Who Marries Whom and Why

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This paper proposes and estimates a static transferable utility model of the marriage market. The model generates a nonparametric marriage matching function with spillover effects. It rationalizes the standard interpretation of marriage rate regressions and points out its limitations. The model was used to estimate U.S. marital behavior in 1971/72 and 1981/82. The marriage matching function estimates show that the gains to marriage for young adults fell substantially over the decade. Unlike contradictory marriage rate regression results, the marriage matching function estimates showed that the legalization of abortion had a significant quantitative impact on the fall in the gains to marriage for young men and women.

I. Introduction

Thirty years ago, Gary Becker (1973, 1974) exposited a static transferable utility model of the marriage market. While implications of his model have been tested and applied (South and Trent 1988; South and Lloyd 1992; Grossbard-Shechtman 1993; Rao 1993; Seitz 1999; Edlund 2000; Hamilton and Siow 2000; Angrist 2002; Chiappori, Fortin, and Lacroix

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¹ This is also summarized in Becker (1991).

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2002; Botticini and Siow 2003),² it has seldom been estimated.³ There are two problems that have to be solved before a transferable utility model of the marriage market can be estimated. First, equilibrium transfers in modern marriages are seldom observed. Second, individuals may differ by age, religion, education, wealth, ethnicity, and so on. Different types of individuals may not agree on the rankings of individuals of the opposite gender as spouses. Thus an empirical model of the marriage market should not impose too much a priori structure on the nature of preferences for marriage partners. However, without a priori structure, it is unclear what can be identified from the data.

To understand the identification problem, consider a society with I types of men and I types of women participating in the marriage market. A type is defined by an age range, ethnicity, education, geographic location, and so on. Each individual chooses whom to marry or to remain unmarried. For each type of man (woman), there are potentially I(I)preference parameters to characterize his (her) choice of whether to marry and whom to marry. In total, there are as many as $2 \times I \times J$ preference parameters. What is observable to a researcher? In principle, the researcher observes the quantity of each type of man in the marriage market, m_i for type i men (I observations); the quantity of each type of woman, f_i for type j women (J observations); and the quantity of type *i* men married to type *j* women, μ_{ij} ($I \times J$ observations). So the total number of observables is $I + J + I \times J$. For I, J > 2, the number of observables is less than the number of unknown preference parameters. Thus any behavioral empirical model will need to make identifying assumptions to reduce the number of unknown parameters.⁴

To finesse the identification problem, demographers use a reduced-form approach in the form of *marriage matching functions* to estimate the behavior of the entire marriage market.⁵ A marriage matching function is defined as follows. Let M be the vector of available men by types, $i = 1, \ldots, I$, at that time. The ith element of the vector M is denoted by m_i . Let F be the vector of available women by types, $j = 1, \ldots, J$, where the jth element of the vector is denoted by f_j . Let \mathbf{II} be a matrix of parameters. A marriage matching function is an $I \times J$ matrix $\mu(M, F; \mathbf{II})$, whose i, jth element is μ_{ij} . Denote the number of unmarried

² Bergstrom (1997) and Weiss (1997) provide surveys of the economics literature up to the mid-1990s. Waite et al. (2000) and Casper and Bianchi (2002) show his influence outside economics.

³ Bergstrom and Lam (1994) and Suen and Lui (1999) are exceptions.

⁴ The identification problem is not well known because economists calibrate or estimate models with strong a priori identifying assumptions (examples include Bergstrom and Lam [1994], Seitz [1999], Suen and Lui [1999], Aiyagari, Greenwood, and Guner [2000], Hamilton and Siow [2000], Wong [2003*a*, 2003*b*], and Fernandez, Guner, and Knowles [2005]).

⁵ Our discussion borrows heavily from Pollak (1990a, 1990b).

men of type i as μ_{i0} and the number of unmarried women of type j as μ_{0j} . The marriage matching function $\mu(M, F; \Pi)$ must satisfy the following accounting constraints:

$$\mu_{0j} + \sum_{i=1}^{I} \mu_{ij} = f_j \quad \forall j, \tag{1}$$

$$\mu_{i0} + \sum_{i=1}^{J} \mu_{ij} = m_i \quad \forall i, \tag{2}$$

$$\mu_{0i}, \ \mu_{i0}, \ \mu_{ii} \ge 0 \quad \forall i, \ j. \tag{3}$$

Demographers have mostly estimated marriage matching functions without spillover effects. This paper proposes and estimates a static transferable utility model of the marriage market. The model produces a simple nonparametric marriage matching function with spillover effects that will fit any cross-section marriage distribution.

There are three conceptual benefits for considering transferable utility models of the marriage market. First, marriage market-clearing equilibrium must satisfy all the accounting constraints, (1), (2), and (3). Second, the reduced form for equilibrium quantities of a market-clearing model does not include prices, that is, equilibrium transfers. Thus the absence of observable transfers to the researcher may not be a problem. Third, transferable utility models provide a solution to the identification problem discussed above. To see how the identification problem may be resolved, let the marital output of a type i man and a type j woman depend only on i and j. Then there are $I \times J$ marital outputs plus I+I outputs of the types remaining unmarried. If the behavior of the marriage market is characterized by these outputs alone, then we may be able to estimate all the parameters that are necessary to determine marital behavior. In particular, we do not have to estimate separate male and female preferences for spouses. A well-known property of transferable utility models of the marriage market is that they maximize the sum of marital output in the society (e.g., Roth and Sotomayor 1990, chap. 8; Gretsky, Ostroy, and Zame 1992). Thus behavior in transferable utility models can be characterized by knowledge about marital output alone, and separate knowledge about male and female preferences is not necessary. The novelty of this paper is to exploit this

⁶ The functions $\mu_{ij} = g(m_p f_p \pi_{ij})$ do not have spillover or substitution effects (Schoen 1981). Variations in $m_{≠i}$ or $f_{≠j}$ do not affect μ_{ij} . This deficiency is known (Pollak 1990*b*; Pollard and Höhn 1993–94; Pollard 1997).

⁷ It also satisfies the conditions in Pollak (1990*a*) sufficient to generate a well-posed two-sex model of population growth.

property to specify a just-identified econometric model of the marriage market and minimize a priori restrictions on preferences for spousal types.⁸

An important theoretical antecedent to our work is the article by Dagsvik (2000), who also derived a behavioral marriage matching function (see also Johansen and Dagsvik 1999; Dagsvik, Brunborg, and Flaatten 2001; Logan, Hoff, and Newton 2001). We follow his lead in using McFadden's (1974) extreme-value random utility functions. We use a transferable utility framework, whereas he uses a nontransferable utilities model (for a fuller comparison, see our working paper [Choo and Siow 2003]).

The current paper has two limitations. While our model admits spill-over or substitution effects, we do not know how restrictive our substitution patterns are on the marriage matching function. Another limitation of our static approach is that it ignores dynamic considerations. Choo and Siow (2005) extend this model into a dynamic framework.

Using ages as the only types for men and women in the benchmark model, the second part of the paper estimates the model using data from the 1970 and 1980 U.S. Census and 1971/72 and 1981/82 vital statistics. The baby boom generation came into marriageable age between the two decades, and thus there were substantial changes in the population vectors between the decades. Our marriage matching function can capture some changes in marital patterns in the United States between 1971/72 and 1981/82 due to changes in population vectors between the two periods. However, our benchmark model could not capture the drastic fall in the marriage rate among young adults over the decade.⁹

There were many social changes between 1970 and 1980 that could have affected the gains to marriage over the decade. A major change was the national legalization of abortion in 1973. Legal abortions were partially available in some states by 1970. If the partial legalization of abortions in a state reduced the gains to marriage in that state, we would expect to see lower gains to marriage in the early legalizing states relative to later legalizing states in 1970 but not in 1980. Moreover, this difference in difference in the gains to marriage should be concentrated among women of childbearing age. Using marriage rate regressions, Angrist and Evans (1999) showed that the marriage rates of young men and women were lower in early legalizing states relative to later legalizing states in the early 1970s. We show that the estimates of the number of marriages affected are sensitive to whether we use male or female mar-

 $^{^8\,\}mathrm{Fox}\ (2005)$ extends our insight for estimating other matching models with transferable utilities.

⁹ The drop is well known (Qian and Preston 1993; Qian 1998).

riage rate regressions. We extend the benchmark model to include whether an individual resided in a state that allowed legal abortions or not as part of the definition of the type of an individual. Methodologically, we extend the standard difference in differences estimator to estimate the effect of a policy change on bivariate distributions. Estimating this extended model, we show that the partial legalization of abortion in some states can explain up to 20 percent of the drop in the gains to marriage among young adults in the 1970s.

II. The Model

We begin by describing a transferable utility model of the marriage market. There are I types of men and J types of women. For a type i man to marry a type j woman, he must transfer τ_{ij} amount of income to her. There are $I \times J$ sub-marriage markets for every combination of types of men and women. The marriage market clears when, given equilibrium transfers τ_{ij} , the demand by type i men for type j spouses is equal to the supply of type j women for type i men for all i, j.

To implement the above framework empirically, we adopt the extreme-value (logit) random utility model of McFadden (1974) to generate market demands for marriage partners. Each individual considers matching with a member of the opposite gender. Let the utility of type i man g who marries a type j woman be

$$V_{ijg} = \tilde{\alpha}_{ij} - \tau_{ij} + \varepsilon_{ijg}, \tag{4}$$

where $\tilde{\alpha}_{ij}$ is the systematic gross return to a type i man married to a type j woman; τ_{ij} is the equilibrium transfer made by a type i man to a type j spouse; and ε_{ijg} is an independently and identically distributed random variable with a type I extreme-value distribution. ¹⁰ Equation (4) says that the payoff to a man g from marrying a type j woman consists of two components: a systematic and an idiosyncratic component. The systematic component, $\tilde{\alpha}_{ij} - \tau_{ij}$, is common to all type i men married to type j women. This systematic return is reduced when τ_{ij} , the equilibrium transfer, is increased.

The idiosyncratic component, ε_{ijg} , measures the departure of g's individual-specific match payoff, V_{ijg} , from the systematic component. We assume that the distribution of ε_{ijg} does not depend on the specific type j woman that he may marry. The payoff to g from remaining unmarried, denoted by j=0, is

$$V_{i0g} = \tilde{\alpha}_{i0} + \varepsilon_{i0g}, \tag{5}$$

 $^{^{10}}$ The random variable ε_{ijg} has the cumulative distribution given by $F(\varepsilon)=\exp{[-\exp{(-\varepsilon)}]}.$

where ε_{i0g} is also an independently and identically distributed random variable with a type I extreme-value distribution.

Individual g will choose according to

$$V_{ig} = \max_{j} \{V_{i0g}, \dots, V_{ijg}, \dots, V_{iJg}\}.$$
 (6)

We assume that the number of men and women of each type is large. Let μ_{ij}^d be the number of i, j marriages demanded by type i men and μ_{i0}^d be the number of unmarried type i men. Then McFadden (1974) showed that (App. A includes a proof for convenience)

$$\ln \mu_{ij}^{d} = \ln \mu_{i0}^{d} + \tilde{\alpha}_{ij} - \tilde{\alpha}_{i0} - \tau_{ij}$$

$$= \ln \mu_{i0}^{d} + \alpha_{ij} - \tau_{ij}.$$
(7)

The term $\alpha_{ij} = \tilde{\alpha}_{ij} - \tilde{\alpha}_{i0}$ is the systematic gross return to a type i man from an i, j marriage relative to being unmarried. The above equation is a quasi demand equation by type i men for type j spouses.

Let $\it c$ be Euler's constant. Appendix A shows another well-known result:

$$\mathbb{E}V_{ig} = c + \tilde{\alpha}_{i0} + \ln\left(\frac{m_i}{\mu_{i0}^d}\right),\tag{8}$$

where $\mathbb{E}V_{ig}$ is the expected utility of a type i man before he sees his realizations of his ε_{ijg} for all j. Equation (8) shows that it is proportional to the log of the ratio of the number of available type i men relative to the number of type i men who choose to remain unmarried. The expected payoff of being unmarried is given by $\mathbb{E}V_{i0g} = c + \tilde{\alpha}_{i0}$. Let $q_i = \ln{(m_i/\mu_{i0}^d)}$. It measures the expected gains or benefit from being able to participate in the marriage market for a type i man. As shown in Appendix A and Section III, the expected gains depend on preference parameters, $\tilde{\alpha}_{ij}$ and $\tilde{\alpha}_{i0}$, as well as transfers, τ_{ij} .

The random utility function for women has a similar form except that in marriage with a type i man, a type j woman receives a transfer τ_{ij} . Let $\tilde{\gamma}_{ij}$ denote the systematic gross gain that type j women get from marrying type i men and $\tilde{\gamma}_{0j}$ be the systematic payoff that type j women get from remaining unmarried. The term $\gamma_{ij} = \tilde{\gamma}_{ij} - \tilde{\gamma}_{0j}$ is the systematic gross gain that type j women get from marrying type i men relative to not marrying.

Let μ_{ij}^s be the number of i, j marriages demanded by type j women and μ_{0j}^s the number of type j women who want to remain unmarried. The quasi supply equation of type j women who marry type i men is given by

$$\ln \mu_{ij}^s = \ln \mu_{0j}^s + \gamma_{ij} + \tau_{ij}. \tag{9}$$

From (8), the expected gain to entering the marriage market for a type j woman is $Q_i = \ln (f_i/\mu_{0i}^s)$.

The $I \times J$ sub-marriage market clears when, given equilibrium transfers τ_{ii} , the demand by type i men for type j spouses is equal to the supply of type j women for type i men for all i, j. That is, for all i, jpairs, $\mu_{ij} = \mu_{ij}^d = \mu_{ij}^s$. Substituting this into equations (7) and (9) and adding the two equations yields

$$\ln \mu_{ij} - \frac{\ln \mu_{i0} + \ln \mu_{0j}}{2} = \frac{\alpha_{ij} + \gamma_{ij}}{2}.$$
 (10)

If we let $\pi_{ij} = \ln \Pi_{ij} = (\alpha_{ij} + \gamma_{ij})/2$, we can rewrite equation (10) as

$$\Pi_{ij} = \frac{\mu_{ij}}{\sqrt{\mu_{i0}\mu_{0j}}},\tag{11}$$

which is our marriage matching function.

Equation (11) has an intuitive interpretation. The right-hand side of (11) is the ratio of the number of i, j marriages to the geometric average of those types who are unmarried. The log of the left-hand side, $\ln \Pi_{ii} = \pi_{ii}$, is interpreted as the total systematic gain to marriage per partner for any i, j pair relative to the total systematic gain per partner from remaining unmarried. One expects the systematic gains to marriage to be large for i, j pairs if one observes many i, j marriages. However, there are two other explanations for numerous i, j marriages. First, there are lots of type i men and type j women in the population. Second, there are relatively more type i men and type j women in the population than other types of participants. Scaling the number of i, j marriages by the geometric average of the numbers of unmarrieds of those types controls for these effects. Equation (11) is homogeneous of degree zero in population vectors and the number of marriages. Thus our marriage matching function has no scale effect in population vectors.

A. Identification

A point estimate for Π_{ij} is given by $\mu_{ij}/\sqrt{\mu_{i0}\mu_{0j}}$. Equation (11) is nonparametric in the sense that it fits any observed marriage distribution. Observing Π_{ij} however, is not sufficient for us to identify the individualspecific systematic returns, α_{ij} and γ_{ij} . 12

¹¹ See Roth and Sotomayor (1990) and Gretsky et al. (1992) for the existence of equi-

libria in transferable utility market assignment models.

12 It is also not sufficient to estimate $\alpha_{ij} - \gamma_{ij}$, which is needed to identify the equilibrium transfers.

In addition to π_{ij} equations (7) and (9) allow us to identify $\alpha_{ij} = \tau_{ij}$ and $\gamma_{ij} + \tau_{ij}$:

$$\ln\left(\frac{\mu_{ij}}{\mu_{i0}}\right) = \alpha_{ij} - \tau_{ij} = n_{ij},$$

$$\ln\left(\frac{\mu_{ij}}{\mu_{0i}}\right) = \gamma_{ij} + \tau_{ij} = N_{ij}.$$
(12)

We refer to n_{ij} as the systematic net gain to marriage for a type i man in an i, j marriage relative to not marrying, and N_{ij} as the systematic net gain to marriage for a type j woman in an i, j marriage relative to not marrying.

B. Comparative Statics and Policy Evaluations

Given the preference parameters of the system, Π_{ij} , we are often interested in how variations in the supply population vectors, M and F, affect the distribution of marriages as represented by μ . Our marriage matching function may be rewritten as

$$\mu_{ij} = \Pi_{ij} \sqrt{\mu_{i0} \times \mu_{0j}}$$

$$= \Pi_{ij} \sqrt{\left(m_i - \sum_{k=1}^{J} \mu_{ik}\right) \left(f_j - \sum_{g=1}^{I} \mu_{gj}\right)}.$$
(13)

If we take Π_{ij} , M, and F as exogenously given, the second line of equation (13) defines an $I \times J$ system of quadratic equations with the $I \times J$ elements of μ as unknowns. Given population quantities M, F, μ , and Π as defined in equation (11), local uniqueness of μ^* for new values of $M^* \neq M$ and $F^* \neq F$ and Π held fixed is given by the following result.

Proposition 1. Let

$$\Pi_{ij} = \mu_{ij} \left[\left(m_i - \sum_{k=1}^{I} \mu_{ik} \right) \left(f_j - \sum_{g=1}^{J} \mu_{gj} \right) \right]^{-1/2}$$

and M and F be the vectors of m_i and f_p respectively. For M^* and F^* close to M and F, μ^* is uniquely determined.

The proof using the implicit function theorem is given in Appendix B.

Our marriage matching function does not suffer from the zero spillover or substitution restrictions that plague many marriage matching functions in this literature. For example, with preferences Π_{ij} held constant, the marginal effect on μ_{ij} from an increase in m_r is given by

$$\frac{\partial \mu_{ij}}{\partial m_r} = \frac{1}{2} \Pi_{ij} \left[\left(\frac{\mu_{0j}}{\mu_{i0}} \right)^{1/2} \frac{\partial \mu_{i0}}{\partial m_r} + \left(\frac{\mu_{i0}}{\mu_{0j}} \right)^{1/2} \frac{\partial \mu_{0j}}{\partial m_r} \right].$$

The actual forms of $\partial \mu_{i0}/\partial m_r$ and $\partial \mu_{0j}/\partial m_r$ are given by equations (B3) and (B4) in Appendix B. These derivatives are not zero.

The expected gain to entering the marriage market for a type j woman, denoted by Q_p is related to the marriage rate by

$$Q_j = \ln\left(\frac{f_j}{\mu_{0j}}\right) = -\ln\left(1 - \frac{\sum_i \mu_{ij}}{f_j}\right) \approx \frac{\sum_i \mu_{ij}}{f_j} = \rho_j^f.$$
 (14)

This approximation is accurate for small marriage rates. The marriage rate for type j women is also related to the systematic net gains N_{ij} in (12) according to

$$\rho_j^f \approx Q_j = \ln\left(1 + \sum_i \frac{\mu_{ij}}{\mu_{0j}}\right) = \ln\left[1 + \sum_i \exp\left(\gamma_{ij} + \tau_{ij}\right)\right]$$

$$= \ln\left[1 + \sum_i \exp\left(N_{ij}\right)\right]. \tag{15}$$

Equation (15) says that the marriage rate of type j women depends positively on the systematic gross gains to marriage, γ_{ij} , and equilibrium transfers, τ_{ij} . Thus (15) provides a formal justification for the standard interpretation of marriage rate regressions, where the marriage rate of type j women is assumed to vary positively with factors that increase the gains to marriage for these women.

We can also do policy evaluations with π_{ij} . Consider the following regression model for the total systematic gains to an i, j marriage:

$$\pi_{ij} = X'_{ij}\beta + u_{ij}, \tag{16}$$

where X_{ij} denotes the vector of variables (including policy variables) that affect the total systematic gains to an i, j marriage, and u_{ij} is an error term with mean zero and is uncorrelated with X_{ij} . Since we can construct π_{ij} from equation (11), we can estimate β in equation (16). Policy changes will induce changes in π_{ij} as captured by (16). Changes in π_{ij} will affect marital behavior via the marriage matching function described in equation (11). So given estimates of β , one can predict the effect of changes in X_{ij} on marriage behavior including marriage rates

TABLE 1 Data Summary A. U.S. Census Data

	1970	1980	Δ
Available men (M')	16.0 million	23.4 million	46%
Available women (F')	19.6 million	27.2 million	39%
Average age of available men	30.4	29.6	
Average age of available women	39.1	37.1	
B. Vit	AL STATISTICS DATA		
	1971/72	1981/82	Δ
Marrieds (μ')	3.24 million	3.45 million	6.5%
Average age of married men	27.1	29.1	
Average age of married women	24.5	26.4	

III. Changes in the Gains to Marriage in the 1970s

To estimate the marriage distributions by ages in 1971/72 and 1981/82, we use data from the 1970 and 1980 U.S. Census to construct population vectors. Marriage records from the 1971/72 and 1981/82 vital statistics were used to construct the bivariate distributions of marriages. A state has to report the number of marriages to the National Center for Health Statistics to be in the sample. This requirement eliminated 10 states in 1971/72 and nine states in 1981/82.¹³

For each period, we investigate a two-year rather than a one-year marriage distribution because the two-year distribution has thicker cells. For each period, we examine the marital behavior of individuals between the ages of 16 and 75 implied by the population vectors and preference parameters estimated from our model. Details of the construction of the data used are left to Appendix C.

In our sample (table 1), there were 16.0 million and 19.6 million available (unmarried) men and women, respectively, between the ages of 16 and 75 in 1970. There were 3.24 million marriages in 1971/72. There were 23.4 million and 27.2 million available men and women, respectively, in 1980. Although the available population had increased by more than 39 percent over the decade, there were only 3.45 million marriages in 1981/82, an increase of 6.5 percent.

Figures 1a and b show the bivariate age distributions of the marrieds in 1971/72 and 1981/82, respectively. In both years, most marriages occurred between young adults, and there was strong, positive assortative matching by age.

In figure 2, we graph the 1970 and 1980 age distributions of the

¹³ Arizona, Arkansas, Colorado, Nevada, New Mexico, New York, North Dakota, Oklahoma, Texas, and Washington were excluded in 1971/72 and 1981/82. Colorado was added in 1981/82.

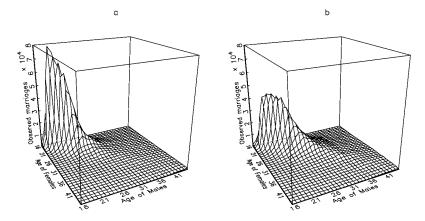


Fig. 1.—Surface of observed μ_{ii} : a, 1971/72; b, 1981/82

population vectors.¹⁴ For both decades, there are more available men than women in the early ages, and the reverse is true in the later ages. These gender differences arise from the fact that there are relatively more widows and lower remarriage rates of divorced women. The higher remarriage rate of divorced men reduced the availability of younger women. The arrival of the baby boomers to the marriage market in 1980 is readily visible from the increase in the population of availables. This arrival should have had a substantial impact on the marriage market. However, as noted in table 1, the number of marrieds in 1980 marginally increased.

Estimating the Net Gains to Marriage by Gender

Our model allows us to estimate the systematic net gain relative to not marrying for each party in any i, j marriage. The 1971/72 estimates for type i men, given by $n_{ij}^{71} = \ln{(\mu_{ij}^{71}/\mu_{i0}^{71})}$, and type j women, given by $N_{ij}^{71} = \ln{(\mu_{ij}^{71}/\mu_{i0}^{71})}$, are compared in figure 3.¹⁵
Figure 3 plots n_{ij}^{71} and N_{ij}^{71} for 20- and 40-year-old men and women by the ages of their spouses. The distributions of N_{i20}^{71} and N_{i20j}^{71} are right-

 $^{^{14}}$ The average ages of available men and women in 1970 were 30.4 and 39.1, respectively. This gender difference reflected the larger fraction of available older women. The average ages of the married men and women in 1971/72 were 27.1 and 24.5, respectively, reflecting the usual gender difference in ages of marriage. The statistics for 1980 are similar, as shown in table 1.

 $^{^{15}}$ In the 1971/72 and 1981/82 marital records, there were many age pairs that had no marriage. This is a common problem in empirical discrete-choice applications and is encountered throughout the empirical section of this paper. We employ kernel smoothers to deal with this thin cell problem. For details, see Choo and Siow (2003). Yatchew (2003) provides an excellent overview of these techniques.

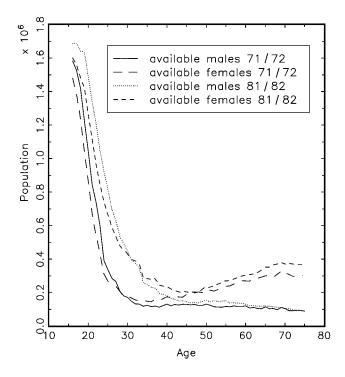


Fig. 2.—Availables in 1971/72 and 1981/82

skewed, with 20-year-old women receiving the largest systematic net gain when marrying a slightly older man and the 20-year-old men receiving the largest systematic net gain when marrying a slightly younger woman. Comparing the distribution of systematic net gain for a 40-year-old woman, N_{i40}^{71} , with that for her 20-year-old counterpart, we find that the distribution for a 40-year-old woman is more dispersed. Again she receives the largest net gain when she marries someone slightly older. If we consider the distribution for 40-year-old men, n_{40}^{71} , we also find the distribution to be more dispersed than for 20-year-old men. Again a man receives the largest net gain by marrying someone slightly younger.

According to equations (14) and (15), the area below the transformed net gains, $\exp(n_{ij})$ and $\exp(N_{ij})$, is proportional to the type-specific marriage rates. From figure 3, we also observe that the estimated net gains are negative, which reflects the fact that the systematic net gains to marriage are smaller than those from not marrying. This is not surprising since at any age, most individuals do not marry.¹⁶ The model

¹⁶ The net gain $n_{ij} > 0$ implies $\mu_{ij} > \mu_{i0}$, which is counterfactual for all i, j.

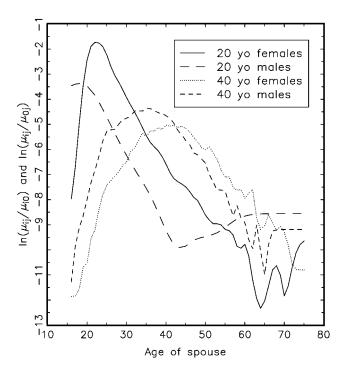


Fig. 3.—Systematic net returns for 20- and 40-year-olds

predicts that a match occurs only when the match-specific idiosyncratic utility is large. Most of the features of the empirical distributions in figure 3 are expected. What is new is that our model provides a normative interpretation of these empirical distributions.

B. Estimating the Total Gains to Marriage

Using age as the only type differentiating individuals, figure 4 shows the smoothed nonparametric plot of $\widehat{\pi_{ij}^{71}}$ for the period 1971/72. In a comparison with figure 1*a*, the distribution of the estimated total gains is less peaked and less concentrated. In particular, the total gains are larger off the age diagonal and for older individuals than would be predicted from the bivariate marriage distribution of figure 1*a*. As with the estimates of the net gains from marriage in the previous section, we observe that the estimated total gains relative to remaining unmarried are also negative. This reflects the empirical fact that most available individuals do not marry. Figure 4 also shows the standard result that there is strong, positive assortative matching by age.¹⁷

¹⁷ Choo and Siow (2005) provide an explanation based on dynamic considerations.

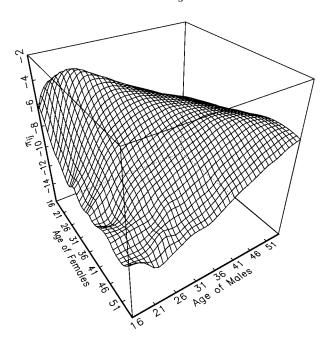


Fig. 4.—Smoothed π_{ii} for 1971/72

C. Drop in the Gains to Marriage

Figure 5 plots the change in the gains to marriage over the decade, $\Delta \pi_{ij} = \pi_{ij}^{\$1} - \pi_{ij}^{71}$, for spouses who are close in age (where most of the data lie). The striking feature of the data is the sharp drop in the estimated total gains to marriage to young adults between the ages of 16 and 30 in 1981/82. Technological innovations and social changes such as the invention of the birth control pill and the legalization of abortion in the 1970s affected the gains to marriage by changing the opportunities available to women. In this subsection, we explore the role of differential access to legal abortions across states in the 1970s in affecting the gains to marriage. Before 1967, legal abortion was generally unavailable. Between 1967 and 1973, legal abortion became easier to obtain in several states (reform states). The reform states included in our analysis are Alaska, California, Delaware, Florida, Geor-

¹⁸ Refer to the working paper version (Choo and Siow 2003) for illustrations.

¹⁹ Akerlof, Yellen, and Katz (1996) argued that the legalization of abortion may substantially reduce the gains to marriage. See also Goldin and Katz (2002) and Siow (2002).

²⁰ Thirteen states passed Model Penal Code legislation. Alaska, Florida, Hawaii, New York, and Washington enacted even more liberal laws. California's restrictive abortion laws were struck down by the state courts. See Merz, Jackson, and Klerman (1995) for details.

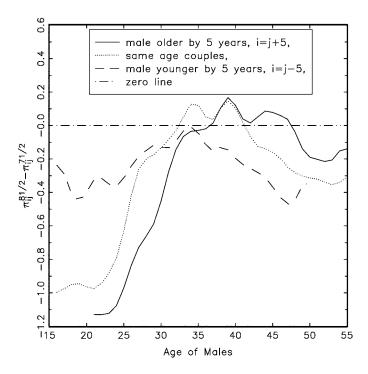


Fig. 5.—Smoothed difference in aggregate π , i.e., $\pi_{ii}^{81/82} - \pi_{ii}^{71/72}$ among married couples

gia, Hawaii, Kansas, Maryland, North Carolina, Oregon, South Carolina, and Virginia.

On January 22, 1973, as a result of the U.S. Supreme Court's ruling in *Roe v. Wade*, legal abortions became available in the entire country. This ruling was less restrictive on access to abortion than what was available previously in the reform states.

If partial availability of legal abortions in a state reduced the gains to marriage in that state, we would expect to see lower gains to marriage in reform states relative to nonreform states in 1971/72 but not in 1981/82. Moreover, this difference in difference in the gains to marriage should be concentrated among women of childbearing age and the men who marry them.

In order to empirically study the impact of the partial legalization of abortions on the gains to marriage, consider an expansion of the type space of individuals. A type of an individual is now defined by his or her age, whether the individual lives in a reform state (r for male and R for female) or a nonreform state (n for male and N for female), and time, t. We shall use the convention s and S to denote the states of residence for a man and a woman, respectively, where $s \in \{r, n\}$ and

 $S \in \{R, N\}$. We assume that all individuals at time t, living in a reform state or a nonreform state, are available to other individuals at the same time t in one national marriage market. So an individual living in a reform state at time t may marry someone living at the same time in either a reform or a nonreform state and vice versa. Let t = 71 refer to the marriage market in the years 1971 and 1972 and t = 81 refer to 1981 and 1982. The number of i, j marriages between male and female individuals from states (s, S), respectively, at time t is denoted by μ_{ij}^{sS}

To provide a benchmark for our analysis, consider the marriage rate regression, where ρ_{ji}^{s} is the marriage rate of age j women living in state S at time t:

$$\rho_{it}^{S} = h(j) + h_{t}(j) \cdot (1 - D_{it}) + h^{R}(j) \cdot D_{i}^{R} + h_{t}^{R}(j) \cdot D_{i}^{R} \cdot D_{it} + v_{it}^{S}.$$
 (17)

We use D to denote dummy variables and h(x) to denote some general nonparametric function that has x as its argument. The variable D_{jt} takes a value of one for t = 1971/72 and zero otherwise; D_j^R takes a value of one if the woman is from a reform state, and zero otherwise; and v_{jt}^S is an error term with mean zero.

The terms $h_i(j)$ and $h^R(j)$ allow for an age-specific time trend and an age-specific state effect, respectively. Then $h_i^R(j)$ measures the impact of living in a reform state at t = 71 on the marriage rate of type j women. The function $h_i^R(j)$ can be estimated nonparametrically by appropriately smoothing the difference in differences estimator:

$$\Delta^2 \rho_i^f = (\rho_{i71}^R - \rho_{i81}^R) - (\rho_{i71}^N - \rho_{i81}^N). \tag{18}$$

The total gains to marriage can be parameterized in a similar manner. Let the gains to marriage of an age i man living in state s with an age j woman living in state S at time t, π_{ij}^{sS} , be given by

$$\pi_{ijt}^{sS} = g(i, j) + g_{i}(i, j)(1 - D_{ijt}) + g^{rR}(i, j)D_{ij}^{rR} + g^{nR}(i, j)D_{ij}^{nR}$$

$$+ g^{rN}(i, j)D_{ij}^{rN} + g_{t}^{rR}(i, j)D_{ij}^{rR}D_{ijt} + g_{t}^{nR}(i, j)D_{ij}^{nR}D_{ijt}$$

$$+ g_{t}^{rN}(i, j)D_{ij}^{rN}D_{ijt} + \varepsilon_{ijt}^{sS}.$$

$$(19)$$

The notational convention adopted in equation (17) applies. The dummy variable D_{ij} takes a value of one for age combinations in years t=1971/72 and zero otherwise; the variable D_{ij}^{rN} takes a value of one for couples in which the man resides in a reform state, r, and the woman in a nonreform state, N, and zero otherwise, and so on. The function g(i, j) captures the systematic gain to marriage for an age i man in a nonreform state with an age j woman in a nonreform state in 1971/72. It forms the base gains to marriage that vary according to the ages of the couples, (i, j). The functions $g^{rR}(i, j)$, $g^{nR}(i, j)$, and $g^{rN}(i, j)$ capture the remaining fixed effects arising from the state of residence of the

couple. For example, $g^{rR}(i, j)$ is the increment in systematic gains added to the base g(i, j) if the couples are both from the reform states.

The increment to the gains to marriage in years 1981/82 for an (i, j) pair is captured by the function $g_i(i, j)$. This time effect is assumed to be independent of the state of residence. The function $g_i^{sS}(i, j)$ is the increment to the gains to marriage in t = 1971/72 between a man in state s and a woman in state s for state combinations $sS \neq nN$. If we expect the legalization of abortion in the reform states to have lowered the gains to marriages among young adults who both reside in those states, then $g_i^{rR}(i, j) < 0$ for young couples. The mean zero error term is denoted by ε_{iii}^{sS} .

Our model for the systematic gains to marriage in equation (19) has some advantages over the marriage rate formulation in equation (17). First, the formulation using the systematic gains satisfies all the restrictions of a marriage matching function, whereas the marriage rate models of the form in equation (17) do not impose any restriction between different marriage rates. Second, our model can distinguish between the effect of the legalization of abortion on the systematic gains to marriage for age i men with different types of women. For example, $g_i^{R}(i, j)$ need not be the same as $g_i^{R}(i, j')$.

For any age combination (i, j) with observed marriages, the systematic gains, π_{ijt}^{ss} , are estimated by $\widehat{\pi_{ijt}^{ss}} = \ln(\mu_{ijt}^{ss}/\sqrt{\mu_{i0t}^{ss}\mu_{0jt}^{ss}})$. The increment in the gain to marriage for an i, j pair in 1971/72 who lived in reform states, $g_t^{rR}(i, j)$, can be estimated by the difference in differences estimator:

$$\Delta^2 \pi_{ij}^{R} = (\widehat{\pi_{ij71}^{R}} - \widehat{\pi_{ij81}^{R}}) - (\widehat{\pi_{ij71}^{R}} - \widehat{\pi_{ij81}^{R}}). \tag{20}$$

Note the similarity between $\Delta^2 \pi_{ij}^{sS}$ and the standard difference in differences marriage rate estimator, $\Delta^2 \rho_l^f$, for l = i, j. Although $\Delta^2 \pi_{ij}^{sS}$ is defined for an age pair (i, j) and state pair (s, S) rather than for male or female ages alone, it is as easy to estimate as equation (18).

D. Data

Using information on the place of residence from the U.S. Census and the marriage records from the vital statistics, we classify the data described in the beginning of this section according to whether the place of residence of an individual is a reform or a nonreform state. Table 2 provides a summary of the data used.

The sample of available men and women on the marriage market from the nonreform states is considerably larger than that from the reform states. The increases in the population observed over the decade in the two groups of states also differ in magnitude. In the reform states, the population of available men and women increased by 60.4 percent

TABLE 2
Data Summary Based on Place of Residence
A. U.S. Census Data

	1970	1980	Δ
Available men in reform states (M')	5.76 million	9.24 million	60.41%
Available women in reform states (F')	6.70 million	10.36 million	54.63%
Available men in nonreform states			
(M^n)	10.25 million	14.17 million	38.24%
Available women in nonreform states			
(F^n)	12.90 million	16.86 million	30.70%
Average age of available men in reform			
states	30.00	29.62	
Average age of available women in re-			
form states	38.93	36.93	
Average age of available men in nonre-			
form states	30.64	29.53	
Average age of available women in			
nonreform states	39.22	37.24	
B. VITAL S	TATISTICS DATA		
	1971/72	1981/82	Δ
Marriages in rR states (μ^{rR})	1.05 million	1.26 million	17.17%
Marriages in rN states (μ^{rN})	45,456	38,730	-14.80%
Marriages in nR states (μ^{nR})	39,367	30,358	-29.68%
Marriages in nN states (μ^{nN})	2.10 million	2.11 million	.56%
Average age of married men in reform			
states	27.5	29.6	
Average age of married men in nonre-			
form states	26.9	28.9	
Average age of married women in re-			
form states	24.8	26.8	
Average age of married women in non-			
reform states	24.4	26.2	

and 54.6 percent, respectively, compared to more modest increases of 38.2 percent and 30.7 percent for available men and women, respectively, in the nonreform states. The average ages of men and women in the two groups of states are comparable to those of the entire sample reported in table 1.

As expected, marriages between individuals in the same state of residence are more likely relative to marriages between individuals living in different states. There are 2.1 million marriages between couples in the nonreform states (μ^{nN}), compared to 1.05 million between couples in the reform states (μ^{rR}), in 1971/72. The number of cross-marriages in 1971/72 (μ^{rN} , μ^{nR}) is around 85,000.

The changes in the total number of marriages across the four groups over the decade differ in magnitude and sign. Marriages between reform state men and nonreform state women decreased by 14.8 percent, whereas marriages between men from nonreform states and women

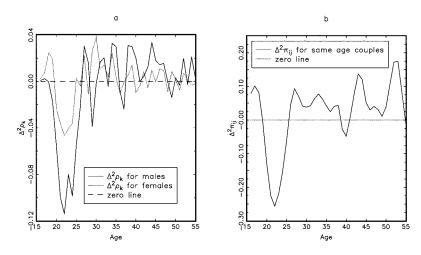


FIG. 6.—a, Comparing the difference in differences in the marriage rate. b, The difference in differences in marriage gains for same-age couples.

from reform states decreased by almost 30 percent. In the reform states, where there was little change in access to legalized abortion over the decade, we find that total marriages increased by 17 percent, whereas in the nonreform states, where legalized abortion became more accessible, total marriages increased by only 0.56 percent. It is this differential change in marriage patterns in the four groups and the changes in the population of marriage market participants that provide identification of the fall in marriage gains due to legalizing abortion.

E. Results

Figure 6a shows estimates of the decrease in marriage rates in the reform states from the difference in differences marriage rate estimators, $\Delta^2 \rho_l^k$ (k = m, f). Consistent with the findings in Angrist and Evans (1999), $\Delta^2 \rho_l^k$ are negative for both young men and young women. There is evidence of a small increase in the marriage rates of men and a smaller increase in the marriage rates of women between the ages of 30 and 40. As explained later, it is problematic that the estimated effects for men are significantly larger than those for women.

Figure 6b shows estimates of $\Delta^2 \pi_{ij}^{R}$ for same-age spouses, where i=j. This slice of the distribution is informative because there are many same-age spouses. The drop in the gains to marriage for same-age spouses, between the ages of 19 and 26, living in reform states in 1971/72 relative to those living in nonreform states is substantial. We also see a small increase in the gains to marriage for same-age spouses between

the ages of 27 and 40. An explanation of these gains is that these are young individuals who would have gotten married young had abortion not been legalized. This social change allows these individuals to delay marriage to an older age.

We interpret the effects on marriage rates and the gains to marriage, displayed in figure 6, as due to the partial legalization of abortion. The standard difference in differences argument for identification is based on the claim that the policy intervention of interest generates year- and location-specific interaction effects that would otherwise not be there. In addition to the standard argument, we also expect partial legalization to affect young adults more than older adults, which is consistent with the evidence in figure 6.

In order to quantify the effect of the partial legalization of abortion on marriage rates, we use the two estimators, $\Delta^2 \pi_{ij}^{sS}$ and $\Delta^2 \rho_l^k$ (k = m, f), to do a counterfactual experiment. Consider an experiment in which the nonreform states also partially legalize abortion in 1971/72 as the reform states do. The estimates from the difference in differences marriage rate equation (18) allow us to construct a counterfactual marriage rate for men and women in the nonreform states. ²¹ Using the counterfactual marriage rates in the nonreform states and the observed rates in the reform states, we construct an aggregate male and female marriage rate in the scenario in which there was no differential access to abortion in 1971/72.

A comparable counterfactual marriage rate can be constructed using the difference in differences marriage gains estimator. Using the estimates from equation (20), we first construct gains to marriage in the nonreform states in the counterfactual scenario that abortion was partially legalized in these states in 1971/72.²² We then compute the number of marriages (and the implied male and female marriage rates) that would have been observed using these counterfactual marriage gains and the observed marriage gains for the reform states.

Let the counterfactual aggregate marriage rates in 1971/72 constructed using the difference in differences marriage rate and difference in differences marriage gains estimator be denoted by C_{j71}^{ρ} and C_{j71}^{π} , respectively. Figures 7a and b compare the total observed change in marriage rates for age k over the decade, $\Delta \rho_k^l = \rho_{k81}^l - \rho_{k71}^l$ (where l = m, f is an index for gender), with the change in marriage rate in 1971/72 in the above counterfactual scenario as suggested by the difference

²¹ Using our estimate of $\widehat{h_{71}}(k)$, let the counterfactual marriage rates for men and women be denoted by $\widetilde{\rho_{71}}$ and $\widetilde{\rho_{71}}$, respectively, where $\widetilde{\rho_{k71}} = \rho_{k71}^s - \widehat{h_{71}}(k)$, with $(s, k) \in \{(n, i), (N, i)\}$

⁽N, j)}.

²² We first estimate $\widehat{g_{71}^R}(i, j)$, $\widehat{g_{71}^N}(i, j)$, and $\widehat{g_{71}^R}(i, j)$ according to eq. (20). These counterfactual marriage gains, $\widehat{\pi_{j71}^N}$, $\widehat{\pi_{j71}^N}$, and $\widehat{\pi_{j71}^R}$, are estimated according to the equation $\widehat{\pi_{j71}^N} = \widehat{\pi_{j71}^N} - \widehat{g_{71}^N}(i, j)$, where $sS \in \{nN, rN, Rn\}$, for all i, j.

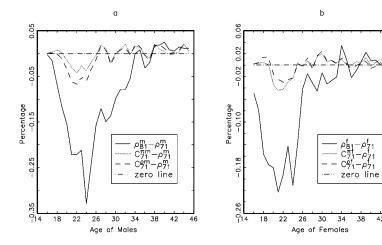


Fig. 7.—Comparing observed total change in marriage rates with change attributed to the legalization of abortion from the two estimators: a, men; b, women.

in differences marriage rate estimator, $\Delta^{\rho}\rho_{l}^{l} = C_{k71}^{\rho l} - \rho_{k71}^{l}$, and by the marriage gain estimator, $\Delta^{\pi}\rho_{k}^{l} = C_{k71}^{\pi l} - \rho_{k71}^{l}$. As discussed earlier, marriage rates for men and women fell over the decade. Both $\Delta^{\rho}\rho_{k}^{l}$ and $\Delta^{\pi}\rho_{k}^{l}$ suggest that a quantitatively significant part of the fall in aggregate marriage rates for young adults over the decade is attributable to the lack of partial legalization in the nonreform states in 1970. The term $\Delta^{\pi}\rho_{22}^{m}$ suggests that 20 percent of the observed fall in the marriage rates of 22-year-old men can be attributed to partial legalization compared to the 31 percent estimate from $\Delta^{\rho}\rho_{22}^{m}$. The estimate of the decrease in the female marriage rate attributable to the partial legalization of abortion is more modest.

While both estimators provide qualitatively similar results, the quantitative predictions of the two estimators are very different. The estimate from the female difference in differences marriage rates estimator suggests that legalizing abortion in the nonreform states would have resulted in 7,080 fewer marriages in 1971/72, whereas the estimate using the male marriage rates is 196,270.²⁴ This estimate is larger by a factor of 27!²⁵ This kind of discrepancy from male and female marriage rate

²³ Nonreform states went from no legalization to full legalization between 1970 and 1980. This change can be conceptually decomposed into (1) no legalization to partial legalization and (2) partial legalization to full legalization. We are asking how much of the change in marriage rates over the decade can be attributed to the conceptual change from no legalization to partial legalization.

²⁴ The total number of recorded marriages in 1971/72 is 3,235,806.

²⁵ Smaller, significant disparity remains if we limit ourselves to marriages for individuals younger than age 30.

regressions is not unusual. So while marriage regressions are easy to use and interpret, the biases in these estimators can be substantial. The estimate from the difference in differences marriage gains estimator is around 45,440 fewer marriages among individuals aged 16–75. In other words, a partial legalization of abortion in the nonreform states in 1971/72 would have resulted in 1.4 percent fewer total U.S. marriages in that period. Among young individuals the decrease is more pronounced. For men aged 16–25, partial legalization of abortion in the nonreform states would have lowered the number of total marriages in this age group by 4.2 percent, whereas among 16–25-year-old women, the decrease is around 3.6 percent. For men older than 26 years of age, this social change would have increased the total number of marriages in this age group by 3.8 percent, and for women older than 26 years of age, the increase is around 5.2 percent.

IV. Conclusion

We provide brief suggestions for further research. Empirically, because of space constraints, we have only briefly investigated the effect of legalized abortions on the gains to marriage. Other social changes also need to be examined. Theoretically, dynamic considerations are needed (see Choo and Siow 2005). This model of marriage matching should be integrated with models of intrahousehold allocations such as those of Lundberg and Pollak (1993) and Chiappori et al. (2002). Finally, substitution patterns of the current model need to be better understood.

Appendix A

A. Derivation of (7)

Equation (4) may be rewritten as $V_{ijg} = \tilde{\alpha}_{ij} - \tau_{ij} + \varepsilon_{ijg} = \eta_{ij} + \varepsilon_{ijg}$, where ε_{ijg} has an extreme-value distribution with cumulative distribution function $F(\varepsilon) = \exp{[\exp{(-\varepsilon)}]}$. As specified by (6), individual g solves $V_{ig} = \max_{j} \{V_{i0g}, \ldots, V_{ijg}, \ldots, V_{ijg}\}$. The probability that an option j is chosen is

$$\begin{split} \mathbb{P} \bigg[j &= \argmax_{k=0,...,J} V_{ikg} \bigg] \\ &= \mathbb{P} \{ \boldsymbol{\varepsilon}_{ikg} \! < \boldsymbol{\eta}_{ij} - \boldsymbol{\eta}_{ik} + \boldsymbol{\varepsilon}_{ijg} \ \forall \ k \neq j \} \\ &= \int_{-\infty}^{\infty} \prod_{k \neq j} F(\boldsymbol{\eta}_{ij} - \boldsymbol{\eta}_{ik} + \boldsymbol{\varepsilon}_{ijg}) f(\boldsymbol{\varepsilon}_{ijg}) d\boldsymbol{\varepsilon}_{ijg} \\ &= \int_{-\infty}^{\infty} \prod_{k \neq j} \exp \{ -\exp \left[-(\boldsymbol{\eta}_{ij} - \boldsymbol{\eta}_{ik} + \boldsymbol{\varepsilon}_{ijg}) \right] \} \exp \left[-\boldsymbol{\varepsilon}_{ijg} - \exp \left(-\boldsymbol{\varepsilon}_{ijg} \right) \right] d\boldsymbol{\varepsilon}_{ijg}. \end{split}$$

If we let $z_k = \exp{[-(\eta_{ij} - \eta_{ik})]}$ and apply a change of variable $\psi = \exp{(-\varepsilon_{ijg})}$, we get

$$\mathbb{P}(j) = \int_{0}^{\infty} \exp\left[-\psi \left(1 + \sum_{k \neq j} z_{k}\right)\right] d\psi = \frac{1}{1 + \sum_{k \neq j} z_{k}} = \frac{\exp \eta_{ij}}{\sum_{k=0}^{J} \exp \eta_{ik}}$$

The maximum likelihood estimator of $\mathbb{P}\{j=\arg\max_{k=0,\dots,J}V_{ikg}\}\$ is μ_{ij}/m_{i^*} Then (7) follows.

B. Derivation of (8)

Conditional on a type i man g choosing to match with a type j woman, the expected utility of that man is

$$\begin{split} \mathbb{E} & \left(V_{ijg} \,\middle|\, j = \operatorname*{max}_{k=0,...,J} V_{ikg} \right) \\ & = \eta_{ij} + \mathbb{E} (\varepsilon_{ijg} \,|\, \varepsilon_{ijg} + \eta_{ij} \! > \! \eta_{ik} + \varepsilon_{ikg} \,\, \forall \,\, k \neq j) \\ & = \eta_{ij} + \left[\mathbb{P} \!\!\left\{ j = \operatorname*{max}_{k=0,...,J} V_{ikg} \right\} \right]^{-1} \int_{-\infty}^{\infty} \varepsilon_{ijg} \exp \left[-\varepsilon_{ijg} - e^{-\varepsilon_{ijg}} \!\!\left(1 + \sum_{k \neq j} z_k \right) \!\!\right] \!\! d\varepsilon_{ijg}, \end{split}$$

where $z_k = \exp\left[-(\eta_{ij} - \eta_{ik})\right]$. From the fact that $\int_{-\infty}^{\infty} x \exp\left(x - \phi e^x\right) dx = -(c + \ln \phi)/\phi$, where c is Euler's constant, ≈ 0.577215 , thus

$$\mathbb{E}\left(V_{ijg}\middle|j = \arg\max_{k=0,\dots,I} V_{ikg}\right) = c + \ln\left(\sum_{k} \exp\eta_{ik}\right),\tag{A1}$$

which is independent of j. Then (A1) and (7) imply

$$\mathbb{E}V_{ig} = c + \ln\left[\sum_{k} \exp\left(\tilde{\alpha}_{ik} - \tau_{ik}\right)\right] = c + \tilde{\alpha}_{i0} + \ln m_i - \ln \mu_{i0}. \tag{A2}$$

Appendix B

Proof of Proposition 1

The system (13) can be reduced to an I+J system with I+J number of unmarrieds of each type, μ_{i0} and μ_{0j} , as unknowns. This reduced system, defined by equations (B1) and (B2), is derived by summing equation (13) over all i's and j's, respectively:

$$f_j - \mu_{0j} = \sum_{i=1}^{I} \Pi_{ij} \sqrt{\mu_{i0} \times \mu_{0j}}$$
 (B1)

and

$$m_i - \mu_{i0} = \sum_{j=1}^{J} \prod_{ij} \sqrt{\mu_{i0} \times \mu_{0j}}.$$
 (B2)

If we can solve for μ_{i0} and μ_{0p} , then the μ_{ij} 's are fully determined by equation (13). We apply the implicit function theorem to the system (B1) and (B2). Consider taking derivatives with respect to m_r . Then we get the linear system

$$\begin{bmatrix} \mathbf{D}_{J} & \mathbf{B} \\ \mathbf{C} & \mathbf{D}_{J} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{\Delta}_{r} f \\ \boldsymbol{\Delta}_{r} m \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \boldsymbol{e}_{r} \end{bmatrix},$$

where \mathbf{D}_I is a $J \times J$ diagonal matrix whose jjth element is $(\mu_{0j} + f_j)/2\mu_{0j}$, \mathbf{D}_I is an $I \times I$ diagonal matrix whose iith element is $(\mu_{i0} + m_i)/2\mu_{i0}$, \mathbf{B} is a $J \times I$ matrix whose jith element is $\mu_{ij}/2\mu_{i0}$, \mathbf{C} is an $I \times J$ matrix whose jith element is $\partial \mu_{ij}/\partial \mu_{0j}$, $\Delta_r m$ is an $I \times 1$ vector whose jth element is $\partial \mu_{0j}/\partial m_r$, and e_r is an $I \times 1$ zero vector with 1 as the rth element. We need to show that the Jacobian of the system is nonsingular. As long as $\mu_{i0} \neq 0$ and $\mu_{0j} \neq 0$, we know that \mathbf{D}_I^{-1} and \mathbf{D}_J^{-1} exist. Using the formula for a partition inverse, we get

$$\boldsymbol{\Delta}_{r}m = \mathbf{D}_{I}^{-1}[\mathbf{I}_{I} - \mathbf{C}\mathbf{D}_{I}^{-1}\mathbf{B}\mathbf{D}_{I}^{-1}]^{-1}\boldsymbol{e}_{r}$$
(B3)

and

$$\mathbf{\Delta}_{r}f = -\mathbf{D}_{I}^{-1}\mathbf{B}\mathbf{\Delta}_{r}m. \tag{B4}$$

The Jacobian is nonsingular as long as $[\mathbf{I}_I - \mathbf{C}\mathbf{D}_I^{-1}\mathbf{B}\mathbf{D}_I^{-1}]^{-1}$ exists. Let $\mathbf{A} = \mathbf{C}\mathbf{D}_I^{-1}\mathbf{B}\mathbf{D}_I^{-1}$; then $(\mathbf{I}_J - \mathbf{A})$ is invertible if there is a matrix norm $\|\cdot\|$ such that $\|\mathbf{A}\| < 1$. The product $\mathbf{C}\mathbf{D}_I^{-1}$ generates an $I \times J$ matrix whose ijth element is $\mu_{ij}/(\mu_{i0} + f_j)$ and $\mathbf{B}\mathbf{D}_I^{-1}$ generates a $J \times I$ matrix whose jith element is $\mu_{ij}/(\mu_{i0} + m_i)$. Consider the maximum column sum matrix norm defined by $\|\mathbf{A}\| = \max_j \sum_{i=1}^n |a_{ij}|$. Then

$$\|\mathbf{C}\mathbf{D}_{J}^{-1}\| = \max_{j} \frac{\sum_{i} \mu_{ij}}{\mu_{0j} + f_{j}} < 1,$$

$$\|\mathbf{B}\mathbf{D}_{I}^{-1}\| = \max_{i} \frac{\sum_{j} \mu_{ij}}{\mu_{i0} + m_{i}} < 1.$$

By definition of a matrix norm, $\|\mathbf{C}\mathbf{D}_J^{-1}\mathbf{B}\mathbf{D}_I^{-1}\| \le \|\mathbf{C}\mathbf{D}_J^{-1}\| \cdot \|\mathbf{B}\mathbf{D}_I^{-1}\| < 1$, and hence $(\mathbf{I}_I - \mathbf{A})^{-1}$ exists. QED

Appendix C

Data Construction

Data used were extracted from the Integrated Public-Use Microdata Sample files of the U.S. Census. The samples used were the 5 percent state samples for 1980

²⁶ If we take a derivative with respect to f_r , the system of first derivatives will have a similar form, except the position of $\mathbf{0}$ and e_r is reversed.

and the 1 percent Form 1 and Form 2 samples for 1970. The 1970 data sets were appropriately scaled to be comparable with the 1980 files. To maintain consistency between states reporting marriages to the National Center for Health Statistics (vital statistics) and the data collected from the respective U.S. Census, some states were excluded. This results in the use of data from the following states: Alabama, Alaska, California, Connecticut, Delaware, District of Columbia, Florida, Georgia, Hawaii, Idaho, Illinois, Indiana, Kansas, Kentucky, Louisiana, Maine, Maryland, Massachusetts, Michigan, Mississippi, Missouri, Montana, Nebraska, New Hampshire, New Jersey, New York, North Carolina, Ohio, Oregon, Pennsylvania, Rhode Island, South Dakota, Tennessee, Utah, Vermont, Virginia, West Virginia, Wisconsin, and Wyoming. The age range studied was 16–75 years of age. We use the *marst* variable in the census to identify a person as either never married, currently married, or previously married (divorced or widowed). To calculate the number of available individuals, we simply add the never marrieds and previously married.

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- ²⁷ State of residence in the 1970 census files can be identified only in the state samples (Form 1 and Form 2 samples, both of which are 1 percent samples). This is the reason that the other samples were not used for 1970 calculations. Further, the age of marriage variable is available only in Form 1 samples in 1970, which meant that only one sample, the Form 1 state sample, was used for calculations involving married couples in the 1970 census.
- ²⁸ In other words, the excluded states (cities) are Arizona, Arkansas, Colorado, Iowa, Minnesota, Nevada, New Mexico, New York City, North Dakota, Oklahoma, South Carolina, Texas, and Washington.

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