A “presentation” on Hyper-Inverse Wishart Distributions”

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Gaussian Graphical Models

Let $\mathcal{G} := (V, E)$ be a graph consisting of

- vertices $V := v_1, \dots v_G$, and
- edges $E := e_1, \dots e_k$.

We can also represent $\mathcal{G}$ as an adjacency matrix $\mathcal{A} := (a_{ij})_{i,j=1}^{G}$.

In a Gaussian Graphical Model (GGM), each node v_i is then associated with a random vector $Y_i \in \mathbb{R}^n$ and a predictor vector $X_i \in \mathbb{R}^p$ and the assumption is made that

$$\mathbf{Y}_{n \times G} = \mathbf{X}_{n \times p} \mathbf{B}_{p \times G} + \mathbf{E}_{n \times G},$$

where

$$\mathbf{E} \sim \mathcal{MN}_{n \times G}(0, \mathbf{\Lambda}, \mathbf{\Sigma}),$$

the Matrix Normal Distribution with mean 0, row-wise correlation $\mathbf{\Lambda}$, and column-wise correlation $\mathbf{\Sigma}$, where, for $i \neq j$, $a_{ij} = 0 \implies (\mathbf{\Sigma})_{ij} = 0$.

Hyper-Inverse Wishart

Let $M(\mathcal{G})$ denote the set of positive-definite symmetric matrices with elements equal to zeros for all $(i, j) \notin E$.