Visualizing the Quantum Fourier Transform in Qiskit

A Short Introduction to Both

June 30, 2021

Intent

- Introduce you all to some tools in Qiskit
- Define and give some visual intuition for the QFT
- Build a foundation for further exploration of quantum algorithims and QML

What is Qiskit?

Qiskit is a python module that allows users to code and simulate quantum circuits offering several advantages:

• Currently implementable on IBM's quantum computers with certain backends.

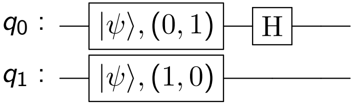
• Useful for experimentation with and visualization of quantum circuits.

• Recently added a QML package[8] including Quantum Kernels[4] and Quantum Neural Nets[5]!

Using Qiskit: Circuit Building

First, make sure qiskit is installed using pip and then import qiskit and qiskit.visualization in your favorite python environment. Jupyter notebooks work well. Now, for some simple code:

```
from qiskit import *
qc = QuantumCircuit(2)
qc.initialize([0,1], 0)
qc.initialize([1,0], 1)
qc.h(0)
qc.draw()
```



Using Qiskit: Simulating a Circuit

Qiskit provides several options for simulation of quantum circuits[2]. Here, we'll just use an idealized quantum circuit (no noise) that lets us peek at the state vector as it evolves.

```
backend = BasicAer.get_backend('statevector_simulator')
result = execute(qc, backend).result()
endvector = result.get_statevector()
print(endvector, decimals = 3)
```

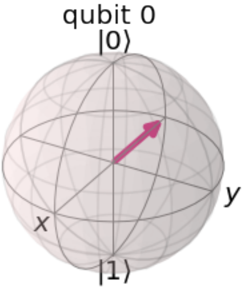
[0.707+0.j -0.707+0.j 0. +0.j 0. +0.j]

Here we have simulated the circuit's output, a two qubit quantum state, and given it the name 'endvector'. Now, to visualize it!

Using Qiskit: Visualizing States

Qiskit gives several options for the visualization of quantum states[3]. A local favorite is the Bloch sphere.

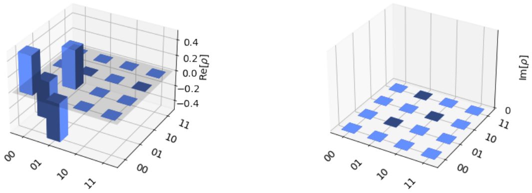
```
from qiskit.visualization import *
plot_bloch_multivector(endvector)
```



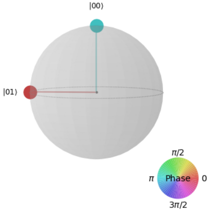
Using Qiskit: Visualizing States cont.

Other options include:

`plot_state_city(endvector)`



`plot_state_qsphere(endvector)`



The Quantum Fourier Transform

The QFT generalizes the discrete Fourier transform via:

$$|j\rangle \to \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} e^{2\pi ijk/N} |k\rangle$$

.

Which is unitary, and thus has some quantum circuit describing it.

1-qubit QFT

The QFT for the 1-qubit case should look familiar:

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$$

.

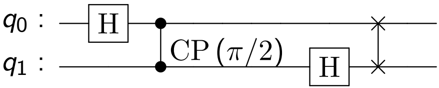
$$q_0 : \; -\boxed{\mathrm{H}}-$$

2-qubit QFT

In the computational basis, the QFT for two qubits takes the form:

$$\frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix}$$

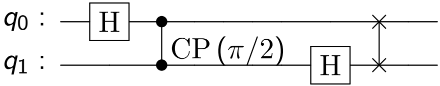
And is represented by the circuit:



where $CP(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{i\theta} \end{bmatrix}$

2-qubit QFT cont.

For your consideration,



$$\begin{bmatrix}1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}1&1&0&0\\1&-1&0&0\\0&0&1&1\\0&0&1&-1\end{bmatrix}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&e^{i\pi/2}\end{bmatrix}\begin{bmatrix}1&0&1&0\\0&1&0&1\\1&0&-1&0\\0&1&0&-1\end{bmatrix}$$

$$=\begin{bmatrix}1&1&1&1\\1&i&-1&-i\\1&-1&1&-1\\1&-i&-1&i\end{bmatrix}$$

3-qubit QFT

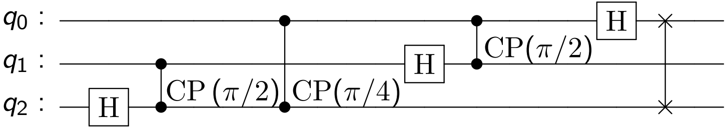
Written out in terms of the 8th roots of unity, the matrix form of the 3 qubit QFT is:

$$\frac{1}{2\sqrt{2}}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \omega & \omega^2 & \omega^3 & \omega^4 & \omega^5 & \omega^6 & \omega^7 \\ 1 & \omega^2 & \omega^4 & \omega^6 & 1 & \omega^2 & \omega^4 & \omega^6 \\ 1 & \omega^3 & \omega^6 & \omega & \omega^4 & \omega^7 & \omega^2 & \omega^5 \\ 1 & \omega^4 & 1 & \omega^4 & 1 & \omega^4 & 1 & \omega^4 \\ 1 & \omega^5 & \omega^2 & \omega^7 & \omega^4 & \omega & \omega^6 & \omega^3 \\ 1 & \omega^6 & \omega^4 & \omega^2 & 1 & \omega^6 & \omega^4 & \omega^2 \\ 1 & \omega^7 & \omega^6 & \omega^5 & \omega^4 & \omega^3 & \omega^2 & \omega \end{bmatrix}$$

.

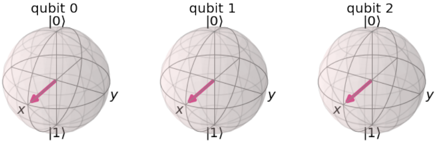
3-qubit QFT cont.

With a circuit diagram:



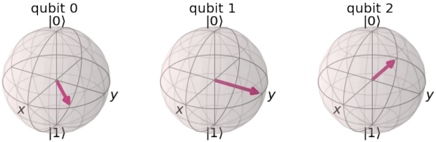
Let's see what it does to some qubits on the bloch sphere:

```
from qiskit.circuit.library import QFT
qft = QFT(3)
qft3_000 = execute(qft, backend).result()
plot_bloch_multivector(qft3_000.get_statevector())
```

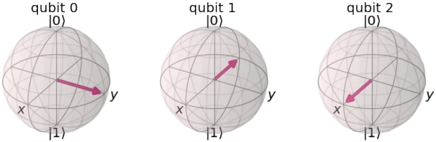


3-qubit QFT cont.

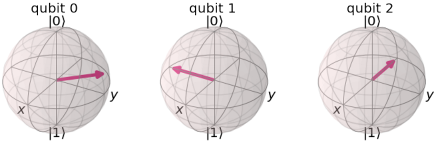
```
plot_bloch_multivector(qft3_001.get_statevector())
```




```
plot_bloch_multivector(qft3_010.get_statevector())
```




```
plot_bloch_multivector(qft3_011.get_statevector())
```



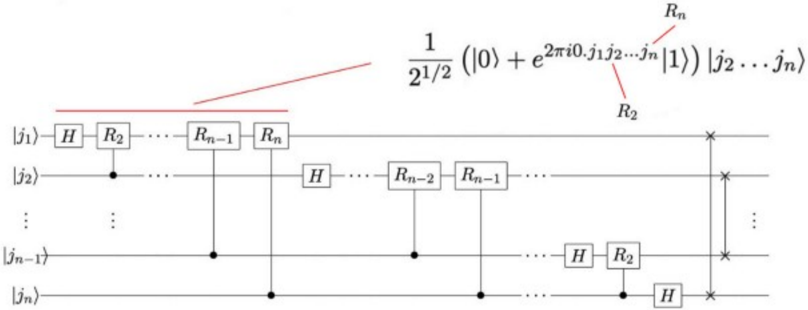
n-qubit QFT

This relative rotation of the output bits by some angle θ^{2n} hints at the identity proven in [7], where $0.b_1..b_n = b_1/2 + ... + b_n/2^n$

$|j_1, ..., j_n\rangle =$

$$\frac{\left(|0\rangle + e^{2\pi i 0.j_n}|1\rangle\right) \otimes \left(\left(|0\rangle + e^{2\pi i 0.j_{n-1}j_n}|1\rangle\right) \otimes ... \otimes \left(|0\rangle + e^{2\pi i 0.j_1 \dots j_n}|1\rangle\right)}{2^{n/2}}$$

.



Outlook

- Improve skills
- Use Qiskit to simulate phase estimation
- Long term, extend to QML?

THANK YOU!

–OUTLINE–

Intro to QisKit

How to build a circuit

How to run a simulation

Motivating the QFT

Building the QFT

Running/Visualizing QFT [6]

n-qubit qft diagram source.
https://jonathan-hui.medium.com/qc-quantum-fourier-transform-45436f90a43.

Qiskit simulator documentation.
https://qiskit.org/documentation/tutorials/simulators/1_aer_provider.html.

Qiskit visualization documentation.
https://qiskit.org/documentation/tutorials/circuits/2_plotting_data_in_qiskit.html.

Quantum kernel machine learning tutorial on github.
https://github.com/Qiskit/qiskit-machine-learning/blob/master/docs/tutorials/03_quantum_kernel.ipynb.

Quantum neural network tutorial on github.
https://github.com/Qiskit/qiskit-machine-learning/blob/master/docs/tutorials/01_neural_networks.ipynb.

Héctor Abraham, AduOffei, Rochisha Agarwal, Ismail Yunus Akhalwaya, Gadi Aleksandrowicz, Thomas Alexander, Matthew Amy, Eli Arbel, Arijit02, Abraham Asfaw, Artur Avkhadiev, Carlos Azaustre, AzizNgoueya, Abhik Banerjee, Aman Bansal, Panagiotis Barkoutsos, Ashish Barnawal, George Barron, George S. Barron, Luciano Bello, Yael Ben-Haim, Daniel Bevenius, Arjun Bhobe, Lev S. Bishop, Carsten Blank, Sorin Bolos, Samuel Bosch, Brandon, Sergey Bravyi, Bryce-Fuller, David Bucher, Artemiy Burov, Fran Cabrera, Padraic Calpin, Lauren Capelluto, Jorge Carballo, Ginés Carrascal, Adrian Chen, Chun-Fu Chen, Edward Chen, Jielun (Chris) Chen, Richard Chen, Jerry M. Chow, Spencer Churchill, Christian Claus, Christian Clauss, Romilly Cocking, Filipe Correa, Abigail J. Cross, Andrew W. Cross, Simon Cross, Juan Cruz-Benito, Chris Culver, Antonio D. Córcoles-Gonzales, Sean Dague, Tareq El Dandachi, Marcus Daniels, Matthieu Dartiailh, DavideFrr, Abdón Rodríguez Davila, Anton Dekusar, Delton Ding, Jun Doi, Eric Drechsler, Drew, Eugene Dumitrescu, Karel Dumon, Ivan Duran, Kareem EL-Safty, Eric

Eastman, Grant Eberle, Pieter Eendebak, Daniel Egger, Mark Everitt, Paco Martín Fernández, Axel Hernández Ferrera, Romain Fouilland, FranckChevallier, Albert Frisch, Andreas Fuhrer, Bryce Fuller, MELVIN GEORGE, Julien Gacon, Borja Godoy Gago, Claudio Gambella, Jay M. Gambetta, Adhisha Gammanpila, Luis Garcia, Tanya Garg, Shelly Garion, Austin Gilliam, Aditya Giridharan, Juan Gomez-Mosquera, Gonzalo, Salvador de la Puente González, Jesse Gorzinski, Ian Gould, Donny Greenberg, Dmitry Grinko, Wen Guan, John A. Gunnels, Mikael Haglund, Isabel Haide, Ikko Hamamura, Omar Costa Hamido, Frank Harkins, Vojtech Havlicek, Joe Hellmers, Łukasz Herok, Stefan Hillmich, Hiroshi Horii, Connor Howington, Shaohan Hu, Wei Hu, Junye Huang, Rolf Huisman, Haruki Imai, Takashi Imamichi, Kazuaki Ishizaki, Raban Iten, Toshinari Itoko, JamesSeaward, Ali Javadi, Ali Javadi-Abhari, Wahaj Javed, Jessica, Madhav Jivrajani, Kiran Johns, Scott Johnstun, Jonathan-Shoemaker, Vismai K, Tal Kachmann, Akshay Kale, Naoki Kanazawa, Kang-Bae, Anton Karazeev, Paul Kassebaum, Josh Kelso, Spencer King, Knabberjoe, Yuri Kobayashi, Arseny Kovyrshin, Rajiv Krishnakumar, Vivek Krishnan, Kevin Krsulich, Prasad Kumkar, Gawel Kus, Ryan LaRose, Enrique Lacal, Raphaël Lambert, John Lapeyre, Joe Latone, Scott Lawrence, Christina Lee, Gushu Li, Dennis Liu, Peng Liu, Yunho Maeng, Kahan Majmudar, Aleksei Malyshev, Joshua Manela, Jakub Marecek, Manoel Marques, Dmitri Maslov, Dolph Mathews, Atsushi Matsuo, Douglas T. McClure, Cameron McGarry, David McKay, Dan McPherson, Srujan Meesala, Thomas Metcalfe, Martin Mevissen, Andrew Meyer, Antonio Mezzacapo, Rohit Midha, Zlatko Minev, Abby Mitchell, Nikolaj Moll, Jhon Montanez, Gabriel Monteiro, Michael Duane Mooring, Renier Morales, Niall Moran, Mario Motta, MrF, Prakash Murali, Jan Müggenburg, David Nadlinger, Ken Nakanishi, Giacomo Nannicini, Paul Nation, Edwin Navarro, Yehuda Naveh, Scott Wyman Neagle, Patrick Neuweiler, Johan Nicander, Pradeep Niroula, Hassi Norlen, NuoWenLei, Lee James O'Riordan, Oluwatobi Ogunbayo, Pauline Ollitrault, Raul Otaolea, Steven Oud, Dan Padilha, Hanhee Paik, Soham Pal, Yuchen Pang, Vincent R. Pascuzzi, Simone Perriello, Anna Phan, Francesco Piro, Marco Pistoia, Christophe Piveteau, Pierre Pocreau, Alejandro Pozas-Kerstjens, Milos Prokop, Viktor Prutyanov, Daniel Puzzuoli, Jesús Pérez, Quintiii, Rafey Iqbal Rahman, Arun Raja, Nipun Ramagiri, Anirudh Rao, Rudy Raymond, Rafael Martín-Cuevas Redondo, Max Reuter, Julia Rice, Matt Riedemann, Marcello La Rocca, Diego M. Rodríguez, RohithKarur, Max Rossmannek, Mingi Ryu, Tharrmashastha SAPV, SamFerracin, Martin Sandberg, Hirmay Sandesara, Ritvik Sapra, Hayk Sargsyan, Aniruddha Sarkar, Ninad Sathaye, Bruno Schmitt, Chris Schnabel, Zachary Schoenfeld, Travis L. Scholten, Eddie Schoute, Joachim Schwarm, Ismael Faro Sertage, Kanav Setia, Nathan Shammah, Yunong Shi, Adenilton Silva, Andrea Simonetto, Nick Singstock, Yukio Siraichi, Iskandar Sitdikov, Seyon Sivarajah, Magnus Berg Sletfjerding, John A. Smolin, Mathias Soeken, Igor Olegovich Sokolov, Igor Sokolov, SooluThomas, Starfish, Dominik Steenken, Matt Stypulkoski, Shaojun Sun, Kevin J. Sung, Hitomi Takahashi, Tanvesh Takawale, Ivano

Tavernelli, Charles Taylor, Pete Taylour, Soolu Thomas, Mathieu Tillet, Maddy Tod, Miroslav Tomasik, Enrique de la Torre, Kenso Trabing, Matthew Treinish, TrishaPe, Davindra Tulsi, Wes Turner, Yotam Vaknin, Carmen Recio Valcarce, Francois Varchon, Almudena Carrera Vazquez, Victor Villar, Desiree Vogt-Lee, Christophe Vuillot, James Weaver, Johannes Weidenfeller, Rafal Wieczorek, Jonathan A. Wildstrom, Erick Winston, Jack J. Woehr, Stefan Woerner, Ryan Woo, Christopher J. Wood, Ryan Wood, Stephen Wood, Steve Wood, James Wootton, Daniyar Yeralin, David Yonge-Mallo, Richard Young, Jessie Yu, Christopher Zachow, Laura Zdanski, Helena Zhang, Christa Zoufal, Zoufalc, a kapila, a matsuo, bcamorrison, brandhsn, nick bronn, brosand, chlorophyll zz, csseifms, dekel.meirom, dekelmeirom, dekool, dime10, drholmie, dtrenev, ehchen, elfrocampeador, faisaldebouni, fanizzamarco, gabrieleagl, gadial, galeinston, georgios ts, gruu, hhorii, hykavitha, jagunther, jliu45, jscott2, kanejess, klinvill, krutik2966, kurarrr, lerongil, ma5x, merav aharoni, michelle4654, ordmoj, sagar pahwa, rmoyard, saswati qiskit,

scottkelso, sethmerkel, shaashwat, sternparky, strickroman, sumitpuri, tigerjack, toural, tsura crisaldo, vvilpas, welien, willhbang, yang.luh, yotamvakninibm, and Mantas Čepulkovskis.
Qiskit: An open-source framework for quantum computing, 2019.

Michael A. Nielsen and Isaac L. Chuang.
Quantum Computation and Quantum Information: 10th Anniversary Edition.
Cambridge University Press, USA, 10th edition, 2011.

Qiskit Applications Team.
Introducing qiskit machine learning.
https://medium.com/qiskit/introducing-qiskit-machine-learning-5f06b6597526.