

5112

10-24

Models

Why algorithms are [not] useful?

RAM Model

An array of memory 

Have some registers 

Operations: Load a word from memory into a register
Store " " from a register into memory
Arithmetic: $+$, $-$, \times , \div , maybe other things. } each cost $\Theta(1)$

What make a good model?

A good model predict performance.

1. Accurate
 2. Useable
- } in tension with each other

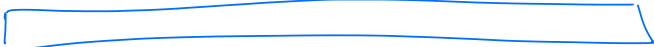
RAM Model and Memory



Small / very fast  L1 cache

 L2 cache

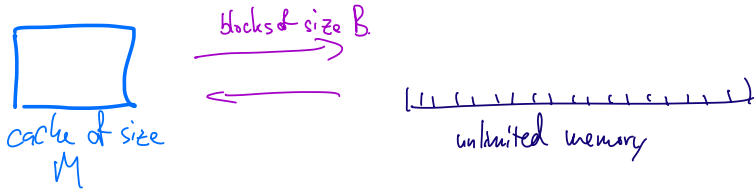
large and "slow"  L3 cache

 DRAM

Cell Probe Model

Same as the RAM model, but all operations on registers are free.

External Memory Model



Everything in cache is free.

Cost is the # of transfers.

Asymptotic Analysis.

Accurate? Useful?

Is it accurate? It says something about performance of "large enough" inputs.

$$\begin{matrix} w_1^T \\ w_2^T \\ w_3^T \end{matrix} \begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \end{bmatrix} \begin{matrix} v_1 & v_2 & v_3 \\ \begin{bmatrix} | & | & | \end{bmatrix} \end{matrix} = \begin{bmatrix} v_1 \cdot w_1 & v_2 \cdot w_1 & v_3 \cdot w_1 \\ v_1 \cdot w_2 & v_2 \cdot w_2 & v_3 \cdot w_2 \\ v_1 \cdot w_3 & v_2 \cdot w_3 & v_3 \cdot w_3 \end{bmatrix}$$

cost of $\Theta(n)$

schoolbook algorithm $\Theta(n^3)$.

Models of Data

Example: sorting

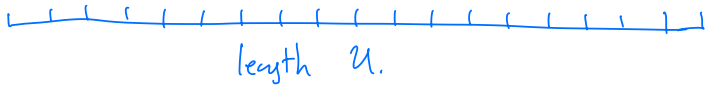
"Sorting costs $\Theta(n \log n)$ "

Implicitly in the comparison model.

But what if you are sorting integers?

Give me n integers $\leq U$.

A



For each key k , put it into the k^{th} entry ($A[k]$)

Then read through the array.

Cost : $O(n + U)$

Counting Sort

Radix Sort

3 decimal integers

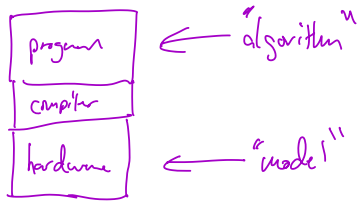
Start with least significant digit.

	109	351	464	521	700	718	
sort by	700	351	521	464	718	109	sort stably by LSD
→	700	109	718	521	351	464	" " " middle digit
2nd digit	109	351	464	521	700	718	" " " LSD

Suppose the radix is b . n/b per round and $\log_b n$ rounds

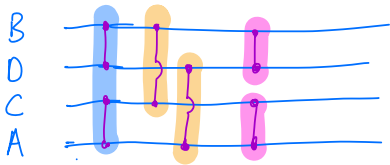
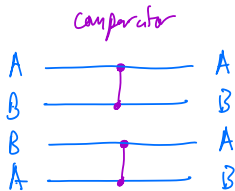
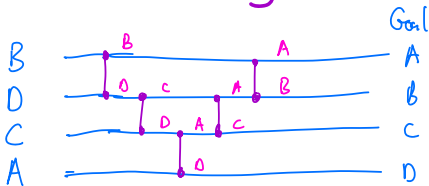
Cost: $O(n \log_b n)$

"Usual Algorithms Universe"



Can also look at hardware algorithms

Sorting Networks



↑
output in
sorted order