CS 5112

Algorithms

Slack

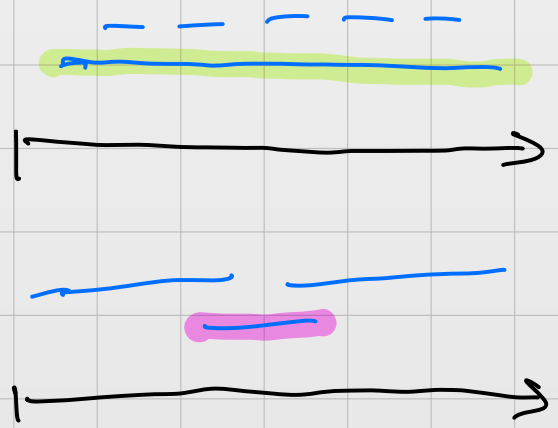
Website

Interval Scheduling

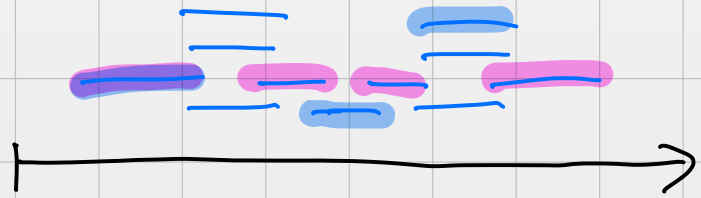
Greedy Algorithms

1. Earliest start time (FCFS)
   (not maximal)

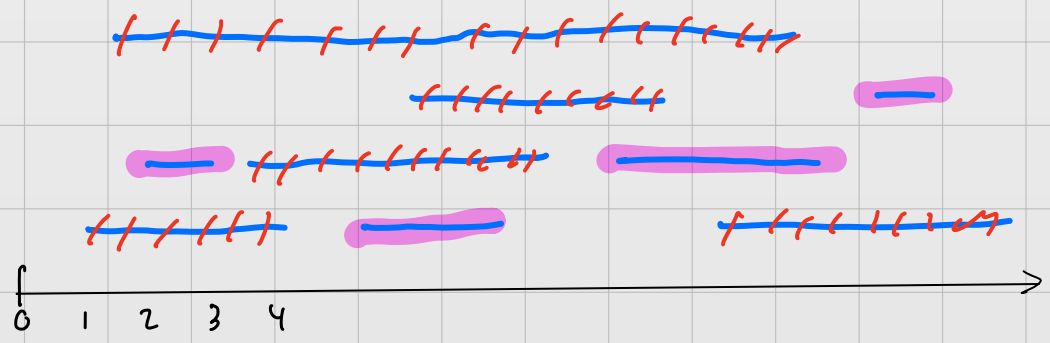
2. Shortest job
   (not maximal)



3. Fewest conflicts (not maximal)



4. Earliest finish time (looks promising)



Intuition: Choose the job that frees the resource as soon as possible

EFT Greedy Algorithm

FT order

$O(n \log n)$ → Sort all the jobs by finish time. $(J_1, \ldots, J_n)$

Loop is $O(n)$

- $\emptyset$ → Start w/ J_1
- Then for J_2, if $(s_2 < f_1)$, then discard. $(J_i = (s_i, f_i))$
- Repeat until we don't discard some J_i, replace J_1 w/ J_i, and start over at $\emptyset$.

Running time is $O(n \log n)$

EFT Greedy Algorithm

Is the output compatible (no overlapping jobs)?

YES!

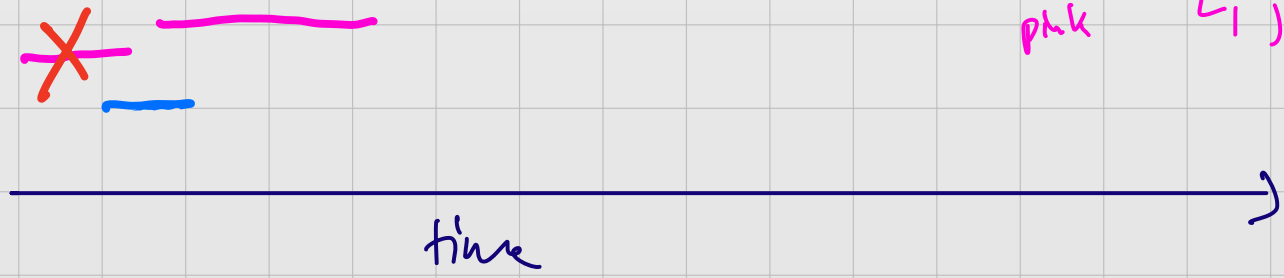
Is it maximal?

Can't happen b/c EFT

J_1 finishes before L_1

EFT blue: $J_1, J_2, \ldots, J_k$

Other pink: $L_1, L_2, \ldots, L_m$

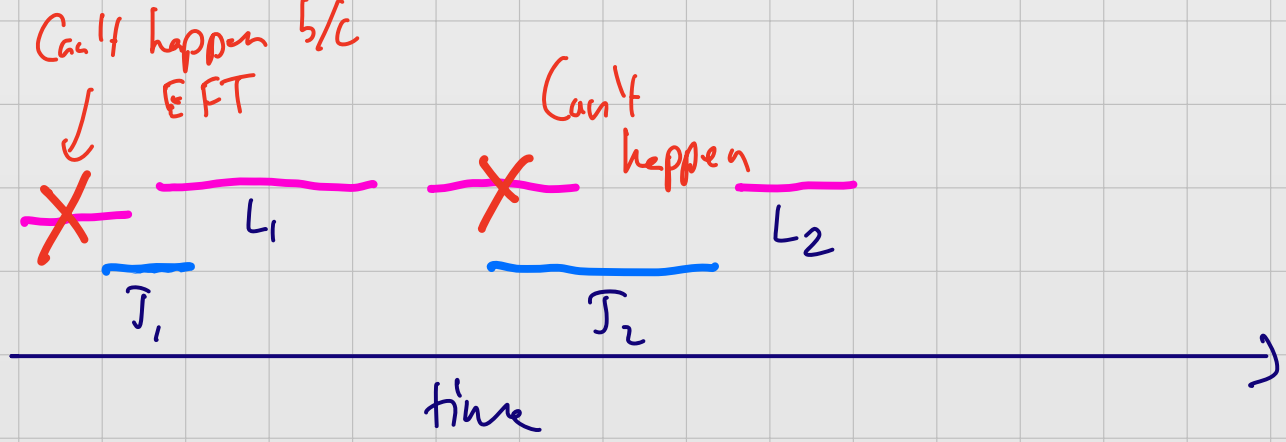


EFT Greedy Algorithm

J_1 finishes before L_1
J_2 finishes before L_2
J_3 finishes before L_3
⋮

EFT blue: $J_1, J_2, \ldots, J_k$

Other pink: $L_1, L_2, \ldots, L_m$



EFT Greedy Algorithm

J_1 finishes before L_1
J_2 finishes before L_2
J_3 finishes before L_3
⋮

EFT alg: $J_1, J_2, \dots, J_k$

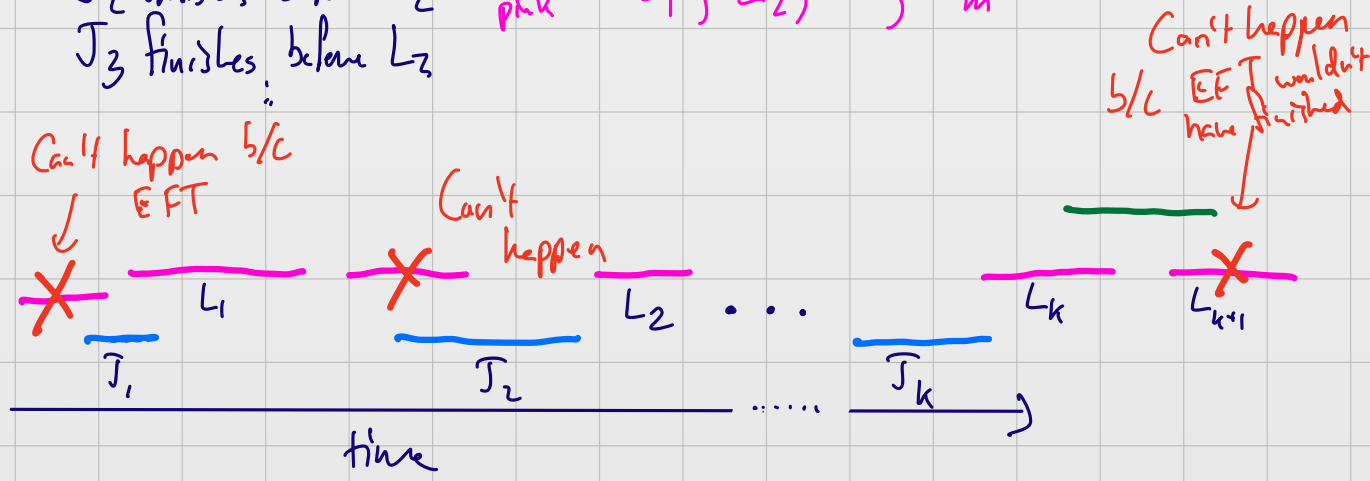
Other pick: $L_1, L_2, \dots, L_m$

EFT Greedy Algorithm

EFT blue: $J_1, J_2, \ldots, J_k$

Other pink: $L_1, L_2, \ldots, L_m$

J_1 finishes before L_1
J_2 finishes before L_2
J_3 finishes before L_3
⋮



$\Rightarrow$ There are not more jobs in other set than EFT.

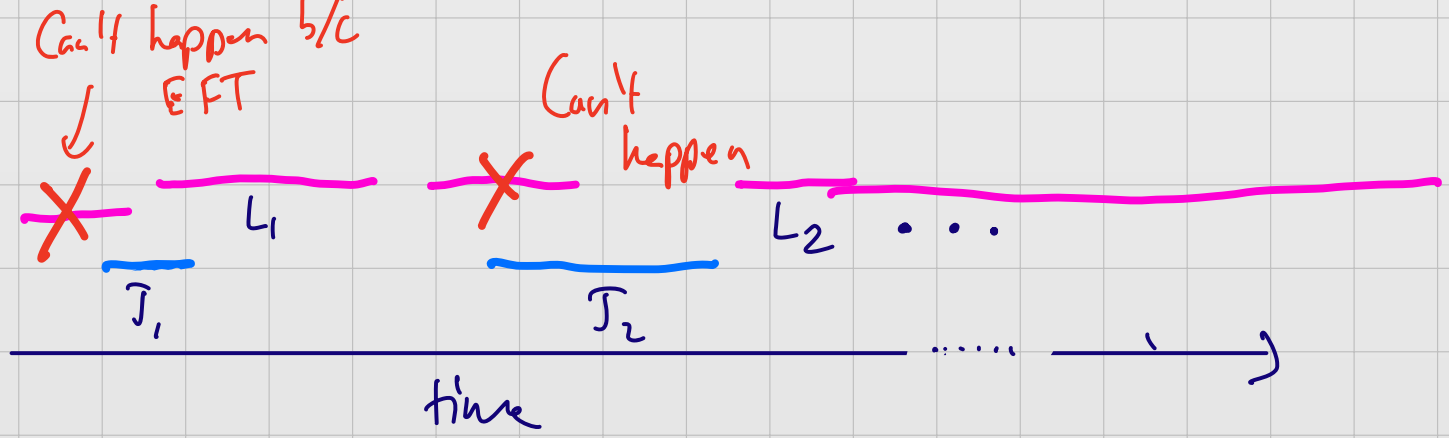
$\Rightarrow$ EFT is maximal.

EFT Greedy Algorithm

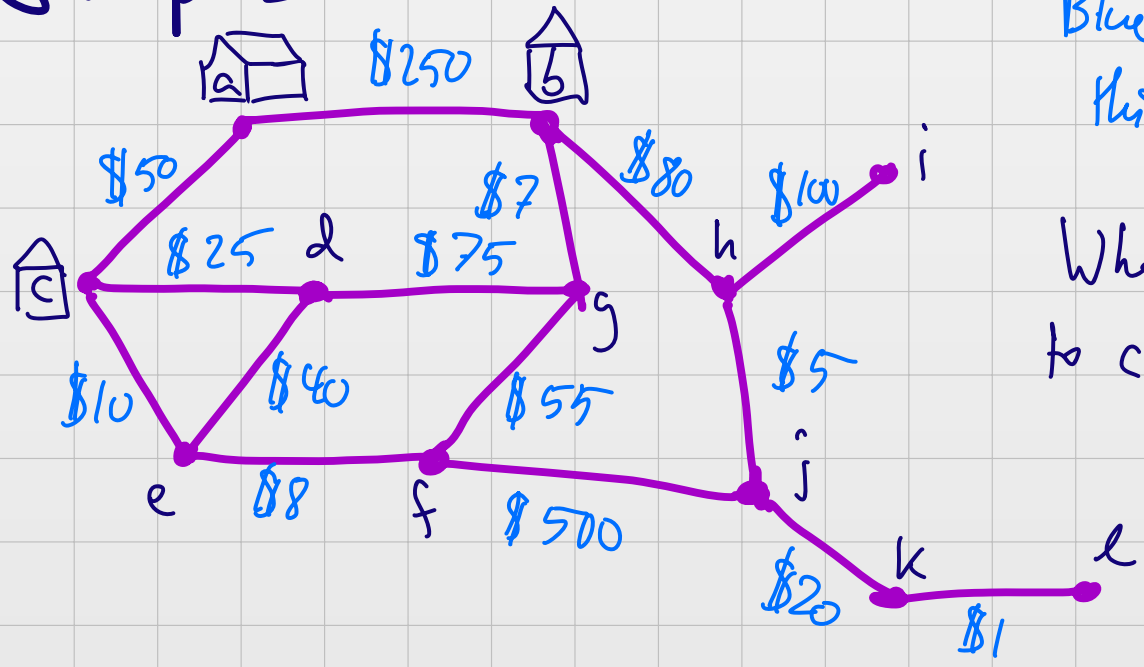
J_1 finishes before L_1
J_2 finishes before L_2
J_3 finishes before L_3
⋮

EFT blue: $J_1, J_2, \ldots, J_k$

Other pink: $L_1, L_2, \ldots, L_m$



Graphs



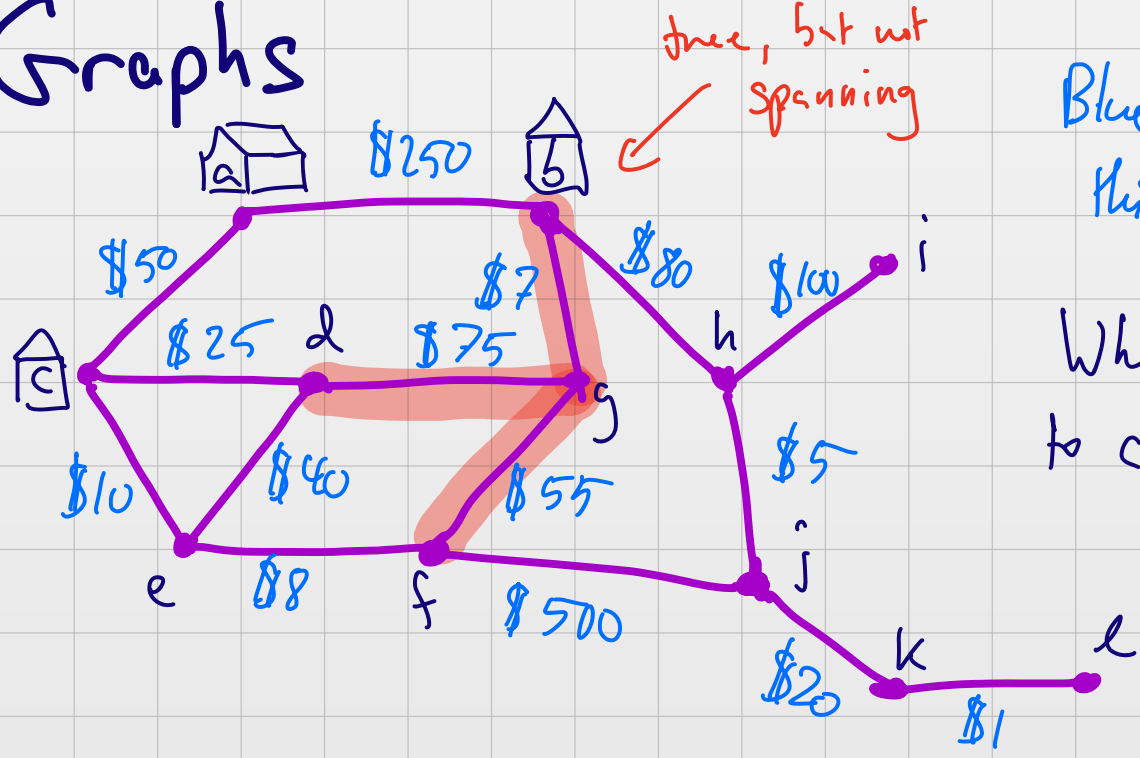
Blue are the weights of this graph.

What is the cheapest way to connect all the nodes?

Minimum Spanning Tree

A graph $G = (V, E)$ $\quad E \subseteq V^2$

Graphs



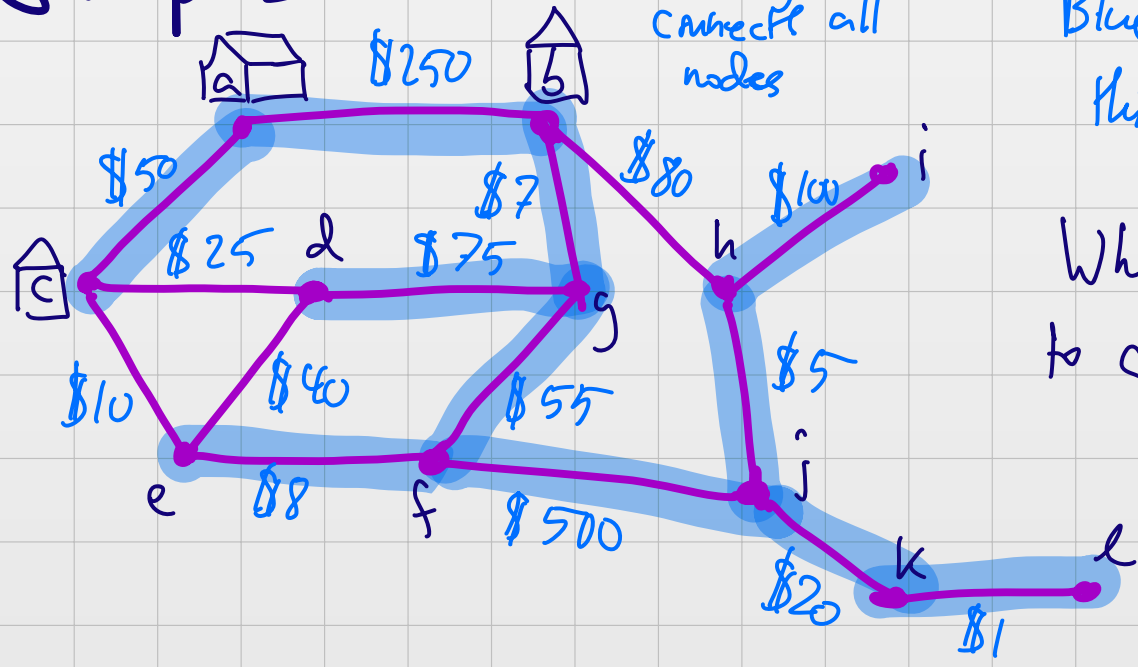
Blue are the weights of this graph.

What is the cheapest way to connect all the nodes?

Minimum Spanning Tree

A graph $G = (V, E)$ $\quad E \subseteq V^2$

Graphs



Spanning tree connects all nodes

Blue are the weights of this graph.

What is the cheapest way to connect all the nodes?

Minimum Spanning Tree

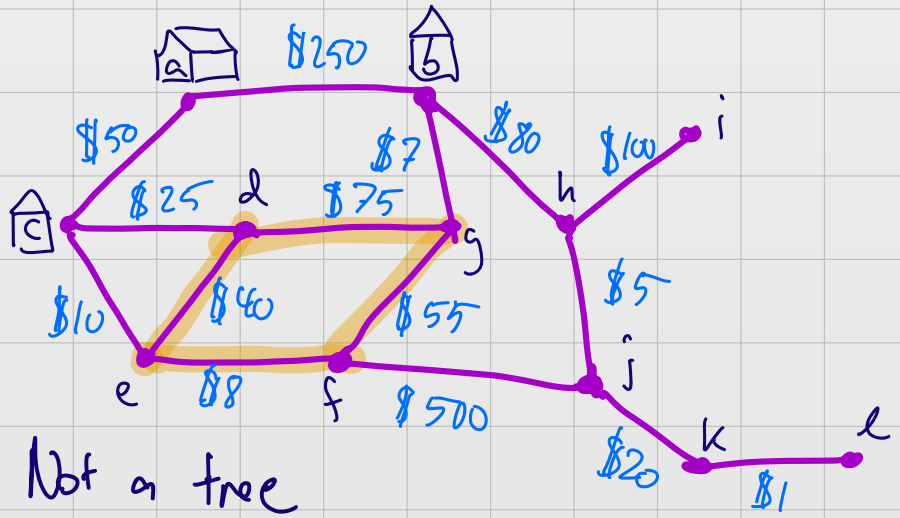
A graph $G = (V, E)$ $\quad E \subseteq V^2$

Assumptions: 1. Positive integer weights

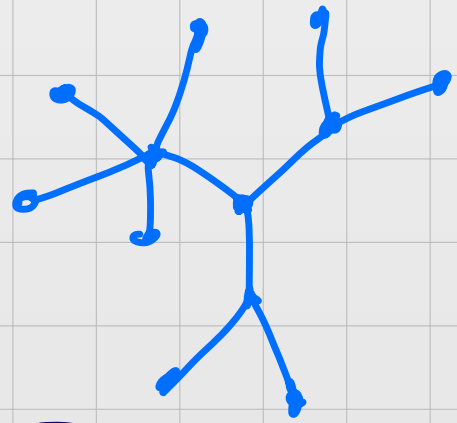
2. Distinct weights

Minimum Spanning Tree (MST)

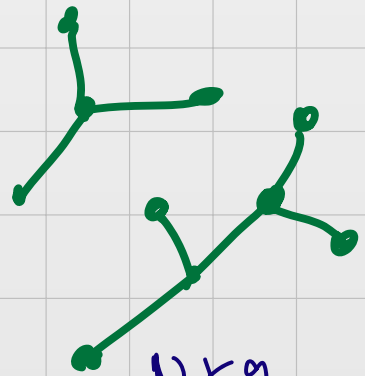
A tree is a connected graph w/o cycles.
A cycle is a path that begins and ends at the same node



Not a tree



Is a tree



Not a tree

not connected.