

Discrete Structures for Computer Science

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Lecture #3: Logic Puzzles and Propositional
Equivalence





Remember...

Slides on the web are **incomplete!**

- Provided for those who want to follow along
- Note that simply filling in the blanks isn't good enough

Please ask questions!

- If I'm talking too fast, let me know
- If you get confused, ask for clarification—some of your classmates probably have the same question!
- I can't read minds 😊



Today's topics

- Logic puzzles
- Propositional equivalences



A technical support conundrum

Alice and Bob are technical support agents. If an agent is having a bad day, he or she will always lie to you. If an agent is having a good day, he or she will always tell you the truth. Alice tells you that Bob is having a bad day. Bob tells you that he and Alice are both having the same type of day. Can you trust the advice you receive from Alice during your call?

**How do we solve this
type of problem?**



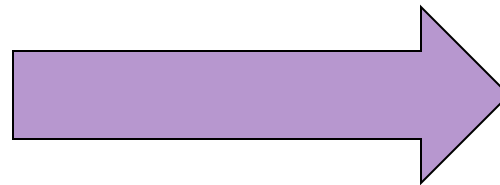
Solving logic puzzles is easy!



**Step 1: Identify rules
and constraints**



**Step 2: Assign propositions
to key concepts**



**Step 3: Make assumptions
and reason logically!**



Technical support revisited

Alice and Bob are technical support agents. If an agent is having a bad day, he or she will always lie to you. If an agent is having a good day, he or she will always tell you the truth. Alice tells you that Bob is having a bad day. Bob tells you that he and Alice are both having the same type of day. Can you trust the advice you receive from Alice during your call?

Step 1: Identify the rules of the puzzle

Step 2: Assign propositions to the key concepts in the puzzle

Step 3: Make assumptions and reason logically





Another example

Consider a group of friends: Frank, Anna, and Chris. If Frank is not the oldest, then Anna is. If Anna is not the youngest, then Chris is the oldest. Determine the relative ages of Frank, Anna, and Chris.

Propositions:

Rules:

Step 3: Make assumptions and reason logically





Sometimes no solution is a solution!

Alice and Bob are technical support agents. Alice says “I am having a good day.” Bob says “I am having a good day.” Can you trust either Alice or Bob?

Step 1: Identify rules

Step 2: Assign propositions

Step 3: Make assumptions and reason logically





In-class exercises

Problem 1: Alice and Bob are technical support agents working to fix your computer. Alice tells you that Bob is having a bad day today and that you should expect a long wait before your computer is fixed. Bob tells you not to worry, Alice is just having a bad day—your computer will be ready in no time.

Question: Can you draw any conclusions about when your computer will be fixed? If so, what can you learn?

Propositional equivalences: preliminaries



Definition: A **tautology** is a compound proposition that is always **true**, regardless of the truth values of the propositions occurring within it.

Definition: A **contradiction** is a compound proposition that is always **false**, regardless of the truth values of the propositions occurring within it.

Definition: A **contingency** is a compound proposition whose truth value is dependent on the propositions occurring within it.



Examples

Are the following compound propositions tautologies, contradictions, or contingencies?

■ $p \vee \neg p$ tautology

■ $\neg p \wedge p$ contradiction

■ $p \vee q$ contingency

What are logical equivalences and why are they useful?



Definition: Compound propositions p and q are **logically equivalent** if $p \leftrightarrow q$ is a tautology. The notation $p \equiv q$ means that p and q are logically equivalent.

Logical equivalences are extremely useful!

- Aid in the construction of proofs
- Allow us to simplify compound propositions

Example: $p \rightarrow q \equiv \neg p \vee q$

*How do we
prove this type
of statement?*

It is easy to prove propositional equivalences



We can prove simple logical equivalences using our good friend the truth table!

Prove: $p \rightarrow q \equiv \neg p \vee q$

p	q	$\neg p$	$\neg p \vee q$	$p \rightarrow q$
T	T			
T	F			
F	T			
F	F			

DeMorgan's laws allow us to distribute negation over compound propositions



Two laws:

- $\neg(p \vee q) \equiv \neg p \wedge \neg q$

- $\neg(p \wedge q) \equiv \neg p \vee \neg q$

If "p or q" isn't true, then neither p nor q is true

If "p and q" isn't true, then at least one of p or q is false

Prove: $\neg(p \vee q) \equiv \neg p \wedge \neg q$

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T					
T	F					
F	T					
F	F					



Using DeMorgan's laws

Use DeMorgan's laws to negate the following expressions:

- “Bob is wearing blue pants and a sweatshirt”
 - $b \wedge s$
 - $\neg(b \wedge s) \equiv \neg b \vee \neg s$
 - Bob is not wearing blue pants or is not wearing a sweatshirt

- “I will drive or I will walk”
 - $d \vee w$
 - $\neg(d \vee w) \equiv \neg d \wedge \neg w$
 - I will not drive and I will not walk



In-class exercises

Problem 2: Prove that $\neg(p \wedge q)$ and $\neg p \vee \neg q$ are logically equivalent, i.e., $\neg(p \wedge q) \equiv \neg p \vee \neg q$. This is the second DeMorgan's law.

Problem 3: Use DeMorgan's laws to negate the following propositions:

- Today I will go running or ride my bike
- Tom likes both pizza and beer



Sometimes using truth tables to prove logical equivalencies can become cumbersome

Recall that for an equivalence with n propositions, we need to build a truth table with 2^n rows

- Fine for tables with $n = 2, 3$, or 4
- Consider $n = 30$ —we would need 1,073,741,824 rows in the truth table!

Another option: Direct manipulation of compound propositions using known logical equivalencies





There are many useful logical equivalences

Equivalence	Name



More useful logical equivalences

Equivalence	Name

More equivalencies in the book!

Prove that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology



Prove: $(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$





Final Thoughts

- Logic can help us solve real world problems and play challenging games
- Logical equivalences help us simplify complex propositions and construct proofs
 - More on proofs later in the course
- Next:
 - Predicate logic and quantification
 - Please read section 1.4