac{1}{\sqrt{6}}\begin{bmatrix}1\\-1\\2\end{bmatrix}$$

Check $q_1^T q_2 = 0$

$$e = c - (q_1^T c)q_1 - (q_2^T c)q_2 = \begin{bmatrix}1\\1\\1\end{bmatrix} - \left(\frac{1}{\sqrt{2}}[-1 \quad -1 \quad 0]\begin{bmatrix}1\\1\\1\end{bmatrix}\right)\frac{1}{\sqrt{2}}\begin{bmatrix}-1\\-1\\0\end{bmatrix} - \left(\frac{1}{\sqrt{6}}[1 \quad -1 \quad 2]\begin{bmatrix}1\\1\\1\end{bmatrix}\right)\frac{1}{\sqrt{6}}\begin{bmatrix}1\\-1\\2\end{bmatrix}$$

$$= \begin{bmatrix}1\\1\\1\end{bmatrix} - \begin{bmatrix}1\\1\\0\end{bmatrix} - \frac{1}{3}\begin{bmatrix}1\\-1\\2\end{bmatrix} = \frac{1}{3}\begin{bmatrix}-1\\1\\1\end{bmatrix}$$

$$q_3 = \frac{e}{||e||} = \frac{1}{\sqrt{3}}\begin{bmatrix}-1\\1\\1\end{bmatrix}$$

Check $q_1^T q_3 = 0$ and $q_2^T q_3 = 0$