

$$\operatorname{Tr}_{int} \Big[ U_2^{out} U_1^{out} \Big( \operatorname{Tr}_{in} \Big[ U_3^{int} U_2^{int} U_1^{int} (\rho_{in} \otimes |000\rangle \langle 000|_{int}) U_1^{int\dagger} U_2^{int\dagger} U_3^{int\dagger} \Big] \Big) U_1^{out\dagger} U_2^{out\dagger} \Big] = \rho_{out}$$