Exercises

I The following questions test your understanding of relationships between sets and their membership.

1 Let $A \subseteq B$.

a Given $x \in A$, x _____.

b Suppose $x \notin B$, then _____.

c If $x \notin A$, what can you conclude or not about the membership of x in B.

d Suppose $x \in A$, then x must not be in _____.

e Given $x \in B \backslash A$, x cannot be in _____.

f If B is finite, then A must be _____.

g If $A \neq \emptyset$, then B must be _____.

h ~~If~~ Suppose $A = \emptyset$, then B _____.

i Suppose every elements of A is even, what can you conclude about elements in B?

j ~~Let~~ Suppose $B \subseteq C$, then when $x \in A$ x is also in _____ & _____.

x. Let $A \subseteq B$ and $B \subseteq A$. What can you say about sets A & B in relation to each other.

2a Let $A = \{x \in \mathbb{Z} : x \text{ is even}\}$. If $y \in A$, then y is _____.

b; Let $A = \{x : \text{property } P \text{ holds}\}$. If $z \in A$, then _____.

" Suppose $w \in A^c$, then _____.

3a Let $A = \{x \in \mathbb{R} : x \in (2,5]\}$ and let $y = \frac{z}{6}$

a Mention an odd integer z such that $y \in A$.

b "   "   "   " z "   " $y \in A^c$.

4 Let $\mathcal{U} = \mathbb{R}$ and $A = (1,4)$. ~~If~~ Suppose $x \notin A$, then ~~which~~ x ~~is in~~ the two largest disjoints subsets of $\mathbb{R}$ that can contain x are:

5 Let $|A| = n < \infty$ and $B \subset P(A)$. What can you say about cardinality of B.

II. Use the method of direct proof to show that the following conditional statements are true.

1. If $x \notin A$, then $x \notin A \cap B$
2. If $x \in A \cap (B \cup C)$, then $x \in A$.
3. If $A \subseteq B^c$, then A and B are disjoint
4. If $x \in A$, then $x \in A \cup (B \cap C)$
5. If $A \subseteq B$ and $x \in A \cap C$, then $x \in B$.
6. If $x \in (A \cup B)$ and $x \notin A$, then $x \in B$.

III. Let $\{m\mathbb{Z} : m \in \mathbb{Z}^{>0}\}$ be a family sets where $m\mathbb{Z}$ is the set of integers that are multiples of m, that is
$$m\mathbb{Z} = \{mq : q \in \mathbb{Z}\}$$
$$= \{\ldots, -4m, -3m, -2m, -m, 0, m, 2m, 3m, 4m, 5m, 6m, \ldots\}$$

Find

1. $\bigcup\limits_{n \in \{5, 10, 20\}} n\mathbb{Z}$

2. $\bigcap\limits_{n \in \{5, 10, 20\}} n\mathbb{Z}$

3. $\bigcup\limits_{n \in \{2, 4, 8, 16\}} n\mathbb{Z}$

4. $\bigcap\limits_{n \in \{2, 4, 8, 16\}} n\mathbb{Z}$

5. $\bigcup\limits_{n \in \{2, 3, 6\}} n\mathbb{Z}$

6. $\bigcap\limits_{n \in \{2, 3, 6\}} n\mathbb{Z}$

7. $\bigcup\limits_{n \in \{3, 5\}} n\mathbb{Z}$

8. $\bigcap\limits_{n \in \{3, 5\}} m\mathbb{Z}$

9. $\bigcup\limits_{m \in \mathbb{Z}} m\mathbb{Z}$

10. $\bigcap\limits_{m \in \mathbb{Z}} m\mathbb{Z}$