Quantum Algorithims & Qiskit

Alexander J. Heilman

August 18, 2021

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But what does that look like?

QFT: The gory details

The explict action of the n-qubit QFT on some given basis vector is

$$|j_1, ..., j_n\rangle \longrightarrow \frac{\left(|0\rangle + e^{2\pi i 0.j_n}|1\rangle\right) \otimes \left(|0\rangle + e^{2\pi i 0.j_{n-1}j_n}|1\rangle\right) \otimes ... \otimes \left(|0\rangle + e^{2\pi i 0.j_1 \dots j_n}|1\rangle\right)}{2^{n/2}}.$$

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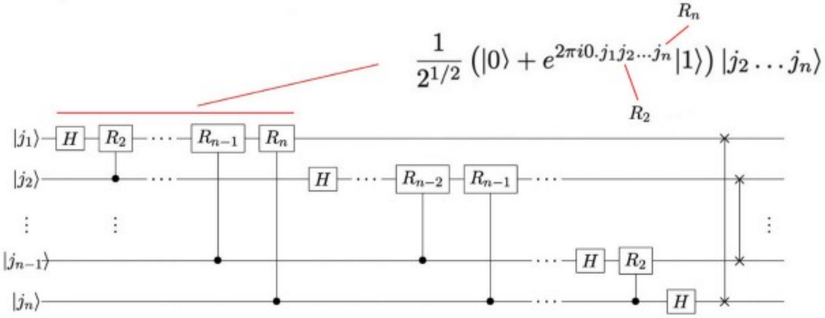

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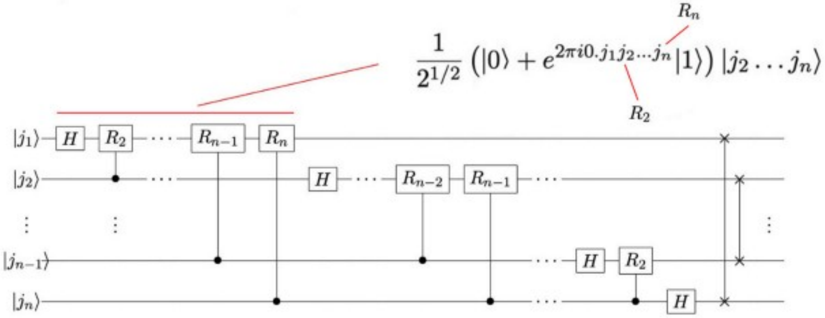

Sorry for asking? Don't be it's easier to see in matrix form

QFT: The gory details II

For example with $n = 3$ the QFT is

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Not so interesting yet, but it's usefulness will soon be found in phase estimation

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So how do we do this? Easy!

Phase Estimation: Not so bad!

show circuit here with annotated state vector evolution, compare resulting statevector to QFT output

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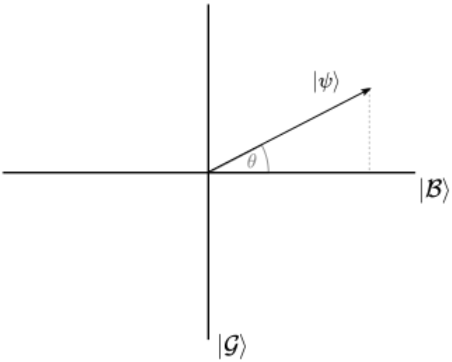

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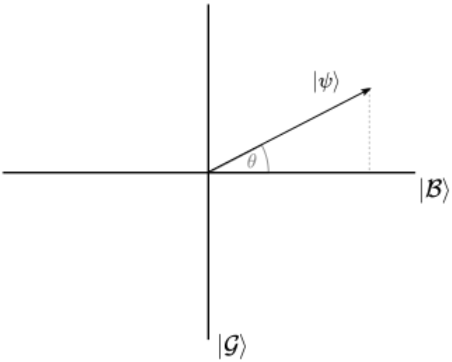
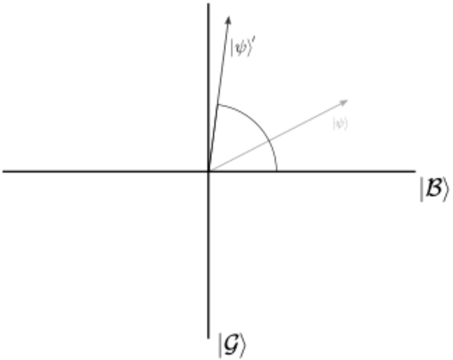


$$|\psi\rangle = sin(\theta)|\mathcal{G}\rangle + cos(\theta)|\mathcal{B}\rangle.$$

Amplitude Amplification

And returns a statevector nudged towards the good subspace.



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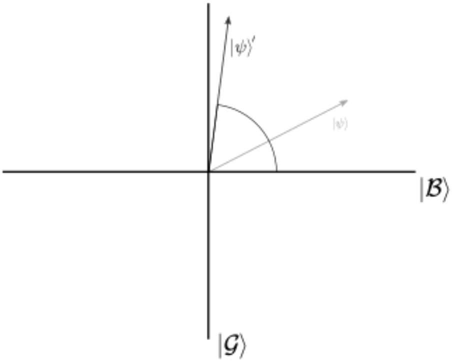

This is accomplished with consecutive applications of a special operator

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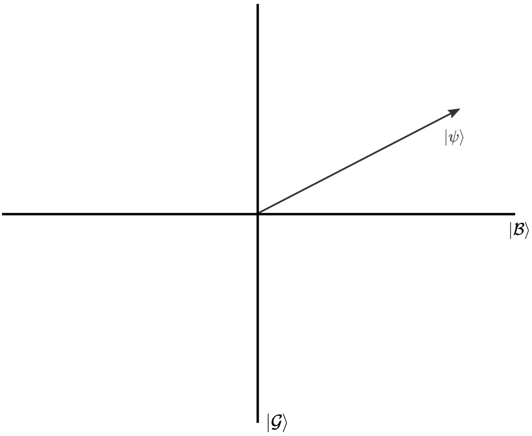
Let's see what they do!

Amplitude Amplification: Visualization I

First apply the operator $\mathcal{S}_{\mathcal{G}}$,



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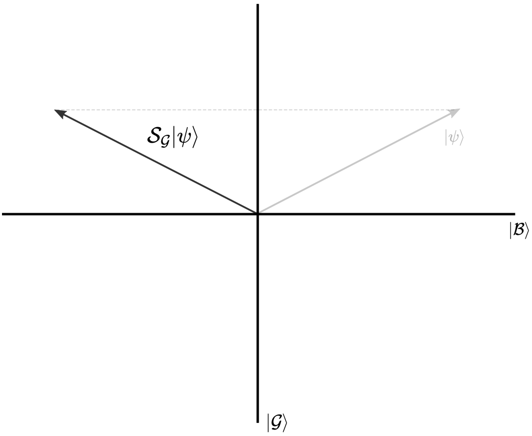

Amplitude Amplification: Visualization II

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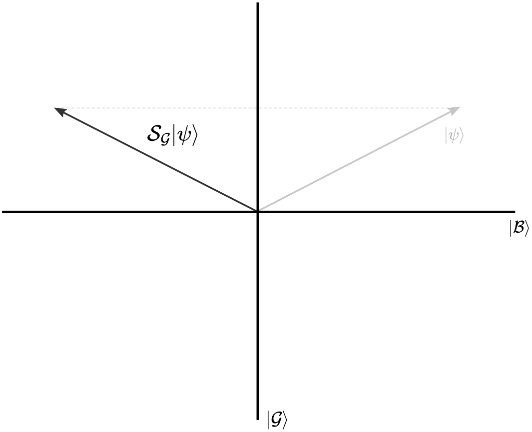

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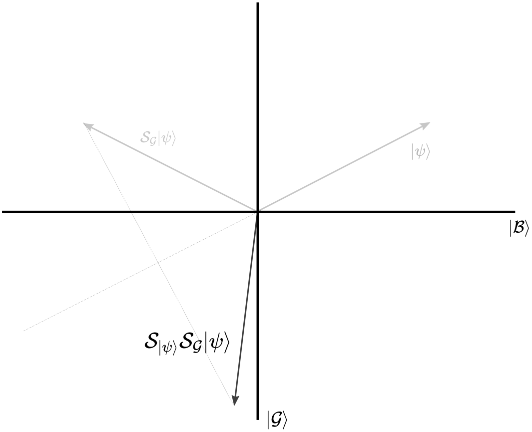

Amplitude Amplification: Visualization III

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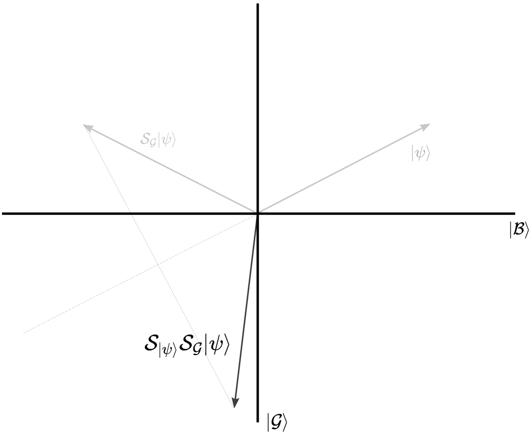

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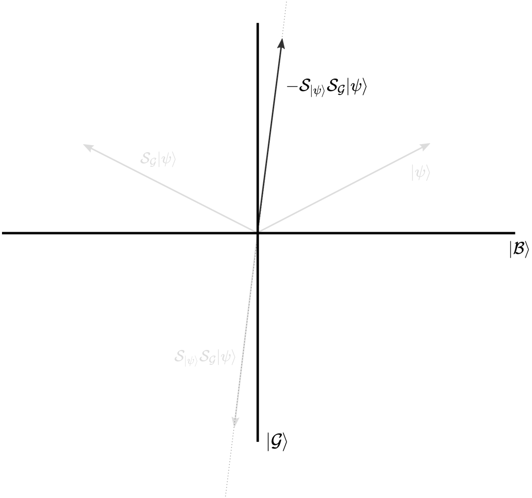
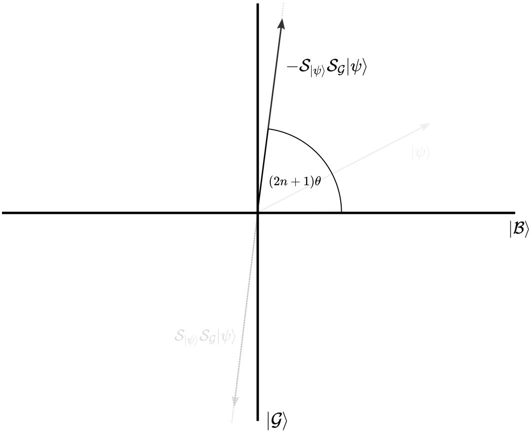


Amplitude Amplification: Visualization IV

And voilá!



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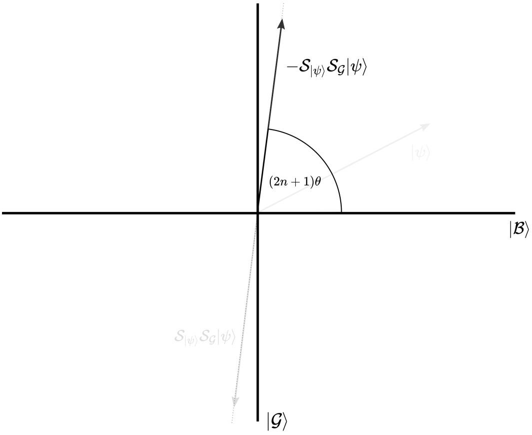

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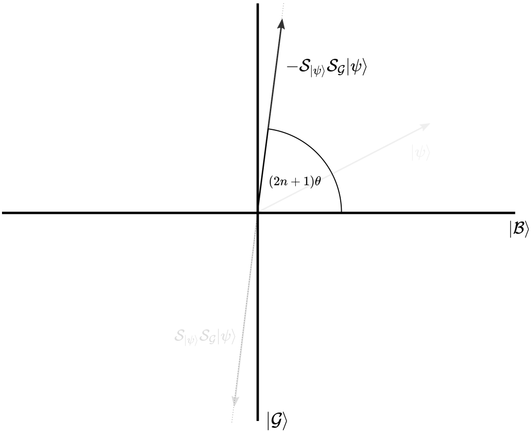

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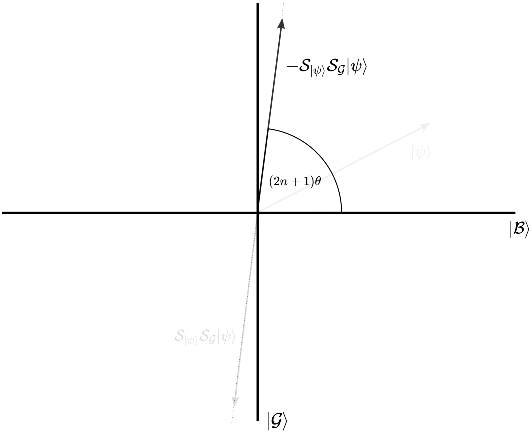

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Now, let's do some coding!

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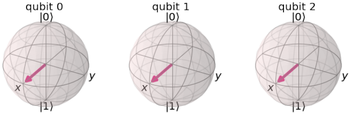

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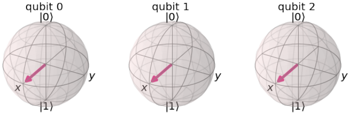

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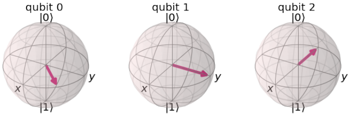

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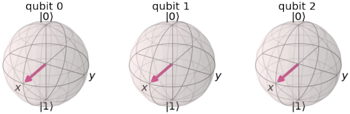

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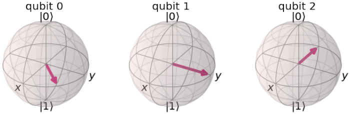

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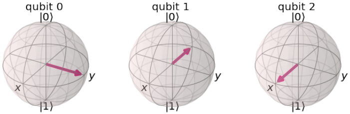

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Moving forward, I'll just link the code online.

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We already know that $|1\rangle$ is an eigenvector of basic phase gates.

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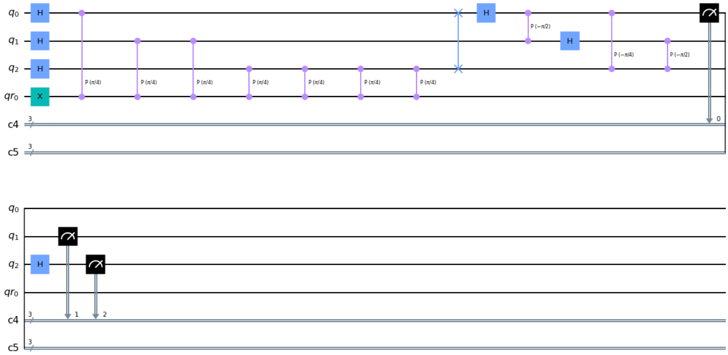

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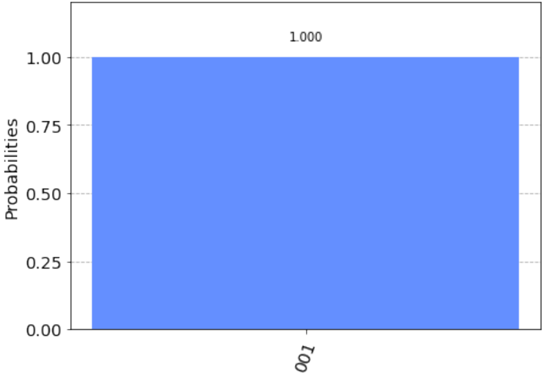

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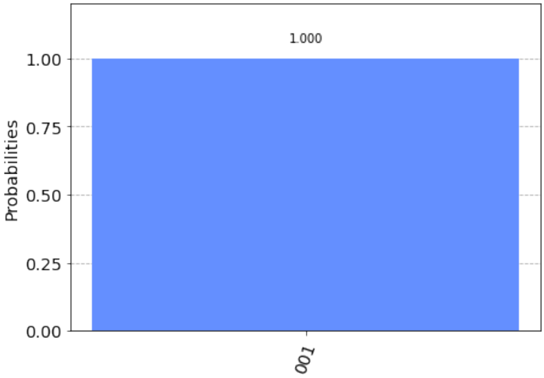

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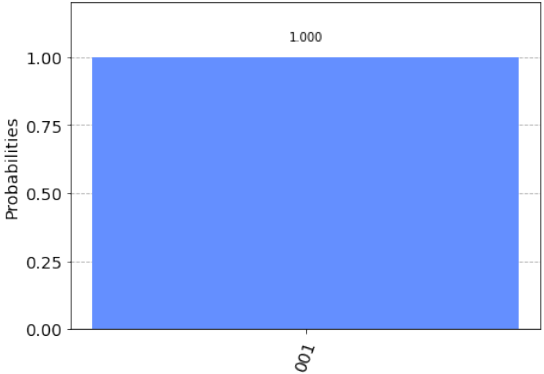

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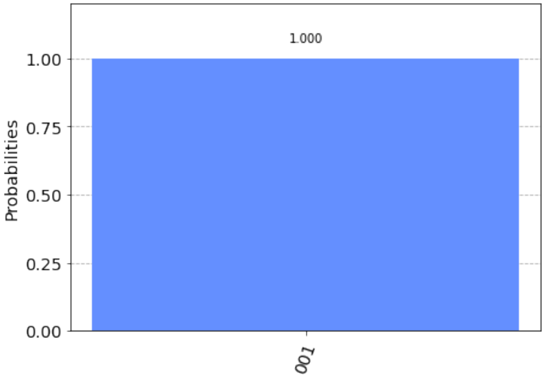

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As the phase can be encoded perfectly in 3 qubits, we should expect the output vector to be $|k\rangle = |\rangle$

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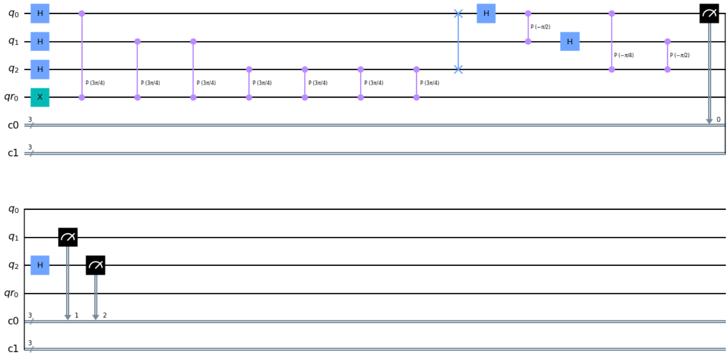

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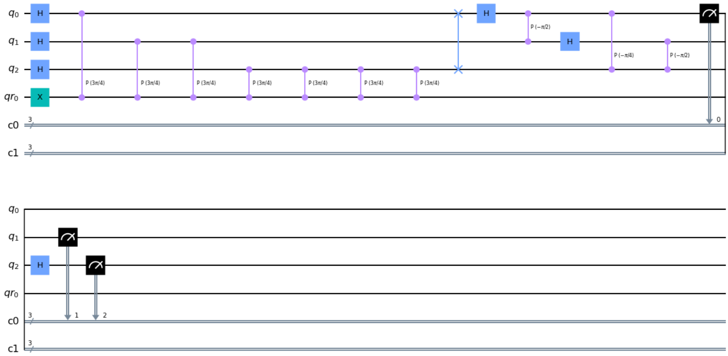

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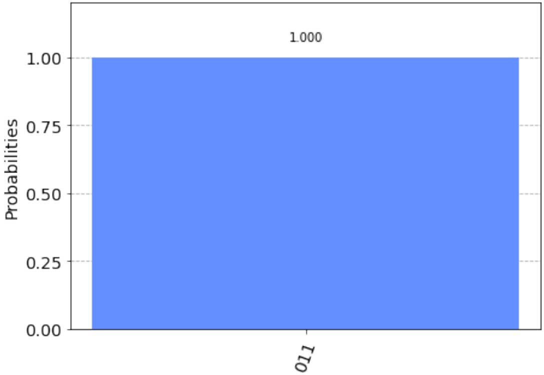

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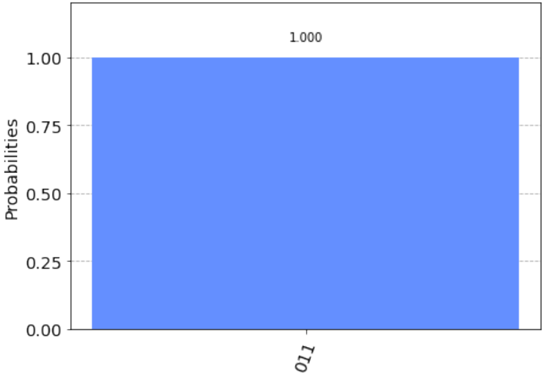

References I

Thanks