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ORIGINAL ARTICLE

Static and free vibration analysis of sandwich shells with double curvature considering the effects of transverse normal strain

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ABSTRACT
Higher-order closed-form solutions for the static and free vibration problems of sandwich shells with double curvature are presented in the current study based on a new hyperbolic shell theory considering the effects of transverse normal strain. A theory involves six unknowns and satisfies traction-free boundary conditions at the top and the bottom surfaces of the shell. The theory does not require a shear correction factor to account for the strain energy due to the shear deformation effect. A shell consists of three layers, wherein the top and the bottom layers, i.e. face sheets are made up of hard material and the middle layer, i.e. core is made up of soft material. The governing equations and associated boundary conditions of the theory are produced by employing Hamilton's principle. Semi-analytical closed-form solutions for the static and free vibration problems are produced by the Navier technique for simply supported boundary conditions of the shell. The present results are compared with results that have already been published to confirm the accuracy and efficacy of the current higher-order hyperbolic shell theory.

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KEYWORDS
Static and free vibration analysis; sandwich shells; double-curvature; hyperbolic shell theory; transverse normal strain

1. Introduction

Fiber-reinforced polymer composites are the most important kind of composite material. The most significant characteristics of fiber-reinforced polymer composite materials are their high strength-to-weight and stiffness-to-weight properties. Consequently, these are being employed more frequently in a variety of technical applications. Lightweight composite panels called laminated sandwich shells have a soft inner core between two thin, stronger face sheets. Its function is to transfer loads from the face sheets to the core structure, and if it fails, the structure will no longer function. Inflicted stresses on sandwich structures are distributed between the face sheets and the core structure according to their material properties and thicknesses. Sandwich panels are frequently utilized in a variety of engineering fields, including civil, mechanical, aerospace, marine, and offshore. The classical shell theory (CST) and Mindlin's first-order shear deformation theory [1] (FSDT) are not capable to capture the accurate bending and vibration behavior of sandwich structures due to their limitations. Therefore, researchers have developed two-dimensional approximate shear deformation theories which consider the effects of transverse shear and normal deformations which are more pronounced in the sandwich structures. Pagano [2] has developed the three-dimensional elasticity solutions for the bi-directional bending analysis of sandwich plates which is further used by many researchers as a reference solution for the comparison of the numerical results obtained using approximate theories. Bhimaraddi [3] presented polynomial-type higher-order shear deformation theory for the vibration analysis of cylindrical shells. Reddy [4] developed the well-known third-order plate theory for the analysis of laminated composite plates and shells which satisfies the zero tangential traction boundary conditions on the top and the bottom surfaces of the plate/shell. Reddy [5] presented the exact solutions for cross-ply laminated shells using numerical methods like the finite element method and the finite difference method. Mallikarjuna and Kant [6] provided a critical review and some results of recently developed refined theories of fiber-reinforced laminated composites and sandwiches and this review is limited to linear free vibration and transient dynamic analyses, and geometric nonlinear transient response of multilayer sandwich/fiber-reinforced composite plates. Similarly, Bhimaraddi [7] presented the three-dimensional elasticity solution for the static analysis of doubly curved shells. Soldatos and Timarci [8] presented certain general functions of the transverse coordinates into the shell displacement approximation, thereby accounting for the transverse shear deformation effects. Rikards and Chate [9] presented a finite element method based on single-layer theory for frequency analysis of sandwich shells as well as laminated composite shells. Ferreira et al. [10] presented a Non-linear analysis of sandwich shells and the effect of core plasticity using first-order shear deformation theory based on the finite element method and using the Ahmad shell element with five degrees of freedom per node. Mourtiz et al. [11]

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described composite shell structures are widely used in different engineering sectors for many years, including the naval, aerospace, automotive, and construction sectors, as well as sporting goods, medical devices, and many other areas.

Kant and Swaminathan [12] presented analytical solutions for the static analysis of laminated sandwich plates using higher-order refined theory based on Navier's solution technique. The theoretical model presented by the author incorporates laminate deformations which account for the effects of transverse shear deformation. Hohe and Librescu [13] presented a nonlinear theory for doubly curved anisotropic sandwich shells with a transversely compressible core using an advanced geometrically nonlinear shell theory of doubly curved structural sandwich panels with the transversely compressible core is presented based on Kirchhoff's theory. This theory accounts for dynamic effects as well as for initial geometric imperfections. Zhong and Reimerdes [14] presented the stability behavior of cylindrical and conical sandwich shells with flexible cores using a higher-order theory based on a three-layer model and solved by numerical integration. Khare et al. [15] presented solutions for thick laminated sandwich shells using higher-order theory based on closed-form solutions. Closed-form formulations of 2D higher-order shear deformation theory are presented for the thermo-mechanical and free vibration analysis of simply supported, cross-ply, laminated sandwich, doubly thick curved shells. Results on static and dynamic problems of double-core sandwich shells are not presented in the paper. Garg et al. [16] presented Solutions for free vibration of laminated composite and sandwich shells using higher-order closed-form. It described free vibration characteristics of simply supported, laminated cross-ply, composite, and sandwich shell panels using the various higher-order theories, which account for the effects of transverse shear strains/stresses and the transverse normal strain/stress. The results of a multi-layered sandwich shell analysis are not presented by the authors. Turkin [17] presented a technique for calculating the rational design parameters of a sandwich shell with an account of thermal loading using the nonlinear theory of thin elastic shells. Baba [18] presented how debonding and curvature change the natural frequency and related mode forms of FG sandwich curved beams. Ghugal and Sayyad [19] investigated flexural stress analysis of square laminated plates subjected to the line and parabolic loads using trigonometric shear deformation theory with transverse normal strain effects. Sayyad and Ghugal [20] presented transverse shear deformation, transverse normal strain, and localized stress concentration that affect in-plane normal and transverse shear stresses through the thickness of simply supported orthotropic and laminated plates. Khalili et al. [21] presented an impact analysis of the cylindrical composite sandwich shells using a high-order sandwich shell theory where the interaction between the impactor body and the sandwich shell is described by using a spring-mass model. Sayyad and Ghugal [22] presented a new trigonometric shear and normal deformation theory to study the buckling behavior of orthotropic, transversely orthotropic, and cross-ply laminated composite plates. Dey and Ramachandra [23] presented an analysis of cylindrical sandwich panels exposed to combined static and dynamic non-uniform in-plane loading under linear and non-linear dynamic instability. Sayyad and Ghugal [24] presented a simple four-variable trigonometric shear deformation theory for the free vibration analysis of soft-core sandwich plates. Shinde and Sayyad [25–27] presented an analysis of laminated and sandwich spherical shells considering the effects of transverse shear and normal deformations. The authors have developed a fifth-order shear deformation theory involving nine unknowns. Zaitoun et al. [28] presented a free vibration analysis of a functionally graded sandwich plate resting on a viscoelastic foundation under a hygrothermal environment using a high-order shear deformation theory. The authors studied the effects of the damping coefficient, aspect ratio, volume fraction density, moisture and temperature variation, and thickness on the frequencies of sandwich plates. Vinh and Tounsi [29] presented the effects of nonlocal parameters on free vibration analysis of the functionally graded sandwich nanoplates using modified nonlocal elasticity theory. Bounouara et al. [30] presented a free vibration analysis of exponentially graded sandwich plates resting on Visco-Pasternak foundations using an improved integral trigonometric shear deformation theory. Bennedjadi et al. [31] presented the impact of visco-elastic foundation on the buckling response of exponentially-gradient sandwich plates under various boundary conditions using a refined shear deformation theory. Hadji et al. [32] presented the influence of porosity and elastic foundation parameters on the bending of functionally graded sandwich plates using the quasi-3D sinusoidal shear deformation theory. Tahir et al. [33] presented the effects of a three-variable viscoelastic foundation on the wave propagation analysis of functionally graded sandwich plates using a quasi-3D higher-order shear deformation theory involving four unknown variables. Zaitoun et al. [34] presented a hygrothermal buckling analysis of a functionally graded sandwich plate resting on a viscoelastic foundation using a higher-order shear deformation theory. Kouider et al. [35] presented a novel high-order shear and normal deformation theory for the static and free vibration analysis of sandwich plates considering the effects of transverse shear and normal strains. Xia et al. [36] presented a free vibration analysis of functionally graded porous truncated conical shell panels reinforced by graphene platelets (GPLs). The authors have studied the effects of three different types of porosity distribution and five different types of GPLs patterns on the dynamic response of the shell. Vinh and Tounsi [37] presented the nonlocal first-order shear deformation theory with variable nonlocal parameters for the free vibration analysis of the functionally graded doubly curved nanoshells. Huang et al. [38] presented the static stability analysis of doubly curved micro-panels resting on an elastic foundation by using a size-dependent model. Liu et al. [39] developed a nonlinear coupled mechanical model to investigate the nonlinear forced vibration behavior of rotating shells exposed to multi-harmonic excitation in a temperature environment. Liu et al. [40, 41] presented the nonlinear forced vibration analysis of functionally graded sandwich cylindrical shells with porosities resting on an elastic foundation. Liu et al. [42] presented the nonlinear forced vibration analysis of composite cylindrical shells under magneto-electro-thermo-mechanical loads. Liu et al. [43] investigated the impact response of shear deformable functionally graded sandwich cylindrical porous shells using the first-order shear deformation shell theory. Gao et al. [44] presented wave propagation in graphene platelets reinforced functionally graded plates integrated with piezoelectric actuator and sensor layers resting on an orthotropic visco-Pasternak medium in a magneto-electro-thermo environment. Li et al. [45] investigated amplitude-dependent damping characteristics of sandwich plates with a foam-filled hexagon honeycomb core.

1.1. Literature on CUF

The unified formulation developed by Carrera is now well-established in the literature and widely used by many researchers for the validation or verification of their studies. Sciuva and Carrera [46] presented the displacement model that has five generalized variables to describe the deformation of the laminated structures and to satisfy the static and geometric continuity constraints at the interfaces between the layers. Carrera [47] has developed mixed layer-wise theories for the analysis of symmetrically and unsymmetrically laminated, as well as sandwich, plates. The author employed RMVT to derive governing equations in terms of displacement and stress variables. Carrera [48] examined the effects of transverse normal stress on the vibration behavior of multilayered laminated and sandwich structures. The author has used a mixed plate model initially introduced by Toledano and Murakami. These models allow continuous interlaminar transverse shear and normal stresses as well as the zig-zag form of displacement distribution in the shell thickness directions. Carrera [49] presented modeling of single-layer and multilayer plate theories based on Reissner's mixed variational theorem (RMVT). The author has presented transverse shear stress distributions in layered plates using three-dimensional equations of equilibrium of the theory of elasticity. Carrera and Demasi [50] presented solutions to two benchmark problems of sandwich plates. (1) A sandwich plate loaded by harmonic transverse pressure distribution (2) a rectangular sandwich plate loaded by a transverse pressure located at the center. The theoretical formulation is done using different equivalent single-layer and layerwise theories based on CUF. Carrera [51] presented a historical review of the zig-zag theories that have been developed for the analysis of multilayered structures. Zig-zag theories describe a piecewise continuous displacement field in the plate thickness direction and fulfill Interlaminar continuity of transverse stresses at each layer interface. Carrera [52] in his study recommended that the analysis of any layered composite structures is meaningless unless and until the effects of the transverse normal strain are considered. The author presented the effects of transverse normal strain on the static thermoelastic analysis of homogeneous and multilayered plates using classical, refined, and advanced zig-zag plate theories developed using Carrera's unified formulation (CUF). Carrera and Petrolo [53] presented refined theories for beams with an increasing order of unknown variables in the displacement field based on CUF. The effectiveness of each expansion term, that is, of each displacement variable, has been established numerically considering various problems such as traction, bending, and torsion, as well as several beam sections such as square, annular, and airfoil-type.

2. Novelty and the objectives of the present study

It is well-known that the CST and FSDT cannot be effectively used for the analysis of sandwich structures, especially doubly-curved sandwich shells. Also, higher-order shell theories ignoring the effects of transverse normal strain are not accurate enough to capture the bending behavior of sandwich shells. The objectives of the present study are framed based on the shortcomings on sandwich shells with double curvature.

1. It is found that plenty of research articles have been published on the analysis of sandwich plates. However, it is found that limited attempts were made for the analysis of doubly-curved sandwich shells with laminated composite face sheets and the transversely isotropic core. Therefore, the present study focused on the static and vibration analysis of sandwich shells with transversely isotropic cores.
2. Based on the aforementioned literature review, it is found that the literature on the analysis of sandwich shells considering the effects of the transverse normal strain is limited. Carrera [52] recommended that the analysis of any layered composite structures is meaningless unless and until the effects of the transverse normal strain are considered. Hence, the objective of the present study is to carry out an analysis of sandwich shells with double curvature using a new hyperbolic shell theory considering the effects of transverse normal strain.
3. The present theory is developed using a new hyperbolic shearing strain function. A hyperbolic shell theory of Soldatos and Timarci [8] can be recovered from the present formulation by setting $\xi = 1.0$. The in-plane displacements use a hyperbolic function. This new theory has six degrees of freedom, provides parabolic transverse shear strains across the thickness direction, and hence, does not need a shear correction factor. Moreover, zero-traction boundary conditions on the top and bottom surfaces of the shell are satisfied.
4. The key differences between the present hyperbolic theory and the theory of Reddy [4] are; a) the present theory uses the non-polynomial type shape function in the displacement field, whereas Reddy's theory uses the polynomial type shape function. b) The effects of transverse normal strain are considered in the transverse displacement field of the present theory which is even neglected by the theory of Reddy.
5. The static and free vibration responses of hyperbolic and elliptical sandwich shells are studied for the first

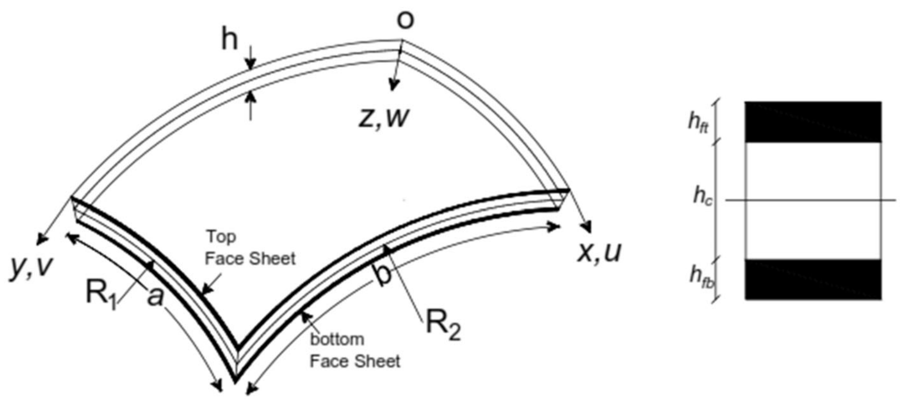

Figure 1. Geometry and coordinates of the sandwich shell under consideration.

time in this study. This can be considered an important contribution to the present study.

3. Kinematic assumption and formulation

Figure 1 shows a differential shell element considered in the (x, y, z) coordinate system. The x and y curves depicted here are lines of substantial curvature on the mid-plane of the laminate. The downward z-direction is seen to be positive. R_1 and R_2, respectively, stand for the primary radii of curvature of the mid-plane along the x and y axes. On the top surface of a laminate, that is, $z = -h/2$, a transverse load of $q\ (x,\ y)$ is applied. The following kinematic assumptions are made in the theoretical formulation of the present theory.

1. Face sheets of the sandwich shell are made up of orthotropic material and the middle core is made up of transversely isotropic material.
2. It is assumed that all the layers of the sandwich shell are perfectly bonded together.
3. In the displacement field, in-plane displacements (u, v) consist of extension, bending, and shear components, whereas transverse displacement (w) considers the effects of transverse normal strain.
   a. The extension components (u_0, v_0) are middle surface displacements in x- and y- directions.
   b. The bending components (w_0) are analogous to displacement in classical shell theory.
   c. Shear components are assumed to be hyperbolic functions in terms of thickness coordinates.
   d. The transverse displacements are a function of x-, y- and z-coordinates to account for the effects of transverse normal strain
4. The body forces are ignored in the static analysis. (body forces, if required, can be considered in the free vibration analysis).

The present theory considers the effects of both transverse shear and normal strains. There are many papers to discuss the effects of transverse normal strain on the bending of laminated composite and sandwich plates [12, 15, 16, 19, 20, 22, 25, 27, 33, 35, 47–53]. However, the assessment of these theories for the doubly-curved sandwich shells is limited. Also, the present theory uses a new hyperbolic shearing strain function to represent the in-plane as well as transverse displacements. Following is the displacement field assumed for the current hyperbolic shell theory.

$$\begin{aligned} u(x,y,z) &= \left(1+\frac{z}{R_1}\right)u_0(x,y) - z\frac{\partial w_0}{\partial x} + f(z)\theta_x(x,y) \\ v(x,y,z) &= \left(1+\frac{z}{R_2}\right)v_0(x,y) - z\frac{\partial w_0}{\partial y} + f(z)\theta_y(x,y) \\ w(x,y,z) &= w_0(x,y) + f'(z)\theta_z(x,y) \end{aligned} \tag{1}$$

where $u,\ v,\ w$ are the displacements in $x,\ y,\ z$ directions, respectively; $\theta_x, \theta_y, \theta_z$ are the shear slopes in x, y and z direction, respectively; $u_0,\ v_0,\ w_0$ are the mid-plane displacements in x, y, z direction, respectively; and $f(z)$ represents the shearing strain function. A hyperbolic shell theory of Soldatos and Timarci [8] can be recovered from the present formulation by setting $= 1.0$. Using the linear theory of elasticity [54], the normal and shear strains associated with the present displacement field stated in Eq. (1) can be calculated as follows:

$$\begin{aligned} \varepsilon_x &= \left(\frac{\partial u_0}{\partial x} + \frac{w_0}{R_1}\right) - z\frac{\partial^2 w_0}{\partial x^2} + f(z)\frac{\partial \theta_x}{\partial x} + \frac{f'(z)}{R_1}\theta_z \\ \varepsilon_y &= \left(\frac{\partial v_0}{\partial y} + \frac{w_0}{R_2}\right) - z\frac{\partial^2 w_0}{\partial y^2} + f(z)\frac{\partial \theta_y}{\partial y} + \frac{f'(z)}{R_2}\theta_z \\ \varepsilon_z &= f''(z)\theta_z \\ \gamma_{xy} &= \left(\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x}\right) - 2z\frac{\partial^2 w_0}{\partial x \partial y} + f(z)\left(\frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x}\right) \\ \gamma_{xz} &= f'(z)\theta_x + f'(z)\frac{\partial \theta_x}{\partial x} \\ \gamma_{yz} &= f'(z)\theta_y + f'(z)\frac{\partial \theta_z}{\partial y} \end{aligned} \tag{2}$$

where

$$\begin{aligned} f(z) &= \left[z\cosh\left(\frac{\xi}{2}\right)\right] - \left[\left(\frac{h}{\xi}\right)\sinh\left(\frac{\xi z}{h}\right)\right] \\ f'(z) &= \left[\cosh\left(\frac{\xi}{2}\right)\right] - \left[\cosh\left(\frac{\xi z}{h}\right)\right] \\ \xi &= 2.634 \end{aligned} \tag{3}$$

Stresses of a k^{th} layer of sandwich laminated shells are calculated using Hooke's law from the 3D elasticity problem [54].

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix}^k = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & Q_{23} & 0 & 0 & 0 \\ Q_{13} & Q_{23} & Q_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{44} \end{bmatrix}^k \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix}^k \tag{4}$$

where $(\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{xz}, \tau_{yz})$ are the normal and shear stresses; $(\varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{xy}, \gamma_{yz}, \gamma_{xz})$ are the normal and shear strains; and $(Q_{11}, Q_{12}, Q_{13}, Q_{22}, Q_{23}, Q_{33}, Q_{44}, Q_{55}, Q_{66})^k$ are the reduced stiffness coefficients. The values of these stiffness coefficients in terms of engineering constants are as follows [54].

$$\begin{gathered} Q_{11} = \frac{E_1(1-\mu_{23}\mu_{32})}{\Delta}; Q_{12} = \frac{E_1(\mu_{21}+\mu_{31}\mu_{23})}{\Delta}; Q_{13} = \frac{E_1(\mu_{31}+\mu_{21}\mu_{32})}{\Delta}; \\ Q_{22} = \frac{E_2(1-\mu_{13}\mu_{31})}{\Delta}; Q_{23} = \frac{E_2(\mu_{32}+\mu_{12}\mu_{31})}{\Delta}; Q_{33} = \frac{E_3(1-\mu_{12}\mu_{21})}{\Delta}; \\ Q_{44} = G_{23}; Q_{55} = G_{13}; Q_{66} = G_{12}; \\ \Delta = 1 - \mu_{12}\mu_{21} - \mu_{23}\mu_{32} - \mu_{13}\mu_{31} - 2\mu_{21}\mu_{32}\mu_{13} \end{gathered} \tag{5}$$

Hamilton's principle is employed to derive the governing equations of motion associated with the current hyperbolic shell theory [54].

$$\int_{t1}^{t2} (\delta U - \delta V + \delta K) \tag{6}$$

where δ is the variational operator, t_1 and t_2 are the initial time and final time; δU represents total strain energy due to internal forces, δV represents potential energy due to external load, and δK represents kinetic energy due to inertia force. Equation (6) leads to the following form after the substitution of values of these energies.

$$\begin{aligned} &\int_0^a\int_0^b\int_{-h/2}^{h/2} (\sigma_x\delta\sigma_x + \sigma_y\delta\sigma_y + \sigma_z\delta\sigma_z + \tau_{xy}\delta\tau_{xy} + \tau_{xz}\delta\tau_{xz} + \tau_{yz}\delta\tau_{yz})\, dzdydx \\ &- \int_0^a\int_0^b\int_{-h/2}^{h/2} q\, dzdydx + \rho \int_0^a\int_0^b\int_{-h/2}^{h/2} \left(\frac{\partial^2 u}{\partial^2 t}\delta u + \frac{\partial^2 v}{\partial^2 t}\delta v + \frac{\partial^2 w}{\partial^2 t}\delta w\right) dzdydx = 0 \end{aligned} \tag{7}$$

where ρ is the density of the material. By substituting the strain components from Eq. (2) into Eq. (7), performing integration by parts; collecting the coefficients of unknown variables and equating them with zero, the following equations of motion are derived.

$$\begin{aligned} \delta u_0 &: \frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} = \left(I_1 + 2\frac{I_2}{R_1} + \frac{I_3}{R_1^2}\right)\frac{\partial^2 u_0}{\partial t^2} - \left(I_2 + \frac{I_3}{R_1}\right)\frac{\partial^3 w_0}{\partial x \partial t^2} + \left(I_4 + \frac{I_6}{R_1}\right)\frac{\partial^2 \theta_x}{\partial t^2} \\ \delta v_0 &: \frac{\partial N_{yy}}{\partial y} + \frac{\partial N_{xy}}{\partial x} = \left(I_1 + 2\frac{I_2}{R_2} + \frac{I_3}{R_2^2}\right)\frac{\partial^2 v_0}{\partial t^2} - \left(I_2 + \frac{I_3}{R_2}\right)\frac{\partial^3 w_0}{\partial y \partial t^2} + \left(I_4 + \frac{I_6}{R_2}\right)\frac{\partial^2 \theta_y}{\partial t^2} \\ \delta w_0 &: \frac{\partial^2 M^b_{xx}}{\partial x^2} + \frac{\partial^2 M^b_{yy}}{\partial y^2} + 2\frac{\partial^2 M^b_{xy}}{\partial x \partial y} - \frac{N_{xx}}{R_1} - \frac{N_{yy}}{R_2} + q = \left(I_2 + \frac{I_3}{R_1}\right)\frac{\partial^3 u_0}{\partial x \partial^2 t} - I_3\frac{\partial^4 w_0}{\partial x^2 \partial t^2} \\ &\quad + I_6\frac{\partial^3 \theta_x}{\partial x \partial t^2} + \left(I_2 + \frac{I_3}{R_2}\right)\frac{\partial^3 v_0}{\partial y \partial t^2} - I_3\frac{\partial^4 w_0}{\partial y^2 \partial t^2} + I_6\frac{\partial^3 \theta_y}{\partial y \partial t^2} + I_1\frac{\partial^2 w_0}{\partial t^2} + I_7\frac{\partial^2 \theta_z}{\partial t^2} \\ \delta\theta_x &: \frac{\partial M^S_{xx}}{\partial x} + \frac{\partial M^S_{xy}}{\partial y} - Q_{xz} = \left(I_4 + \frac{I_6}{R_1}\right)\frac{\partial^2 u_0}{\partial t^2} - I_6\frac{\partial^3 w_0}{\partial x \partial t^2} + I_5\frac{\partial^2 \theta_x}{\partial t^2} \\ \delta\theta_y &: \frac{\partial M^S_{yy}}{\partial y} + \frac{\partial M^S_{xy}}{\partial x} - Q_{yz} = \left(I_4 + \frac{I_6}{R_2}\right)\frac{\partial^2 v_0}{\partial t^2} - I_6\frac{\partial^3 w_0}{\partial y \partial t^2} + I_5\frac{\partial^2 \theta_y}{\partial t^2} \\ \delta\theta_z &: \frac{\partial Q^s_{xz}}{\partial x} + \frac{\partial Q^s_{yz}}{\partial y} - \frac{V^s_{xx}}{R_1} - \frac{V^s_{yy}}{R_2} - V^s_{zz} = I_7\frac{\partial^2 w_0}{\partial t^2} + I_8\frac{\partial^2 \theta_z}{\partial t^2} \end{aligned} \tag{8}$$

where

$$\begin{gathered} \left(N_{xx}, N_{yy}, N_{xy}, M^b_{xx}, M^b_{yy}, M^b_{xy}\right) = \int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \tau_{xy}, z\sigma_x, z\sigma_y, z\tau_{xy})\, dz; \\ \left(M^S_{xx}, M^S_{yy}, M^S_{xy}\right) = \int_{-h/2}^{h/2} \left[f(z)(\sigma_x, \sigma_y, \tau_{xy})\right] dz; \\ (Q_{xz}, Q_{yz}) = \int_{-h/2}^{h/2} \left[f'(z)(\tau_{xz}, \tau_{yz})\right] dz; \\ \left(V^s_{xx}, V^s_{yy}, Q^s_{xz}, Q^s_{yz}\right) = \int_{-h/2}^{h/2} \left[f'(z)(\sigma_x, \sigma_y, \tau_{xz}, \tau_{yz})\right] dz; \\ V^s_{zz} = \int_{-h/2}^{h/2} \left(\sigma_z f''(z)\right) dz \end{gathered} \tag{9}$$

and

$$(I_1, I_2, I_3, I_4, I_5, I_6, I_7, I_8) = \int_{-h/2}^{h/2} \rho\left[1, z, z^2, f(z), [f(z)]^2, zf(z), f'(z), [f'(z)]^2\right] dz \tag{10}$$

Further substituting the expression of stress resultants from Eq. (9) into Eq. (8), the governing equations can be written in the following forms in terms of unknown displacement variables.

$$\begin{aligned} \delta u_0 : &A_{11}\left(\frac{\partial^2 u_0}{\partial x^2} + \frac{\partial w_0}{R_1 \partial x}\right) - B_{11}\frac{\partial^3 w_0}{\partial x^3} + C_{11}\frac{\partial^2 \theta_x}{\partial x^2} + \frac{F_{11}}{R_1}\frac{\partial \theta_z}{\partial x} + A_{12}\left(\frac{\partial^2 v_0}{\partial x \partial y} + \frac{\partial w_0}{R_2 \partial x}\right) \\ &- B_{12}\frac{\partial^3 w_0}{\partial x \partial y^2} + C_{12}\frac{\partial^2 \theta_y}{\partial x \partial y} + \frac{F_{12}}{R_2}\frac{\partial \theta_z}{\partial x} + D_{13}\frac{\partial \theta_z}{\partial x} + A_{66}\left(\frac{\partial^2 u_0}{\partial y^2} + \frac{\partial^2 v_0}{\partial x \partial y}\right) - 2B_{66}\frac{\partial^3 w_0}{\partial x \partial y^2} \\ &+ C_{66}\left(\frac{\partial^2 \theta_x}{\partial y^2} + \frac{\partial^2 \theta_y}{\partial x \partial y}\right) = \left(I_1 + \frac{2I_2}{R_1} + \frac{I_3}{R_1^2}\right)\frac{\partial^2 u_0}{\partial t^2} - \left(I_2 + \frac{I_3}{R_1}\right)\frac{\partial^3 w_0}{\partial x \partial t^2} + \left(I_4 + \frac{I_6}{R_1}\right)\frac{\partial^2 \theta_x}{\partial t^2} \end{aligned} \tag{11}$$

$$\begin{aligned} \delta v_0 : &A_{21}\left(\frac{\partial^2 u_0}{\partial x \partial y} + \frac{\partial w_0}{R_1 \partial y}\right) - B_{21}\frac{\partial^3 w_0}{\partial x^2 \partial y} + C_{21}\frac{\partial^2 \theta_x}{\partial x \partial y} + \frac{F_{21}}{R_1}\frac{\partial \theta_z}{\partial y} + A_{22}\left(\frac{\partial^2 v_0}{\partial y^2} + \frac{\partial w_0}{R_2 \partial y}\right) \\ &- B_{22}\frac{\partial^3 w_0}{\partial y^3} + C_{22}\frac{\partial^2 \theta_y}{\partial y^2} + \frac{F_{22}}{R_2}\frac{\partial \theta_z}{\partial y} + D_{23}\frac{\partial \theta_z}{\partial y} + A_{66}\left(\frac{\partial^2 u_0}{\partial x \partial y} + \frac{\partial^2 v_0}{\partial x^2}\right) - 2B_{66}\frac{\partial^3 w_0}{\partial x^2 \partial y} \\ &+ C_{66}\left(\frac{\partial^2 \theta_x}{\partial x \partial y} + \frac{\partial^2 \theta_y}{\partial x^2}\right) = \left(I_1 + \frac{2I_2}{R_2} + \frac{I_3}{R_2^2}\right)\frac{\partial^2 v_0}{\partial t^2} - \left(I_2 + \frac{I_3}{R_2}\right)\frac{\partial^3 w_0}{\partial t^2 \partial y} + \left(I_4 + \frac{I_6}{R_2}\right)\frac{\partial^2 \theta_y}{\partial t^2} \end{aligned} \tag{12}$$

$$\begin{aligned} \delta w_0 : &B_{11}\left(\frac{\partial^3 u_0}{\partial x^3} + \frac{\partial^2 w_0}{R_1 \partial x^2}\right) - H_{11}\frac{\partial^4 w_0}{\partial x^4} + I_{11}\frac{\partial^3 \theta_x}{\partial x^3} + \frac{J_{11}}{R_1}\frac{\partial^2 \theta_z}{\partial x^2} + B_{12}\left(\frac{\partial^2 v_0}{\partial x^2 \partial y} + \frac{\partial^2 w_0}{R_2 \partial x^2}\right) \\ &- H_{12}\frac{\partial^4 w_0}{\partial x^2 \partial y^2} + I_{12}\frac{\partial^3 \theta_y}{\partial x^2 \partial y} + \frac{J_{12}}{R_2}\frac{\partial^2 \theta_z}{\partial x^2} + K^S_{13}\frac{\partial^2 \theta_z}{\partial x^2} + B_{12}\left(\frac{\partial^3 u_0}{\partial x \partial y^2} + \frac{\partial^2 w_0}{R_1 \partial y^2}\right) - H_{12}\frac{\partial^4 w_0}{\partial x^2 \partial y^2} \\ &+ I_{12}\frac{\partial^3 \theta_x}{\partial x \partial y^2} + \frac{J_{12}}{R_1}\frac{\partial^2 \theta_z}{\partial y^2} + B_{22}\left(\frac{\partial^3 v_0}{\partial y^3} + \frac{\partial^2 w_0}{R_2 \partial y^2}\right) - H_{22}\frac{\partial^4 w_0}{\partial y^4} + I_{22}\frac{\partial^3 \theta_y}{\partial y^3} + \frac{J_{22}}{R_2}\frac{\partial^2 \theta_z}{\partial y^2} \\ &+ K^S_{23}\frac{\partial^2 \theta_z}{\partial y^2} + 2B_{66}\left(\frac{\partial^3 u_0}{\partial x \partial y^2} + \frac{\partial^3 v_0}{\partial x^2 \partial y}\right) - 4H_{66}\frac{\partial^4 w_0}{\partial x^2 \partial y^2} + 2I_{66}\left(\frac{\partial^3 \theta_x}{\partial x \partial y^2} + \frac{\partial^3 \theta_y}{\partial x^2 \partial y}\right) \\ &- \left[\frac{A_{11}}{R_1}\left(\frac{\partial u_0}{\partial x} + \frac{w_0}{R_1}\right) - \frac{B_{11}}{R_1}\frac{\partial^2 w_0}{\partial x^2} + \frac{C_{11}}{R_1}\frac{\partial \theta_x}{\partial x} + \frac{F_{11}}{R_1^2}\theta_z\right] - \frac{A_{12}}{R_1}\left(\frac{\partial v_0}{\partial y} + \frac{w_0}{R_2}\right) + \frac{B_{12}}{R_1}\frac{\partial^2 w_0}{\partial y^2} \\ &- \frac{C_{12}}{R_1}\frac{\partial \theta_y}{\partial y} - \frac{F_{12}}{R_1 R_2}\theta_z - \frac{D_{13}}{R_1}\theta_z - \frac{A_{12}}{R_2}\left(\frac{\partial u_0}{\partial x} + \frac{w_0}{R_1}\right) + \frac{B_{12}}{R_2}\frac{\partial^2 w_0}{\partial x^2} - \frac{C_{12}}{R_2}\frac{\partial \theta_x}{\partial x} - \frac{F_{12}}{R_1 R_2}\theta_z \\ &- \frac{A_{22}}{R_2}\left(\frac{\partial v_0}{\partial y} + \frac{w_0}{R_2}\right) + \frac{B_{22}}{R_2}\frac{\partial^2 w_0}{\partial y^2} - \frac{C_{22}}{R_2}\frac{\partial \theta_y}{\partial y} - \frac{F_{22}}{R_2^2}\theta_z - \frac{D_{23}}{R_2}\theta_z + q = \left(I_2 + \frac{I_3}{R_1}\right)\frac{\partial^3 u_0}{\partial x \partial t^2} \\ &- I_3\frac{\partial^4 w_0}{\partial x^2 \partial t^2} + I_6\frac{\partial^3 \theta_x}{\partial x \partial t^2} + \left(I_2 + \frac{I_3}{R_2}\right)\frac{\partial^3 v_0}{\partial t^2 \partial y} - I_3\frac{\partial^4 w_0}{\partial y^2 \partial t^2} + I_6\frac{\partial^3 \theta_y}{\partial y \partial t^2} + I_1\frac{\partial^2 w_0}{\partial t^2} + I_7\frac{\partial^2 \theta_z}{\partial t^2} \end{aligned} \tag{13}$$

$$\delta\theta_x : C_{11}\left(\frac{\partial^2 u_0}{\partial x^2}+\frac{\partial w_0}{R_1\partial x}\right)-I_{11}\frac{\partial^3 w_0}{\partial x^3}+L_{11}\frac{\partial^2\theta_x}{\partial x^2}+\frac{M_{11}}{R_1}\frac{\partial\theta_z}{\partial x}+C_{12}\left(\frac{\partial^2 v_0}{\partial x\partial y}+\frac{\partial w_0}{R_2\partial x}\right)$$
$$-I_{12}\frac{\partial^3 w_0}{\partial x\partial y^2}+L_{12}\frac{\partial^2\theta_y}{\partial x\partial y}+\frac{M_{12}}{R_2}\frac{\partial\theta_z}{\partial x}+N_{13}\frac{\partial\theta_z}{\partial x}+C_{66}\left(\frac{\partial^2 u_0}{\partial y^2}+\frac{\partial^2 v_0}{\partial x\partial y}\right)-2I_{66}\frac{\partial^3 w_0}{\partial x\partial y^2}$$
$$+L_{66}\left(\frac{\partial^2\theta_x}{\partial y^2}+\frac{\partial^2\theta_y}{\partial x\partial y}\right)-O_{55}\left(\theta_x+\frac{\partial\theta_z}{\partial x}\right)=\left(I_4+\frac{I_6}{R_1}\right)\frac{\partial^2 u_0}{\partial t^2}-I_6\frac{\partial^3 w_0}{\partial x\partial t^2}+I_5\frac{\partial^2\theta_x}{\partial t^2} \tag{14}$$

$$\delta\theta_y : C_{21}\left(\frac{\partial^2 u_0}{\partial x\partial y}+\frac{\partial w_0}{R_1\partial y}\right)-I_{21}\frac{\partial^3 w_0}{\partial x^2\partial y}+L_{21}\frac{\partial^2\theta_x}{\partial x\partial y}+\frac{M_{21}}{R_1}\frac{\partial\theta_z}{\partial y}+C_{22}\left(\frac{\partial^2 v_0}{\partial y^2}+\frac{\partial w_0}{R_2\partial y}\right)$$
$$-I_{22}\frac{\partial^3 w_0}{\partial y^3}+L_{22}\frac{\partial^2\theta_y}{\partial y^2}+\frac{M_{22}}{R_2}\frac{\partial\theta_z}{\partial y}+N_{23}\frac{\partial\theta_z}{\partial y}+C_{66}\left(\frac{\partial^2 u_0}{\partial x\partial y}+\frac{\partial^2 v_0}{\partial x^2}\right)-2I_{66}\frac{\partial^3 w_0}{\partial x^2\partial y}$$
$$+L_{66}\left(\frac{\partial^2\theta_x}{\partial x\partial y}+\frac{\partial^2\theta_y}{\partial x^2}\right)-O_{44}\left(\theta_y+\frac{\partial\theta_z}{\partial y}\right)=\left(I_4+\frac{I_6}{R_2}\right)\frac{\partial^2 v_0}{\partial t^2}-I_6\frac{\partial^3 w_0}{\partial t^2\partial y}+I_5\frac{\partial^2\theta_y}{\partial t^2} \tag{15}$$

$$\delta\theta_z : -O_{55}\left(\frac{\partial\theta_x}{\partial x}+\frac{\partial^2\theta_z}{\partial x^2}\right)+O_{44}\left(\frac{\partial\theta_y}{\partial y}+\frac{\partial^2\theta_z}{\partial y^2}\right)-\frac{F_{11}}{R_1}\left(\frac{\partial u_0}{\partial x}+\frac{w_0}{R_1}\right)+\frac{J_{11}}{R_1}\frac{\partial^2 w_0}{\partial x^2}$$
$$-\frac{M_{11}}{R_1}\frac{\partial\theta_x}{\partial x}-\frac{O_{11}}{R_1^2}\theta_z-\frac{F_{12}}{R_1}\left(\frac{\partial v_0}{\partial y}+\frac{w_0}{R_2}\right)+\frac{J_{12}}{R_1}\frac{\partial^2 w_0}{\partial y^2}-\frac{M_{12}}{R_1}\frac{\partial\theta_y}{\partial y}-\frac{O_{12}}{R_1R_2}\theta_z-2\frac{P_{13}}{R_1}\theta_z$$
$$-\frac{F_{12}}{R_2}\left(\frac{\partial u_0}{\partial x}+\frac{w_0}{R_1}\right)+\frac{J_{12}}{R_2}\frac{\partial^2 w_0}{\partial x^2}-\frac{M_{12}}{R_2}\frac{\partial\theta_x}{\partial x}-\frac{O_{12}}{R_1R_2}\theta_z-\frac{F_{22}}{R_2}\left(\frac{\partial v_0}{\partial y}+\frac{w_0}{R_2}\right)+\frac{J_{22}}{R_2}\frac{\partial^2 w_0}{\partial y^2}$$
$$-\frac{M_{22}}{R_2}\frac{\partial\theta_y}{\partial y}-\frac{O_{22}}{R_2^2}\theta_z-2\frac{P_{23}}{R_2}\theta_z-D_{13}\left(\frac{\partial u_0}{\partial x}+\frac{w_0}{R_1}\right)+K_{13}^S\frac{\partial^2 w_0}{\partial x^2}-N_{13}\frac{\partial\theta_x}{\partial x}$$
$$-D_{23}\left(\frac{\partial v_0}{\partial y}+\frac{w_0}{R_2}\right)+K_{23}^S\frac{\partial^2 w_0}{\partial y^2}-N_{23}\frac{\partial\theta_y}{\partial y}-S_{33}\theta_z=I_7\frac{\partial^2 w_0}{\partial t^2}+I_8\frac{\partial^2\theta_z}{\partial t^2} \tag{16}$$

where

$$(A_{ij},B_{ij},H_{ij},C_{ij},I_{ij},F_{ij},D_{ij})=Q_{ij}\int_{-h/2}^{h/2}\left[1.0,z,z^2,f(z),zf(z),f'(z),f''(z)\right]dz;$$
$$(L_{ij})=Q_{ij}\int_{-h/2}^{h/2}[f(z)]^2dz;\ (O_{ij})=Q_{ij}\int_{-h/2}^{h/2}\left[f'(z)\right]^2dz;$$
$$(J_{ij},M_{ij},P_{ij})=Q_{ij}\int_{-h/2}^{h/2}f'(z)\left[z,f(z),f''(z)\right]dz;$$
$$(K_{ij}^s,N_{ij})=Q_{ij}\int_{-h/2}^{h/2}f''(z)[z,f(z)]dz;\ S_{ij}=Q_{ij}\int_{-h/2}^{h/2}\left[f''(z)\right]^2dz \tag{17}$$

4. Closed-form solutions

In the present study, closed-formed solutions for the static and free vibration analysis of sandwich shells with double curvature are obtained using the Navier method. Boundary conditions of simply-supported sandwich shells are exactly satisfied using Navier's method. The unknown variables and the transverse load are assumed in the following trigonometric form to obtain the solutions for static and free vibration analysis of sandwich shells [54].

$$\begin{Bmatrix} u_0\\ v_0\\ w_0\\ \theta_x\\ \theta_y\\ \theta_z \end{Bmatrix}=\begin{Bmatrix} u_{mn}\cos\alpha x\sin\beta y\\ v_{mn}\sin\alpha x\cos\beta y\\ w_{mn}\sin\alpha x\sin\beta y\\ \theta_{xmn}\cos\alpha x\sin\beta y\\ \theta_{ymn}\sin\alpha x\cos\beta y\\ \theta_{zmn}\sin\alpha x\sin\beta y \end{Bmatrix}e^{\omega t} \tag{18}$$

and

$$q(x,y)=q_{mn}\sin\alpha x\sin\beta y \tag{19}$$

where $\alpha=m\pi/a,\ \beta=n\pi/b,\ m$ and n are the positive integers, q_{mn} represents the coefficients of transverse load. In the case of sinusoidal transverse load ($q_{mn}=q_0$), the values of positive integers are taken as unity, i.e. ($m=1,\ n=1$); $(u_{mn},v_{mn},w_{mn},\theta_{xmn},\theta_{ymn},\theta_{zmn})$ are the unknown coefficients to be determined, ω is the natural frequency of the shells. At ($m=1,\ n=1$) natural frequencies are called fundamental frequencies. Frequencies corresponding to w_0 are called flexural mode frequencies.

4.1. Static analysis

In the case of the static analysis of sandwich shells, time-dependent terms are neglected from the governing equations as well as Eq. (18). Substitution of solution form from Eq. (18) and transverse load from Eq. (19) into governing Eqs. (11–16) leads to six simultaneous equations which are written in the following matrix form.

$$[K]\{\Delta\}=\{F\} \tag{20}$$

where $[K]$ is the stiffness matrix, $\{\Delta\}$ is the vector of unknown variables and $\{F\}$ represents the force vector. The elements of these matrices are as follows.

$$K_{11}=-A_{11}\alpha^2-A_{66}\beta^2,K_{12}=-A_{12}\alpha\beta-A_{66}\alpha\beta,$$
$$K_{13}=\frac{A_{11}}{R_1}\alpha+\frac{A_{12}}{R_2}\alpha+B_{11}\alpha^3+B_{12}\alpha\beta^2+2B_{66}\alpha\beta^2,$$
$$K_{14}=-C_{11}\alpha^2-C_{66}\beta^2,K_{15}=-C_{12}\alpha\beta-C_{66}\alpha\beta,K_{16}=\left[\frac{F_{11}}{R_1}\alpha+\frac{F_{12}}{R_2}\alpha+D_{13}\alpha\right],$$
$$K_{22}=-A_{22}\beta^2-A_{66}\alpha^2,$$
$$K_{23}=B_{22}\beta^3+[B_{12}+2B_{66}]\alpha^2\beta+\left[\frac{A_{12}}{R_1}+\frac{A_{22}}{R_2}\right]\beta$$
$$K_{24}=-C_{21}\alpha\beta-C_{66}\alpha\beta,K_{25}=-C_{22}\beta^2-C_{66}\alpha^2,K_{26}=\left(D_{23}+\frac{F_{21}}{R_1}+\frac{F_{22}}{R_2}\right)\beta,$$
$$K_{33}=-\left(H_{11}\alpha^4+H_{22}\beta^4\right)-2\alpha^2\beta^2(H_{12}+2H_{66})-2\alpha^2\left(\frac{B_{11}}{R_1}+\frac{B_{12}}{R_2}\right)$$
$$-2\beta^2\left(\frac{B_{12}}{R_1}+\frac{B_{22}}{R_2}\right)-\left(\frac{2A_{12}}{R_1R_2}+\frac{A_{11}}{R_1^2}+\frac{A_{22}}{R_2^2}\right),$$
$$K_{34}=I_{11}\alpha^3+I_{21}\alpha\beta^2+2I_{66}\alpha\beta^2+\frac{C_{11}}{R_1}\alpha+\frac{C_{21}}{R_2}\alpha,$$
$$K_{35}=I_{12}\alpha^2\beta+I_{22}\beta^3+2I_{66}\alpha^2\beta+\frac{C_{12}}{R_1}\beta+\frac{C_{22}}{R_2}\beta,$$
$$K_{36}=-K_{13}\alpha^2-K_{23}\beta^2-\frac{D_{13}}{R_1}-\frac{D_{23}}{R_2}-\left(\frac{J_{11}}{R_1}+\frac{J_{12}}{R_2}\right)\alpha^2-\left(\frac{J_{21}}{R_1}+\frac{J_{22}}{R_2}\right)\beta^2$$
$$-\left(2\frac{F_{12}}{R_1R_2}+\frac{F_{22}}{R_2^2}+\frac{F_{11}}{R_1^2}\right)$$
$$K_{44}=-L_{11}\alpha^2-L_{66}\beta^2-O_{55},\ K_{45}=-(L_{12}+L_{66})\alpha\beta,$$
$$K_{46}=\left(N_{13}-O_{55}+\frac{M_{11}}{R_1}+\frac{M_{12}}{R_2}\right)\alpha,K_{55}=-L_{66}\alpha^2-L_{22}\beta^2-O_{44},$$
$$K_{56}=\left(-O_{44}+N_{23}+\frac{M_{21}}{R_1}+\frac{M_{22}}{R_2}\right)\beta,$$
$$K_{66}=\left(-O_{55}\alpha^2-O_{44}\beta^2-S_{33}+2\frac{P_{23}}{R_2}-2\frac{P_{13}}{R_1}-\frac{O_{11}}{R_1^2}-2\frac{O_{12}}{R_1R_2}-\frac{O_{22}}{R_2^2}\right) \tag{21}$$

$$\{\Delta\}=\left\{u_{mn},v_{mn},w_{mn},\theta_{xmn},\theta_{ymn},\theta_{zmn}\right\}^T \tag{22}$$

$$\{F\}=\{0,0,q_0,0,0,0\}^T \tag{23}$$

where

$$(A_{ij}, B_{ij}, H_{ij}, C_{ij}, F_{ij} I_{ij}) = Q_{ij} \int_{-h/2}^{h/2} \left[1.0, z, z^2, f(z), f'(z), zf(z)\right] dz;$$

$$L_{ij} = Q_{ij} \int_{-h/2}^{h/2} \left\{[f(z)]^2\right\} dz; O_{ij} = Q_{ij} \int_{-h/2}^{h/2} \left\{\left[f'(z)\right]^2\right\} dz;$$

$$(D_{ij}, S_{ij}, P_{ij}) = Q_{ij} \int_{-h/2}^{h/2} \left\{\left[f''(z)\right], \left[f''(z)\right]^2, \left[f'(z)\right]\right\} dz;$$

$$(K_{ij}, N_{ij}) = Q_{ij} \int_{-h/2}^{h/2} f''(z)[z, f(z)] dz;$$

$$(J_{ij}, M_{ij}) = Q_{ij} \int_{-h/2}^{h/2} f'(z)[z, f(z)] dz; \tag{24}$$

It is important to note that the stiffness matrix is always a symmetric ($K_{ij} = K_{ji}$). Solution of Eq. (20) gives the values of unknown coefficients from Eq. (18) which is further used to determine displacements and stresses from Eqs. (1–5).

4.2. Free vibration analysis

For the free vibration analysis, the transverse load acting on the shell (q) is taken as zero. Substitution of Eq. (18) into governing Eqs (11–16) considering the time-dependent terms leads to the Eigenvalue problems stated in Eq. (25).

$$\left([K] - \omega^2 [M]\right)\{\Delta\} = 0 \tag{25}$$

where $[K]$ is the stiffness matrix stated in Eq. (22) and $[M]$ represents the mass matrix. Elements of mass matrices are as follows.

$$M_{11} = \left(I_1 + \frac{2I_2}{R_1} + \frac{I_3}{R_1^2}\right); M_{12} = M_{21} = 0;\ M_{13} = M_{31} = -\left(I_2 + \frac{I_3}{R_1}\right)\alpha;$$
$$M_{14} = M_{41} = \left(I_4 + \frac{I_6}{R_1}\right);\ M_{15} = M_{51} = 0; M_{16} = M_{61} = 0;$$
$$M_{22} = \left(I_1 + \frac{2I_2}{R_2} + \frac{I_3}{R_2^2}\right); M_{23} = M_{32} = -\left(I_2 + \frac{I_3}{R_2}\right)\beta; M_{24} = M_{42} = 0;$$
$$M_{26} = M_{62} = 0; M_{25} = M_{52} = \left(I_4 + \frac{I_6}{R_2}\right); M_{34} = M_{43} = -I_6\alpha;$$
$$M_{33} = \left(I_3\alpha^2 + I_3\beta^2 + I_1\right);\ M_{35} = M_{53} = -I_6\beta; M_{36} = M_{63} = I_7; M_{44} = I_5;$$
$$M_{55} = I_5; M_{45} = M_{54} = 0; M_{46} = M_{64} = 0;\ M_{56} = M_{65} = 0; M_{66} = I_8 \tag{26}$$

Like the stiffness matrix, a mass matrix is also a symmetric matrix ($M_{ij} = M_{ji}$). The Nontrivial solution of the eigenvalue problem stated in Eq. (25) gives natural frequencies for the sandwich shells.

5. Numerical result and discussion

In this study, static and free vibration analysis of sandwich shells with double curvature is presented. The numerical results are obtained for the following material properties.

$$\frac{E_1}{E_2} = 25,\ \frac{E_3}{E_2} = 1,\ \frac{G_{12}}{E_2} = \frac{G_{13}}{E_2} = 0.5,\ \frac{G_{23}}{E_2} = 0.2,\ \mu_{12} = \mu_{13} = \mu_{23} = 0.25 \tag{27}$$

$$E_1 = E_2 = 0.04,\ E_3 = 0.5,\ G_{13} = G_{23} = 0.06, G_{12} = 0.016, \mu_{12} = \mu_{32} = \mu_{31} = 0.25 \tag{28}$$

$$\frac{Q_{22}}{Q_{11}} = 0.543103, \frac{Q_{12}}{Q_{11}} = 0.23319, \frac{Q_{13}}{Q_{11}} = 0.010776, \frac{Q_{23}}{Q_{11}} = 0.098276, \frac{Q_{33}}{Q_{11}} = 0.530172, \frac{Q_{44}}{Q_{11}} = 0.266810, \frac{Q_{55}}{Q_{11}} = 0.159914, \frac{Q_{66}}{Q_{11}} = 0.262931, \rho = \text{Constant} \tag{29}$$

For comparison purposes, the numerical results are presented in the following non-dimensional form.

$$\bar{u}\left(0, \frac{b}{2}, \frac{z}{h}\right) = \frac{h^2 E_3}{q_0 a^3} u, \bar{w}\left(\frac{a}{2}, \frac{b}{2}, \frac{z}{h}\right) = \frac{100 h^3 E_3}{q_0 a^4} w,$$
$$(\bar{\sigma}_x, \bar{\sigma}_y)\left(\frac{a}{2}, \frac{b}{2}, \frac{z}{h}\right) = \frac{h^2}{q_0 a^2}(\sigma_x, \sigma_y), \bar{\tau}_{xy}\left(0, 0, \frac{z}{h}\right) = \frac{h^2}{q_0 a^2}\tau_{xy}$$
$$\bar{\tau}_{zx}\left(0, \frac{b}{2}, \frac{z}{h}\right) = \frac{h}{q_0 a}\tau_{zx}, \bar{\tau}_{yz}\left(\frac{a}{2}, 0, \frac{z}{h}\right) = \frac{h}{q_0 a}\tau_{yz},\ \bar{\omega} = \omega a h\sqrt{\rho/Q_{11}} \tag{30}$$

where E_3 is the modulus of elasticity of the middle layer, i.e. the core of the sandwich shell.

5.1. Discussion on static analysis

A sandwich spherical shell ($R_1 = R_2 = R$) subjected to sinusoidal load is analyzed in this study and the corresponding numerical results are presented in Table 1. A sandwich spherical shell consists of three layers. The top and the bottom face sheets are of thickness $0.1h$, whereas the middle layer is $0.8h$. Here, h is the overall thickness of the sandwich shell. The face sheets are made up of fibrous composite material stated in Eq. (27), whereas the core is made up of transversely isotropic material stated in Eq. (28). These material properties are used by Pagano [2] to determine 3D elasticity solutions for the bi-directional bending of sandwich plates. The present results are compared with Reddy [4] and Mindlin [1]. The numerical results for sandwich plates are compared with those presented by 3D Elasticity provided by Pagano [2]. Table 1 reveals that the present results of transverse displacements and stresses are in excellent agreement with those presented by Pagano [2]. This is due to consideration of the effects of transverse normal strain. The numerical results by Reddy [4] show an error compared with the present results due to ignoring the effects of transverse normal strain. The FSDT of Mindlin

Table 1. Non-dimensional displacements and stresses of three-layer (0°/core/0°) sandwich spherical shells under sinusoidal load ($a = 10h$, $R_1 = R_2 = R$).

R/a	Theory	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\tau}_{xy}$	$\bar{\tau}_{xz}$	$\bar{\tau}_{yz}$
5	Present ($\varepsilon_z \neq 0$)	0.0124	0.9965	1.0655	0.0772	0.0926	0.3108	0.0558
	Reddy [4] ($\varepsilon_z = 0$)	0.0131	1.0063	1.0733	0.0745	0.0932	0.2956	0.0486
	Mindlin [1] ($\varepsilon_z = 0$)	0.0109	0.7122	1.0147	0.0607	0.0715	0.3096	0.0384
10	Present ($\varepsilon_z \neq 0$)	0.0099	1.0152	1.1002	0.0921	0.0805	0.3166	0.0569
	Reddy [4] ($\varepsilon_z = 0$)	0.0102	1.0250	1.1081	0.0891	0.0812	0.3011	0.0495
	Mindlin [1] ($\varepsilon_z = 0$)	0.0088	0.7215	1.0385	0.0708	0.0628	0.3137	0.0389
20	Present ($\varepsilon_z \neq 0$)	0.0085	1.0199	1.1128	0.0993	0.0739	0.3181	0.0571
	Reddy [4] ($\varepsilon_z = 0$)	0.0086	1.0298	1.1207	0.0962	0.0747	0.3025	0.0497
	Mindlin [1] ($\varepsilon_z = 0$)	0.0077	0.7238	1.0471	0.0757	0.0582	0.3147	0.0390
50	Present ($\varepsilon_z \neq 0$)	0.0076	1.0213	1.1247	0.1035	0.0698	0.3185	0.0572
	Reddy [4] ($\varepsilon_z = 0$)	0.0077	1.0312	1.1267	0.1003	0.0707	0.3029	0.0498
	Mindlin [1] ($\varepsilon_z = 0$)	0.0070	0.7245	1.0512	0.0786	0.0553	0.3150	0.0390
100	Present ($\varepsilon_z \neq 0$)	0.0073	1.0215	1.1205	0.1048	0.0685	0.3186	0.0572
	Reddy [4] ($\varepsilon_z = 0$)	0.0073	1.0314	1.1284	0.1017	0.0693	0.3029	0.0498
	Mindlin [1] ($\varepsilon_z = 0$)	0.0068	0.7246	1.0524	0.0795	0.0544	0.3150	0.0390
Plate	Present ($\varepsilon_z \neq 0$)	0.0069	1.0215	1.1220	0.1062	0.0671	0.3186	0.0572
	Reddy [4] ($\varepsilon_z = 0$)	0.0070	1.0315	1.1300	0.1030	0.0679	0.3029	0.0498
	Mindlin [1] ($\varepsilon_z = 0$)	0.0066	0.7246	1.0535	0.0805	0.0534	0.3151	0.0390
	3D Elasticity [2]	0.0071	1.1002	1.1518	0.1098	0.0706	0.2997	0.0526

Table 2. Non-dimensional displacements and stresses of three-layer (0°/core/0°) sandwich shells with double curvature under sinusoidal load ($a = b$, $a = 10h$).

Shell type	(R_1/a,R_2/b)	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\tau}_{xy}$	$\bar{\tau}_{xz}$	$\bar{\tau}_{yz}$
Cylindrical	(5, ∞)	0.0126	1.0152	1.1005	0.0921	0.0805	0.3166	0.0569
	(10, ∞)	0.0100	1.0199	1.1130	0.0993	0.0739	0.3181	0.0571
	(20, ∞)	0.0085	1.0211	1.1179	0.1028	0.0705	0.3184	0.0572
	(50, ∞)	0.0076	1.0215	1.1205	0.1048	0.0685	0.3186	0.0572
	(100, ∞)	0.0073	1.0215	1.1213	0.1055	0.0678	0.3186	0.0572
Hyperbolic	(5,−5)	0.0128	1.0215	1.1225	0.1062	0.0671	0.3186	0.0572
	(10,−10)	0.0100	1.0215	1.1222	0.1062	0.0671	0.3186	0.0572
	(20,−20)	0.0085	1.0215	1.1221	0.1062	0.0671	0.3186	0.0572
	(50,−50)	0.0076	1.0215	1.1221	0.1062	0.0671	0.3186	0.0572
	(100,−100)	0.0073	1.0215	1.1221	0.1062	0.0671	0.3186	0.0572
Elliptical	(5,7.5)	0.0125	1.0040	1.0785	0.0822	0.0887	0.3131	0.0562
	(10, 15)	0.0099	1.0171	1.1049	0.0945	0.0783	0.3172	0.0570
	(20, 30)	0.0085	1.0204	1.1146	0.1004	0.0728	0.3182	0.0571
	(50, 75)	0.0076	1.0214	1.1194	0.1039	0.0694	0.3185	0.0572
	(100,150)	0.0073	1.0215	1.1207	0.1051	0.0682	0.3186	0.0572

underestimates the numerical results of sandwich spherical shells due to first-order variation of thickness coordinates as well as ignoring the effects of transverse normal strain. It is also observed that the displacements and stresses increase with an increase in the radii of curvature. After verification of the present theory for spherical shells, it is extended for the analysis of three different types of shells such as cylindrical shells ($R_1 = R,\ R_2 = \infty$), hyperbolic shells ($R_1 = R, R_2 = -R$), and elliptical shells ($R_1 = R, R_2 = 1.5R$). Table 2 shows the numerical results of displacements and stresses for these sandwich shells under sinusoidal loading. Table 2 reveals that the minimum displacements and stresses are predicted by the elliptical shells, whereas maximum displacements and stresses are predicted by hyperbolic shells. It is also observed that the hyperbolic shells have almost constant values of displacements and stresses for all radii of curvature. Static analysis of doubly-curved shells is presented for the first time in the literature especially hyperbolic and elliptical sandwich shells. Figure 2 shows through the thickness distributions of displacements and stresses for doubly curved sandwich shells under sinusoidal load. Figure 2 reveals that the in-plane stresses in the core layer are very small due to the soft material provided, whereas large values of in-plane stresses are observed in the face sheets due to high-strength fibrous composite materials. From Figure 2, it is also pointed out that the maximum values of in-plane stresses are observed in hyperbolic shells, whereas maximum values of in-plane shear stresses are observed in spherical shells. Transverse shear stresses are obtained using the direct method, i.e. using constitutive relations; therefore, discontinuity in the shear stresses is predicted at the layer interface of sandwich shells. To get the continuity in the shear stresses at the layer interface it is necessary to calculate the shear stresses using equations of equilibrium of the theory of elasticity.

5.2. Discussion on free vibration analysis

Tables 3 and 4 show the comparison of the natural frequencies of sandwich shells with double curvature. The material properties used for the verification of fundamental frequencies are mentioned in Eq. (29). Non-dimensional material properties shown in Eq. (30) are assumed for the core properties, whereas the material properties of face sheets are assumed ‘C’ times the elastic properties of the core, i.e. $(Q_{ij})_f = C(Q_{ij})_c$. The value of C decides the softness of the

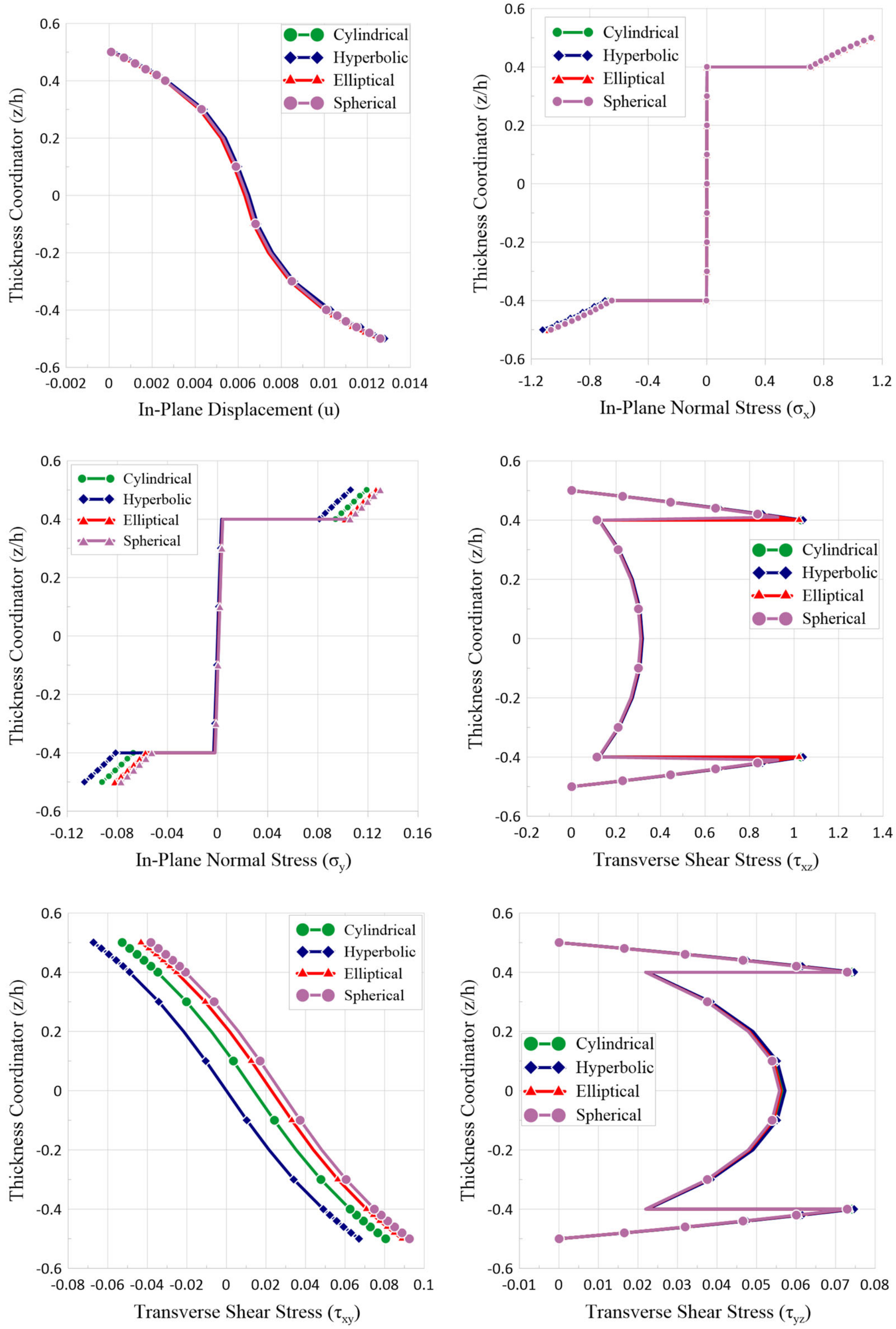

Figure 2. Through-the-thickness distributions of displacements and stresses in (0°/core/0°) sandwich shells of double curvature.

core material. An increase in the value of C increases the softness of the core. The numerical results are obtained for different values of $C = 1$, 2, 5, 10, 15, and the radii of curvature. Table 3 reveals that the increase in the value of C increases the non-dimensional fundamental frequencies of the spherical shells. Also, it is observed that the increase in the values of radii of curvature decreases the fundamental frequencies. This shows that the softcore sandwich shells undergo higher values of non-dimensional frequencies. The effects of curvature on the fundamental frequencies are examined and presented in Table 4. It is pointed out from Table 4 that the hyperbolic shells predict minimum values of fundamental frequencies, whereas spherical shells predict maximum values of fundamental frequencies. It is also observed that the increase in the radii of curvature decreases non-dimensional frequencies.

Table 3. Non-dimensional fundamental frequencies of three-layer (0°/core/0°) sandwich spherical shells ($a = 10h$, $R_1 = R_2 = R$).

		C				
R/a	Model	1	2	5	10	15
5	Present ($\varepsilon_z \neq 0$)	5.0075	5.9698	8.0390	10.3319	12.0045
	Reddy [4] ($\varepsilon_z = 0$)	5.0209	5.9690	8.0090	10.0238	10.2494
	Mindlin [1] ($\varepsilon_z = 0$)	5.0480	5.9841	8.0227	10.0411	10.2695
10	Present ($\varepsilon_z \neq 0$)	4.8131	5.7759	7.8253	10.0738	11.7025
	Reddy [4] ($\varepsilon_z = 0$)	4.8274	5.6883	7.6248	9.90280	11.3320
	Mindlin [1] ($\varepsilon_z = 0$)	4.8556	5.7042	7.6392	9.92090	11.3530
20	Present ($\varepsilon_z \neq 0$)	4.7631	5.7262	7.7707	10.0080	11.6256
	Reddy [4] ($\varepsilon_z = 0$)	4.7776	5.6158	7.5252	9.87120	11.6479
	Mindlin [1] ($\varepsilon_z = 0$)	4.8061	5.6318	7.5399	9.88950	11.6694
50	Present ($\varepsilon_z \neq 0$)	4.7489	5.7122	7.7554	9.98950	11.6039
	Reddy [4] ($\varepsilon_z = 0$)	4.7635	5.5953	7.4971	9.86230	11.7412
	Mindlin [1] ($\varepsilon_z = 0$)	4.7922	5.6114	7.5119	9.88060	11.7628
100	Present ($\varepsilon_z \neq 0$)	4.7469	5.7102	7.7532	9.98680	11.6008
	Reddy [4] ($\varepsilon_z = 0$)	4.7615	5.5923	7.4931	9.86100	11.7547
	Mindlin [1] ($\varepsilon_z = 0$)	4.7902	5.6085	7.5078	9.87930	11.7764
Plate	Present ($\varepsilon_z \neq 0$)	4.7463	5.7095	7.7524	9.98590	11.5998
	Reddy [4] ($\varepsilon_z = 0$)	4.7609	5.5914	7.4918	9.86060	11.7592
	Mindlin [1] ($\varepsilon_z = 0$)	4.7895	5.6075	7.5065	9.87890	11.7809

Table 4. Non-dimensional fundamental frequencies of three-layer (0°/core/0°) sandwich shells with double curvature ($a = b$, $a = 10h$).

		C				
Shell type	(R_1/a,R_2/b)	1	2	5	10	15
Cylindrical	(5, ∞)	4.80657	5.76800	7.81446	10.05978	11.68624
	(10, ∞)	4.76145	5.72423	7.76802	10.00450	11.62150
	(20, ∞)	4.75006	5.71320	7.75632	9.990580	11.60520
	(50, ∞)	4.74686	5.71010	7.75304	9.986670	11.60063
	(100, ∞)	4.74640	5.70966	7.75257	9.986120	11.59998
Hyperbolic	(5,-5)	4.72677	5.68583	7.71975	9.943530	11.55042
	(10,-10)	4.74136	5.70356	7.74421	9.975270	11.58736
	(20,-20)	4.74503	5.70802	7.75036	9.983260	11.59666
	(50,-50)	4.74606	5.70927	7.75209	9.985500	11.59926
	(100,-100)	4.74620	5.70945	7.75234	9.985820	11.59964
Elliptical	(5,7.5)	4.92787	5.88993	7.95042	10.22465	11.87903
	(10, 15)	4.79241	5.75525	7.8025	10.04628	11.67037
	(20, 30)	4.75784	5.72099	7.76498	10.00106	11.61746
	(50, 75)	4.74811	5.71135	7.75443	9.988350	11.60259
	(100,150)	4.74672	5.70997	7.75292	9.986530	11.60047

6. Conclusions

In this paper, sandwich shells of double curvature are analyzed under static and free vibration conditions. The study presents a new hyperbolic shear deformation theory considering the effects of transverse normal strain. Navier's solution technique is used for the analysis of simply-supported sandwich shells. Based on the numerical results and discussion, it is concluded that the present theory predicts the static and free vibration behavior of sandwich spherical shells in close agreement with previously published results. It is concluded that the displacements and stresses increase with an increase in the radii of curvature of the sandwich shells. For the higher values of radii of curvature, non-dimensional values of displacements and stresses for all the types of shells are more or less the same this might be due to soft core sandwich shells. However, for the lower values of radii of curvature, non-dimensional values of displacements and stresses shows the considerable difference among all the types of shells. An increase in the softness of the core layer increases the fundamental frequencies of the sandwich shells. Static and free vibration analysis of sandwich shells with double curvature is rarely available in the literature and can be served as benchmark solutions. Due to consideration of the transverse normal strain effects, the present theory can be applied in the future for the thermal stress analysis of sandwich shells more effectively.

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