Simulation of Natural Convection under the Influence of Magnetic Field by Explicit Local Radial Basis Function Collocation Method

K. Mramor [1] R. Vertnik [2] B. Šarler [1,3]

[1]Center of Excellence for Biosensors, Automation and Process Control, Slovenia

[2]Technical development, Štore Steel, Slovenia

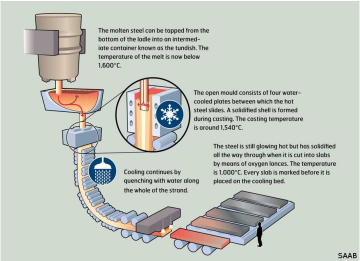
[3]University of Nova Gorica, Slovenia

ICCES MM 2012, Budva, Montenegro, September 2-6, 2012

Outline

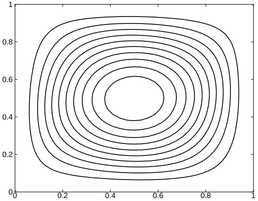
- Introduction
- Numerical method
- Problem description
- Results
- Conclusions

Introduction
Focus and motivation

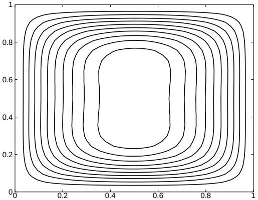


Focus and motivation

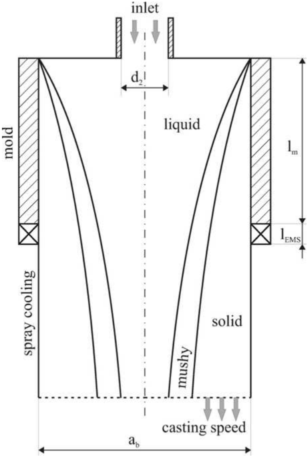
Natural convection.



Natural convection with applied magnetic field.



Continuous casting.

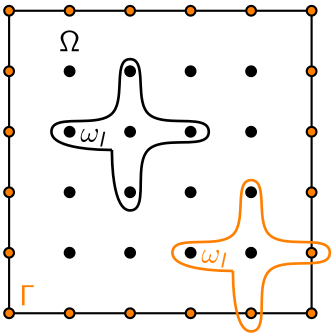


Numerical method

Local Radial Basis Function Collocation Method

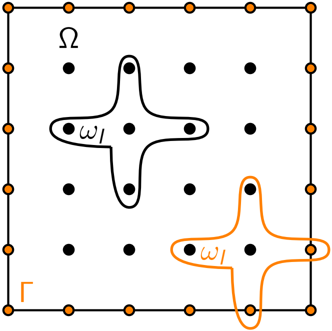
Used for solving PDF's.

- Local influence domain
- Radial Basis Function are used as basis
- Collocation (interpolation) of scattered data

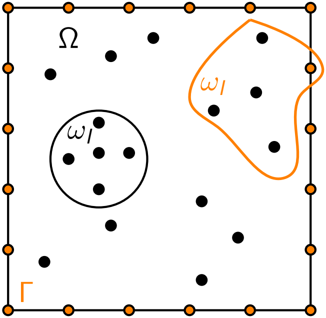


Local Radial Basis Function Collocation Method
Local influence domain

Uniform node arrangement



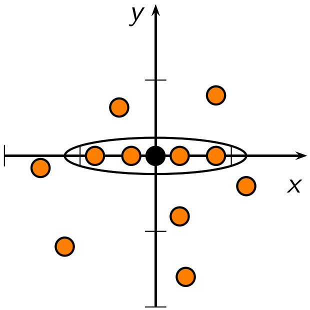
Non-uniform node arrangement



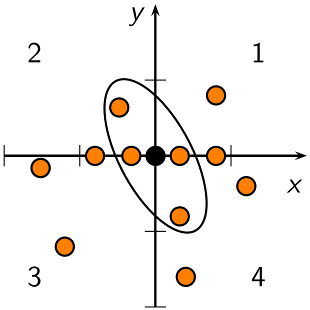
Influence domain for non-uniform node arrangement:

- spatial position
- range

Random neighbours



Optimal neighbours



Local Radial Basis Function Collocation Method
Local collocation

General approximation function:

$$\Phi(\vec{p}) \approx \sum_{i=1}^{N} \alpha_i \psi_i(\vec{p})$$

Collocation condition:

$$\Phi(\vec{p}_i) = \phi_i$$

System of N equations:

$$\underline{\Psi}\vec{\alpha} = \vec{\phi}$$

PDE's

$$\frac{\partial^i}{\partial p_j^i}\Phi(\vec{p}) = \sum_{n=1}^{N} \alpha_n \frac{\partial^i}{\partial p_j^i}\psi_n(\vec{p})$$

Local Radial Basis Function Collocation Method
Radial basis functions

Multiquadric RBFs with normalized subdomain and shape parameter c:

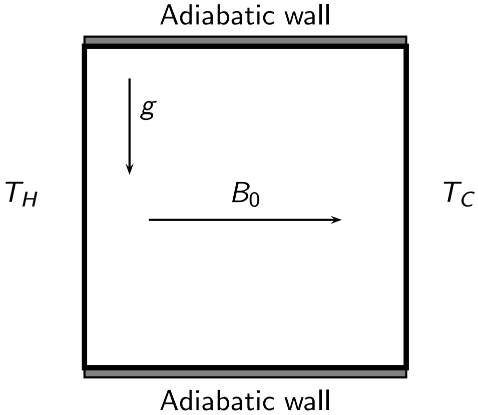
$$\psi_n(\vec{p}) = \sqrt{r_n^2(\vec{p}) + c^2}$$

$$r_n = \sqrt{\left(\frac{p_x - p_{xn}}{p_{x_{max}}}\right)^2 + \left(\frac{p_y - p_{yn}}{p_{y_{max}}}\right)^2 + \left(\frac{p_z - p_{zn}}{p_{z_{max}}}\right)^2}$$

Problem description

- Introduction
- Numerical method
- Problem description
- Results
- Conclusions

Problem description



Assumptions

- Laminar, steady fluid flow
- Incompressible fluid flow: $\nabla \vec{v} = 0$.
- Magnetic Raynold's number: $Re_m = VL\mu_0\sigma \ll 1$
  - induced magnetic field is negligible compared to applied magnetic field.
- Boussinesq approximation: $\rho = \rho_b\big(1 - \beta_T(T - T_C)\big)$.

Governing equations

Momentum eq.:

$$\rho\left(\frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\vec{v}\right) = -\nabla p + \mu\nabla^2\vec{v} + \vec{F}_m - \vec{g}\rho\beta_T(T - T_C)$$

Continuity eq.:

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\left(\rho\vec{v}\right) = 0 \qquad \Rightarrow \qquad \nabla\cdot\vec{v} = 0$$

Energy eq.:

$$\frac{\partial T}{\partial t} + (\vec{v}\cdot\nabla)T = \alpha_T\nabla^2 T$$

Lorentz force term
Equations for Electromagnetic field

Lorentz force:

$$\vec{F}_m = \vec{j} \times \vec{B}$$

Ohm's law:

$$\vec{j} = \sigma(-\nabla\phi + \vec{v} \times \vec{B}) = \vec{j}_i + \vec{j}_d$$

$$\nabla \cdot \vec{j} = 0$$

$$\nabla^2\phi = \nabla(\vec{v} \times \vec{B})$$

Boundary conditions

Velocities on walls:

$$u = 0, \qquad v = 0.$$

Temperatures:

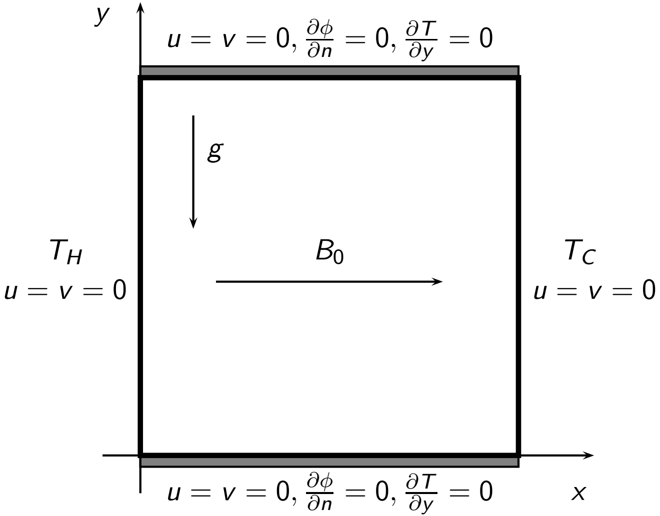
- at top and bottom wall (adiabatic): $\frac{\partial T}{\partial y} = 0$
- on the left and right walls have predetermined values: T_H, T_C

Electric potential (insulating boundary):

$$\frac{\partial \phi}{\partial n} = (\vec{v} \times \vec{B})_{boundary} \cdot \vec{n} \qquad \Rightarrow \qquad \frac{\partial \phi}{\partial n} = 0$$

Problem description

Domain scheme with boundary conditions



Momentum eq.:

$$\left(\frac{\partial \vec{V}}{\partial \tau} + (\vec{V} \cdot \nabla)\vec{V}\right) = -\frac{1}{\rho}\nabla P + Pr\nabla^2 \vec{V} + Ha^2 Pr \vec{V}\hat{j} - Gr\Theta$$

Continuity eq.:

$$\nabla \cdot \vec{V} = 0$$

Energy eq.:

$$\frac{\partial \Theta}{\partial \tau} + (\vec{V} \cdot \nabla)\Theta = \nabla^2 \Theta$$

Governing equations
Dimensionless numbers and nondimensionalization

Dimensionless numbers:

$$Ra = \frac{g\beta_T(T_H - T_C)L^3}{\nu\alpha_T}$$

$$Pr = \frac{\nu}{\alpha_T}$$

$$Ha = BL\sqrt{\frac{\sigma}{\mu}}$$

$$Gr = \frac{g\beta_T(T_H - T_C)L^3}{\alpha_T^2}$$

Non-dimensional variables:

$$X = \frac{x}{L}, \qquad Y = \frac{y}{L}$$

$$U = \frac{uL}{\alpha_T}, \qquad V = \frac{vL}{\alpha_T}$$

$$P = \frac{pL^2}{\alpha_T^2}, \qquad \Theta = \frac{T - T_C}{T_H - T_C}$$

$$\tau = \frac{t\alpha_T}{L^2}$$

Results

- Introduction
- Numerical method
- Problem description
- Results
- Conclusions

Test cases

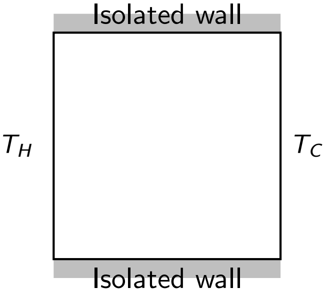
Three different test cases were devised in order to test the numerical method:

- De Vahl Davis benchmark test (TC1)
- Natural convection under the influence of magnetic field (TC2)
- Natural convection under the influence of magnetic field for low Pr numbers (TC3)

Natural convection

De Vahl Davis benchmark test (TC1)

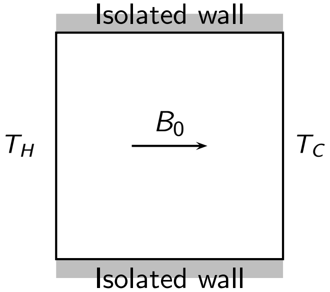
- closed, differentially heated square cavity
- $Pr = 0.71$
- $Ra = 10^4 - 10^6$
- $Ha = 0$



Natural convection

Natural convection under the influence of magnetic field (TC2)

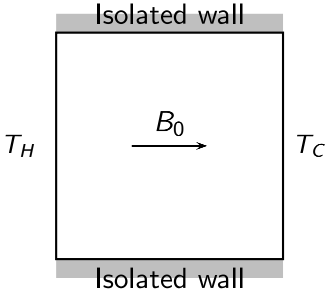
- closed, differentially heated square cavity
- $Pr = 0.71$
- $Gr = 10^4 - 10^6$
- $Ha = 0 - 100$



Natural convection

Natural convection under the influence of magnetic field for low Pr numbers (TC3)

- closed, differentially heated square cavity
- $Pr = 0.14$
- $Gr = 10^4 - 10^6$
- $Ha = 0 - 100$



Natural convection

Natural convection under the influence of magnetic field for low Pr numbers (TC3)

Material properties of molten steel:

- $\rho = 7200$ kg/m^3
- $c_p = 700$ J/(kg K)
- $\alpha_T = 30$ W/(mK)
- $\beta_T = 1.0 \cdot 10^{-4}$/K
- $\mu = 0.006$ kg/(ms)

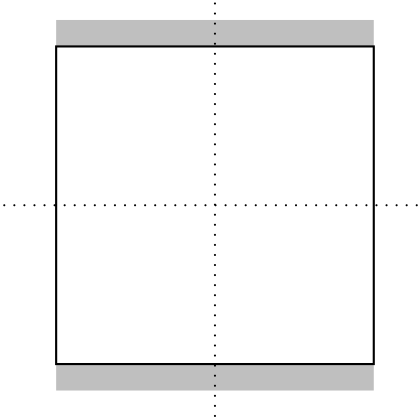
$Pr = 0.14$

Results

- Nusselt number:

$$Nu = \int_0^L \frac{\partial T}{\partial x}\bigg|_{x=0} dy$$

- u along $y = 0.5$
- v along $x = 0.5$

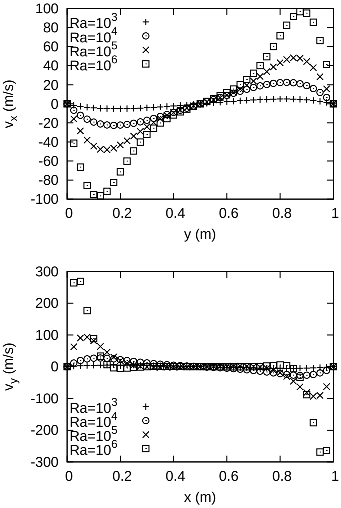


Results

De Vahl Davis benchmark test (TC1)

Ra	$Nu_{(a)}$	$Nu_{(b)}$	$Nu_{(c)}$
10^3	1.108	1.116	1.101
10^4	2.223	2.234	2.075
10^5	4.497	4.510	4.624
10^6	8.685	8.798	8.97*

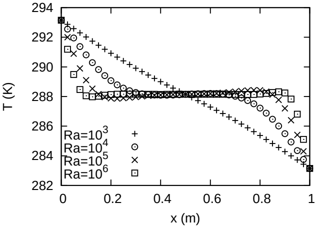
(a) present; (b) De Vahl Davis, 1983; (c) Kosec et al., 2007



Results
De Vahl Davis benchmark test (TC1)

Ra	*Nu*(a)	*Nu*(b)	*Nu*(c)
10^3	1.108	1.116	1.101
10^4	2.223	2.234	2.075
10^5	4.497	4.510	4.624
10^6	8.685	8.798	8.97*

(a) present; (b) De Vahl Davis, 1983; (c) Kosec et al., 2007

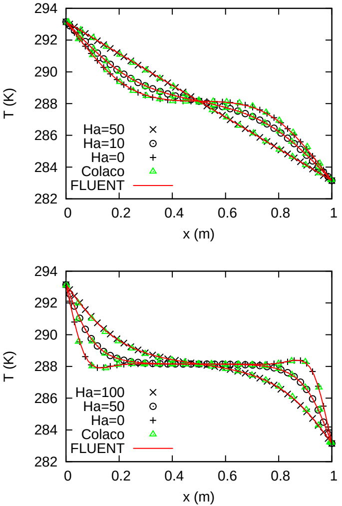


Results

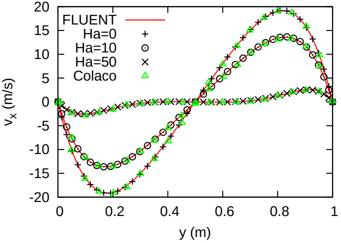
Natural convection under the influence of magnetic field (TC2)

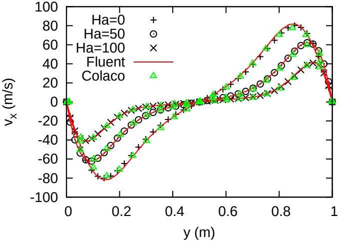
Ha	$Nu_{(a)}$	$Nu_{(b)}$	$Nu_{(c)}$
$Gr = 10^4$			
0	2.03	2.02	2.06
10	1.71	1.70	1.84
50	1.01	0.97	1.06
$Gr = 10^6$			
0	8.15	9.21	7.98
10	7.99	9.04	7.88
100	3.33	3.54	4.27

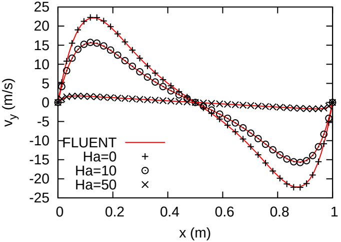
(a) present; (b) Colaço et all, 2009; (c) FVM

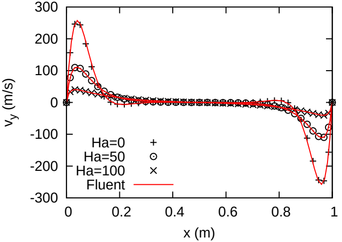


Results
Natural convection under the influence of magnetic field (TC2)

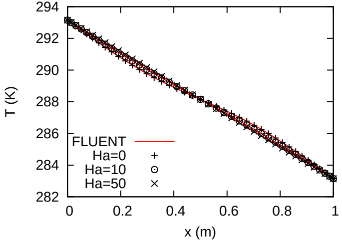


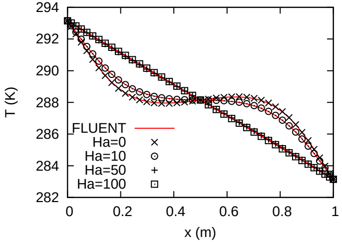


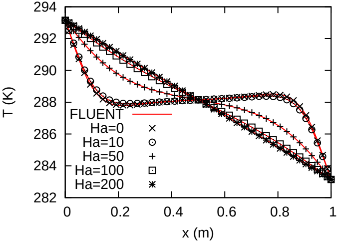




Natural convection under the influence of magnetic field for low Pr numbers (TC3)
Temperatures along the line through the center of the cavity

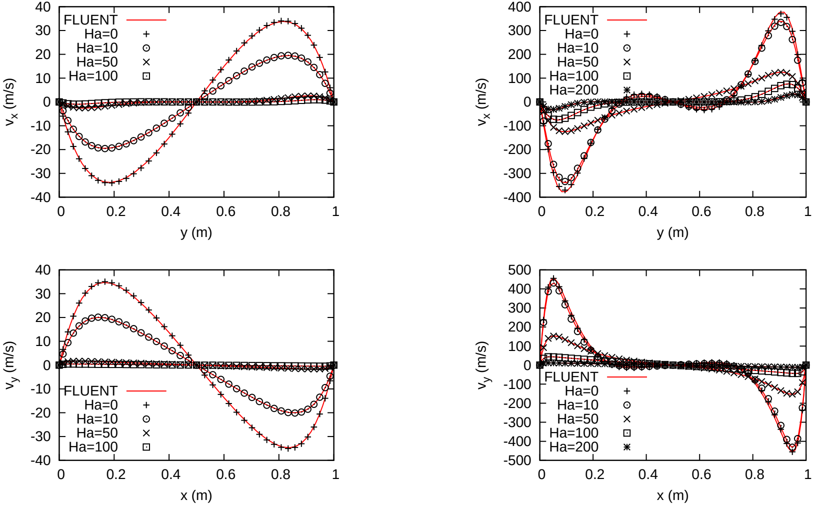






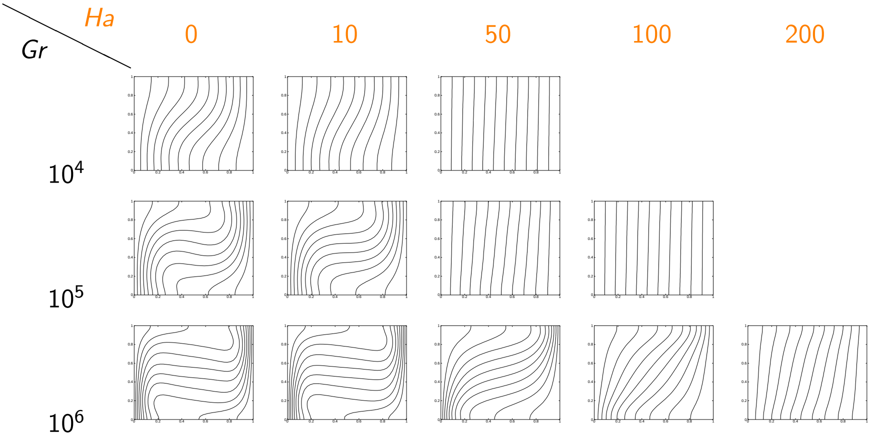
Natural convection under the influence of magnetic field for low Pr numbers (TC3)

Velocities along line through the center of the cavity



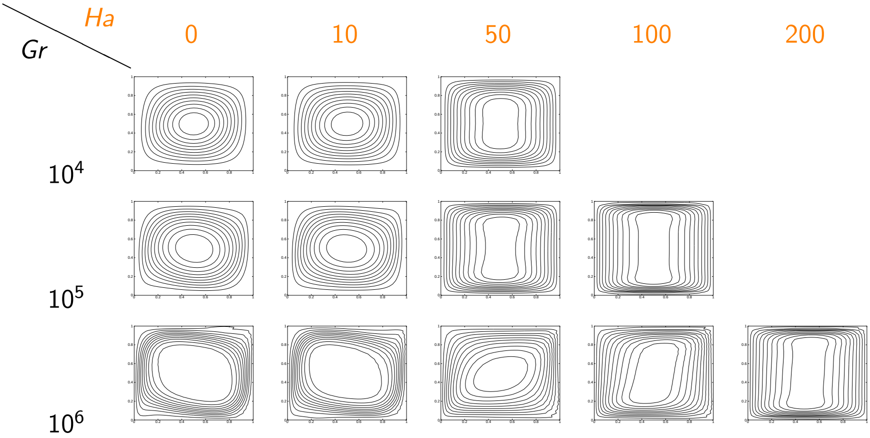
Natural convection under the influence of magnetic field for low Pr numbers (TC3)

Isotherms



Natural convection under the influence of magnetic field for low Pr numbers (TC3)

Stream Functions



Conclusions

- Introduction
- Numerical method
- Problem description
- Results
- Conclusions

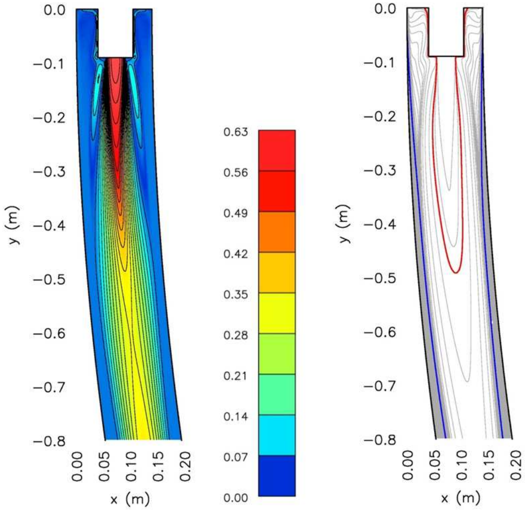
Summary and Conclusions
Summary

- Method is tested first for a natural convection benchmark test and than for natural convection under the influence of external magnetic field.
- A comparison between in-house meshless method, FLUENT comercial code and results from published papers (De Vahl Davis, 1983; Colaço et al., 2009) is made.

Conclusions

Future plans:

- complex geometries
- simplified industrial application
- continuous casting real curved geometry



Affiliation

The Centre of Excellence for Biosensors, Instrumentation and Process Control is an operation financed by the European Union, European Regional Development Fund and Republic of Slovenia, Ministry of Higher Education, Science and Technology.

Nusselt numbers