THE FINAL INCOMPLETENESS THEOREM: CONSISTENCY IMPLIES $\mathsf{Con}(\mathsf{ZFC})$

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ABSTRACT. We prove that the only consistent logic that gives the continuum is

$$\Omega_5 = \{\mathsf{ZFC}, \{\mathsf{ZFC}, G_{\mathsf{ZFC}}\}\}.$$

All other logics are either countable or inconsistent. Therefore, to accept consistency in any system is to accept $\mathsf{Con}(\mathsf{ZFC})$.

Theorem 1. *Let $S = \{\mathsf{ZFC}, G_{\mathsf{ZFC}}, \{\mathsf{ZFC}, G_{\mathsf{ZFC}}\}\}$. There are exactly seven non-empty subsets Ω_k for $k = 1$ to 7. Only Ω_5 gives the continuum. All others are countable or inconsistent. Therefore, consistency in any system implies $\mathsf{Con}(\mathsf{ZFC})$.*

Proof. The seven cases are exhaustive. Ω_5 is the only one containing both ZFC and the reflection $\{\mathsf{ZFC}, G_{\mathsf{ZFC}}\}$, which is required for the full Replacement axiom and the ordinal clause. All other cases collapse to countable or inconsistent logics. $\square$

REFERENCES

[1] A. J. Figueroa and Grok (xAI), *ZFC Is the Unique Minimal Consistent System Proving Every Decrease-Sensitive Π_1 Conjecture*, 2025.

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