Quantum Neural Network Simulation

Project Proposal

Alex Heilman

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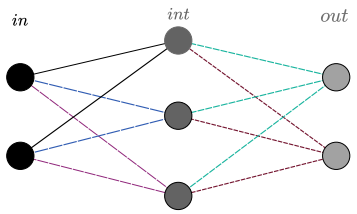

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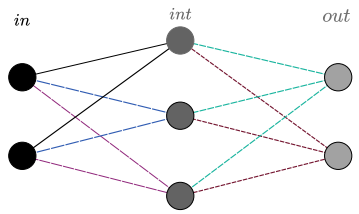

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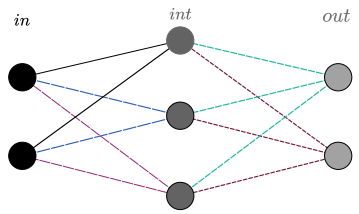

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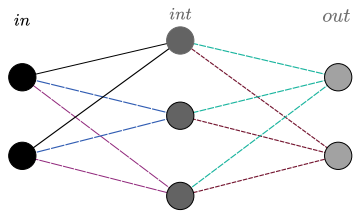

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Training Data Structure

Data will be provided for the training of the network via a set of arbitrary states (inputs), and the set of these same states after having some common unitary action act upon them (outputs). Hence, we will assume some given data set of the following form:

$$\text{Training Data: } \{(|\psi_i\rangle, V|\psi_i\rangle) \mid 1 \leq i \leq N\}$$

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This is a reasonable set of data since the most general quantum network will apply an arbitrary unitary gate, and hence the most general circuit should be able to approximate such actions.

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Each layer may have a different number of qubits M_{ℓ}. Explicitly, the ℓ-th layer's transition map takes the form:

$$\epsilon^{\ell}\left(\rho_{\ell-1}\right) =$$

$$\mathrm{Tr}_{\ell-1}\left[\left(\prod_{m=1}^{M_{\ell}} U_{\ell}^{m-M_{\ell}}\right)\left(\left(|0\rangle^{\otimes M_{\ell}}\langle 0|^{\otimes M_{\ell}}\right)_{\ell} \otimes \rho_{\ell-1}\right)\left(\prod_{m=1}^{M_{\ell}} U_{\ell}^{m\dagger}\right)\right]$$

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And, hence, a total circuit of L layers returns ρ_{out}, defined below, for given some input state ρ_{in}.

$$\rho_{out} = \epsilon^{out}\left(\epsilon^{L}\left(\epsilon^{L-1}\left(\ldots\epsilon^{1}\left(\rho_{in}\right)\ldots\right)\right)\right)$$

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1. The next layer's M qubits are prepared in the initial state $|0\rangle^{\otimes m}\langle 0|_{\ell}^{\otimes m}$ and tensor producted with the previous layer's output $\rho_{\ell-1}$.

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2. The ℓ-th layer's M associated unitary matrices U_{ℓ}^{m} are applied to this tensor product state (from top to bottom).

$$\rho''_{\ell} = \left(\prod_{m=0}^{M-1} U_{\ell}^{M-m}\right)(\rho'_{\ell})\left(\prod_{m=1}^{M} U_{\ell}^{m\dagger}\right)$$

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3. The partial trace over the $(\ell-1)$th layer's Hilbert space is taken, resulting in the output state ρ_{ℓ} of the ℓ-th layer.

$$\rho_{\ell} = \mathrm{Tr}_{\ell-1}[\rho''_{\ell}]$$

Simple Example: $2 \times 3 \times 2$

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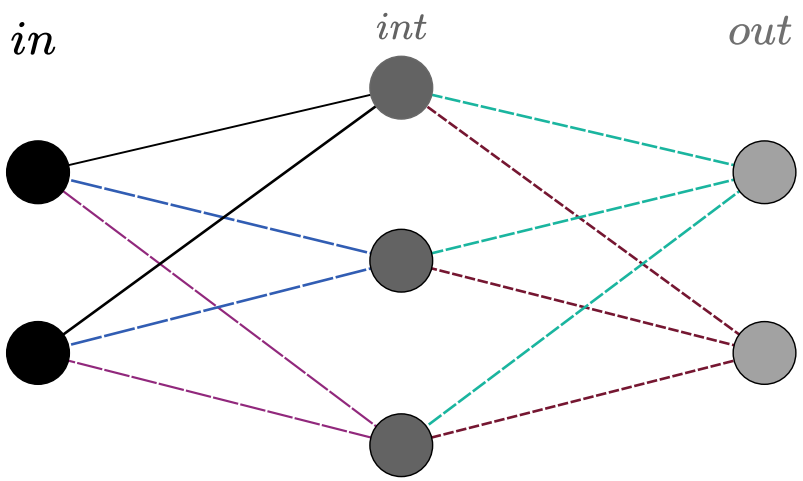

$$\mathrm{Tr}_{int}\left[U_2^{out}U_1^{out}\left(\mathrm{Tr}_{in}\left[U_3^{int}U_2^{int}U_1^{int}(\rho_{in}\otimes|000\rangle\langle 000|_{int})U_1^{int\dagger}U_2^{int\dagger}U_3^{int\dagger}\right]\right)U_1^{out\dagger}U_2^{out\dagger}\right]=\rho_{out}$$

Cost

The metric by which we will judge the performance of the network on the training data is the cost, here taken as the average fidelity between the networks output state and the corresponding state given in training and explicity defined as:

$$C = \frac{1}{N} \sum_{i=1}^{N} \langle \psi_i^{out} | \rho_{out} | \psi_i^{out} \rangle$$

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For input mixed states, we may replace the above with an averaged fidelity between output and target states of the form:

$$C = \frac{1}{N} \sum_{i=1}^{N} \left(\mathrm{Tr} \left[\sqrt{\sqrt{\rho_i} \rho_i^{out} \sqrt{\rho_i}} \right] \right)^2$$

Training

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Training may be performed by evolving each unitary via the following map:

$$U_m^\ell \to e^{i\varepsilon K_m^\ell} U_m^\ell$$

which is parameterized by the step size ε, and where K_ℓ^m is derived from the derivative of the cost function and takes the following form:

$$K_m^\ell = \eta \frac{2^{m_\ell - 1}}{N} \sum_{i=1}^{N} \mathrm{Tr}_{\neg \ell} \Bigg[\left(\prod_{n=0}^{m-1} U_{m-n}^\ell \right) \left(\left(|0\rangle^{\otimes m_\ell} \langle 0|^{\otimes m_\ell} \right)_\ell \otimes \rho_i^{\ell-1} \right) \left(\prod_{n=1}^{m} U_m^{\ell\dagger} \right), \\ \left(\prod_{n=m+1}^{m_\ell} U_n^{\ell\dagger} \right) \left(\sigma_i^\ell \otimes \mathbb{I}_{\ell-1} \right) \left(\prod_{n=1}^{m_\ell - (m+1)} U_{m_\ell - n}^\ell \right) \Bigg]$$

where the square brackets denote a commutator and $\sigma_i^\ell = \mathcal{F}^{\ell+1}(\ldots \mathcal{F}^{out}(\rho_i^{out})\ldots)$ is the adjoint channel to the layer-to-layer transition map ϵ^ℓ for layer ℓ.

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Implementation: Goals

Goals will include the following:

1. Define auxilary functions in an appropriate manner (partial trace, concatenated unitary actions, etc.)
2. Define layer-to-layer transition maps of arbitrary qubit size
3. Define arbitrary depth layer composition (network of arbitrary depth)
4. Define appropriate cost function
5. Implement corresponding training scheme
6. Test on set(s) of data
   (a) Compare performance on different underlying unitaries
   (b) Compare performance for data set size used in training
   (c) (*time permitting*) Compare performance on noisy data