Retro-Causal Optimization in Expanding Graph Topologies: Anomalies of Apparent Intent and Precocious Maturity

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Abstract

We present a formal mathematical framework for a "Retro-Selective" causal network: a directed tree graph where the root acts as a fixed observational anchor (the Present) and the leaves represent a growing frontier of initial conditions (the Past). Unlike standard forward-chaining stochastic processes, this model employs a root-centric optimization that effectively resamples the entire history at each step of expansion. We derive two primary geometric anomalies inherent to this topology: (1) *The Collapse of Historical Entropy*, where the surviving history exhibits asymptotic efficiency indistinguishable from determinism, and (2) *The Linear Growth of Teleological Bias*, where the initial conditions appear statistically fine-tuned to target the observer with a precision that scales linearly with the depth of the system.

1 Introduction

Standard causal models typically assume a fixed initial condition t_0 and evolve forward into a branching future. We propose an inverted architecture: a fixed terminal state (the Root, ρ) and a branching set of potential origins (Leaves, Λ). The system evolves by extending the depth of the leaves, effectively adding "past" to the structure.

This paper formalizes the selection mechanism within this structure as a thermodynamic optimization problem. We demonstrate that fixing the observer (ρ) while expanding the causal horizon ($t \to \infty$) forces the selected history to occupy an increasingly negligible volume of the phase space, generating artifacts of "instant evolution" and "fine-tuning."

2 Formal Framework

2.1 Topological Definitions

Let the universe at iteration t be defined as a directed tree graph $\mathcal{T}_t = (V_t, E_t)$.

Definition 1 (The Fixed Observer). *The root node $\rho \in V_t$ is the unique terminal node of all paths. It represents the static "Present" or the state of observation. Its position is invariant.*

Definition 2 (The Expanding Frontier). *Let $\Lambda_t \subset V_t$ be the set of leaf nodes at depth t. These represent the manifold of possible initial conditions (the "Big Bang") at iteration t.*

Definition 3 (Causal History). *For any leaf $u \in \Lambda_t$, there exists a unique directed path γ_u connecting u to ρ. The set of all possible histories is $\Gamma_t = \{\gamma_u \mid u \in \Lambda_t\}$.*

2.2 The Measure of Action

We assign a scalar cost (or action) $S(e) \in \mathbb{R}^+$ to every edge $e \in E_t$. The total action of a history is the sum of its edge costs:

$$\mathcal{S}(\gamma_u) = \sum_{e \in \gamma_u} S(e) \tag{1}$$

We assume edge costs are independent and identically distributed (i.i.d.) random variables drawn from a distribution with mean μ and variance σ^2.

2.3 The Selection Mechanism

The selection of the "actual" history is governed by a Boltzmann distribution conditioned on the fixed root. The weight of a history is given by the propagator:

$$W(\gamma_u) = e^{-\beta \mathcal{S}(\gamma_u)} \tag{2}$$

where $\beta > 0$ represents the selection pressure (inverse temperature).

The partition function Z_t represents the total probability amplitude arriving at the root:

$$Z_t = \sum_{u \in \Lambda_t} e^{-\beta \mathcal{S}(\gamma_u)} \tag{3}$$

The probability that a specific leaf u is the true initial condition is given by the Gibbs measure:

$$P_t(u) = \frac{1}{Z_t} e^{-\beta \mathcal{S}(\gamma_u)} \tag{4}$$

3 Dynamics and Evolution

The system evolves via a recursive process $\mathcal{T}_t \to \mathcal{T}_{t+1}$.

1. **Global Extension:** Every leaf $u \in \Lambda_t$ branches into k new nodes $\{v_1, \ldots, v_k\}$. The depth of the tree increases by 1.
2. **Resampling:** The edge costs for the new branches are realized. The partition function Z_{t+1} is recalculated.
3. **Optimization:** The "Actual History" H_{t+1} is identified as the Maximum Likelihood Estimate (MLE) of the new distribution:

$$H_{t+1} = \operatorname*{argmax}_{u \in \Lambda_{t+1}} P_{t+1}(u) \tag{5}$$

4 Derivation of Anomalies

4.1 Theorem 1: The Collapse of Historical Entropy

Hypothesis: As the system grows, the ambiguity of the past vanishes, creating an illusion of determinism.

Let $H(P_t)$ be the Shannon entropy of the distribution of initial conditions:

$$H(P_t) = -\sum_{u \in \Lambda_t} P_t(u) \log P_t(u) \tag{6}$$

Theorem 1. *For sufficiently large β, the entropy rate of the history vanishes as $t \to \infty$.*

Proof. Consider the path γ^* with minimal action $\mathcal{S}_{min}$. The probability ratio between the optimal path and any suboptimal path γ' scales as:

$$\frac{P(\gamma^*)}{P(\gamma')} = e^{-\beta(\mathcal{S}_{min}-\mathcal{S}')} \tag{7}$$

Let $\Delta S_t = \mathcal{S}' - \mathcal{S}_{min}$. Since $\mathcal{S}$ is a sum of i.i.d. variables, by the Law of Large Numbers, the accumulated difference ΔS_t grows linearly with t. Thus, the ratio diverges exponentially:

$$\lim_{t\to\infty} \frac{P(\gamma^*)}{P(\gamma')} \to \infty \tag{8}$$

Consequently, the probability mass concentrates entirely on the single optimal trajectory. $P_t(u) \to \delta(u - u^*)$, and $H(P_t) \to 0$. □

Implication (Precocious Maturity): An observer at ρ looking back at $t \gg 1$ sees a history devoid of randomness. The selected path appears to have "evolved" instantly to the optimal configuration, skipping the stochastic exploration phase inherent in the generating process.

4.2 Theorem 2: The Linear Growth of Teleological Bias

Hypothesis: The initial conditions appear fine-tuned to target the root.

We define the **Anomaly Score** $\mathcal{A}(t)$ as the Pointwise Mutual Information (PMI) between the selected origin u^* and the root ρ, relative to a non-selective random walk.

$$\mathcal{A}(t) = \log\left(\frac{P_{\text{select}}(u^*|\rho)}{P_{\text{random}}(u^* \to \rho)}\right) \tag{9}$$

Theorem 2. *The Anomaly Score $\mathcal{A}(t)$ scales linearly with the depth of the system.*

Proof. From Theorem 1, in the limit of high selection, $P_{\text{select}}(u^*|\rho) \approx 1$. In a random walk on a k-ary tree, the probability of a specific leaf connecting to the root is simply the inverse of the number of leaves:

$$P_{\text{random}}(u^* \to \rho) = \frac{1}{|\Lambda_t|} = k^{-t} \tag{10}$$

Substituting these into the Anomaly equation:

$$\mathcal{A}(t) \approx \log\left(\frac{1}{k^{-t}}\right) = \log(k^t) = t \cdot \log(k) \tag{11}$$

□

Implication (Apparent Intent): The "improbability" of the observed history is not constant; it is a linear function of time. A system of depth 1000 does not look 10 times more fine-tuned than a system of depth 100; it looks exponentially more fine-tuned (k^{1000} vs k^{100}). This manifests as an overwhelming "Teleological Bias," where the initial conditions appear to possess prior knowledge of the Root's location.

5 Conclusion

We have shown that a Retro-Selective Causal Network generates a history that is structurally distinct from the process that creates it. By fixing the Root and expanding the Past, the system effectively acts as a **Maxwell's Demon** on its own history, filtering out high-entropy paths. The resulting observable timeline exhibits:

- **Hyper-Efficiency:** The surviving path minimizes action globally, appearing “smarter” than any local decision agent could be.
- **Inverse Luck:** The probability of the observed history occurring by chance vanishes as k^{-t}, creating the illusion of a designed or scripted universe.