Lecture 21: Problem Solving with Recursion

CS 0445: Data Structures

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http://db.cs.pitt.edu/courses/cs0445/current.term/

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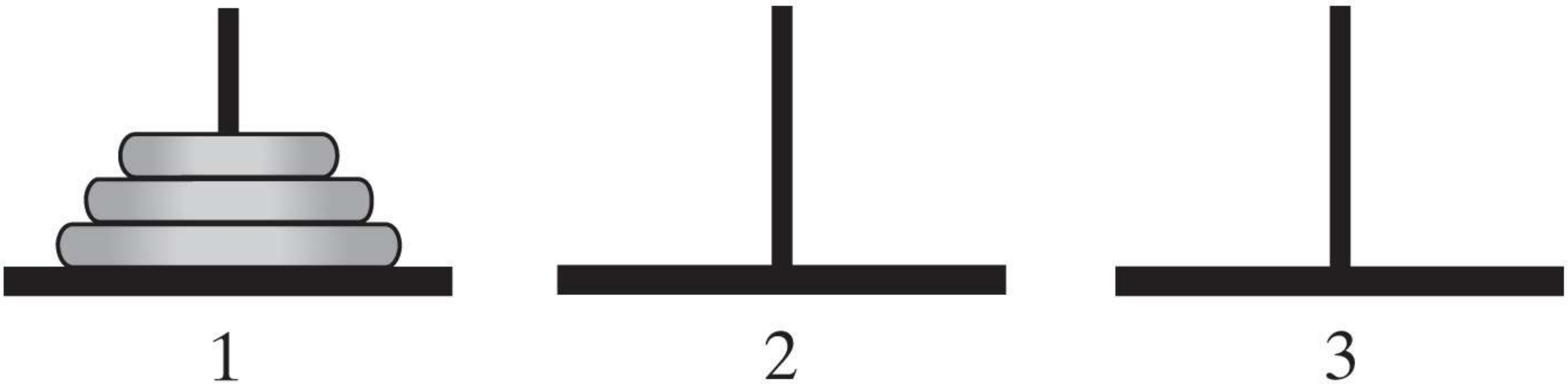
University of Pittsburgh, Pittsburgh, PA

Simple Solution to a Difficult Problem

- The initial configuration of the Towers of Hanoi for three disks



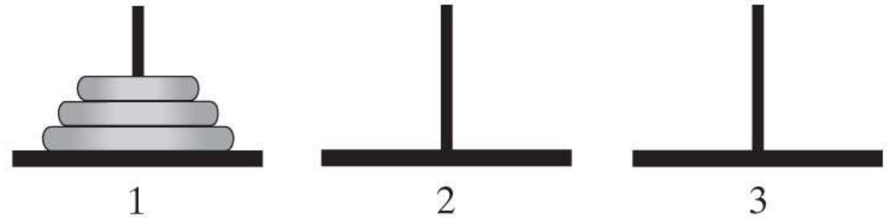
Simple Solution to a Difficult Problem

- Rules:
  - Move one disk at a time. Each disk moved must be the topmost disk.
  - No disk may rest on top of a disk smaller than itself.
  - You can store disks on the second (extra) pole temporarily, as long as you observe the previous two rules.

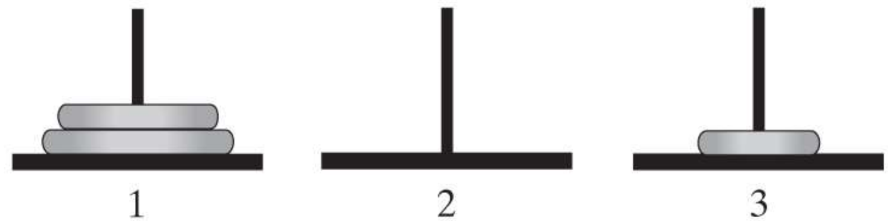
Simple Solution to a Difficult Problem (Part 1)

- Sequence of moves for solving Towers of Hanoi problem with 3 disks

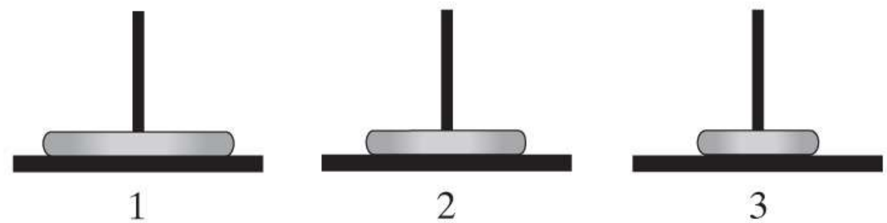
(a) The beginning configuration



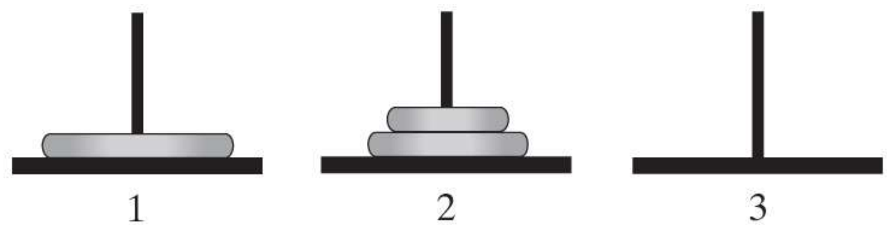
(b) After moving a disk from pole 1 to pole 3



(c) After moving a disk from pole 1 to pole 2



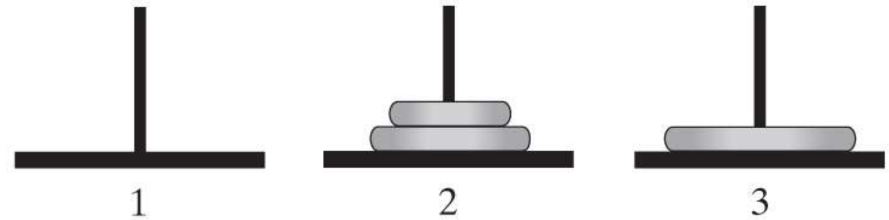
(d) After moving a disk from pole 3 to pole 2



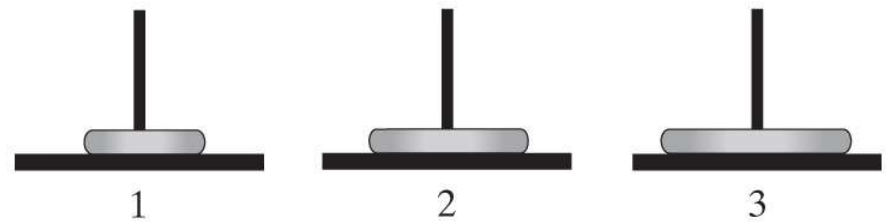
Simple Solution to a Difficult Problem (Part 2)

- Sequence of moves for solving Towers of Hanoi problem with 3 disks

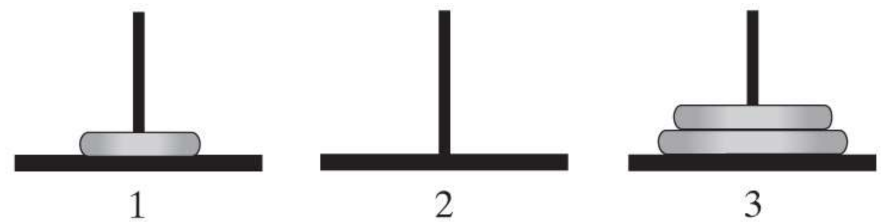
(e) After moving a disk from pole 1 to pole 3



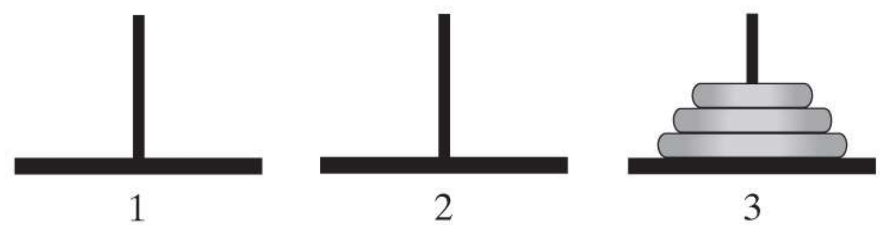
(f) After moving a disk from pole 2 to pole 1



(g) After moving a disk from pole 2 to pole 3

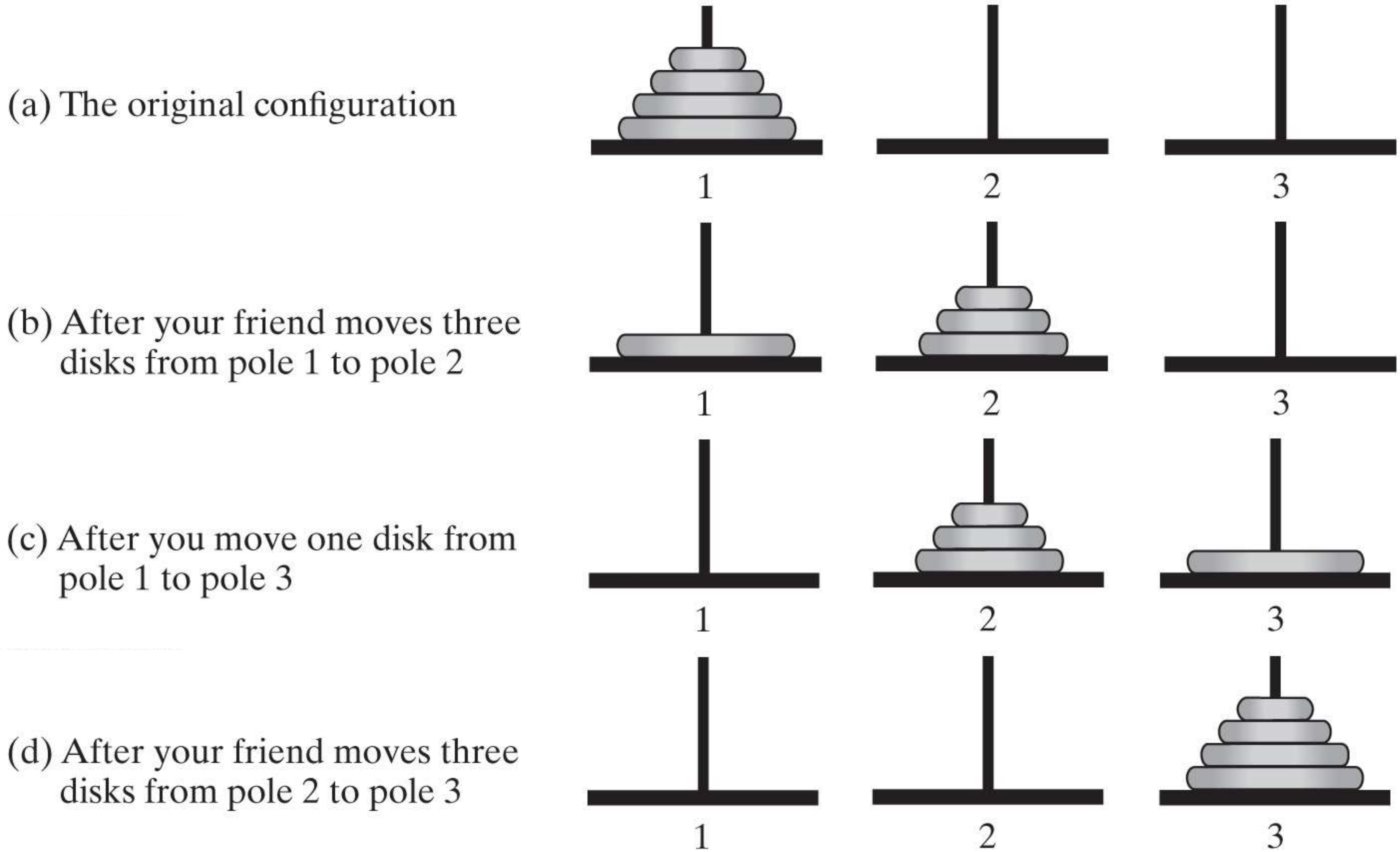


(h) After moving a disk from pole 1 to pole 3



A Smaller Problem

- The smaller problems in a recursive solution for four disks



Solutions

- Recursive algorithm to solve any number of disks.
  Note: for n disks, solution will be $2^n - 1$ moves

Algorithm to move **numberOfDisks** *disks from* **startPole** *to* **endPole** *using* **tempPole** *as a spare according to the rules of the Towers of Hanoi problem*

if (numberOfDisks == 1)

Move disk from **startPole** *to* **endPole**

else

{

Move all but the bottom disk from **startPole** *to* **tempPole**

Move disk from **startPole** *to* **endPole**

Move all disks from **tempPole** *to* **endPole**

}

Solution

```java
// Java recursive program to solve tower of hanoi puzzle
class HanoiPuzzle {

    static String toColor(int i) {
        switch (i) {...}
    }

    // Java recursive function to solve tower of hanoi puzzle
  static void towerOfHanoi(int n, char startPole, char endPole, char tempPole) {
   if (n == 1) {
      System.out.println("Move disk (1) from pole " + startPole + " to pole " + endPole);
      return;
   }
   towerOfHanoi(n - 1, startPole, tempPole, endPole);
   System.out.println("Move disk (" + n + ") from pole " + startPole + " to pole " + endPole);
   towerOfHanoi(n - 1, tempPole, endPole, startPole);
  }

    // Driver method
    public static void main(String args[]) {
        int n = 4; // Number of disks
        towerOfHanoi(n, 'A', 'C', 'B'); // A, B and C are names of poles
    }
}
```

Poor Solution to a Simple Problem

- Algorithm to generate Fibonacci numbers.
- Why is this inefficient?

$$F_0 = 1$$
$$F_1 = 1$$
$$F_n = F_{n-1} + F_{n-2} \text{ when } n \geq 2$$

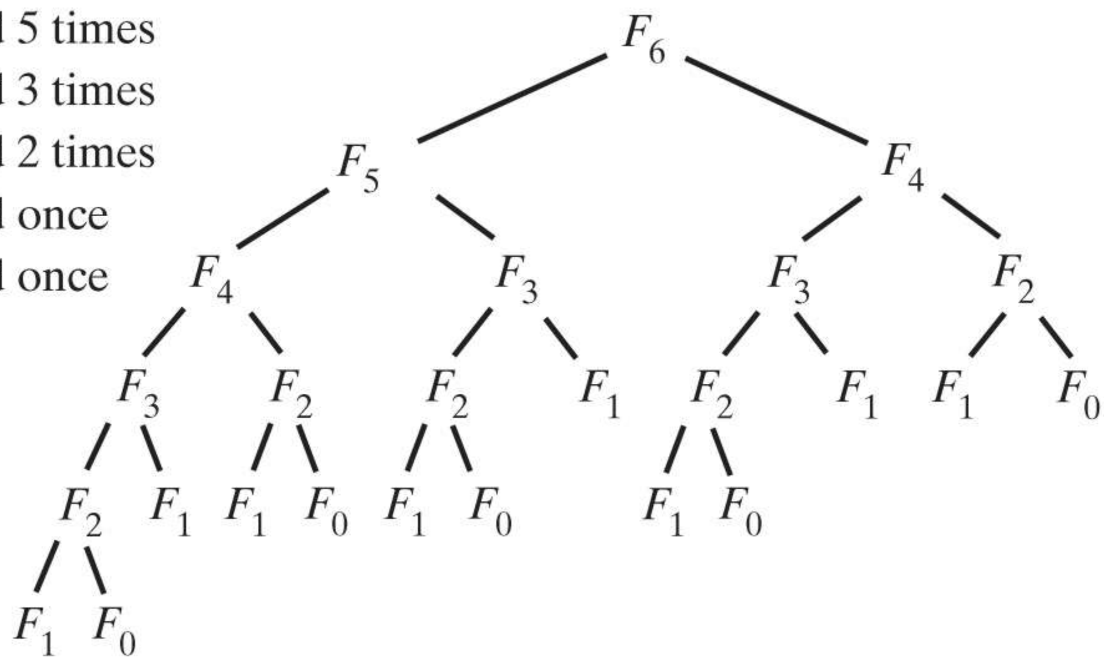
```
Algorithm Fibonacci(n) if (n <= 1)
  return 1
else
  return Fibonacci(n − 1) + Fibonacci(n − 2)
```

Poor Solution to a Simple Problem

- The computation of the Fibonacci number F6

(a) Recursively

F_2 is computed 5 times
F_3 is computed 3 times
F_4 is computed 2 times
F_5 is computed once
F_6 is computed once



Poor Solution to a Simple Problem

- The computation of the Fibonacci number F6

(a) Recursively

F_2 is computed 5 times
F_3 is computed 3 times
F_4 is computed 2 times
F_5 is computed once
F_6 is computed once

(b) Iteratively

$$\begin{aligned}
F_0 &= 1 \\
F_1 &= 1 \\
F_2 &= F_1 + F_0 = 2 \\
F_3 &= F_2 + F_1 = 3 \\
F_4 &= F_3 + F_2 = 5 \\
F_5 &= F_4 + F_3 = 8 \\
F_6 &= F_5 + F_4 = 13
\end{aligned}$$

Indirect Recursion

- Example
  - Method A calls Method B
  - Method B calls Method C
  - Method C calls Method A

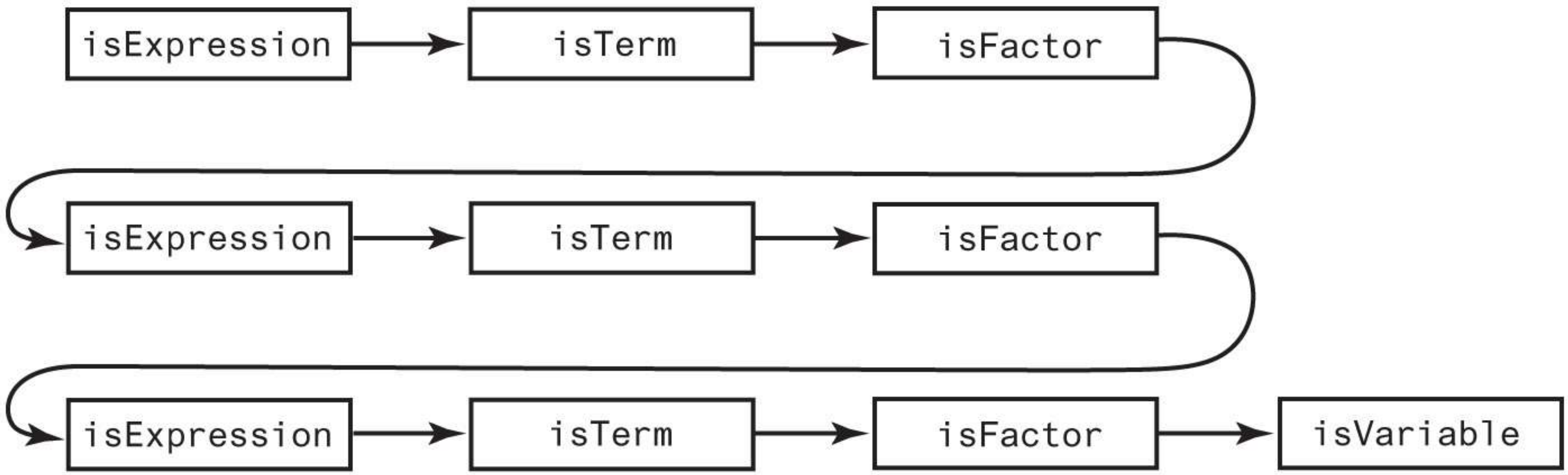
- Difficult to understand and trace
  - But does happen occasionally

Indirect Recursion

- Consider evaluation of validity of an algebraic expression
  - Algebraic expression is either a term or two terms separated by a + or – operator
  - Term is either a factor or two factors separated by a * or / operator
  - Factor is either a variable or an algebraic expression enclosed in parentheses
  - Variable is a single letter

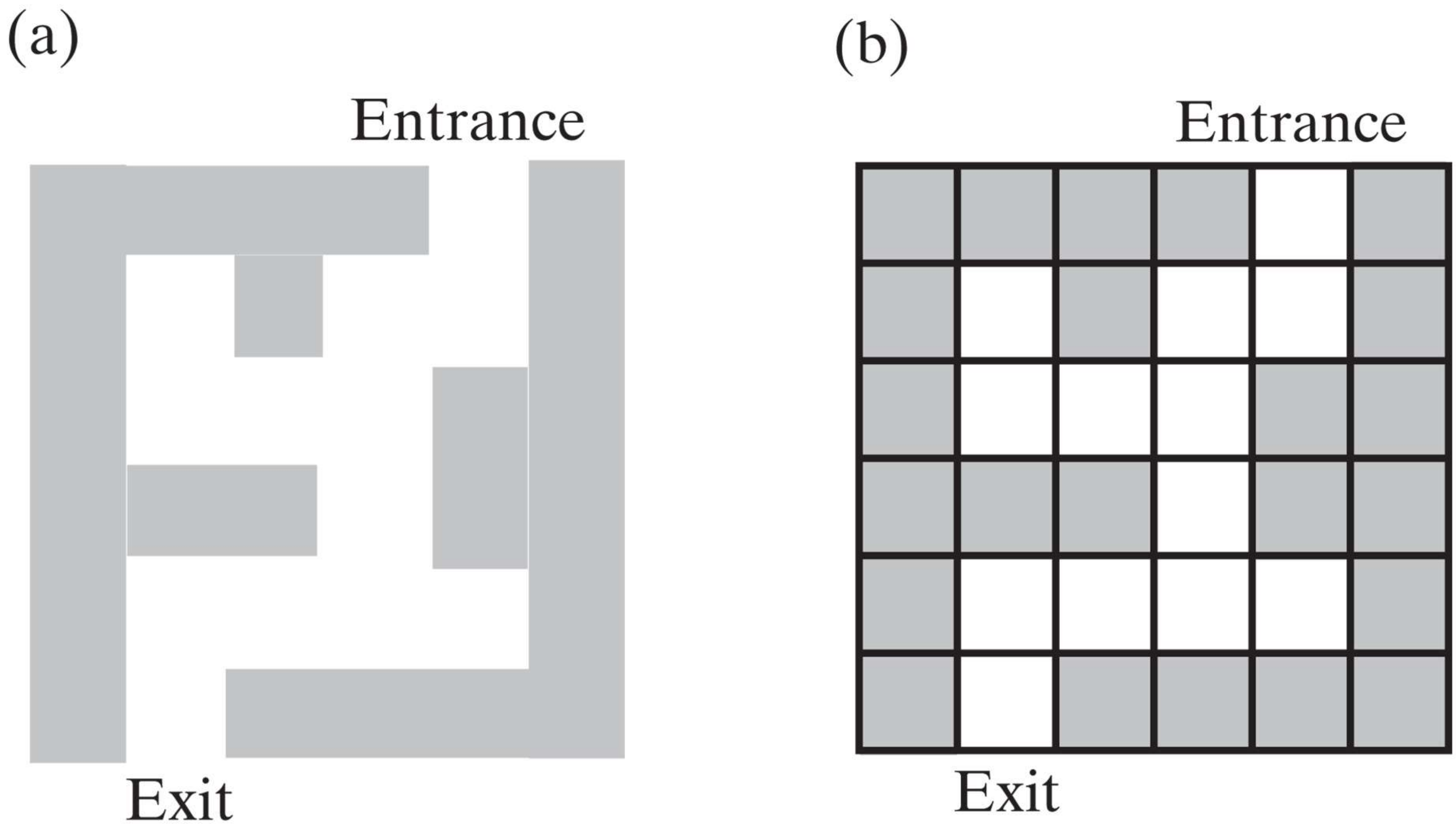
Indirect Recursion

- An example of indirect recursion



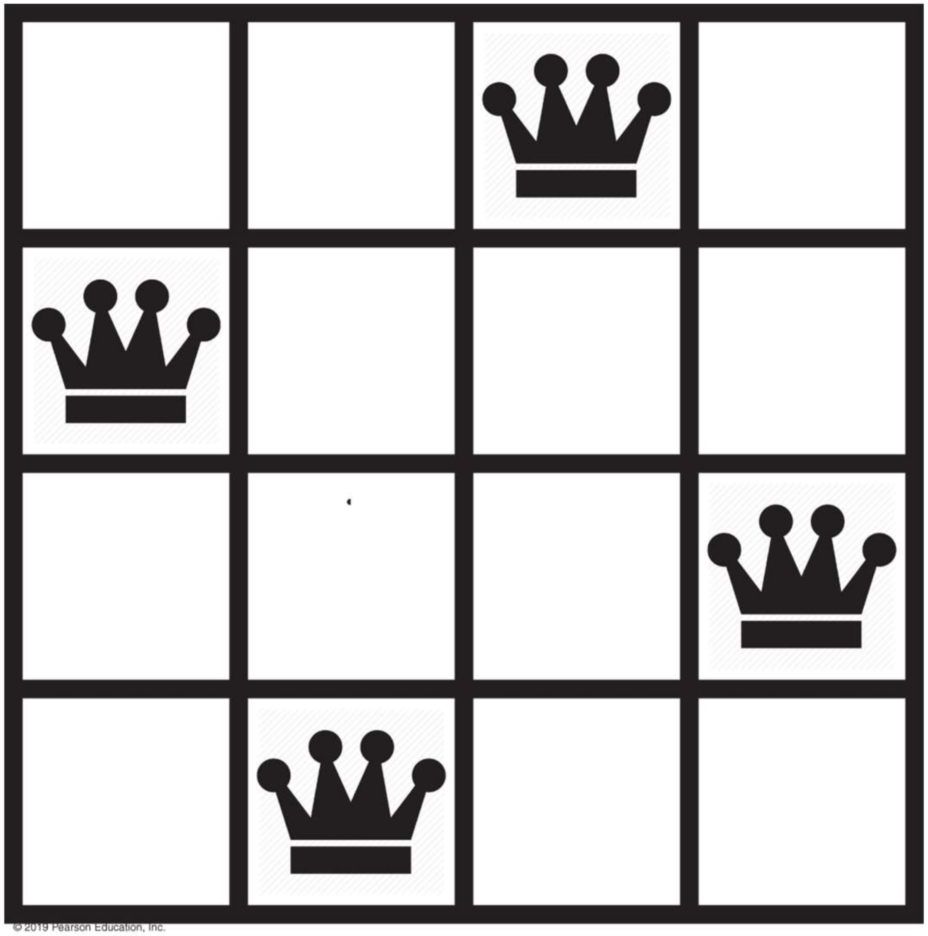
Backtracking

- A two-dimensional maze with one entrance and one exit



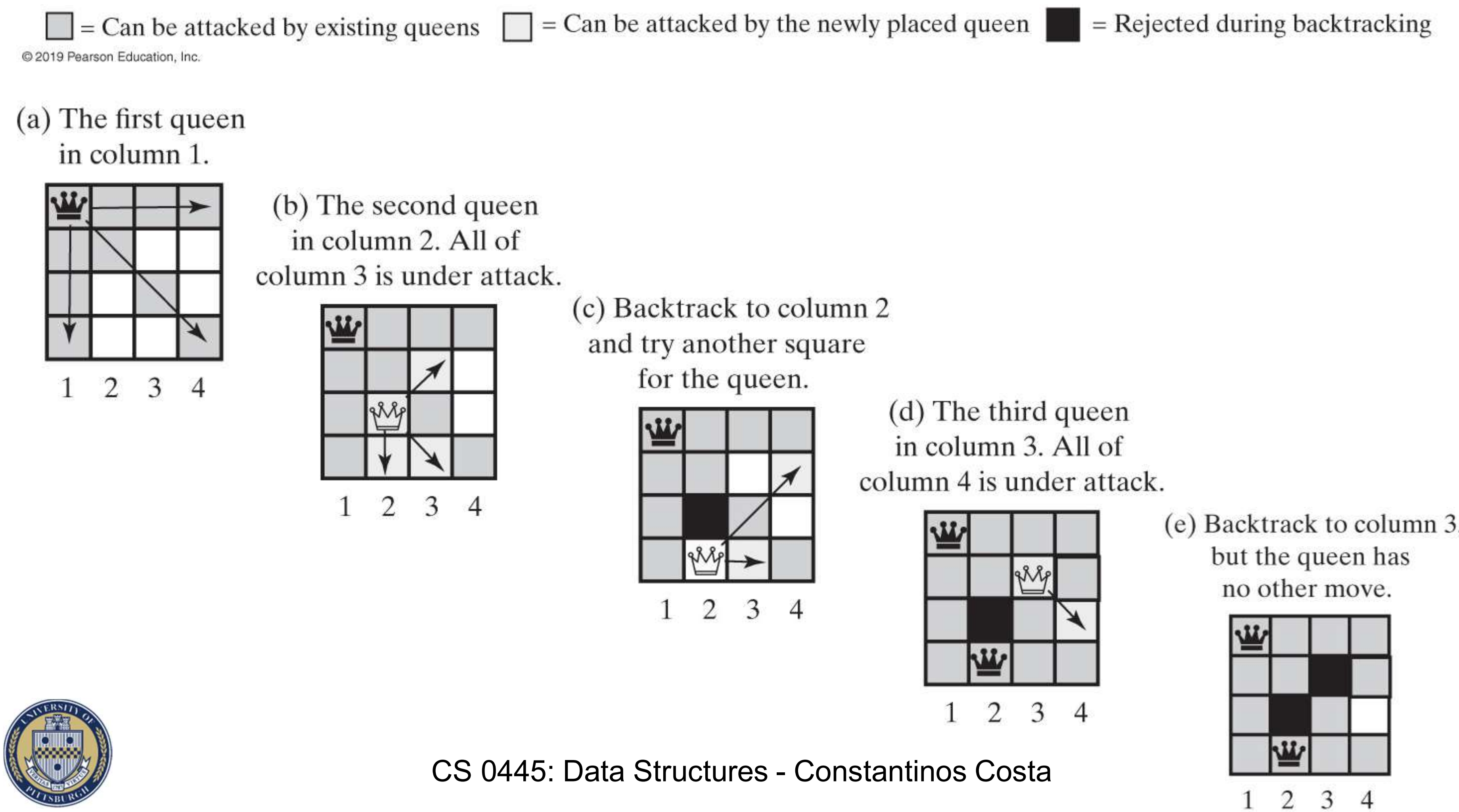
Backtracking

- A solution to the four-queens problem



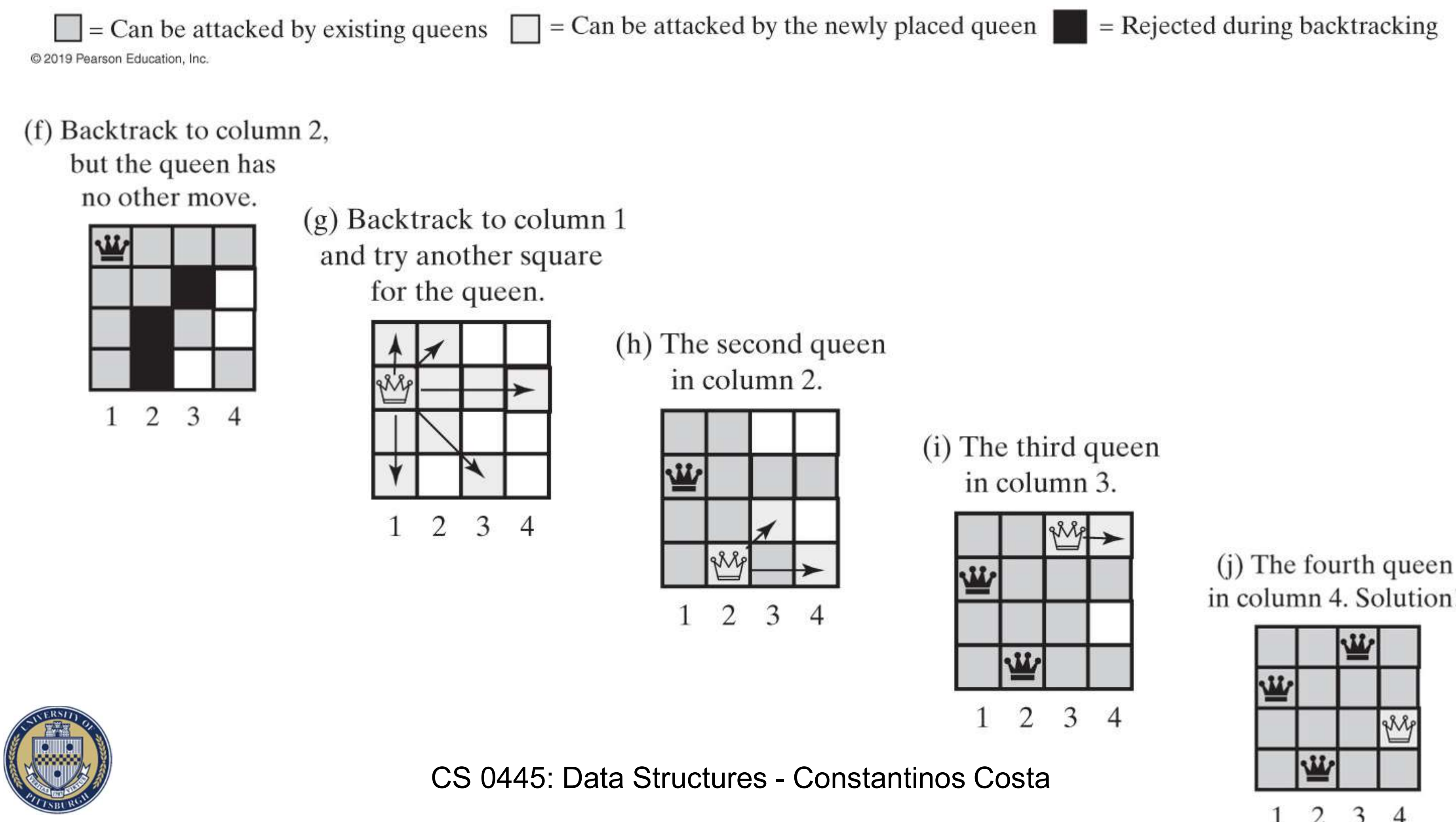
Backtracking - Queens Solution (Part 1)

- Solving the four-queens problem by placing one queen at a time in each column



Backtracking - Queens Solution (Part 2)

- Solving the four-queens problem by placing one queen at a time in each column



Happy **Halloween**!