Notes for Week of 1/28/19 - 2/1/18

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(WARNING: NOTATION IS NOT WELL-DEFINED. NO GUARANTEE OF CORRECTNESS. THESE ARE INTENDED ONLY AS RECORDS. ALL RELEVANT INFORMATION SHOULD BE TRANSFERRED TO A BETTER-WRITTEN DOCUMENT.)

We work with distributions $\mathbb{P}$ and $\mathbb{Q}$ with support over $\mathcal{D} \subset \mathbb{R}$. Recall that our *Laplacian smooth* test statistic can be written as

$$T_2(\boldsymbol{\ell}; G_{n,r}) = \sup_{\mathbf{f} \in \mathbb{R}^n : \|\mathbf{B}\mathbf{f}\|_2^2 \leq C_{n,r}} \frac{1}{n} \sum_{k=1}^{n} \ell_k f_k$$

for $G_{n,r}$ the neighborhood graph of radius r over data,, and $\mathbf{B} \in \mathbb{R}^{m \times n}$ corresponding incidence matrix.

Consider the set of functions

$$\mathcal{H}^{1,2}(\mathcal{D}, \nu) := \left\{ f : \mathcal{D} \to \mathbb{R} |\ f(0) = 0, \text{ and } f \text{ is absolutely continuous with } f' \in L^2(\mathcal{D}) \right\}$$

where f' is the derivative of f, and ν is the Lebesgue measure over $\mathbb{R}$. Equip $\mathcal{H}^{1,2}(\mathcal{D}, \nu)$ with the inner product

$$\langle f, g \rangle_{\mathcal{H},\nu}^{1,2} := \int_{\mathcal{D}} f'(z) g'(z) dz,$$

and denote the corresponding norm $\|\cdot\|_{\mathcal{H},\nu}^{1,2}$

Let us consider the test statistic

$$T_2(\boldsymbol{\ell}; \mathcal{W}^{1,2}) = \sup_{f \in \mathcal{W}^{1,2}(\mathcal{D},\nu)} \frac{1}{n} \sum_{k=1}^{n} \ell_k f(z_k)$$

where

$$\mathcal{W}^{1,2}(\mathcal{D}, \nu) = \left\{ f \in \mathcal{H}^{1,2}(\mathcal{D}, \nu) : \|f\|_{\mathcal{H}} \leq 1 \right\}$$

Hilbert-Sobolev Space. The Hilbert space $\mathcal{H}^{1,2}(\mathcal{D},\nu)$ equipped with inner product $\langle f,g\rangle^{1,2}_{\mathcal{H},\nu}$ can be shown to be an RKHS with associated kernel

$$k(x,z) = \min\{x,z\}$$

As a result, results for two-sample testing in the RKHS setup can be brought to bear.

Summarize results of Gretton.

and eigenfunction / eigenvalue pairs given by

$$\phi_j(t) = \frac{\sin(2j-1)\pi t}{2}, \quad \mu_j = \left(\frac{2}{(2j-1)\pi}\right)^2$$

for $j = 1, 2, \ldots$.