EE 133A — Homework #3 — Discussion 1B — Angel Jimenez — 604 667 905

3.1 *Polynomial interpolation.* In this problem we construct polynomials

$$p(t) = x_1 + x_2 t + \cdots + x_{n-1} t^{n-2} + x_n t^{n-1}$$

of degree 5, 10, and 15 (*i.e.*, for $n = 6, 11, 16$), that interpolate points on the graph of the function $f(t) = 1/(1+25t^2)$ in the interval $[-1, 1]$. For each value of n, we compute the interpolating polynomial as follows. We first generate n pairs (t_i, y_i), using the MATLAB commands

```
t = linspace(-1, 1, n)';
y = 1 ./ (1 + 25*t.^2);
```

This produces two n-vectors: a vector `t` with elements t_i, equally spaced in $[-1, 1]$, and a vector `y` with elements $y_i = f(t_i)$. (See '`help rdivide`' and '`help power`' for the meaning of the operations `./` and `.^`.) We then solve a set of linear equations

$$\begin{bmatrix} 1 & t_1 & \cdots & t_1^{n-2} & t_1^{n-1} \\ 1 & t_2 & \cdots & t_2^{n-2} & t_2^{n-1} \\ \vdots & \vdots & & \vdots & \vdots \\ 1 & t_{n-1} & \cdots & t_{n-1}^{n-2} & t_{n-1}^{n-1} \\ 1 & t_n & \cdots & t_n^{n-2} & t_n^{n-1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{n-1} \\ y_n \end{bmatrix} \tag{1}$$

to find the coefficients x_i.

Calculate the three polynomials (for $n = 6$, $n = 11$, $n = 16$). Plot the three polynomials and the function f on the interval $[-1, 1]$, and attach a printout of the plots to your solutions. What do you conclude about the effect of increasing the degree of the interpolating polynomial?

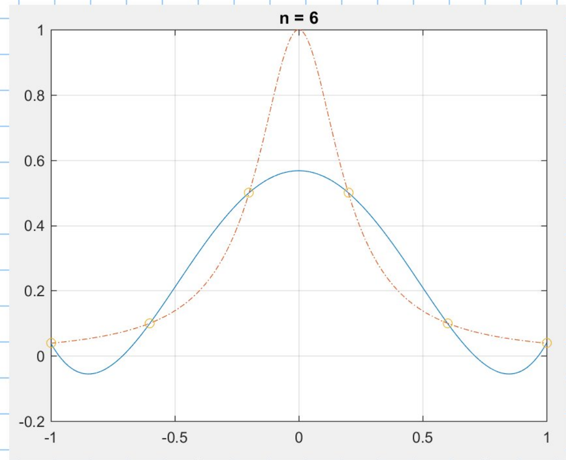
MATLAB hints.

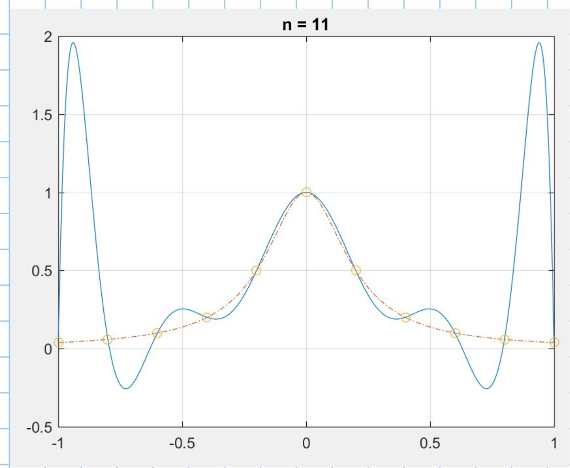
- Use `x = A \ b` to solve a set of n linear equations in n variables $Ax = b$.
- To construct the coefficient matrix in (1), you can write a double for-loop, or use the built-in MATLAB function `vander`, which constructs a matrix of the form

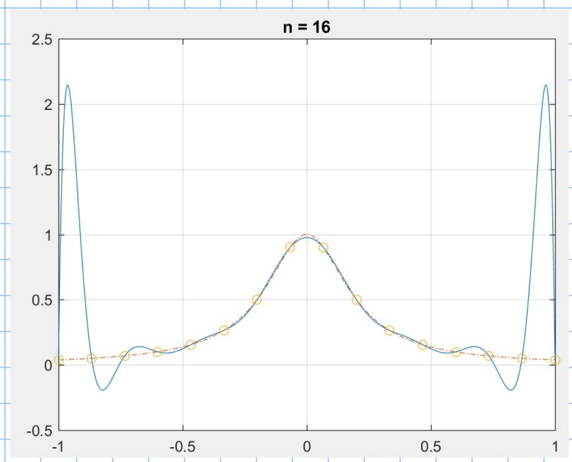
$$\begin{bmatrix} t_1^{n-1} & t_1^{n-2} & \cdots & t_1^2 & t_1 & 1 \\ t_2^{n-1} & t_2^{n-2} & \cdots & t_2^2 & t_2 & 1 \\ \vdots & \vdots & & \vdots & \vdots & \vdots \\ t_n^{n-1} & t_n^{n-2} & \cdots & t_n^2 & t_n & 1 \end{bmatrix}.$$

  Type '`help vander`' for details. This is almost what we need, but you have to 'flip' this matrix from left to right. This operation is also built in in MATLAB (type `help fliplr`).
- We are interested in the behavior of the interpolating polynomials between the points t_i that you used in the construction. Therefore, when you plot the three polynomials, you should use a much denser grid of points (*e.g.*, a few hundred points equally spaced in interval $[-1, 1]$) than the n points that you used to generate the polynomials.

```
n = 16; % degree of polynomial is n-1
K = 400 % set range
t = linspace(-1, 1,n)'; % n vector
y = 1 ./ (1+ 25*t.^2); % n vector
A = fliplr(vander(t)); % we want the ones on the left side

x = A \ y; % solves Ax=b, returns coefficients of polynomial

tp = linspace(-1,1,K)'; % for precision
yf = 1 ./ (1+25*tp.^2);

intpol = x(1)*ones(K,1);
for i=1:n-1 % n-1 is the degree of the polynomial
    intpol = intpol + x(i+1)*tp.^i;
end;


plot(tp,intpol,'-',tp ,yf ,'-.', t,y ,'o');
title('n = 16');
grid on;
```







3.4 Express the following problem as a set of linear equations. Find a quadratic function

$$f(u_1,u_2) = \underbrace{\begin{bmatrix} u_1 & u_2\end{bmatrix}}_{A}\underbrace{\begin{bmatrix} p_{11} & p_{12}\\ p_{12} & p_{22}\end{bmatrix}}_{B}\underbrace{\begin{bmatrix}u_1\\u_2\end{bmatrix}}_{C} + \underbrace{\begin{bmatrix} q_1 & q_2\end{bmatrix}}_{D}\underbrace{\begin{bmatrix}u_1\\u_2\end{bmatrix}}_{C} + r$$

that satisfies the following six conditions:

$$f(0,1)=6, \qquad f(1,0)=6, \qquad f(1,1)=3,$$

$$f(-1,-1)=7, \qquad f(1,2)=2, \qquad f(2,1)=6.$$

The variables in the problem are the parameters p_{11}, p_{12}, p_{22}, q_1, q_2 and r. Write the equations in matrix-vector form $Ax=b$, and solve the equations with MATLAB.

```
syms u1 u2 p11 p12 p22 q1 q2 r

u1 = 2;
u2 = 1;

A = [u1 u2];
B = [p11 p12; p12 p22];
C = [u1; u2];
D = [q1 q2];

E = A*B*C+D*C+r;
disp(E);
```

$$[u_1\ \ u_2]\begin{bmatrix}p_{11} & p_{12}\\ p_{12} & p_{22}\end{bmatrix}\begin{bmatrix}u_1\\u_2\end{bmatrix} + [q_1\ \ q_2]\begin{bmatrix}u_1\\u_2\end{bmatrix} + r$$

$\langle 1\times 2\rangle \quad \langle 2\times 2\rangle \quad \langle 2\times 1\rangle \; + \; \langle 1\times 2\rangle\langle 2\times 1\rangle + \langle 1\times 1\rangle$

$\langle 1\times 2\rangle\langle 2\times 1\rangle$

$\langle 1\times 1\rangle + \langle 1\times 1\rangle + \langle 1\times 1\rangle = \langle 1\times 1\rangle$

$$[u_1p_{11}+u_2p_{12} \quad u_1p_{12}+u_2p_{22}]\begin{bmatrix}u_1\\u_2\end{bmatrix}$$

$\langle 1\times 2\rangle$

$$[(u_1p_{11}+u_2p_{12})u_1 + (u_1p_{12}+u_2p_{22})u_2] + [q_1u_1+q_2u_2] + r$$

$$f(u_1,u_2) = r + q_1u_1 + q_2u_2 + u_1(p_{11}u_1 + p_{12}u_2) + u_2(p_{12}u_1 + p_{22}u_2)$$

$$f(0,1) = p_{22} + q_2 + r = 6$$

$$f(1,0) = p_{11} + q_1 + r = 6$$

$$f(1,1) = p_{11} + 2p_{12} + p_{22} + q_1 + q_2 + r = 3$$

$$f(-1,-1) = p_{11} + 2p_{12} + p_{22} - q_1 - q_2 + r = 7$$

$$f(1,2) = p_{11} + 4p_{12} + 4p_{22} + q_1 + 2q_2 + r = 2$$

$$f(2,1) = 4p_{11} + 4p_{12} + p_{22} + 2q_1 + q_2 + r = 6$$

Is there more efficient MATLAB code?

p_{11}	p_{12}	p_{22}	q_1	q_2	r	
0	0	1	0	1	1	6
1	0	0	1	0	1	6
1	2	1	1	1	1	3
1	2	1	-1	-1	1	7
1	4	4	1	2	1	2
4	4	1	2	1	1	6

$$\underbrace{\begin{bmatrix}0&0&1&0&1&1\\1&0&0&1&0&1\\1&2&1&1&1&1\\1&2&1&-1&-1&1\\1&4&4&1&2&1\\4&4&1&2&1&1\end{bmatrix}}_{A}\underbrace{\begin{bmatrix}p_{11}\\p_{12}\\p_{22}\\q_1\\q_2\\r\end{bmatrix}}_{x} = \underbrace{\begin{bmatrix}6\\6\\3\\7\\2\\6\end{bmatrix}}_{b}$$

```
F = [0 0 1 0 1 1; ...
     1 0 0 1 0 1; ...
     1 2 1 1 1 1; ...
     1 2 1 -1 -1 1; ...
     1 4 4 1 2 1; ...
     4 4 1 2 1 1]
G = [6 6 3 7 2 6]';

H = F \ G;
disp(H);
```

$p_{11} = 3$

$p_{12} = -2$

$p_{22} = 1$

$q_1 = -2$

$q_2 = 0$

$r = 5$

8.8 *Interpolation of rational functions.* A *rational function* of degree two has the form

$$f(t) = \frac{c_1 + c_2 t + c_3 t^2}{1 + d_1 t + d_2 t^2},$$

where c_1, c_2, c_3, d_1, d_2 are coefficients. ('Rational' refers to the fact that f is a ratio of polynomials. Another name for f is *bi-quadratic*.) Consider the interpolation conditions

$$f(t_i) = y_i, \quad i = 1, \ldots, K,$$

where t_i and y_i are given numbers. Express the interpolation conditions as a set of linear equations in the vector of coefficients $\theta = (c_1, c_2, c_3, d_1, d_2)$, as $A\theta = b$. Give A and b, and their dimensions.

Exercise T8.8. Solve the problem for $K = 5$ and interpolation conditions

$$f(1) = 2, \qquad f(2) = 5, \qquad f(3) = 9, \qquad f(4) = -1, \qquad f(5) = -4,$$

and plot the function $f(t)$.

$$b = \begin{bmatrix} f(t_1) = y_1 \\ f(t_2) = y_2 \\ \vdots \\ f(t_K) = y_K \end{bmatrix}$$

$$f(t_i) = y_i = \frac{c_1 + c_2 t_i + c_3 t_i^2}{1 + d_1 t_i + d_2 t_i^2} \Rightarrow (y_i)(1 + d_1 t_i + d_2 t_i^2) = c_1 + c_2 t_i + c_3 t_i^2$$

$$\hookrightarrow y_i + y_i d_1 t_i + y_i d_2 t_i^2 = c_1 + c_2 t_i + c_3 t_i^2$$

$$\hookrightarrow y_i = c_1 + c_2 t_i + c_3 t_i^2 - y_i d_1 t_i - y_i d_2 t_i^2$$

Q

How to analytically do it on matlab?

$$\therefore A = \begin{array}{c} t=1 \\ t=2 \\ \\ t=k \end{array} \begin{bmatrix} 1 & t_1 & t_1^2 & -y_1 t_1 & -y_1 t_1^2 \\ 1 & t_2 & t_2^2 & -y_2 t_2 & -y_2 t_2^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & t_K & t_K^2 & -y_K t_K & -y_K t_K^2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ d_1 \\ d_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 9 \\ -1 \\ -4 \end{bmatrix}$$

$$\hookrightarrow \begin{bmatrix} 1 & 1 & 1 & -2 & -2 \\ 1 & 2 & 4 & -10 & -20 \\ 1 & 3 & 9 & -27 & -81 \\ 1 & 4 & 16 & 4 & 16 \\ 1 & 5 & 25 & 20 & 100 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ d_1 \\ d_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 9 \\ -1 \\ -4 \end{bmatrix}$$

$c_1 = 0.6296$

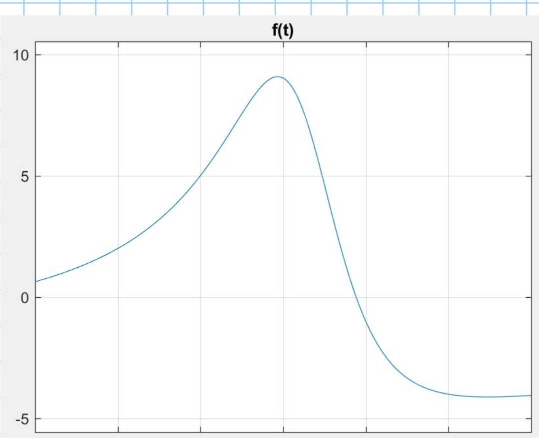
$c_2 = 0.6049$

$c_3 = -0.1975$

$d_1 = -0.5679$

$d_2 = 0.0864$

best way to plot f(t) (Better code?)




```
%Homework 3, problem T8.8
A = [1 1 1 -2 -2; ...
    1 2 4 -10 -20; ...
    1 3 9 -27 -81; ...
    1 4 16 4 16; ...
    1 5 25 20 100];
B = [2 5 9 -1 -4]';
x = A \ B;
% disp(x);

syms t;

y = (0.6296 + 0.6049*t + -0.1975*t.^2) ...
    /(1 -0.5679*t +0.0864*t.^2);
ezplot(y,[0,6]);
title('f(t)');
grid on;
```

8.11 *Location from range measurements.* The 3-vector x represents a location in 3-D. We measure the distances (also called the range) of x to four points at known locations a_1, a_2, a_3, a_4:

$$\rho_1 = \|x - a_1\|, \qquad \rho_2 = \|x - a_2\|, \qquad \rho_3 = \|x - a_3\|, \qquad \rho_4 = \|x - a_4\|.$$

Express these distance conditions as a set of three linear equations in the vector x. *Hint.* Square the distance equations, and subtract one from the others.

Exercise T8.11. Solve the problem for the example in exercise A3.5.

3.5 Solve the problem in exercise 8.11 of the textbook for

$$a_1 = \begin{bmatrix} -10 \\ -10 \\ 10 \end{bmatrix}, \quad a_2 = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}, \quad a_3 = \begin{bmatrix} -10 \\ 10 \\ 0 \end{bmatrix}, \quad a_4 = \begin{bmatrix} -20 \\ -10 \\ -10 \end{bmatrix},$$

and

$$\rho_1 = 18.187, \qquad \rho_2 = 9.4218, \qquad \rho_3 = 14.310, \qquad \rho_4 = 24.955.$$

Generalized $\rho_i = \|x - a_i\|$ for $i = 1, 2, 3, 4$

recall $\|\vec{a}\| = \sqrt{a_1^2 + a_2^2 + \ldots + a_n^2} = \sqrt{a^T a}$

Q : $-x^T a_i - a_i^T x$

$= -2x^T a_i$

or

$-2a_i^T x$?

$\therefore \quad \|\vec{a}\| = \sqrt{(x - a_i)^T (x - a_i)} = \sqrt{x^T x - x^T a_i - a_i^T x + a_i^T a_i}$

$\rho_i = \sqrt{x^T x - 2x^T a_i + a_i^T a_i}$

$\rho_i^2 = x^T x - 2x^T a_i + a_i^T a_i$

$\rho_j^2 - \rho_k^2 = x^T x - 2x^T a_j + a_j^T a_j - (x^T x - 2x^T a_k + a_k^T a_k)$

$\rho_j^2 - \rho_k^2 = \cancel{x^T x} - 2x^T a_j + a_j^T a_j - \cancel{x^T x} - 2x^T a_k - a_k^T a_k$

$= -2x^T a_j + a_j^T a_j - 2x^T a_k - a_k^T a_k$

$= -2x^T a_j + \|a_j\|^2 - 2x^T a_k - \|a_k\|^2$

$= -2x^T (a_j - a_k) + \|a_j\|^2 - \|a_k\|^2$

$\rho_j^2 - \rho_k^2 = -2x^T (a_j - a_k) + \|a_j\|^2 - \|a_k\|^2$ switch from $x^T a$ to $a^T x$

$\rho_j^2 - \rho_k^2 + \|a_k\|^2 - \|a_j\|^2 = 2(a_k - a_j)^T x$

$k = 1 \quad -2(a_j - a_k)^T x = \rho_j^2 - \rho_k^2 + \|a_k\|^2 - \|a_j\|^2$

$$\left.\begin{array}{l} j=2 \quad -2(a_2 - a_1)^T x = \rho_2^2 - \rho_1^2 + \|a_1\|^2 - \|a_2\|^2 \\ j=3 \quad -2(a_3 - a_1)^T x = \rho_3^2 - \rho_1^2 + \|a_1\|^2 - \|a_3\|^2 \\ j=4 \quad -2(a_4 - a_1)^T x = \rho_4^2 - \rho_1^2 + \|a_1\|^2 - \|a_4\|^2 \end{array}\right\} \underbrace{\begin{bmatrix} -2(a_2 - a_1)^T \\ -2(a_3 - a_1)^T \\ -2(a_4 - a_1)^T \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} \rho_2^2 - \rho_1^2 + \|a_1\|^2 - \|a_2\|^2 \\ \rho_3^2 - \rho_1^2 + \|a_1\|^2 - \|a_3\|^2 \\ \rho_4^2 - \rho_1^2 + \|a_1\|^2 - \|a_4\|^2 \end{bmatrix}}_{b}$$

$\|a_1\|^2 = a_1^T a_1 = [\cdots\cdots] \begin{bmatrix} \vdots \end{bmatrix}$

```
% Homework 3: Problem 8.11

p1 = 18.187;    a1 = [-10 -10 10]';
p2 = 9.4218;    a2 = [10 0 0]';
p3 = 14.310;    a3 = [-10 10 0]';
p4 = 24.955;    a4 = [-20 -10 -10]';

A = [ -2*(a2 - a1)'; ...
      -2*(a3 - a1)'; ...
      -2*(a4 - a1)'];
B = [ (p2)^2 - (p1)^2 + a1'*a1 - a2'*a2; ...
      (p3)^2 - (p1)^2 + a1'*a1 - a3'*a3; ...
      (p4)^2 - (p1)^2 + a1'*a1 - a4'*a4]; ...

C = A\B;
disp(C);
```

$X_1 = 0.5999$

$X_2 = 0.3996$

$X_3 = -0.5063$

11.5 *Inverse of a block matrix.* Consider the $(n+1)\times(n+1)$ matrix

$$A = \begin{bmatrix} I & a \\ a^T & 0 \end{bmatrix},$$

where a is an n-vector.

(a) When is A invertible? Give your answer in terms of a. Justify your answer.

(b) Assuming the condition you found in part (a) holds, give an expression for the inverse matrix A^{-1}.

$A = \begin{bmatrix} I & a \\ a^T & 0 \end{bmatrix}$ $\langle (n+1)\times(n+1) \rangle$

a is n vector i.e. $\begin{bmatrix} \vdots \\ n \end{bmatrix}$ ($\langle n\times 1\rangle$ of length n) & $a^T = [\cdots\cdots n]$ ($\langle 1\times n\rangle$ of length n)

$$\therefore A = \begin{array}{c c} & \begin{array}{cc} n & 1 \end{array} \\ \begin{array}{c} n \\ 1 \end{array} & \begin{bmatrix} [\,I\,] & [\,a\,] \\ [\,a^T\,] & [0] \end{bmatrix} \end{array}$$

find conditions for a

$\therefore Ax = 0$ first show it's nonsingular i.e. $Ax=0$ only when $x=0$

$$\therefore \begin{bmatrix} I & a \\ a^T & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\langle 2\times2\rangle\ \langle 2\times1\rangle = \langle 2\times1\rangle$

$$\begin{bmatrix} Ix_1 + ax_2 \\ a^Tx_1 + 0x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{array}{l} x_1 + ax_2 = 0 \Rightarrow \\ a^Tx_1 = 0 \Rightarrow x_1 = 0 \\ \to ax_2 = 0 \quad \therefore x_2 = 0 \text{ as well} \end{array}$$

$\langle 2\times1\rangle$

$\therefore A$ is nonsingular

Since we know A is singular we know $A^{-1}A = I$ or $AA^{-1} = I$

~~Suppoe $A^{-1} = \begin{bmatrix} I & a^T \\ a & 0 \end{bmatrix}$~~

~~$\begin{bmatrix} I & a \\ a^T & 0 \end{bmatrix}\begin{bmatrix} I & a^T \\ a & 0 \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & 1 \end{bmatrix}$~~

~~$\begin{matrix} II + aa & Ia^T + 0a \\ a^TI + 0a & a^Ta^T + 0\cdot 0 \end{matrix} \Rightarrow$~~

why doesn't this work?

Suppose $A^{-1} = \begin{bmatrix} B & C \\ D & E \end{bmatrix}$

$\therefore AA^{-1} = I$

$$\begin{bmatrix} I & a \\ a^T & 0 \end{bmatrix}\begin{bmatrix} B & C \\ D & E \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$$

this has to be specially partitioned... [+][+] unless we multiply by arbitrary vectors B,C,D,E

$$\begin{bmatrix} IB + aD & IC + aE \\ a^TB + 0\cdot D & a^TC + 0\cdot D \end{bmatrix}$$

$$\left.\begin{array}{l} IB + aD = I \\ IC + aE = 0 \\ a^TB + 0\cdot D = 0 \\ a^TC + 0\cdot D = I \end{array}\right\} \Rightarrow \begin{array}{l} B + aD = I \\ C + aE = 0 \\ a^TB = 0 \leftarrow B = 0 \\ a^TC = I \end{array} \Rightarrow \begin{array}{l} aD = I \leftarrow D = a^{-1} \\ C + aE = 0 \\ a^TC = I \leftarrow \text{orthogonal ?} \therefore \quad C = (a^T)^{-1}\ ? \end{array}$$

I'm not sure about this

11.9 *Push-through identity.* Suppose A is $m \times n$, B is $n \times m$, and the $m \times m$ matrix $I + AB$ is invertible.

(a) Show that the $n \times n$ matrix $I + BA$ is invertible. *Hint.* Show that $(I + BA)x = 0$ implies $(I + AB)y = 0$, where $y = Ax$.

(b) Establish the identity

$$B(I + AB)^{-1} = (I + BA)^{-1}B.$$

This is sometimes called the *push-through identity* since the matrix B appearing on the left 'moves' into the inverse, and 'pushes' the B in the inverse out to the right side. *Hint.* Start with the identity

$$B(I + AB) = (I + BA)B,$$

and multiply on the right by $(I + AB)^{-1}$, and on the left by $(I + BA)^{-1}$.

a) $A = \langle m \times n \rangle$, $B = \langle n \times m \rangle$, $(I+AB) = \langle m \times m \rangle$

Show $(I+BA)x = 0 \rightarrow (I+AB)y = 0$, $y = Ax$

$I + BA$ $I + AB$

$\langle n \times m \rangle \langle m \times n \rangle$ $\langle m \times m \rangle + \langle m \times n \rangle \langle n \times m \rangle$

$\langle n \times n \rangle + \langle n \times n \rangle$ $\langle m \times m \rangle + \langle m \times m \rangle$

$\langle n \times n \rangle$ $\langle m \times m \rangle$

for $(I+AB)y = 0$ $y = Ax$

$\langle m \times m \rangle \langle m \times 1 \rangle = \langle m \times 1 \rangle$ $\langle m \times 1 \rangle = \langle m \times n \rangle \langle n \times 1 \rangle$

$\therefore y = \langle m \times 1 \rangle$, $x = \langle n \times 1 \rangle$

if $(I+AB)$ is invertible then it is nonsingular.
which means $(I+AB)x = 0$ when $x = 0$

if $x = 0$ then $y = Ax = 0$

thus $(I + BA)y = 0$ implies $(I + BA)$

is also invertible & non singuler

b) $B(I + AB) = (I + BA)B$

$BI + BAB$

$B + BAB$

$\swarrow \quad \searrow$

$B(I + AB) = (I + BA)B$

$(I+BA)^{-1}B(I+AB) = (I+BA)^{-1}(I+BA)B$

$(I+BA)^{-1}B(I+AB) = B$

$(I+BA)^{-1}B = B(I+AB)^{-1}$

I'm having difficulty Proving things are invertible.

4.2 Suppose A is a nonsingular $n \times n$ matrix, u and v are n-vectors, and $v^T A^{-1} u \neq -1$. Show that $A + uv^T$ is nonsingular with inverse

$$(A + uv^T)^{-1} = A^{-1} - \frac{1}{1 + v^T A^{-1} u} A^{-1} u v^T A^{-1}.$$

A: nonsingular $\langle n \times n \rangle$ $\quad A^{-1} = \langle n \times n \rangle$ $\quad v^T A^{-1} u$

$\langle 1 \times n \rangle \langle n \times n \rangle \langle n \times 1 \rangle$

$\langle 1 \times 1 \rangle$

u: n-vector $\langle n \times 1 \rangle$

v: n-vector $\langle n \times 1 \rangle$ $\quad v^T = \langle 1 \times n \rangle$

$$(A + uv^T)^{-1} = A^{-1} - \frac{A^{-1} u v^T A^{-1}}{1 + v^T A^{-1} u}$$

$A^{-1} u \, v^T A^{-1}$

$\langle n \times n \rangle \langle n \times 1 \rangle \langle 1 \times n \rangle \langle n \times n \rangle$

$\langle n \times 1 \rangle \langle 1 \times n \rangle$

$\langle n \times n \rangle$

$$(A + uv^T)(A + uv^T)^{-1} = (A + uv^T)\left[A^{-1} - \frac{A^{-1} u v^T A^{-1}}{1 + v^T A^{-1} u}\right] \Rightarrow I = (A + uv^T)A^{-1} - \frac{(A + uv^T)A^{-1} u v^T A^{-1}}{1 + v^T A^{-1} u}$$

$$I = AA^{-1} + uv^T A^{-1} - \frac{(AA^{-1} u v^T A^{-1} + u \overbrace{v^T A^{-1} u}^{\text{scalar}} v^T A^{-1})}{1 + v^T A^{-1} u}$$

$v^T A^{-1} u = \text{scalar}$

$$I = I + \underbrace{uv^T A^{-1}}_{\substack{\langle n \times 1 \rangle \langle 1 \times n \rangle \langle n \times n \rangle \\ \langle n \times n \rangle}} - \frac{(uv^T A^{-1} + v^T A^{-1} u \, u v^T A^{-1})}{1 + v^T A^{-1} u}$$

$$I = I + uv^T A^{-1} - \frac{\cancel{(1 + v^T A^{-1} u)}(u v^T A^{-1})}{\cancel{(1 + v^T A^{-1} u)}} = I + uv^T A^{-1} - uv^T A^{-1}$$

$\therefore I = I$

Why does this prove its nonsingular?

nonsingular = interible?

4.7 We consider the problem of localization from range measurements in 3-dimensional space. The 3-vector y represents the unknown location. We measure the distances of the location y to five points at known locations $c_1, \ldots, c_5$. The five distance measurements $\rho_1, \ldots, \rho_5$ are exact, except for an unknown systematic error or offset z (for example, due to a clock offset). We therefore have five equations

$$\|y - c_k\| + z = \rho_k, \quad k = 1, \ldots, 5,$$

with four unknowns y_1, y_2, y_3, z. We assume that the five vectors

$$\begin{bmatrix} c_1 \\ \rho_1 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} c_2 \\ \rho_2 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} c_3 \\ \rho_3 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} c_4 \\ \rho_4 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} c_5 \\ \rho_5 \\ 1 \end{bmatrix}$$

are linearly independent.

Write a set of linear equations $Ax = b$, with a nonsingular matrix A, from which the variable $x = (y_1, y_2, y_3, z)$ can be determined. Explain why A is nonsingular.

$\|y - c_1\| + z = \rho_1$ $\qquad \sqrt{(y-c_i)^T(y-c_i)}$

$\|y - c_2\| + z = \rho_2$ $\qquad \sqrt{y^Ty - 2c_i^Ty + c_i^Tc_i} = \rho_1 - z$

$\|y - c_3\| + z = \rho_3$

$\|y - c_4\| + z = \rho_4$

$\|y - c_5\| + z = \rho_5$

General Case $\|y - c_k\| + z = \rho_k \rightarrow \|y - c_k\|^2 = (\rho_k - z)^2$

$$(y - c_k)^T(y - c_k) = \rho_k^2 - \rho_k z - \rho_k z + z^2$$

$$y^Ty - y^Tc_k - c_k y^T + c_k^T c_k = \rho_k^2 - 2\rho_k z + z^2$$

$$\|y\|^2 - 2c_k^T y + \|c_k\|^2 = \rho_k^2 - 2\rho_k z + z^2$$

I'm not certain how to progress from here

Do I subtract $\|y\|^2$ & $\|c_k\|^2$?

$-2c_k^T y$ $\qquad$ for each vector

$\begin{bmatrix} c_1 \\ \rho_1 \\ 1 \end{bmatrix}$ ← what does the "1" mean?

do I assume

$x_1 = c_1$
$x_2 = \rho_1$
$x_3 = 1$