CSE 252B Assignment 1

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Problem 1 Line plane intersection

A line in 3D is defined by joining the points $X_1 = (x_1, y_1, z_1, t_1)^T$ and $X_2 = (x_2, y_2, z_2, t_2)^T$, and can be represented as $L = X_1X_2^T - X_1^TX_2$ or as a pencil $X(\lambda) = \lambda X_1 + (1-\lambda)X_2$. A plane in 3D is given by $\pi = (a, b, c, d)^T$ and the line L intersects the plane at $X_L = L\pi$ and $L\pi = X(\lambda_\pi)$ and λ_π is determined by $X(\lambda_\pi)\pi = 0$ or that the point $X(\lambda_\pi)$ lies on plane π.

$$\begin{aligned}
X(\lambda_\pi)\pi &= 0 \\
\lambda_\pi X_1^T\pi + (1-\lambda_\pi)X_2^T\pi &= 0 \\
\lambda_\pi(X_1^T - X_2^T)\pi + X_2^T\pi &= 0 \\
\lambda_\pi &= \frac{X_2^T\pi}{(X_2^T - X_1^T)\pi}
\end{aligned}$$

Substituting λ calculated above

$$\begin{aligned}
X_L &= \frac{X_1X_2^T\pi}{(X_2^T - X_1^T)\pi} - \frac{X_1^TX_2\pi}{(X_2^T - X_1^T)\pi} \\
&= \frac{X_1X_2^T\pi - X_1^TX_2\pi}{(X_2^T - X_1^T)\pi} \\
&= \frac{L_\pi}{(X_2^T - X_1^T)\pi} \\
&= \frac{X(\lambda_\pi)}{(X_2^T - X_1^T)\pi}
\end{aligned}$$

Therefore we can see that X_L is defined upto scale in terms of $X(\lambda_\pi)$ where scale term is $\frac{1}{(X_2^T - X_1^T)\pi}$

Problem 2 Line-Quadric Intersection

A 3D line may intersect a 0, 1 or 2 points. Let Q denote the quadric. As discussed above a line can be defined as a pencil $X(\lambda) = \lambda X_1 + (1-\lambda)X_2$. Hence let $X(\lambda_Q)$ be the point intersecting the quadric Q and lies on the line L.

We can reduce the above system to a quadratic equation in λ_Q which may look like $a_2\lambda_Q^2 + a_1\lambda_Q + a_0 = 0$ in the steps that follow. $X(\lambda_Q)$ is determined as following:

$$
\begin{aligned}
X(\lambda_Q)^T Q X(\lambda_Q) &= 0 \\
(\lambda_Q X_1 + (1-\lambda_Q)X_2)^T Q(\lambda_Q X_1 + (1-\lambda_Q)X_2) &= 0 \\
\lambda_Q^2(X_1^T Q X_1 - 2X_1^T Q X_2 + X_2^T Q X_2) + \lambda_Q(2(X_1^T Q X_2 - X2^T Q X_2)) + X_2^T Q X_2 &= 0
\end{aligned}
$$

comparing to $a_2\lambda_Q^2 + a_1\lambda_Q + a_0 = 0$

$$
\begin{aligned}
a_2 &= (X_1^T Q X_1 - 2X_1^T Q X_2 + X_2^T Q X_2) \\
a_1 &= (2(X_1^T Q X_2 - X2^T Q X_2)) \\
a_0 &= X_2^T Q X_2
\end{aligned}
$$

Problem 3 Feature detection

Parameter	**Image1**	**Image2**
Feature detection window size	7	7
Minor eigen value threshold	$0.0002 * w^2 = 0.01$	$0.0002 * w^2 = 0.01$
NMS Window	11	11
Features detected	615	629

Problem 4 Feature matching

Parameter	**Value**
Feature matching window size	11
Correlation Coeff Threshold	0.55
Distance ratio	0.8
Proximity threshold	150
Matches	200