1 Some students asked me about symmetric and antisymmetric relation when $a=b$. These two ~~rle~~ relations tell us how distinct elements relate with one another.

Symmetric relation: Distinct elements that are related in one direction must also be related in the other direction.

* If $a \neq b$ and $a \mathcal{R} b$, then $b \mathcal{R} a$.

Antisym. relation: Distinct elements that are related in one direction can not be related in the other direction

* If $a \neq b$ and $a \mathcal{R} b$, then $b \not\mathcal{R} a$, (equivalently

* If $a \mathcal{R} b$ and $b \mathcal{R} a$, then $a = b$)

2a What if $b=c$ in transitivity?

Suppose $\mathcal{R} = \{(a,b), (b,b)\}$ is $\mathcal{R}$ transitive?

Yes $\mathcal{R}$ is transitive, since (a,b) & $\overset{(b,\,c)}{(b,b)} \Rightarrow (a,c) = (a,b)$ which already exist in $\mathcal{R}$.

2b Is the relation $\mathcal{R} = \{(2,2), (3,1), (4,5)\}$ on $A = \{1,2,3,4,5\}$ transitive?

Yes. Although the hypothesis in the definition of transitive relation (If (a,b) & (b,c) exist in $\mathcal{R}$) is missing, the relation is vacuously (trivially) transitive.

Exercises

1 Let $A = \{1, 2, 3, 6\}$ and $\mathcal{R}$ on A defined as $\mathcal{R} = \{(a,b) : a \text{ divides } b\}$.

a List all the elements of $\mathcal{R}$.

b Determine if $\mathcal{R}$ is i) reflexive, ii) symmetric iii) antisymmetric iv) & transitive. Justify your answers.

2 Let $\mathcal{R}$ be defined on $\mathbb{N}$ as $\mathcal{R} = \{(a,b) : a < b\}$. Is $\mathcal{R}$

i) Is ~~$\mathcal{R}$~~ reflexive ii) symmetric iii) antisym. iv) transitive? Justify your answers

3 Let A be a finite set & $P(A)$ the power set of A. Suppose we define rltn $\mathcal{R}$ on $P(A)$ as $\mathcal{R} = \{(X,Y) : X \subseteq Y\}$ where X, Y are subsets of A. Is $\mathcal{R}$ an equivalence relation

4 Is $\mathcal{R}$ on $\mathbb{Q} \times \mathbb{Q}$ given as $\mathcal{R} = \{(x,y) : x - y \in \mathbb{Z}\}$. Is $\mathcal{R}$ an equivalence rltn.

5 Let A consist of all all straight lines in the cartesian plane $\mathbb{R} \times \mathbb{R}$. We define $\mathcal{R}$ on A as $\mathcal{R} = \{(y_i, y_j) : y_i \parallel y_j\}$ (is parallel to), that is if $y_i = m_i x_i + c_i$ and $y_j = m_j x_j + c_j$, then $y_i \mathcal{R} y_j \iff m_i = m_j$

Is $\mathcal{R}$ an equivalence rltn?

6 Show that the following are equivalence relations on $\mathbb{Z}$.

i $\mathcal{R}_3 = \{(a,b) : a - b \in 3\mathbb{Z}\}$ i.e $a \mathcal{R} b \iff a - b = 3q,\ q \in \mathbb{Z}$.

ii $\mathcal{R}_4 = \{(a,b) : a - b \in 4\mathbb{Z}\}$

iii $\mathcal{R}_5 = \{(a,b) : a - b \in 5\mathbb{Z}\}$

iv $\mathcal{R}_{10} = \{(a,b) : a - b \in 10\mathbb{Z}\}$

7 Determine if $\mathcal{R}$ on $M_n(\mathbb{R}) = \{n \times n \text{ matrices with real entries}\}$ is an equivalence relation when $A \mathcal{R} B$ if there exists invertible matrix P such that $A = P^{-1} B P$.

8 Is $\mathcal{R}$ on $\mathbb{Z}$ such that $m \mathcal{R} n$ iff $m | n$ (i.e $\frac{n}{m} \in \mathbb{Z}$) an equivalence relation?