Quantum Algorithims & Qiskit

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The Quantum Fourier Transform

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But what does that look like?

QFT: The gory details

The explict action of the n-qubit QFT on some given basis vector is

$$|j_1, ..., j_n\rangle \longrightarrow \frac{\left(|0\rangle + e^{2\pi i 0.j_n}|1\rangle\right) \otimes \left(|0\rangle + e^{2\pi i 0.j_{n-1}j_n}|1\rangle\right) \otimes ... \otimes \left(|0\rangle + e^{2\pi i 0.j_1 \cdots j_n}|1\rangle\right)}{2^{n/2}}$$

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Sorry for asking?

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. Show circuir here
Sorry for asking? Don't be it's easier to see in matrix form

QFT: The gory details II

For example with $n = 3$ the QFT is

$$\frac{1}{2\sqrt{2}}\begin{bmatrix}1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\ 1 & \omega & \omega^2 & \omega^3 & \omega^4 & \omega^5 & \omega^6 & \omega^7\\ 1 & \omega^2 & \omega^4 & \omega^6 & 1 & \omega^2 & \omega^4 & \omega^6\\ 1 & \omega^3 & \omega^6 & \omega & \omega^4 & \omega^7 & \omega^2 & \omega^5\\ 1 & \omega^4 & 1 & \omega^4 & 1 & \omega^4 & 1 & \omega^4\\ 1 & \omega^5 & \omega^2 & \omega^7 & \omega^4 & \omega & \omega^6 & \omega^3\\ 1 & \omega^6 & \omega^4 & \omega^2 & 1 & \omega^6 & \omega^4 & \omega^2\\ 1 & \omega^7 & \omega^6 & \omega^5 & \omega^4 & \omega^3 & \omega^2 & \omega\end{bmatrix}$$

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Not so interesting yet, but it's usefulness will soon be found in phase estimation

Phase Estimation

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So how do we do this? Easy!

Phase Estimation: Not so bad!

show circuit here with annotated state vector evolution, compare resulting statevector to QFT output

Amplitude Estimation

Amplitude estimation takes some superposition of states partitioned into a good and a bad subspace, and returns a statevector nudged towards the good subspace.

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This is accomplished with consecutive applications of a special operator Show diagram here

Amplitude Estimation: Working principles

The initial statevector can be decomposed as
$|\psi\rangle = sin(\theta)|\mathcal{G}\rangle + cos(\theta)|\mathcal{B}\rangle$.

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We then construct the operator: $\mathcal{S} = \mathbb{I} - 2|\mathcal{B}\rangle\langle\mathcal{B}|$

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3. Vibrant and active online community

Running the QFT

Let's run the 3-qubit QFT on some states

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Put bloch spheres of QFT inputs + ouputs here

Finding Phases

Let's run the 3-qubit phase estimation algorithim on some gates

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References I

Thanks