NIIT UNIVERSITY

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Project Title : Minimum Steiner Trees

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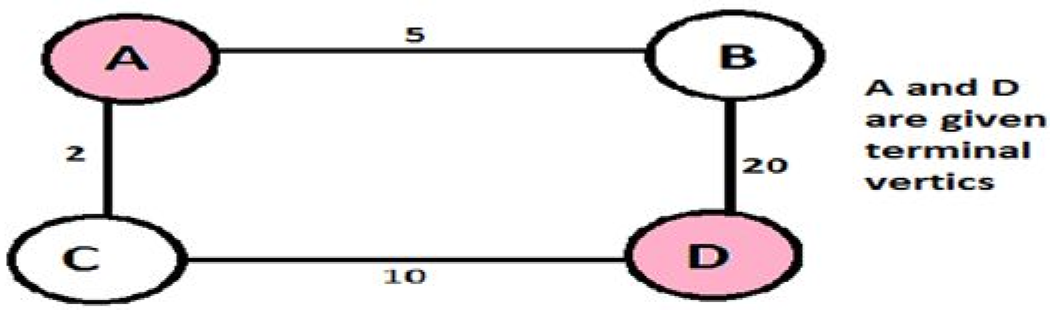
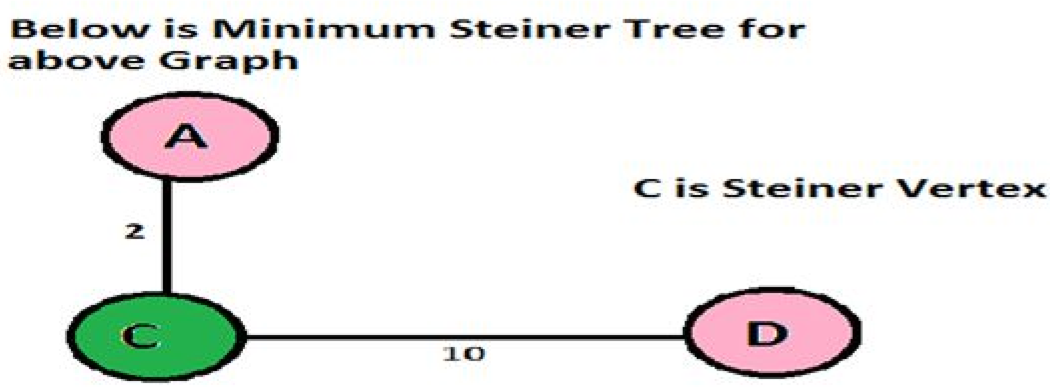
WHAT IS STEINER TREE?

Given a graph and a **subset** of vertices in the graph, a steiner tree spans though the given subset. The Steiner Tree may contain some vertices which are not in given subset but are used to connect the vertices of subset. The given set of vertices is called ***Terminal Vertices*** and other vertices that are used to construct Steiner tree are called ***Steiner vertices***.

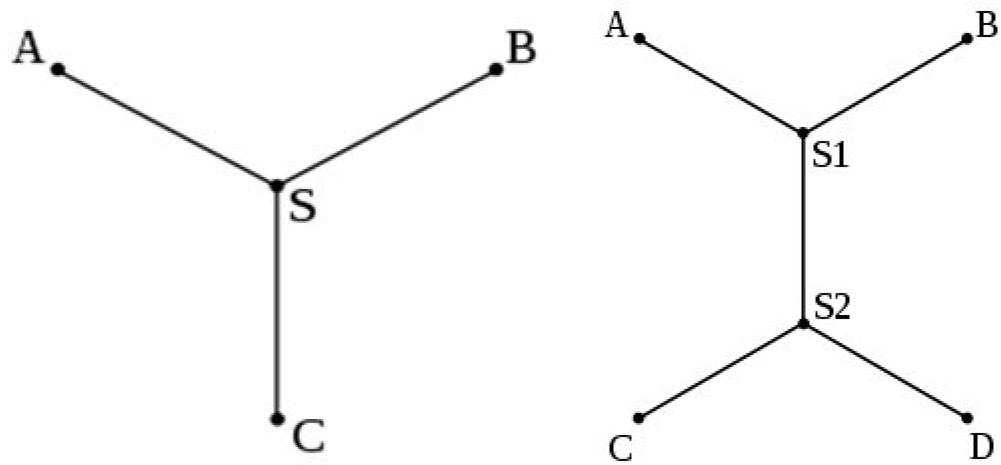




The Steiner tree problem in graphs can be seen as a generalization of two other famous combinatorial optimization problems: the (non-negative) shortest path problem and the minimum spanning tree problem. If a Steiner tree problem in graphs contains exactly two terminals, it reduces to finding a shortest path. If, on the other hand, all vertices are terminals, the Steiner tree problem in graphs is equivalent to the minimum spanning tree. However, while both the non-negative shortest path and the minimum spanning tree problem are solvable in polynomial time, the decision variant of the Steiner tree problem in graphs is NP-complete (which implies that the optimization variant is NP-hard); in fact, the decision variant was among Karp's original 21 NP-complete problems.

The Steiner tree problem in graphs has applications in circuit layout or network design. However, practical applications usually require variations, giving rise to a multitude of Steiner tree problem variants.

Most versions of the Steiner tree problem are NP-hard, but some restricted cases can be solved in polynomial time. Despite the pessimistic worst-case complexity, several Steiner tree problem variants, including the Steiner tree problem in graphs and the rectilinear Steiner tree problem, can be solved efficiently in practice, even for large-scale real-world problems.



PROPERTIES OF MINIMUM STEINER TREES

- Given a graph and a subset of vertices in the graph, a Steiner tree spans through the given subset.
- The Steiner Tree may contain some vertices which are not in given subset but are used to connect the vertices of subset.
- The Steiner Tree Problem is to find the minimum cost Steiner Tree.

- If given subset (or terminal) vertices is equal to set of all vertices in Steiner Tree problem, then the problem becomes Minimum Spanning Tree problem. And if the given subset contains only two vertices, then it shortest path problem between two vertices.
- Minimum Steiner Tree problem is NP Hard.
- Exactly three edges at every Steiner vertex.
- The angles between the edges meeting at a Steiner vertex is 120 degrees.
- If there are N vertices then we can use a maximum of N-2 Steiner vertices.

Shortest Path Based Approximate Algorithm To Solve Steiner Tree Problem

Since Steiner Tree problem is NP-Hard, there are no polynomial time solutions that always give optimal cost. Therefore, there are approximate algorithms to solve the same. Below is one simple approximate algorithm.

1. Start with a subtree T consisting of one given terminal vertex
2. While T does not span all terminals

   a. Select a terminal x not in T that is closest to a vertex in T.

   b. Add to T the shortest path that connects x with T

The above algorithm is (2-2/n) approximate, i.e., it guarantees that solution produced by this algorithm is not more than this ratio of optimized solution for a given graph with n vertices. There are better algorithms also that provide better ratio.

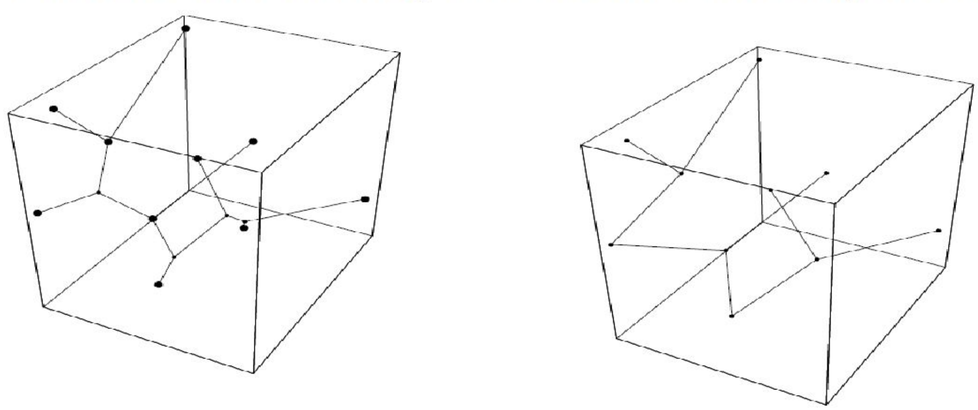
MINIMUM SPANNING TREE VS MINIMUM STEINER TREE

- Minimum Spanning Tree is a minimum weight tree that spans through **all** vertices whereas Steiner tree spans through a given subset of vertices.

- Finding out Minimum Spanning Tree is polynomial time solvable, but Minimum Steiner Tree problem is NP Hard and related decision problem is NP-Complete.

- Spanning trees are just used to remove broadcast loops while Steiner trees ensure smallest common path by adding Steiner vertices.

Minimal Interconnecting Tree for data without any structure

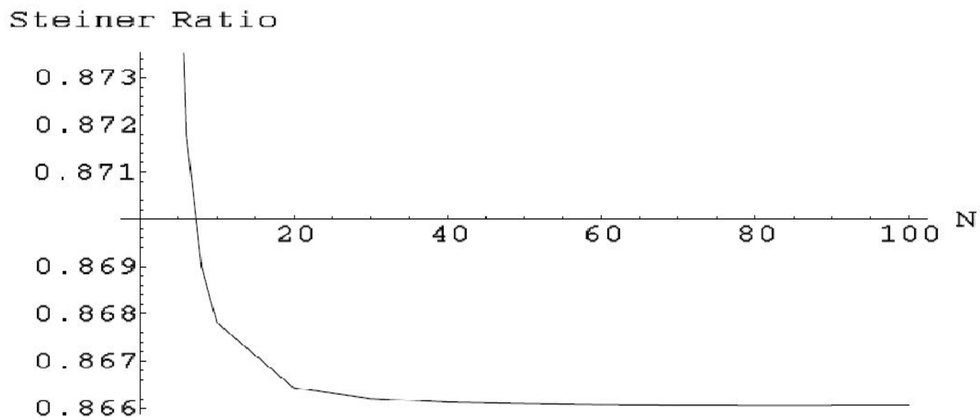


Steiner Tree and Minimum Spanning Tree for
a small random point set in unit cube

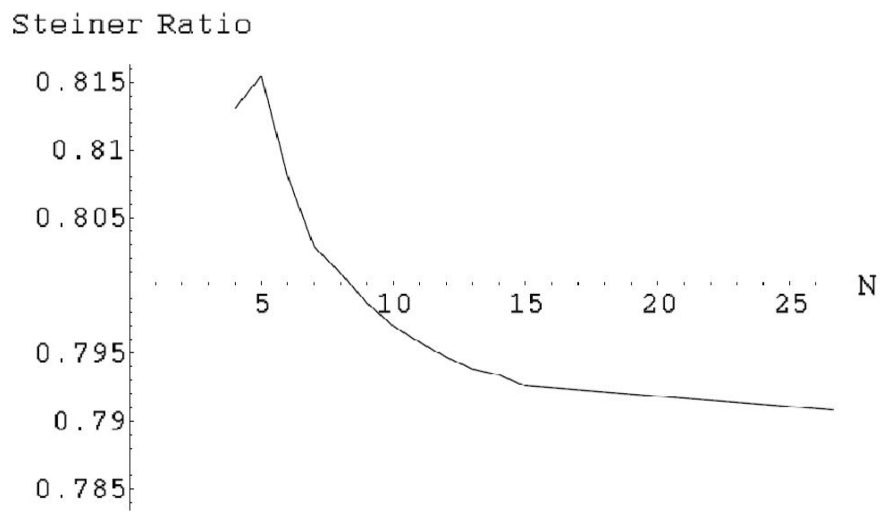
STEINER RATIO

For a given variant of the *Steiner tree* problem, Steiner ratio is the maximum possible ratio of the length of a *minimum spanning tree* of a set of *terminals* to the length of an optimal Steiner tree of the same set of terminals.

It is usually written ρ (rho).



Steiner Ratio in 2d falls to √3/2 as the number of terminal sites increase



Steiner Ratio in 3d falls to ≈ 0.784 as the number of terminal sites increase

Well-known variants of minimum

Steiner trees

- Euclidean Steiner tree
- Rectilinear Steiner tree
- Steiner tree in graphs and variants

- ***k*-edge-connected Steiner network problem**
- ***k*-vertex-connected Steiner network problem**

EUCLIDEAN STEINER TREE

Given *N* points in the plane, the goal is to connect them by lines of minimum total length in such a way that any two points may be interconnected by line segments either directly or via other points and line segments. It may be shown that the connecting line segments do not intersect each other except at the endpoints and form a tree.

The problem for *N* = 3 has long been considered, and quickly extended to the problem of finding a star network with a single hub connecting to all of the *N* given points, of minimum total length

RECTILINEAR STEINER TREE

The rectilinear Steiner tree problem is a variant of the geometric Steiner tree problem in the plane, in which the Euclidean distance is replaced with the rectilinear distance.

The problem arises in the physical design of electronic design automation.

In VLSI circuits, wire routing is carried out by wires that are often constrained by design rules to run only in vertical and horizontal directions, so the rectilinear Steiner tree problem can be used to model the routing of nets with more than two terminals.

APPLICATIONS OF STEINER TREE

Any scenario where the task is to minimize cost of connection among some important locations like

- Modelling of Biomolecular Structures
- VLSI Design
- Computer Networks, etc.....

Exact Algorithms

- The 3 Point Algorithm
- The Melzak Algorithm
- The Numerical Algorithm

CONTRIBUTIONS:

AJAY SHARMA - Introduction, Definition and Examples

AJINKYA - Properties of MST

ABHINAV - Types of Steiner Trees

AMAN GARG - Min. Steiner Trees v/s Min. Spanning Trees, Steiner Ratio

SRI NATH - Algorithms of Steiner Trees

ALTHAF - Applications of Steiner Trees