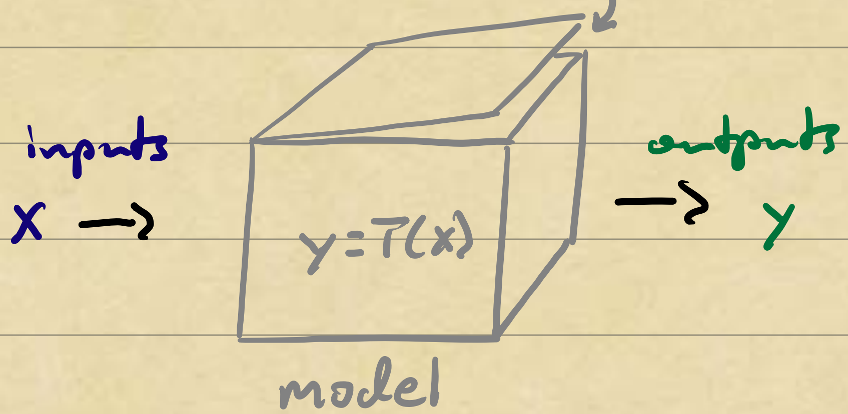
Methods of Applied Math

Fall 2025 Course Info

Linked via LMS:
- github.com/ajhphros23/MATH-6600
- Piazza - Q & A, Discussion, Announcements
- Gradescope - release, view, submit HW

Aim: Develop toolkit to study complex phenomena.



Challenges: In real-world applications, we face

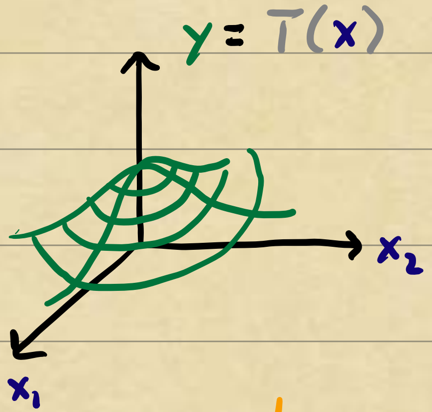
"Slice into simpler pieces"
"Linearize" ⇒ Local ⇒ Iterate ⇒ Embed
"Projection"
"Regularization"
"Filters"

⇒ High-dimensional inputs/outputs/model
⇒ Nonlinear coupling of inputs/outputs
⇒ Partial knowledge of inputs/outputs/model
⇒ Noise & corruption in inputs/outputs/model

Often need to work w/computer

Theme: "Slice" x, T, y into "simpler" pieces.

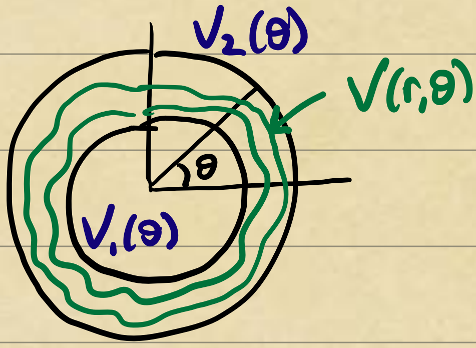
Example 1: "Fit" model to data



Given observations of input/output $(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \ldots, (x^{(m)}, y^{(m)})$ can we "learn" the map T?

Linear & nonlinear approximation

Example 2: Equilibrium Configurations

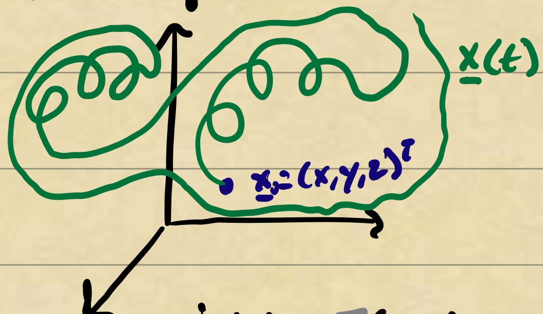


If the inner and outer surfaces of a hollow dielectric are held at electric potentials $V_1(\theta)$ and $V_2(\theta)$, what is the potential $V(r,\theta)$ in the tube?

Laplace Eqn., $\Delta V = 0$
+ boundary conditions

Elliptic PDEs (linear) & Variational Principles (nonlinear)

Example 3: Dynamical Systems



$$\dot{\underline{x}}(t) = \underline{F}(\underline{x}, t)$$
$$\underline{x}(0) = \underline{x}_0$$

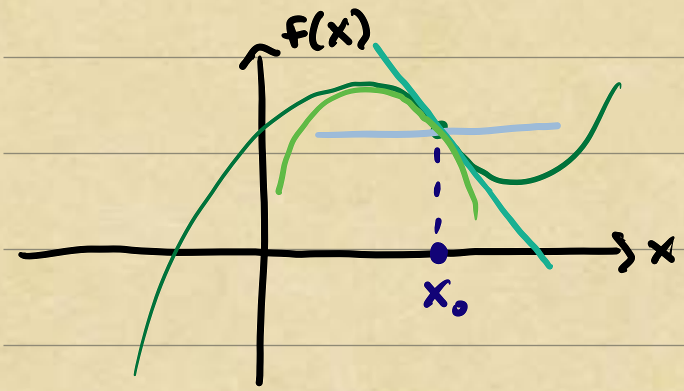
Given initial condition $\underline{x}_0$ and dynamical law $\underline{F}$, how does $\underline{x}(t)$ evolve?

Lin. PDEs (linear inf-dim), Nonlinear Dynamics, Nonlinear PDE

Linear Algebra w/ Functions

One of the most powerful ideas in applied mathematics is that complicated functions can be built up by combining simpler ones.

Example 1: Taylor Polynomials



$$f(x) \approx f(x_0) + f'(x_0)(x-x_0) + \frac{1}{2} f''(x_0)(x-x_0)^2 + \vdots + \frac{1}{n!} f^{(n)}(x_0)(x-x_0)^n$$

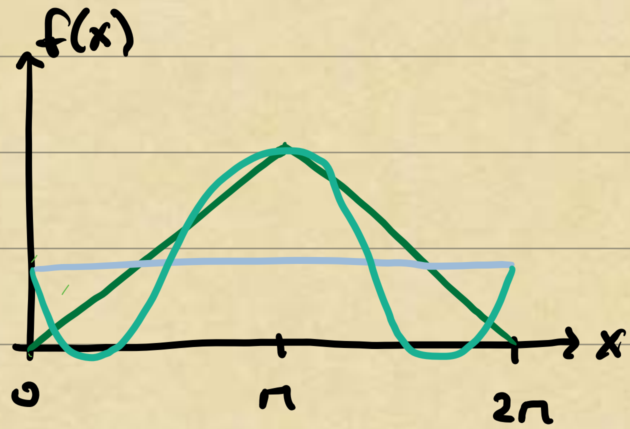
Build up $f(x)$ near x_0 using monomials $(x-x_0)^j$.

$\Rightarrow$ Polynomials are easy to calculate with!

$\Rightarrow$ Consequently, poly. approx. are at the heart of many analytic and numerical techniques.

$\Rightarrow$ Series soln's for ODEs, rootfinders & optimizers, finite difference/element/volume methods.

Example 2: Fourier Series



$$f(x) \approx \frac{a_0}{2} + a_1 \cos(x) + a_2 \cos(2x) + \cdots + a_n \cos(nx)$$

discuss coeffs later

Build up f(x) using trigonometric functions.

⇒ Trig. functions are also easy to work with!

⇒ Trig. approx. is central to many theoretical and computational tasks in signal processing and the solution/analysis of ODE/PDE.

⇒ Image processing, Denoising, Spectral filtering, Stability analysis, wave prop.

Q: What features do Taylor and Fourier series have in common?

Q: In which features do Taylor and Fourier series differ?