$$\text{Tr}_{int}\Big[U_2^{out}U_1^{out}\Big(\text{Tr}_{in}\Big[U_3^{int}U_2^{int}U_1^{int}(\rho_{in}\otimes|000\rangle\langle000|_{int})U_1^{int\dagger}U_2^{int\dagger}U_3^{int\dagger}\Big]\Big)U_1^{out\dagger}U_2^{out\dagger}\Big]=\rho_{out}$$