Fourrer Serres : Ambahre: Applications

Rent Consider a 200 persodre signal f:[4,201]->6.

Fourth $f(\theta) = \sum_{k=0}^{+\infty} \hat{f}_k e^{ik\theta}$

Korrels $\hat{f}_{u} = \frac{1}{2n} \int_{3}^{3n} 5(\theta) e^{ik\theta} d\theta$

If f is histomorphic in 2n-period strip

S= { 8+iy: 8 & [0,2n]per, y & [-a,a]}

end setsfres sup 17(2)1 & M, Hen 265

wells If I & Mealki

Photos sup/f(8) - $\Sigma \hat{S}_{k} e^{ik\theta} / \leq \frac{2Me^{-a(NH)}}{1-e^{-a}}$

(represent | 50) do - 1 Es(20 K) | 5 40M e-an

These exp. accurate approxis head to fast algas.

Example: Solve -u"(x)=f(x), xe[0,2n]

where (u(x) ds =0 and f is pertolle, smooth, mean-zora

Note that if f is snooth, so must be n!

$$u(x) = \sum_{k=-\infty}^{+\infty} \hat{u}_k e^{ikx} \quad \text{and} \quad f(x) = \sum_{k=-\infty}^{+\infty} \hat{f}_k e^{ikx}$$

$$= -(e^{ikx})^{4} = K^{2}e^{iKx}$$

=>
$$N^2 \tilde{u}_N = \hat{S}_N$$
 $u=0$

be careful! => $u(x) = \underbrace{\sum_{k=-\infty}^{\infty} \hat{S}_k}_{N^2} e^{ikx}$ ($\hat{S}_N = 0$)

In practice, we can develop feat and occurred numerical approximations to u(x) by computing Fourier coeffs of F and formany the truncated Fourier series for a.

Fourter Spectre Method

1) Compute
$$\hat{f}_{k} \approx \frac{1}{N} \sum_{j=1}^{N} f(2n_{N}^{j}) e^{ik} \frac{2n_{N}^{j}}{N} = \hat{f}_{k}$$

for N:-N,-,N

2) Compute
$$u(2nix) \approx \frac{N}{5} \frac{x}{k^2} e^{ik^2ni/N}$$

for i=1,...,N

- => Output is approximate samples of u(xi)
 on good 2014, , , 201
- 2) Exponentially accurate for smooth pertocte right-hund sides.
- =) Both steps require only O(NbyN) withheter operations using FF7.

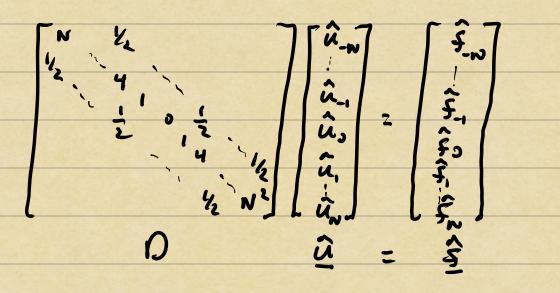
This is fine for constant coefficient problems, but what about variable coefficients?

What is the fourier searces of coscasiulas?

$$= \sum_{k=-\infty}^{+\infty} \frac{\hat{n}_{k}}{2} \left(e^{i(k+1)x} + e^{i(k-1)x} \right)$$

=>
$$\kappa^2 \hat{u}_n + \frac{1}{2} (\hat{u}_{n+1} + \hat{u}_{n-1}) = \hat{f}_n, \quad \kappa_2 = 0, 21, 22, ...$$

For a numerical scheme, we need to truncate and solve 2NH & 2NH system of eggs.



 $u(x) = \sum_{k=-N}^{N} \tilde{u}_k e^{ikx}$ $x \in [0, 2\pi]$

=> Compute (Fu?u=N m O(NbyN) FLOP; mith ffi-bused trap rule.

=> Solve totologonal system in O(N) FLOBs with bunded Cholesky (Gauss. ethn.)

=) Evaluate uls) on equispaced good in Oldleger) with FF7.

The key & HAB scheme betry first is that the linear system for the Fourter weth was bunded (Artotragenet). "fest" = regumes FLDPS ~ NbozN

What about more general coefficients, like C(x): exp(-cos²(x))?

Idea: If coeffs are smooth and pertodie, theor system never gets "too expensive."

Multiphrendon in Fourter Space"

Consider the multipolisation of his fundaments of and a given by Fourter series:

What is the Fourier serves for [59] (B)?

$$[\mathcal{G}_{g}](\theta) = (\mathcal{E}_{x=-\infty}^{\mathfrak{S}_{n}} e^{ik\theta}) (\mathcal{E}_{x=-\infty}^{\mathfrak{S}_{n}} e^{ik\theta})$$

$$= \mathcal{E}_{g}^{\mathfrak{S}_{n}} [\mathcal{E}_{x=-\infty}^{\mathfrak{S}_{n}} e^{ik\theta}] e^{ik\theta}$$

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$$z = \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \widehat{g}_{n} e^{i(k+j)\theta}$$

=
$$\mathcal{E}\left(\mathcal{E}, \hat{\mathcal{G}}_{\mathbf{k}}\right) e^{in\theta}$$

= $\mathcal{E}\left(\mathcal{E}, \hat{\mathcal{G}}_{\mathbf{k}}\right) e^{in\theta}$
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"Discrete Consolidon" = $\mathcal{E}\left(\mathcal{E}_{\mathbf{k}}\right)$

To multiphy two fourter series, we perform a discrete combibbles on their Fourier welfs.

"Toeplitz" Operator/Matrix

Note that if Isk & Me-alk!, the matrix entires deany exponentially away from the diagonal.