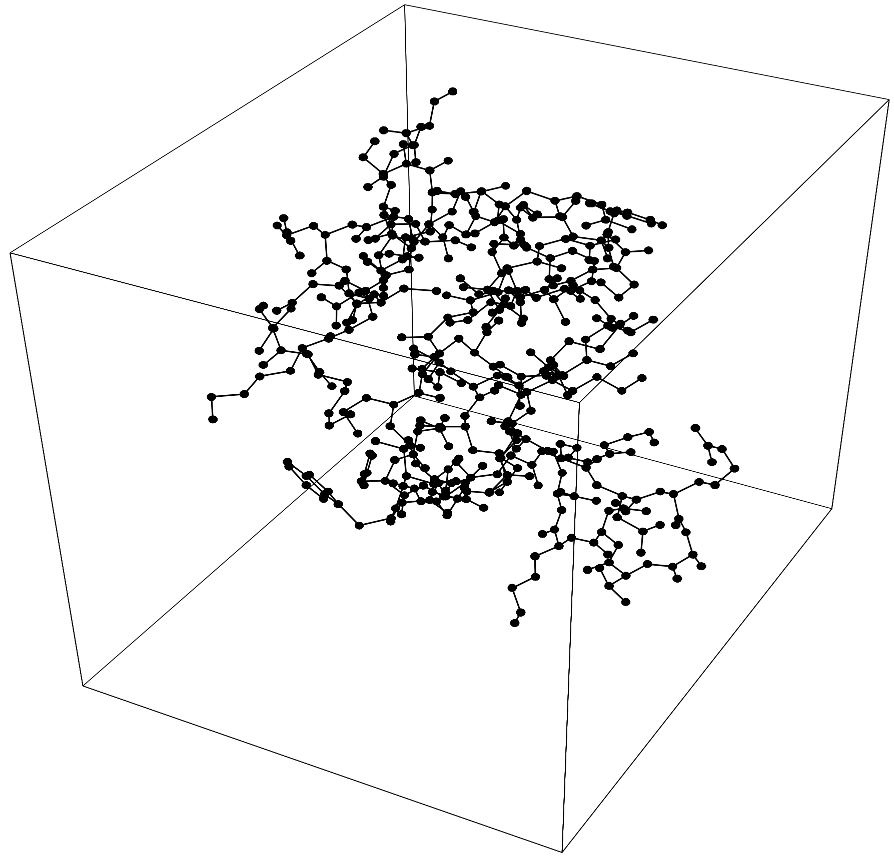
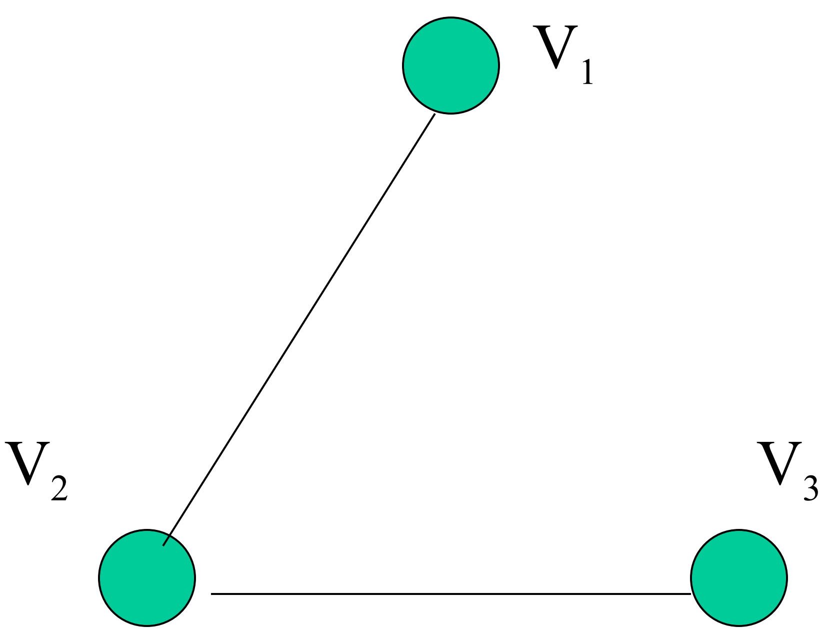


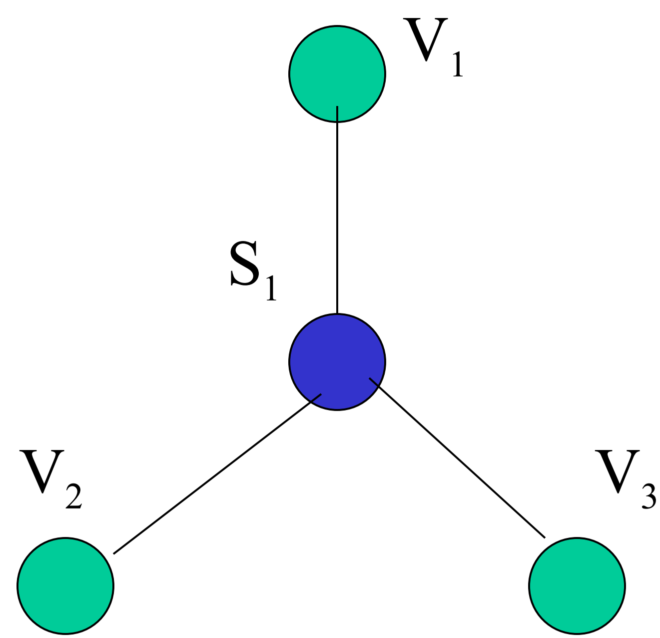
Steiner Tree Applications

Presented by
T. N. Badri

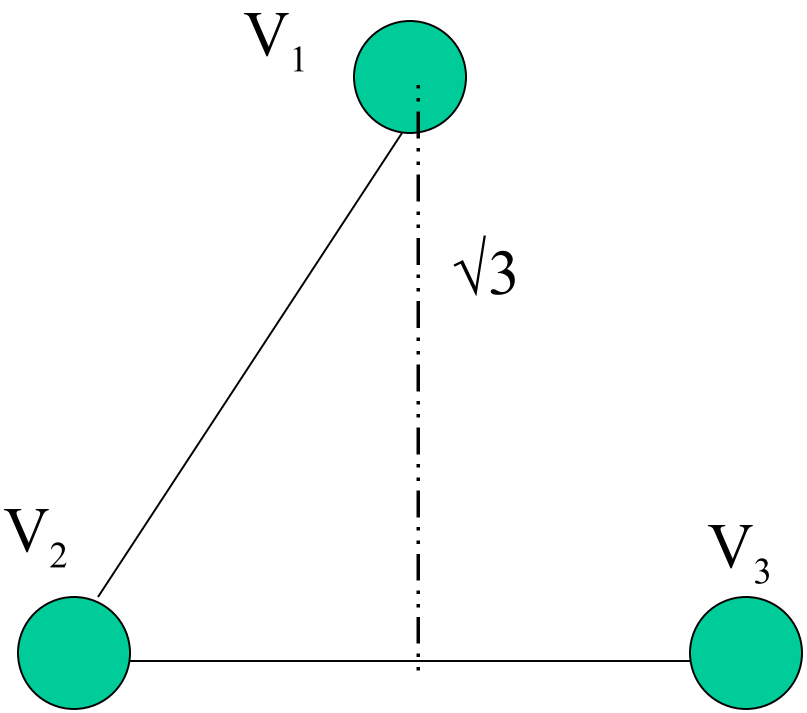
The Steiner problem dates back to Pierre Fermat. Even Napolean has worked on this problem. Consider three points V_1, V_2, and V_3. The shortest network interconnecting these three points is the sum of the two shortest sides of the triangle.

If one includes a fourth point S_1 in the plane of the triangle, one can find a much shorter interconnecting network.

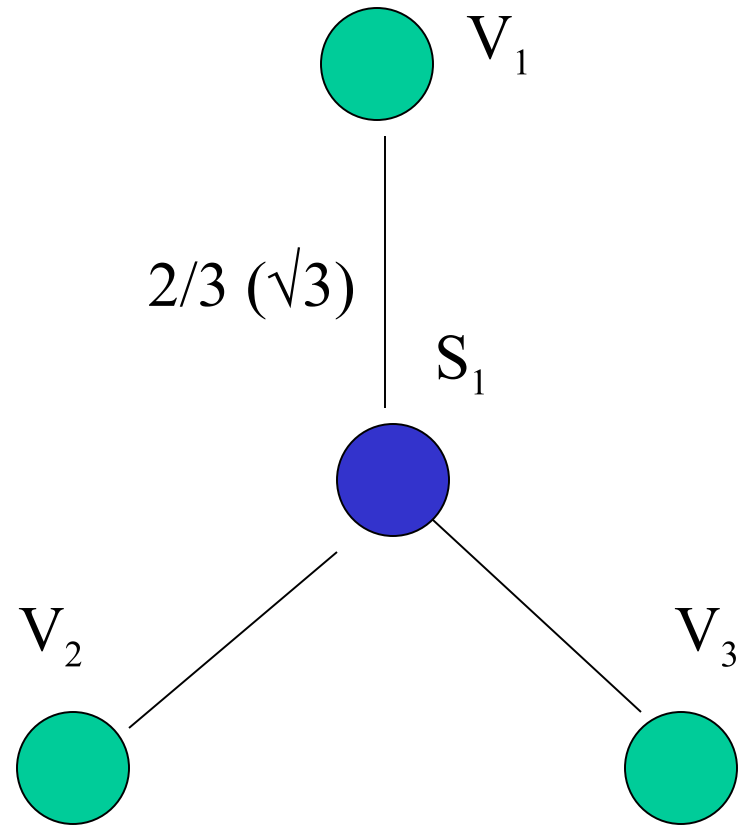




If the three points in question are vertices of an equilateral triangle with side of length 2 kms, one can make some simple calculations such as:



Length = 4 kms



Length = $3(2/3)\sqrt{3}$ = 3.4641 kms

a saving of about 13.4%

If it costs about Rs. 3 lakhs to lay one km of road this is a saving of 0.134 * 300,000 * 4 = Rs. 160,800 on an initial total cost of Rs. 12 lakhs.

Steiner points have some properties

- Exactly three edges meet at every Steiner vertex.

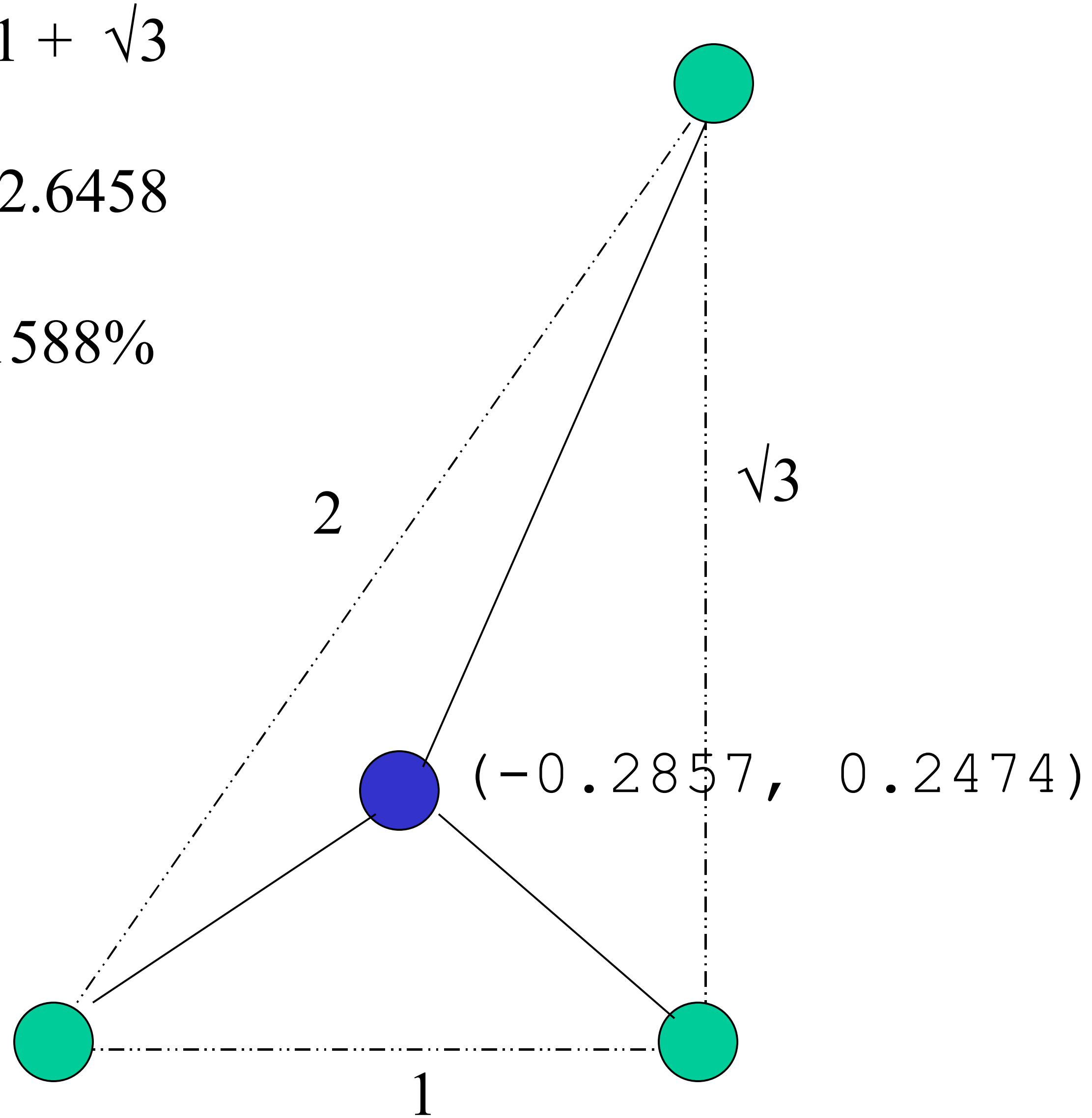
- The angles between the edges meeting at a Steiner vertex is 120 degrees.

- If there are N vertices then we can use a maximum of N-2 Steiner vertices.

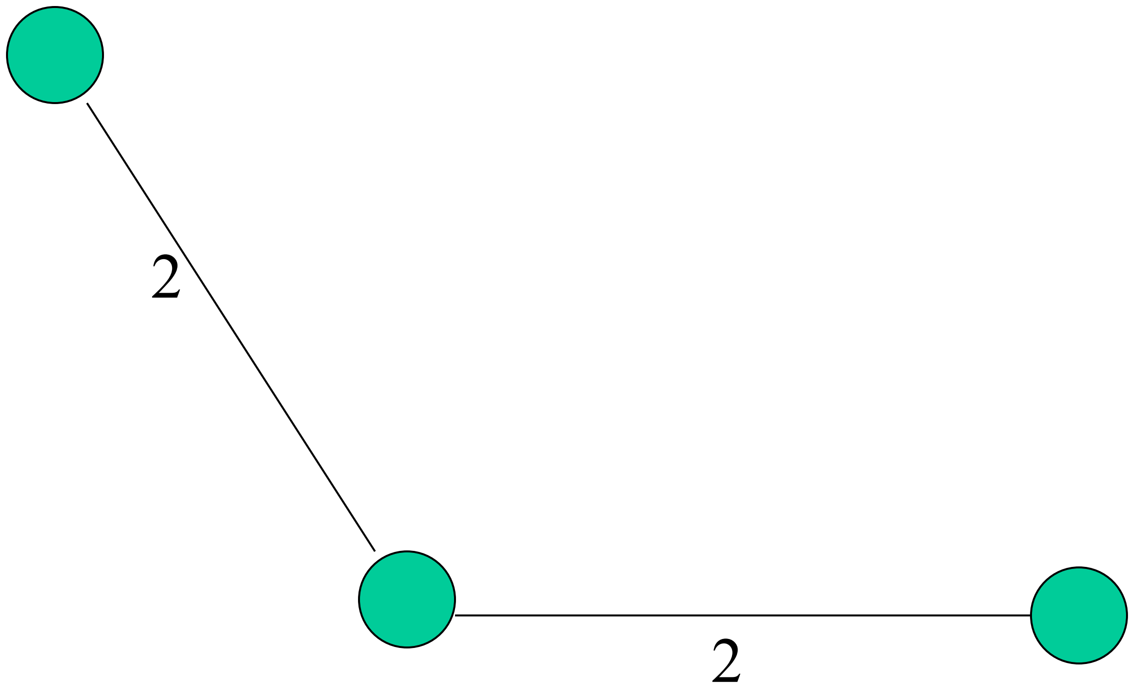
MST length = 1 + √3

SMT length is 2.6458

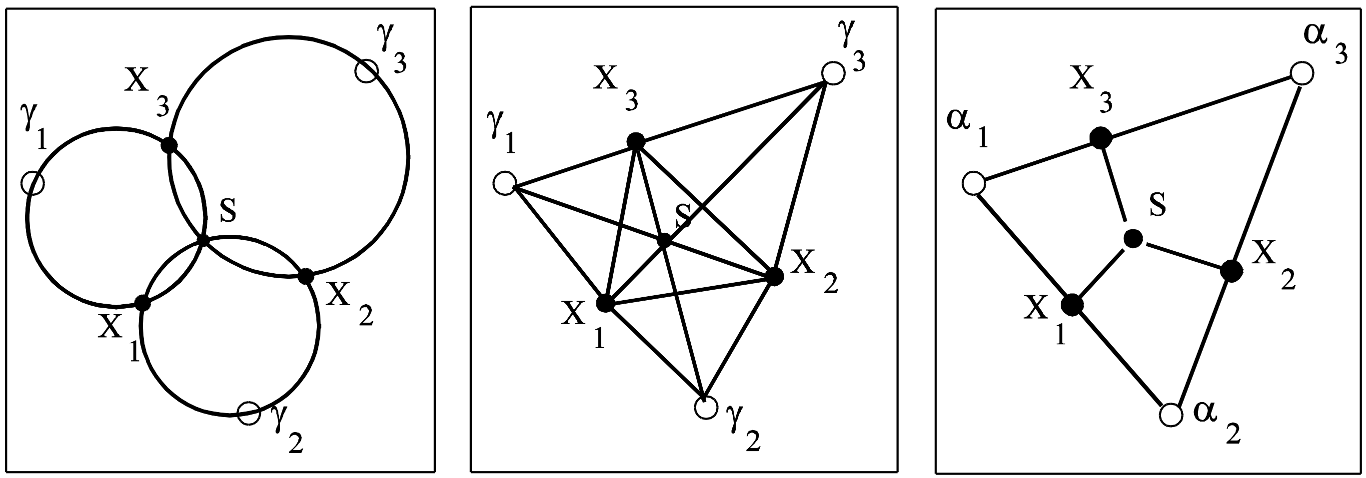
A saving of 3.1588%



Some triangles are such that a Steiner point does not help



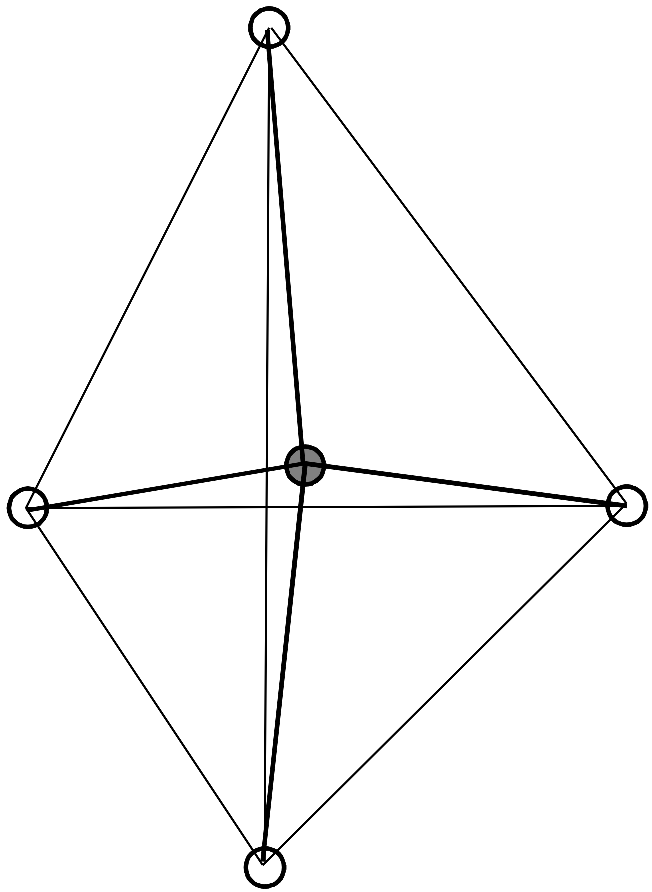
MST length = SMT length = 4
No savings!



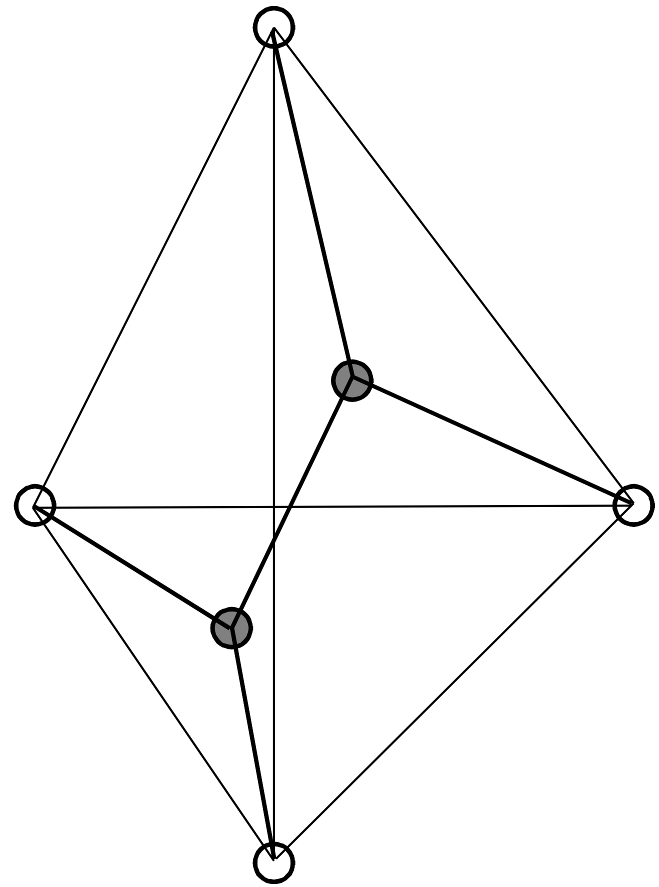
(a) (b) (c)

Figure by Jim Smith

Geometric Construction using compass and straight edge

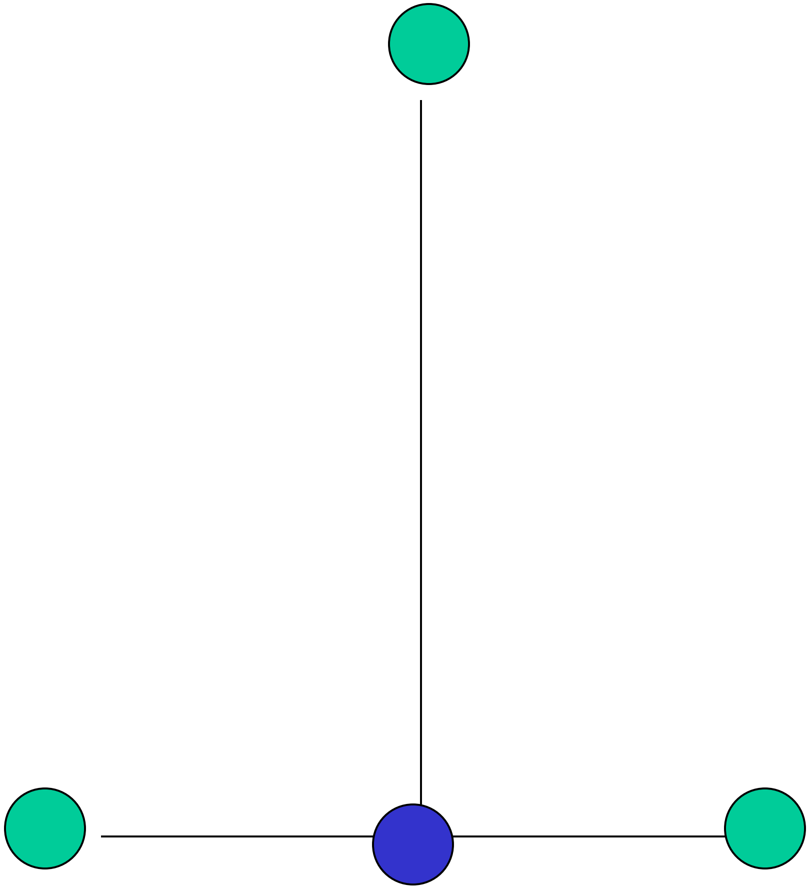


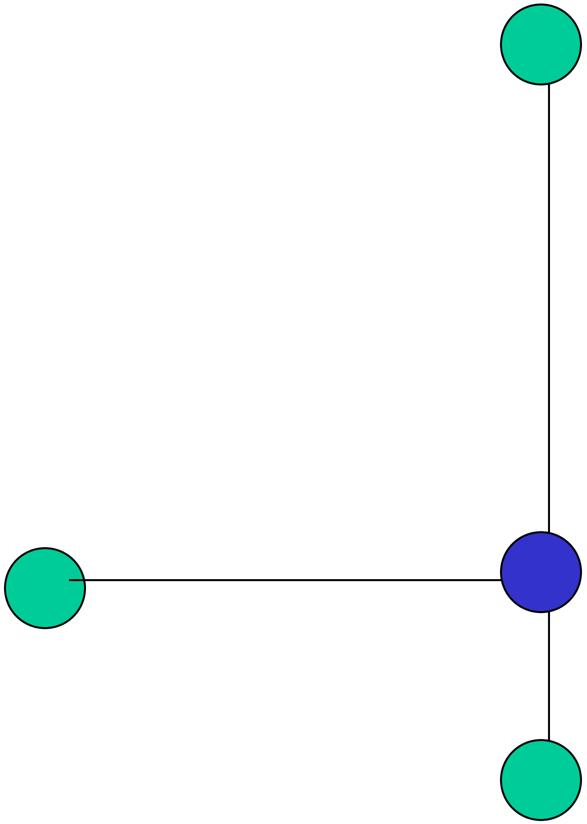
Four terminal sites in 3d
with one Steiner point

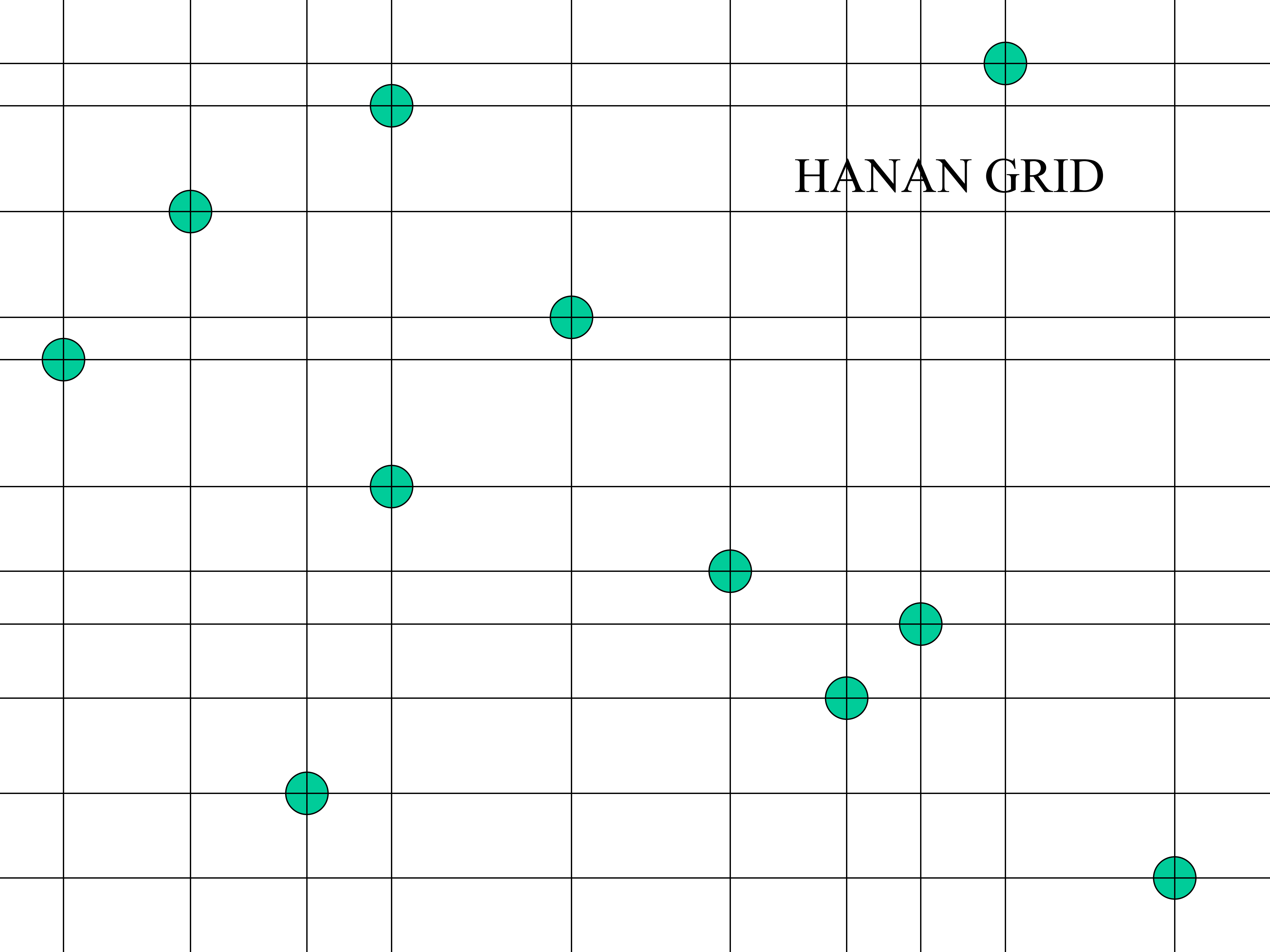


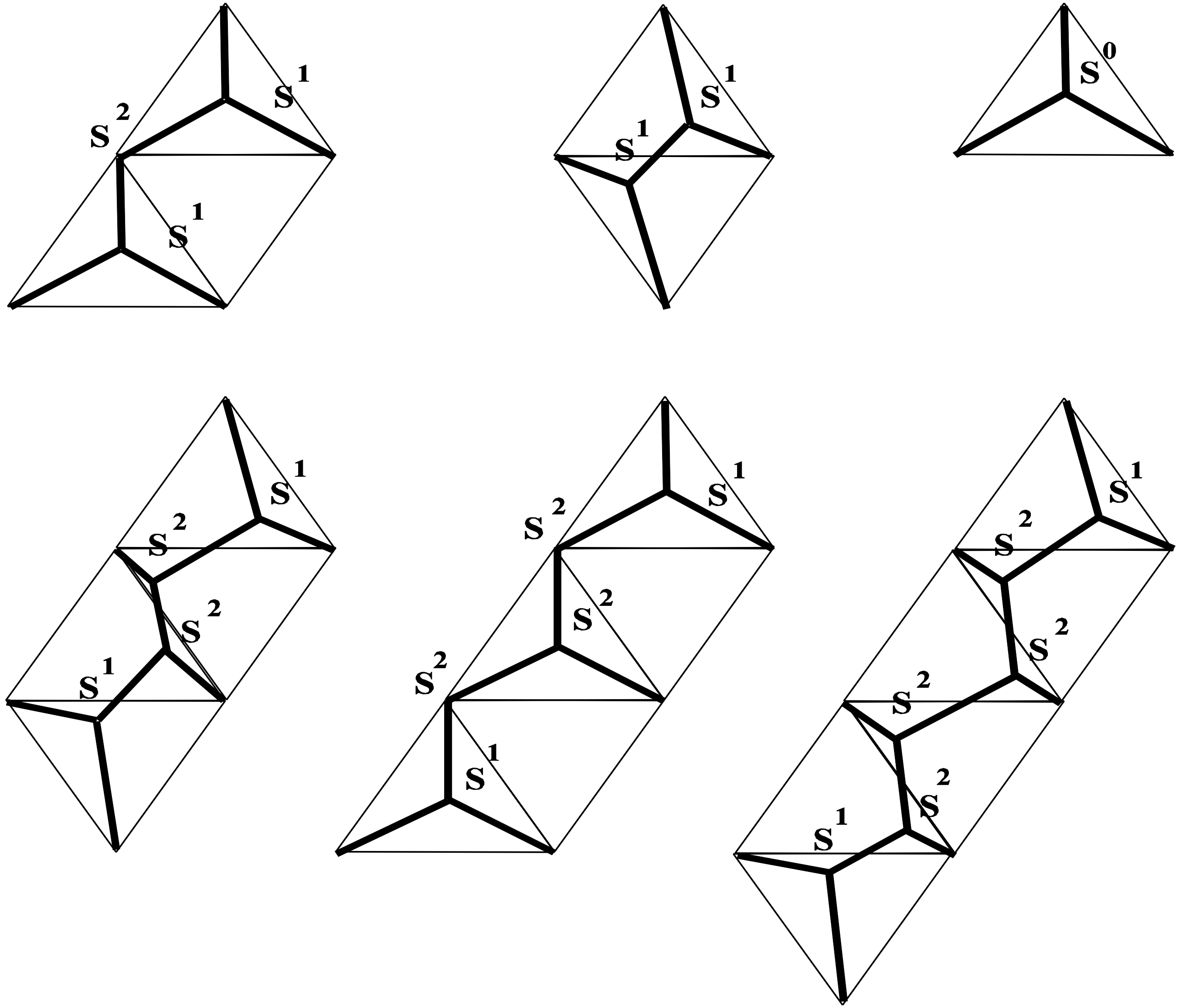
Four terminal sites in 3d
with two Steiner points

Steiner trees with the rectilinear metric appear when laying down conductive pathways on a printed circuit board.

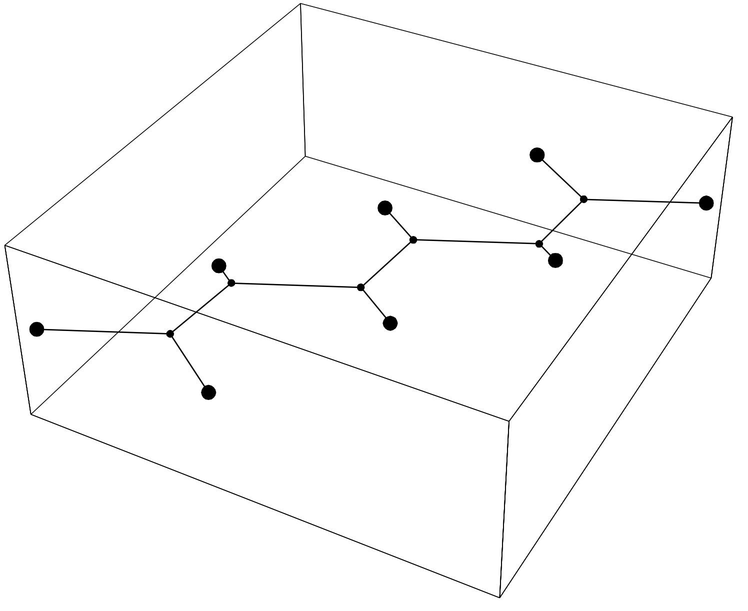





HANAN GRID

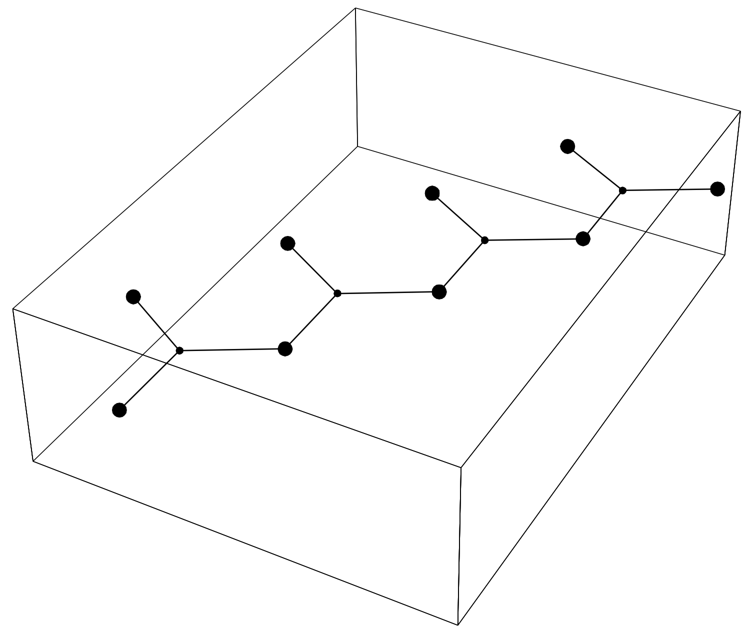


Development of Path Topology in 2d
(Counter-clockwise from top right)



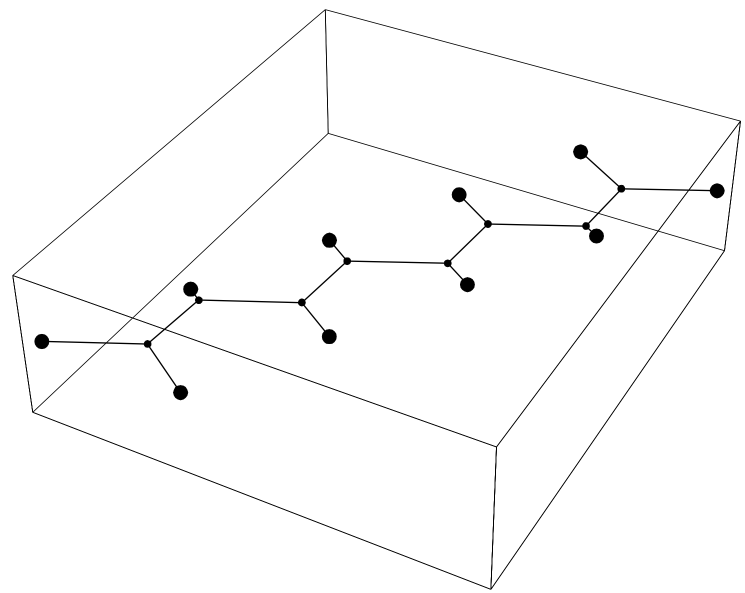
Planar-Sausage with 8 terminal sites

Conjectured infinite 2d structure with best Steiner Ratio $\sqrt{3}/2$



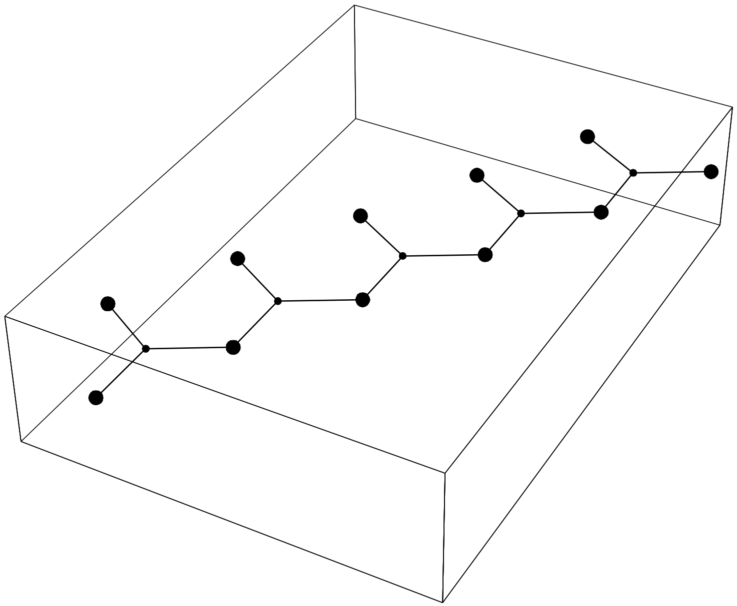
Planar-Sausage with 9 terminal sites

Conjectured infinite 2d structure with best Steiner Ratio √3/2



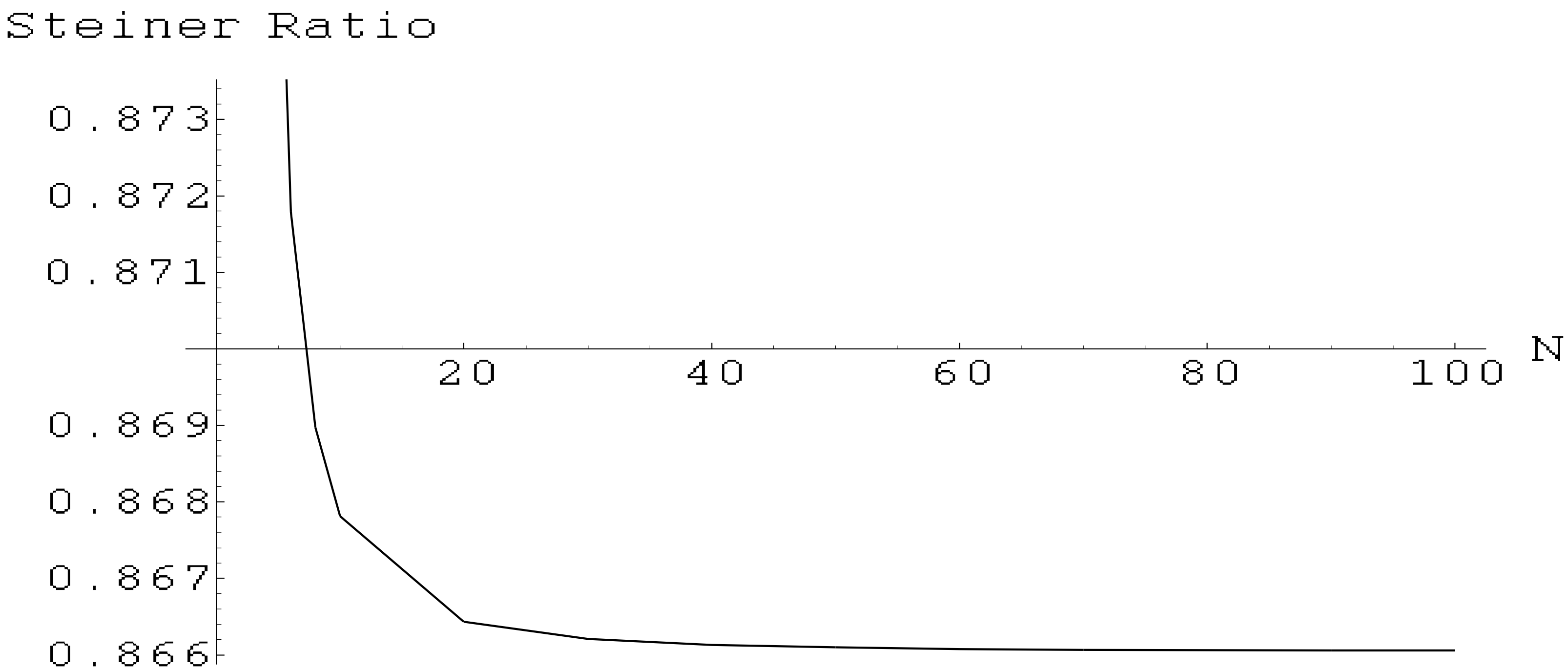
Planar-Sausage with 10 terminal sites

Conjectured 2d structure with best Steiner Ratio √3/2

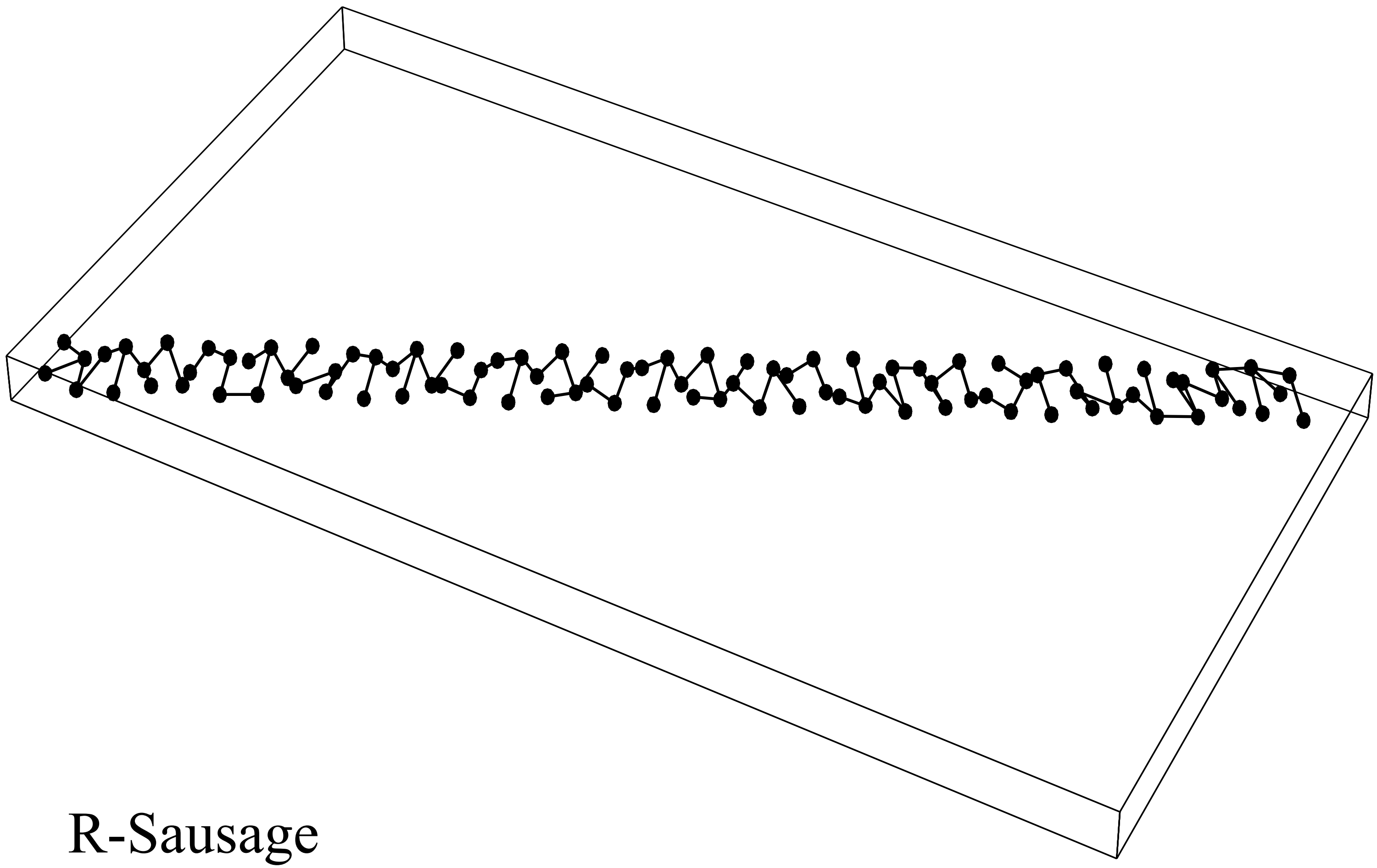


Planar-Sausage with 11 terminal sites

Conjectured infinite 2d structure with best Steiner Ratio = $\sqrt{3}/2$

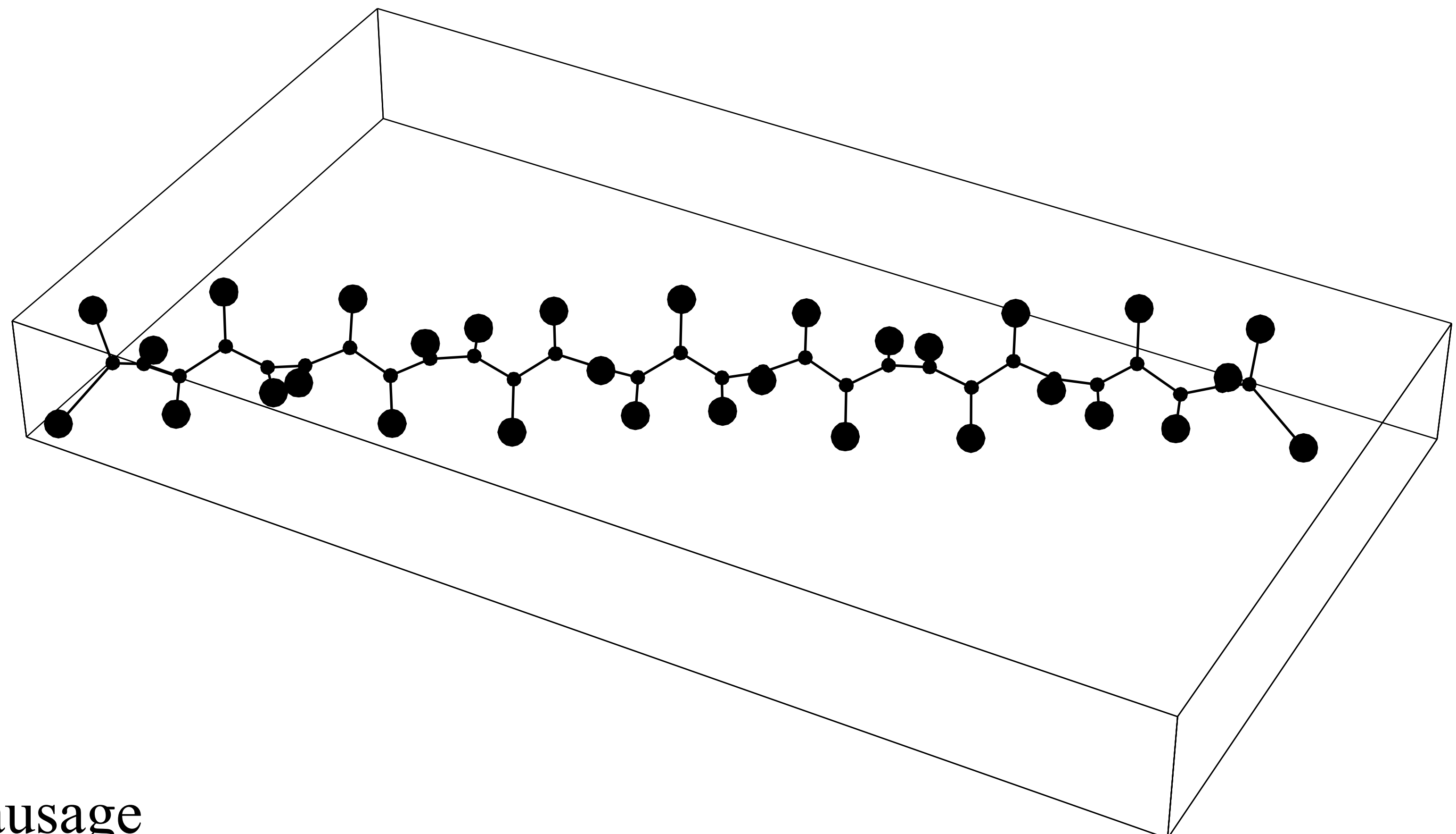


Steiner Ratio in 2d falls to √3/2 as the number of terminal sites increase



R-Sausage

Minimum Spanning Tree

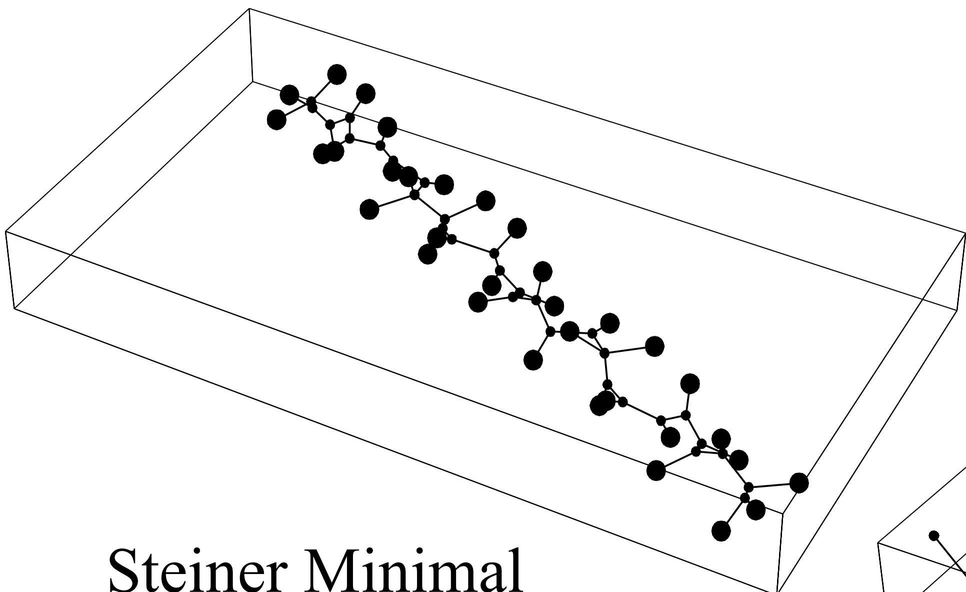


R-Sausage

Conjectured infinite 3d structure with best Steiner Ratio

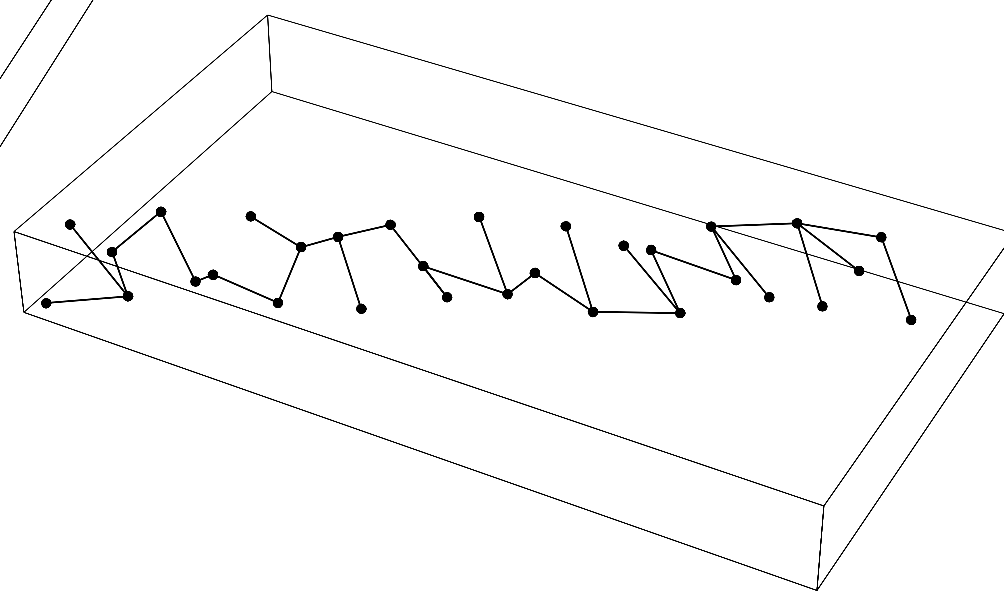
Figure by Jim Smith

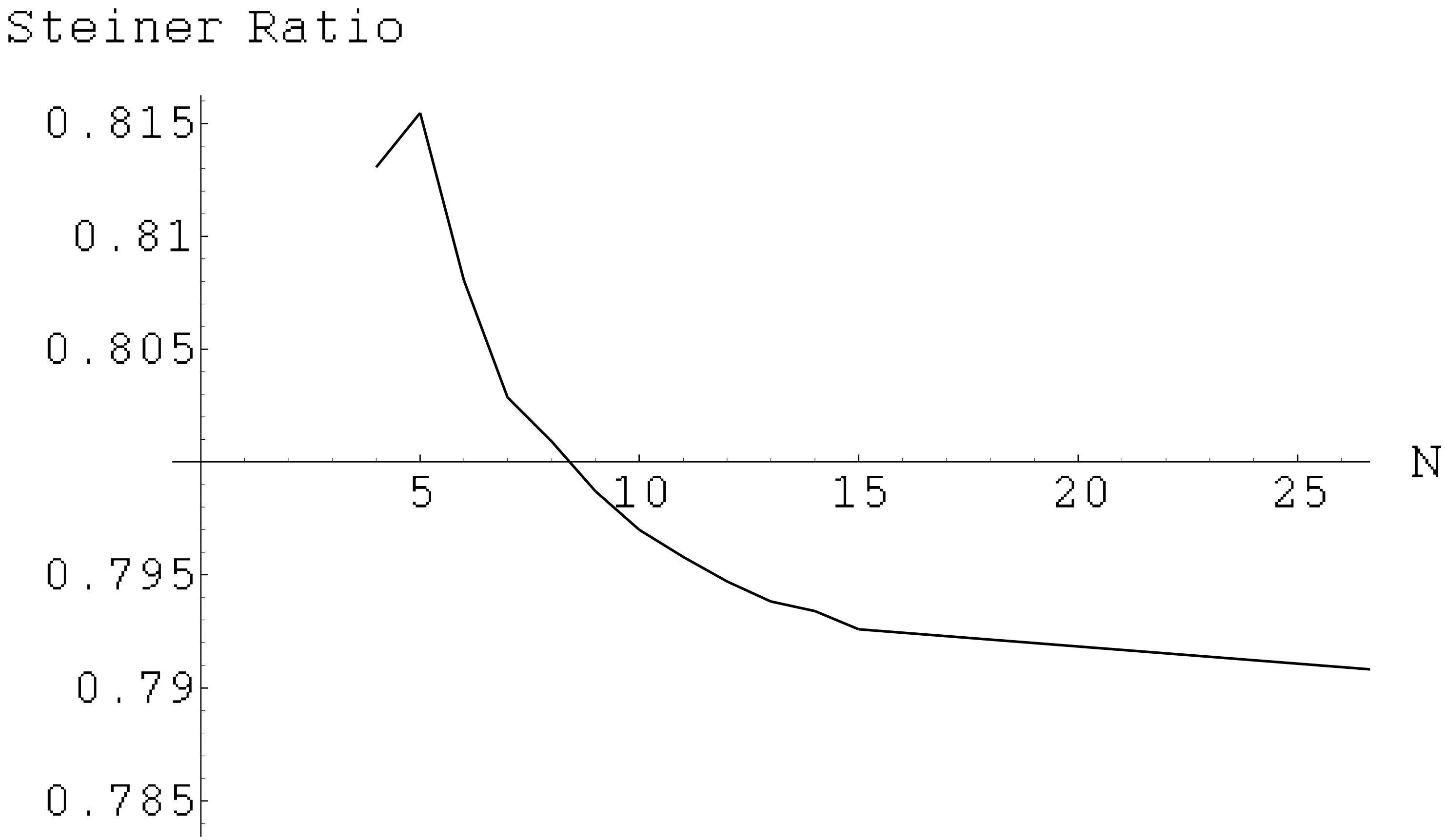
Another view of 3d sausage



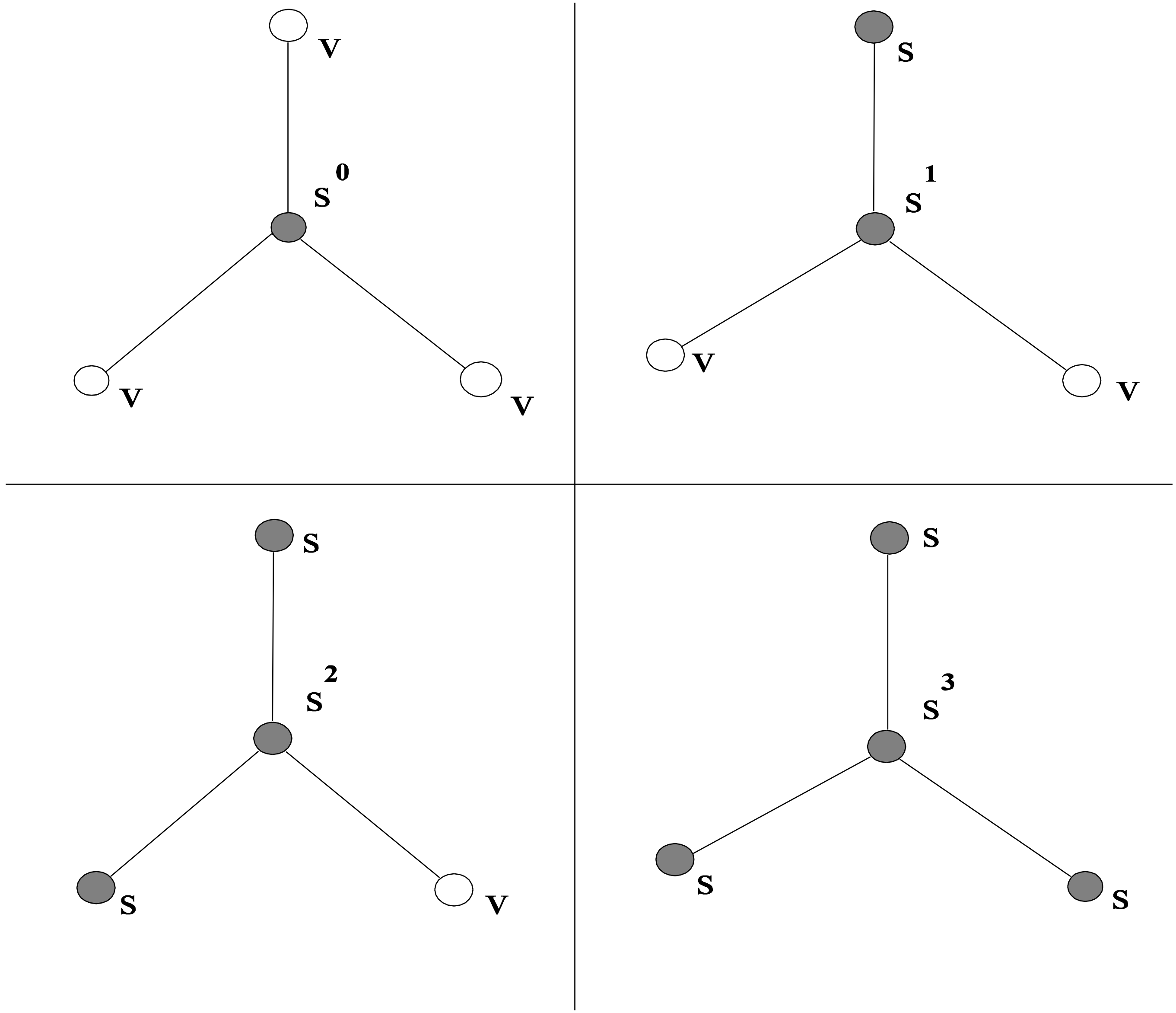
Steiner Minimal
Tree

Minimum Spanning
Tree

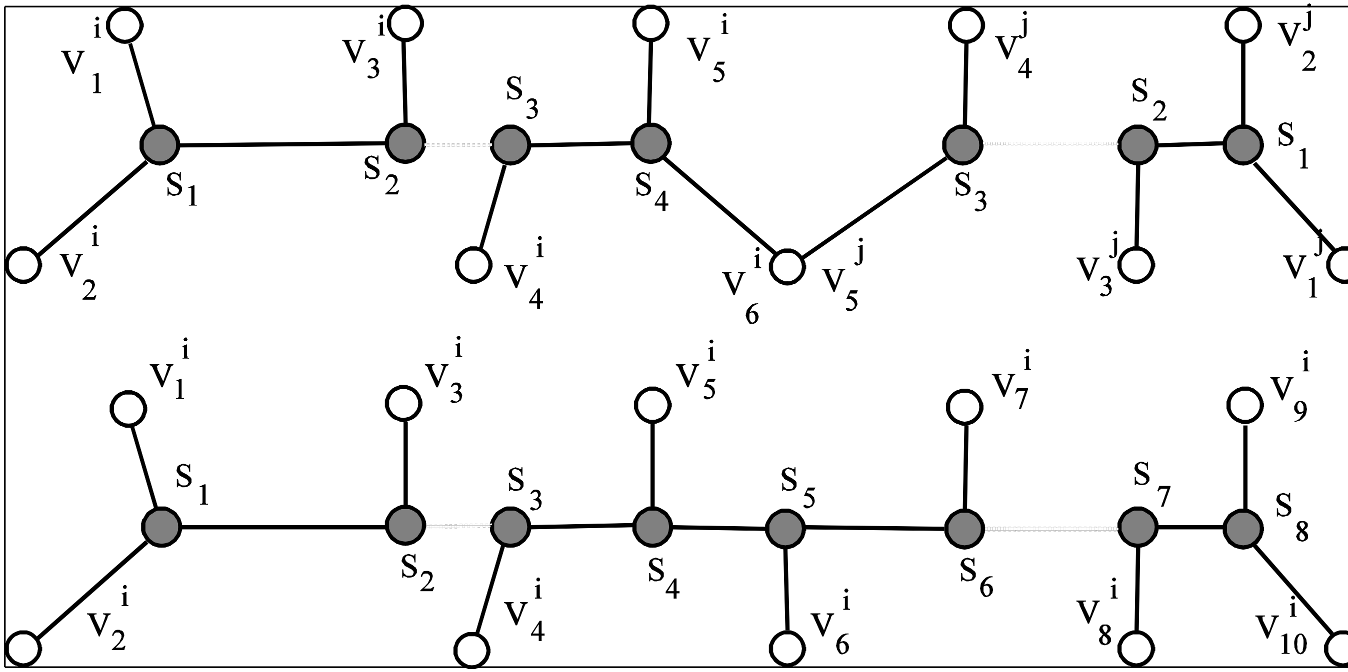




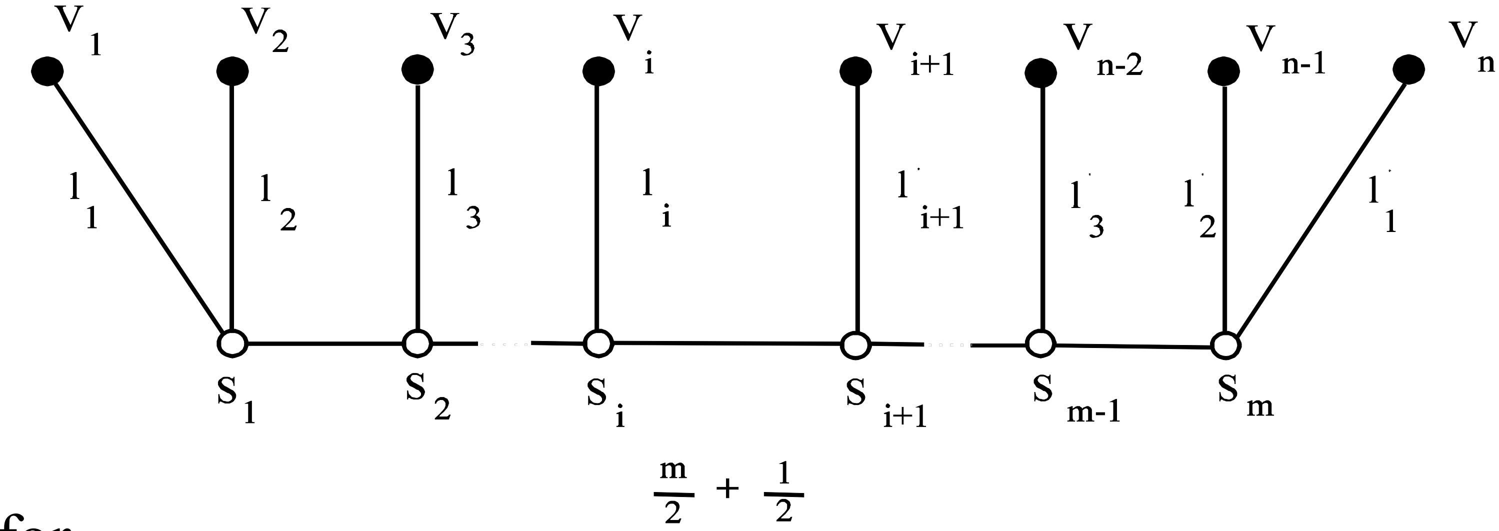
Steiner Ratio in 3d falls to ≈ 0.784 as the number of terminal sites increase



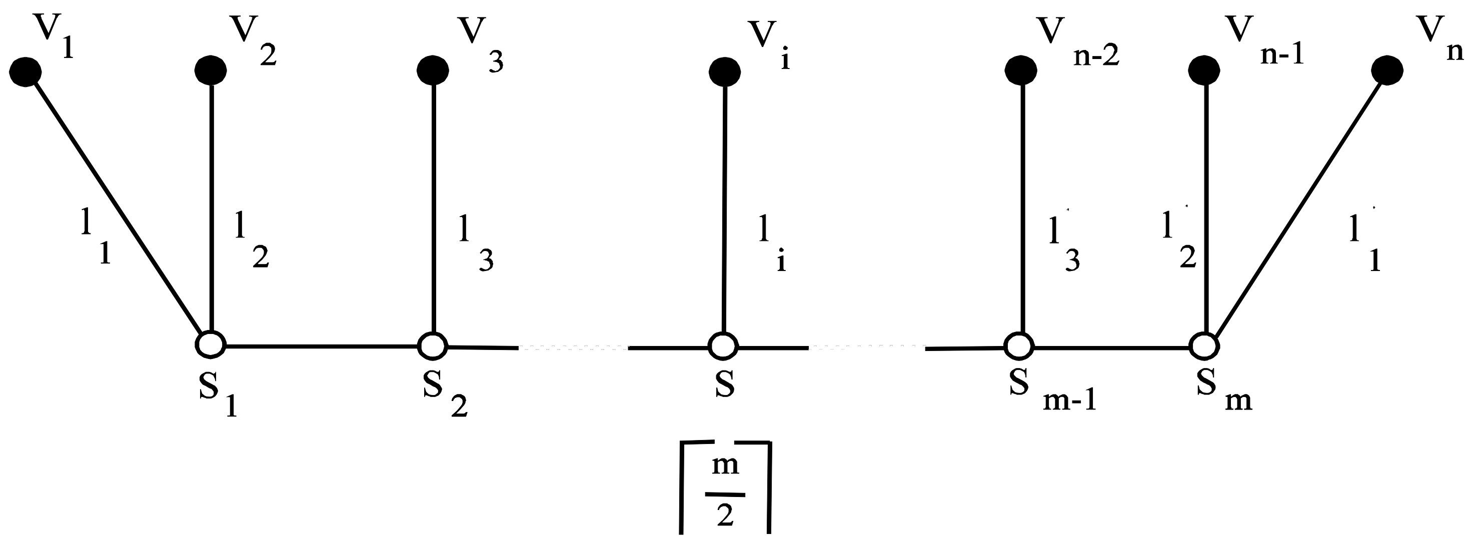
Four types of Steiner points -
according to neighbouring vertices



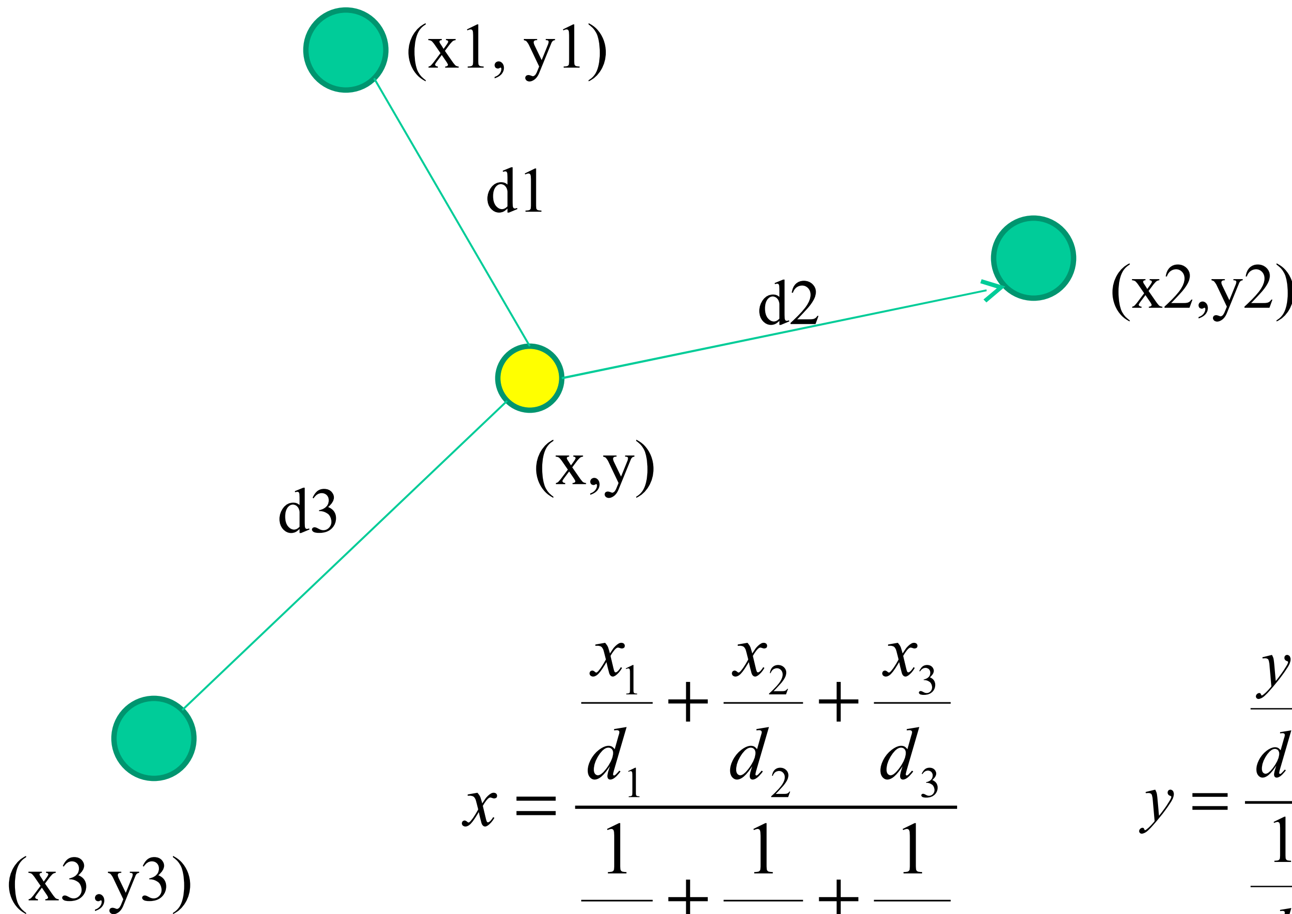
R-Sausage structure is preserved under composition.
It is a semi-group.



Topology for
Even and Odd Sized
R-Sausages in 3d



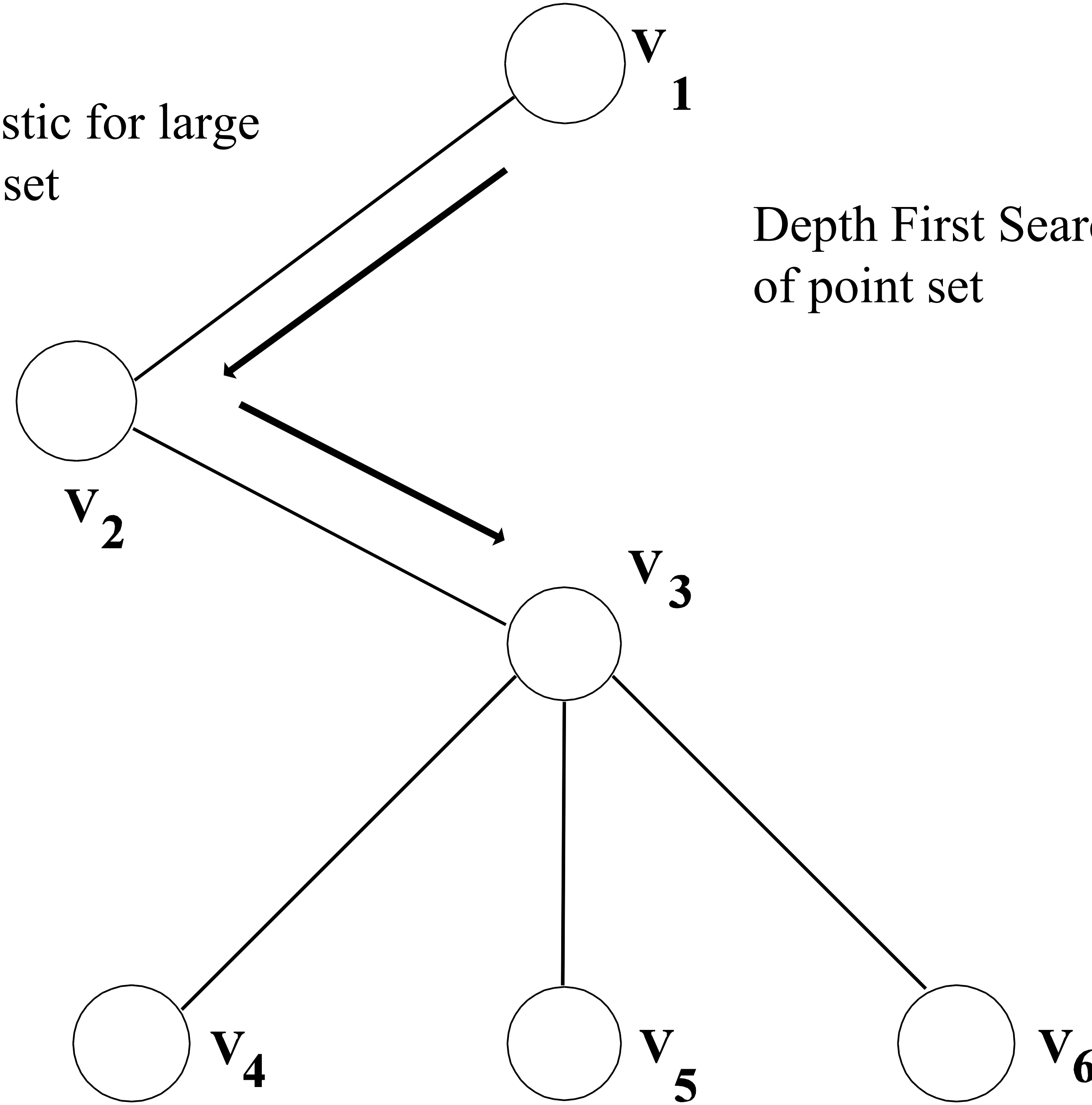
The average of the vertex coordinates weighted by their reciprocal distances from the Steiner point.



$$x = \frac{\frac{x_1}{d_1} + \frac{x_2}{d_2} + \frac{x_3}{d_3}}{\frac{1}{d_1} + \frac{1}{d_2} + \frac{1}{d_3}} \qquad y = \frac{\frac{y_1}{d_1} + \frac{y_2}{d_2} + \frac{y_3}{d_3}}{\frac{1}{d_1} + \frac{1}{d_2} + \frac{1}{d_3}}$$

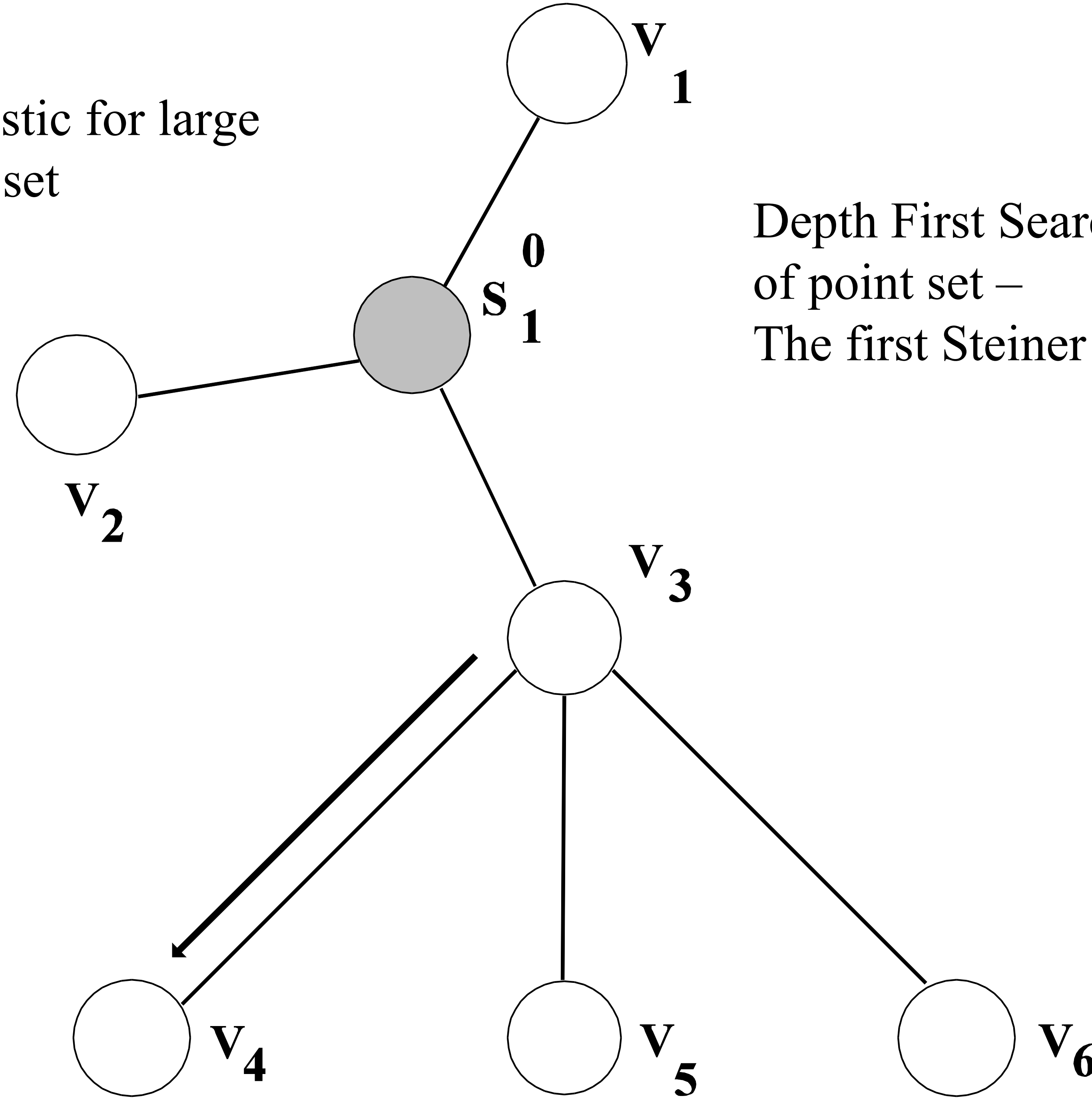
Heuristic for large point set

Depth First Search of point set



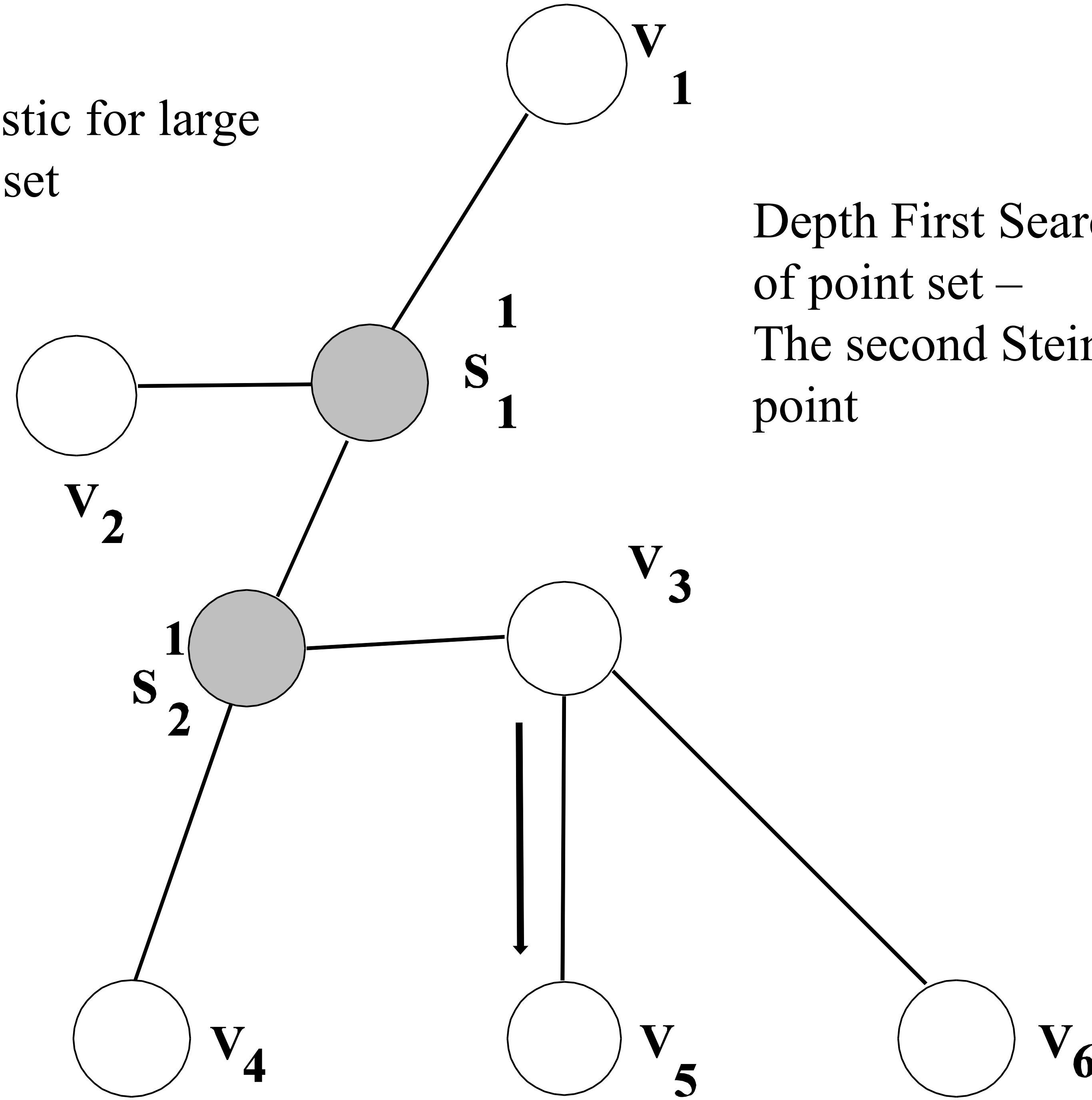
Heuristic for large point set

Depth First Search of point set –
The first Steiner point



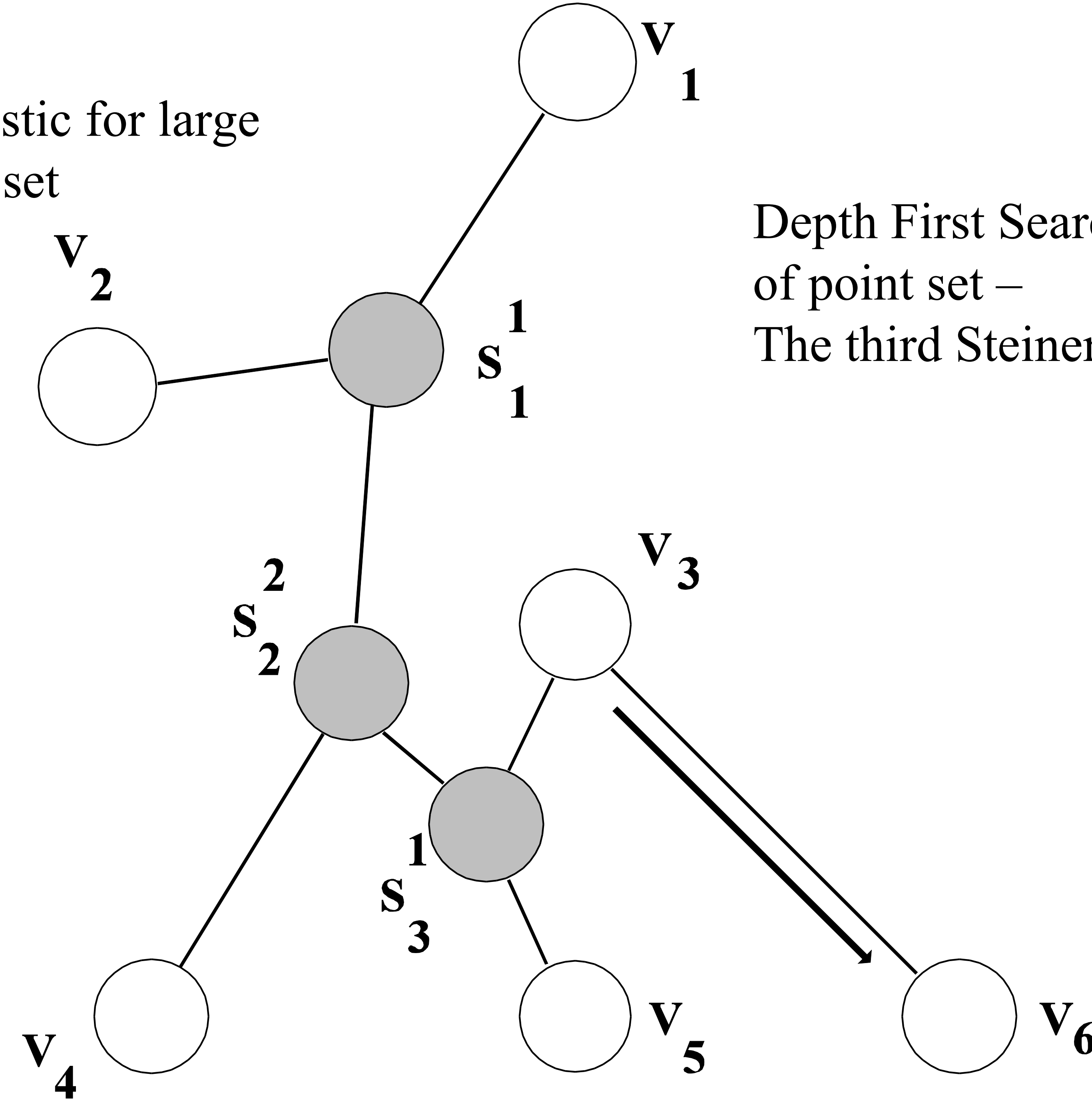
Heuristic for large point set

Depth First Search of point set –
The second Steiner point



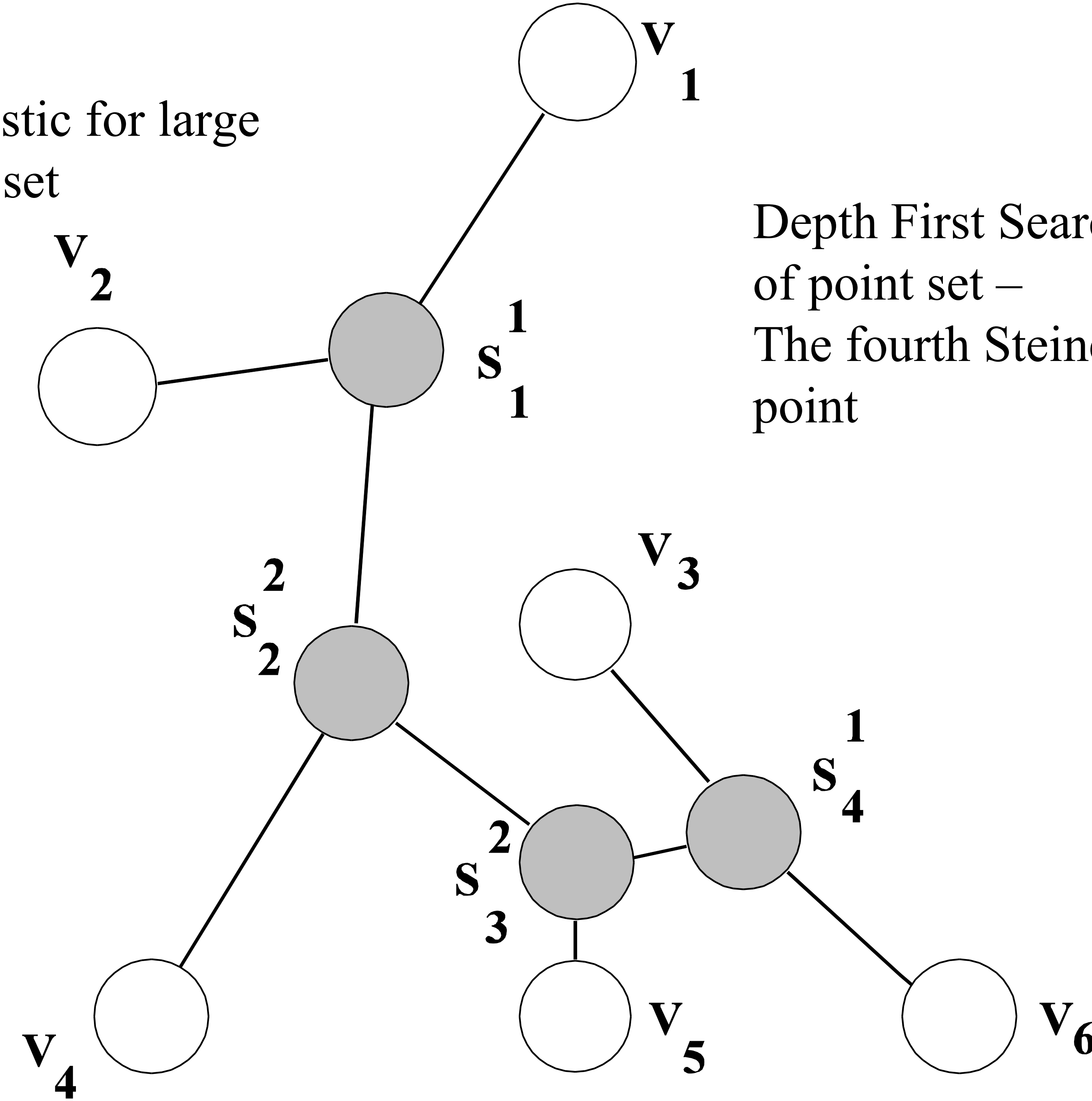
Heuristic for large point set

Depth First Search of point set –
The third Steiner point



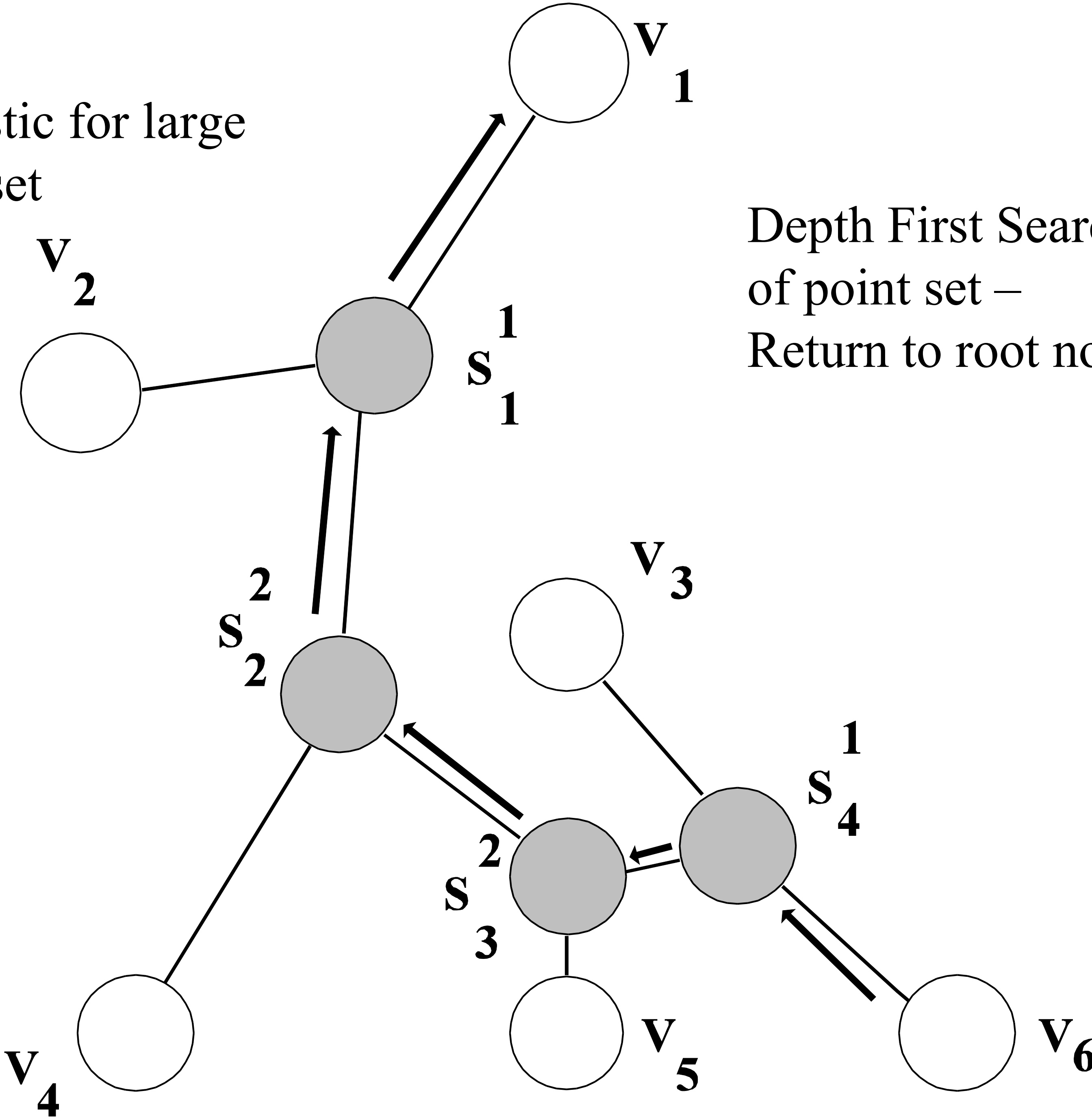
Heuristic for large point set

Depth First Search of point set – The fourth Steiner point



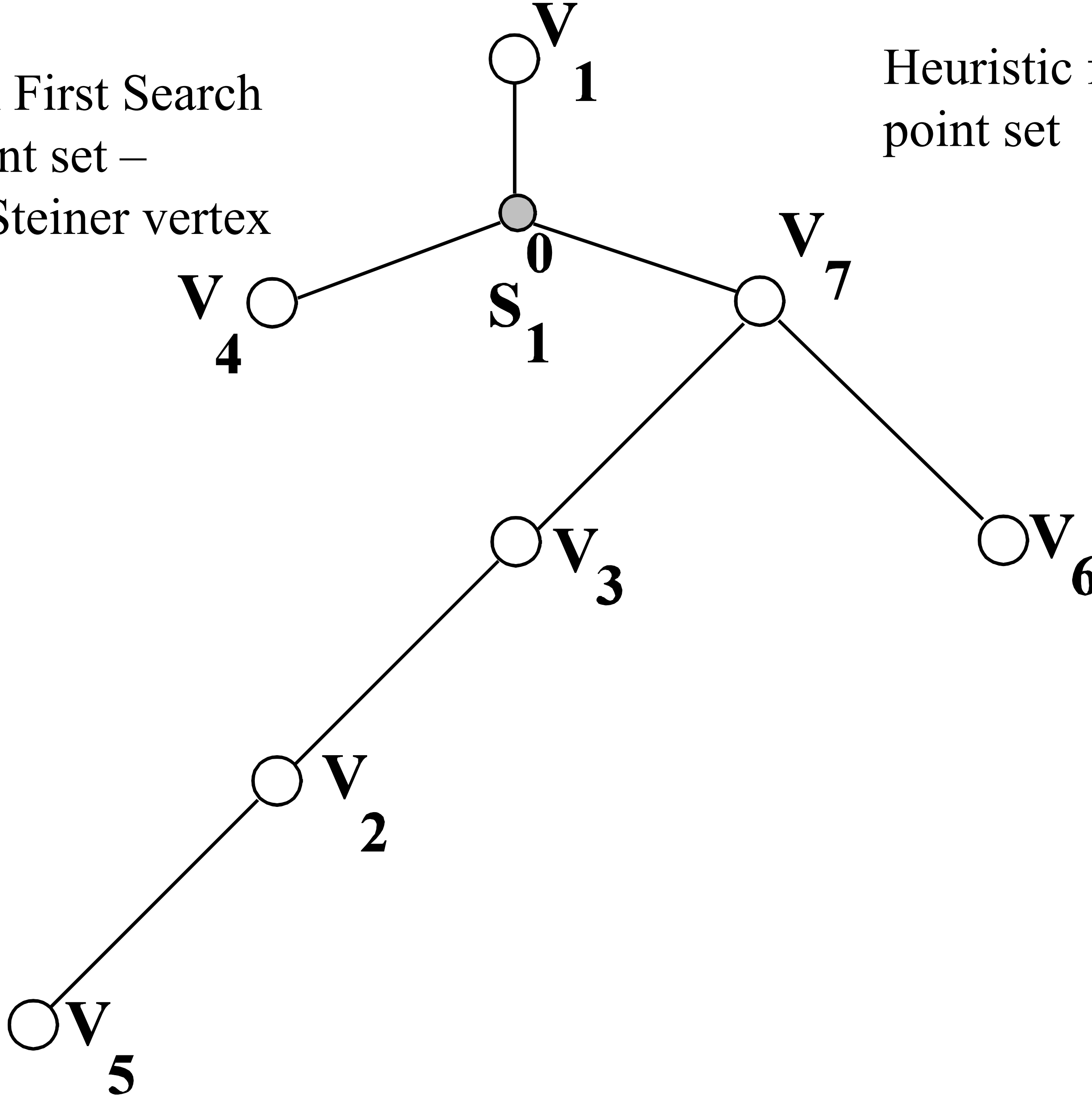
Heuristic for large point set

Depth First Search of point set –
Return to root node



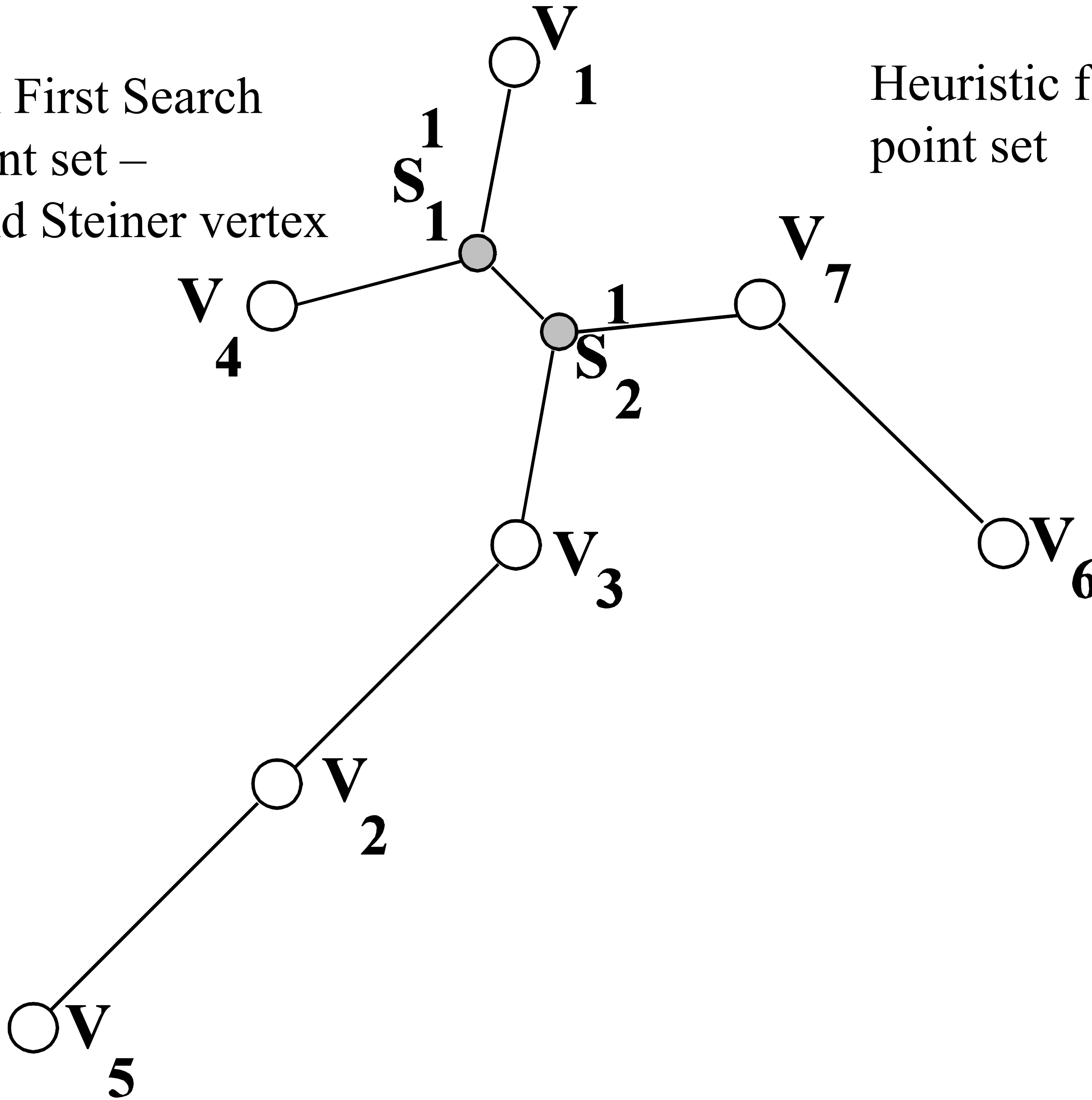
Depth First Search of point set – First Steiner vertex

Heuristic for large point set



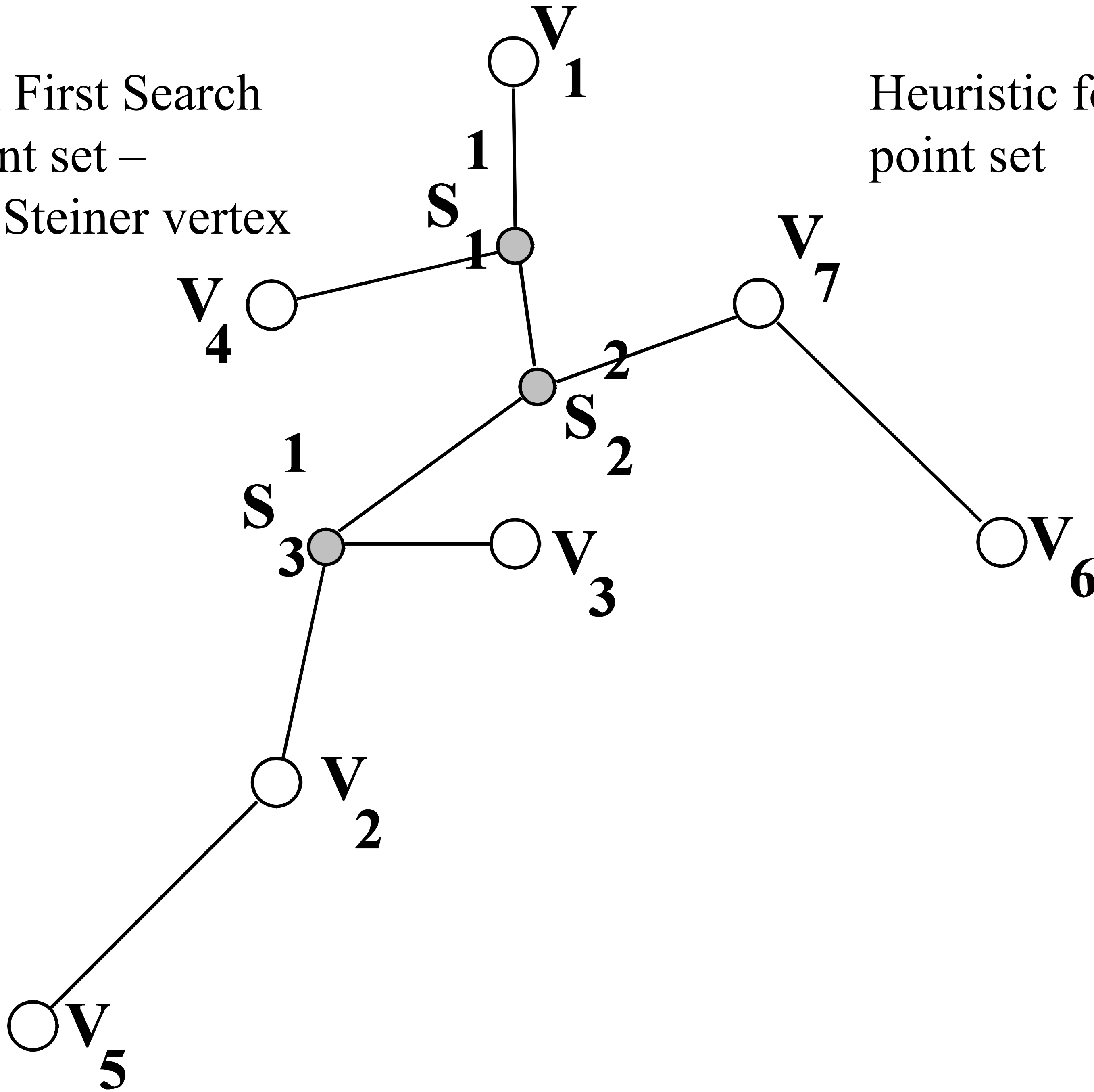
Depth First Search of point set – Second Steiner vertex

Heuristic for large point set



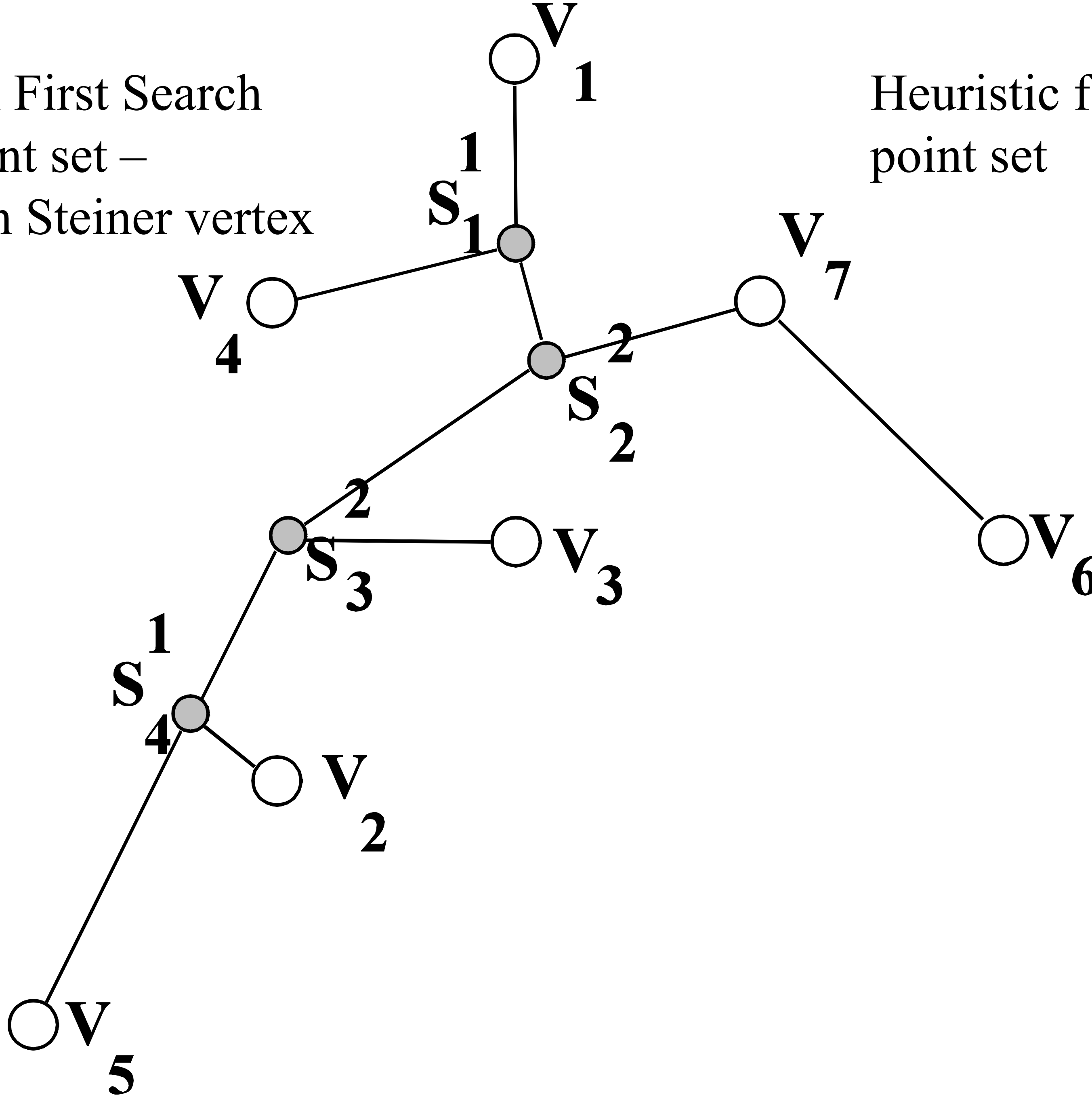
Depth First Search of point set – Third Steiner vertex

Heuristic for large point set



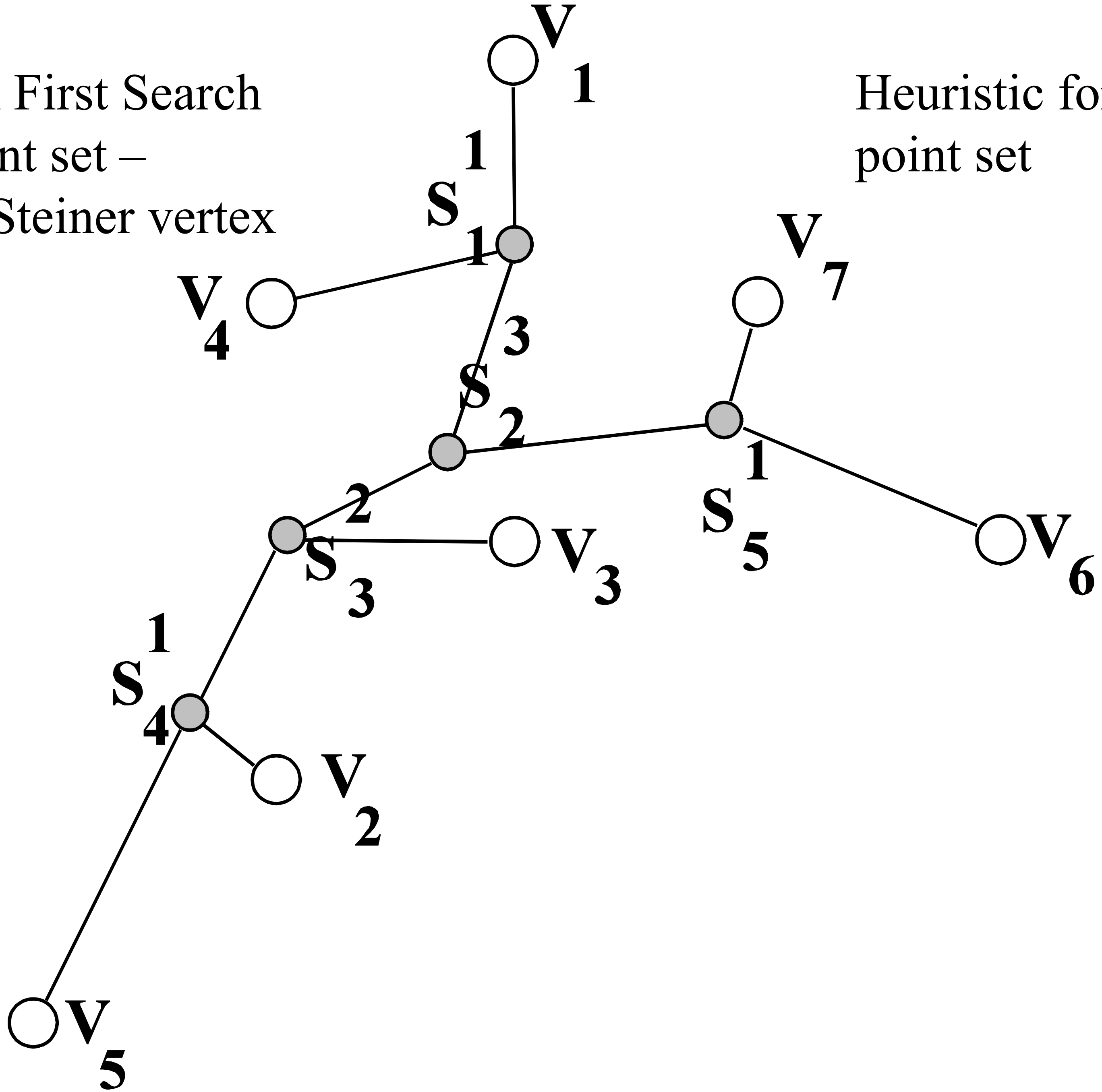
Depth First Search of point set – Fourth Steiner vertex

Heuristic for large point set

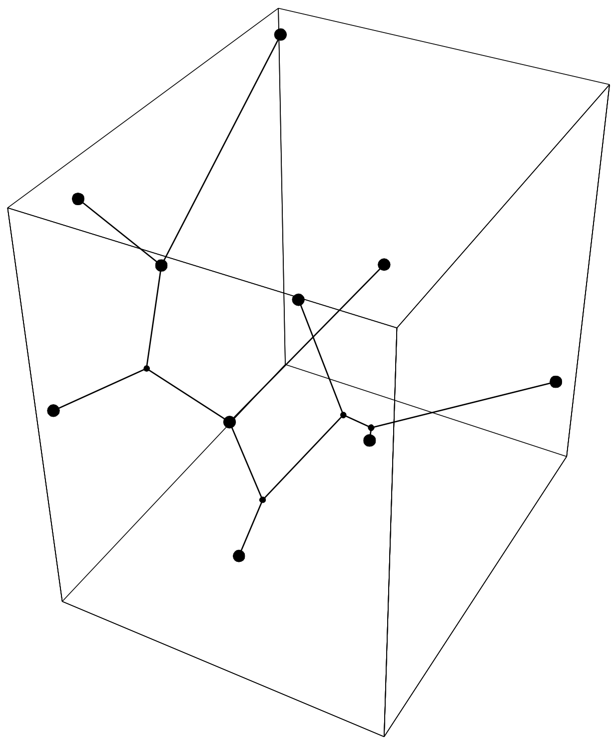


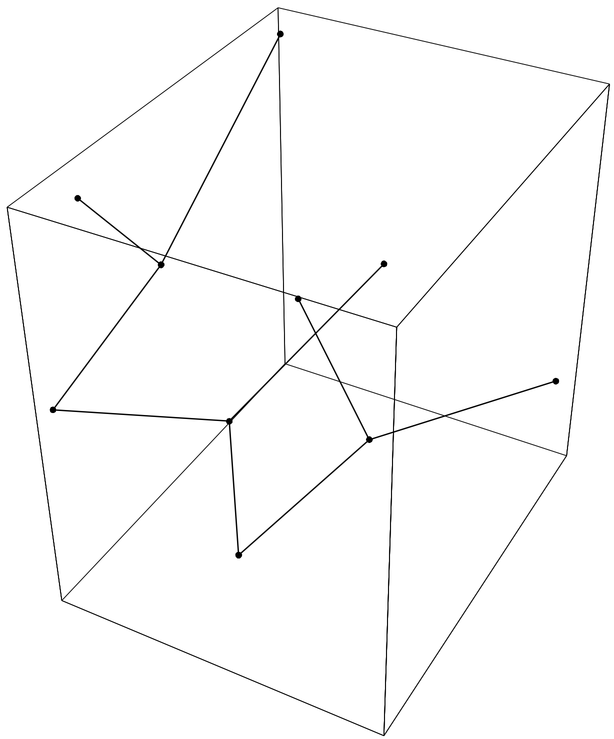
Depth First Search of point set – Fifth Steiner vertex

Heuristic for large point set



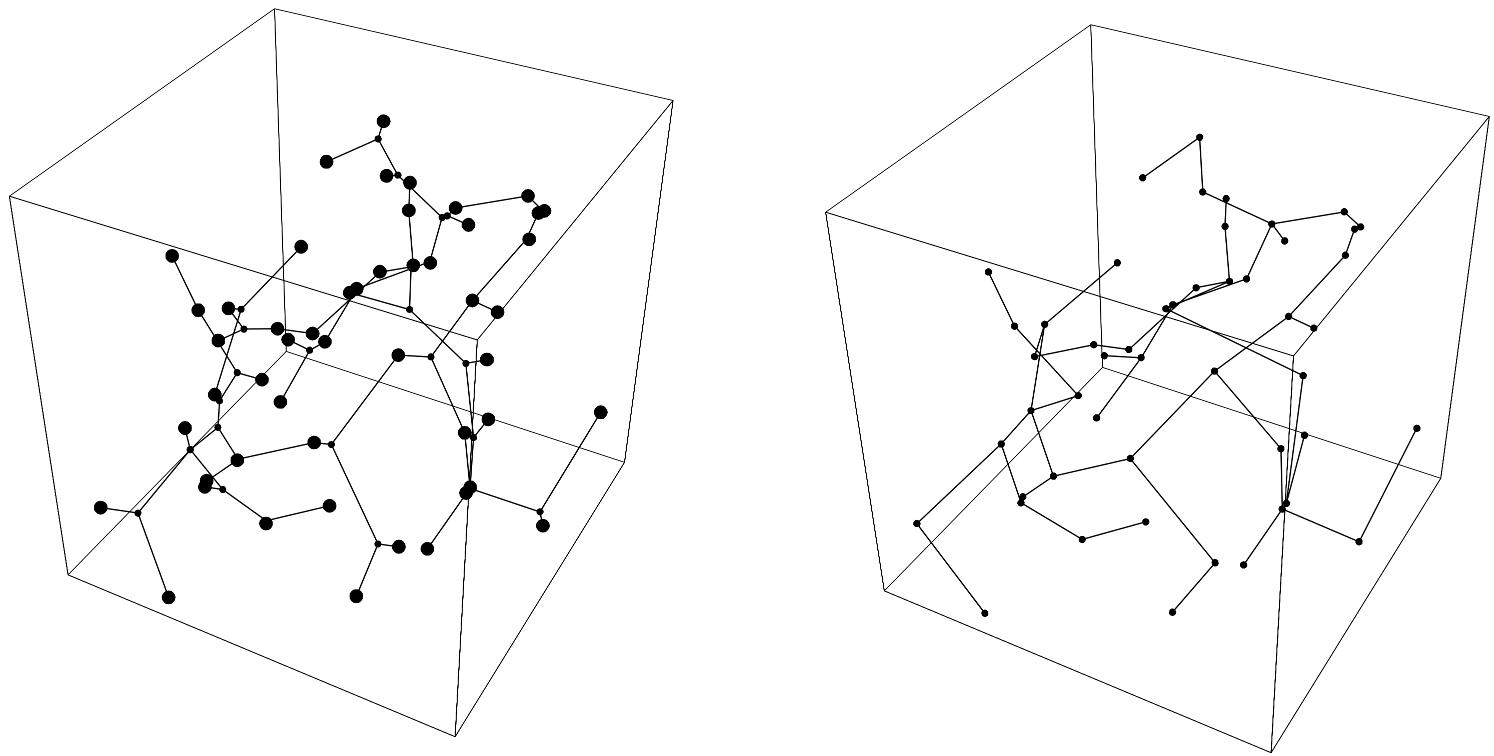
Minimal Interconnecting Tree for data without any structure





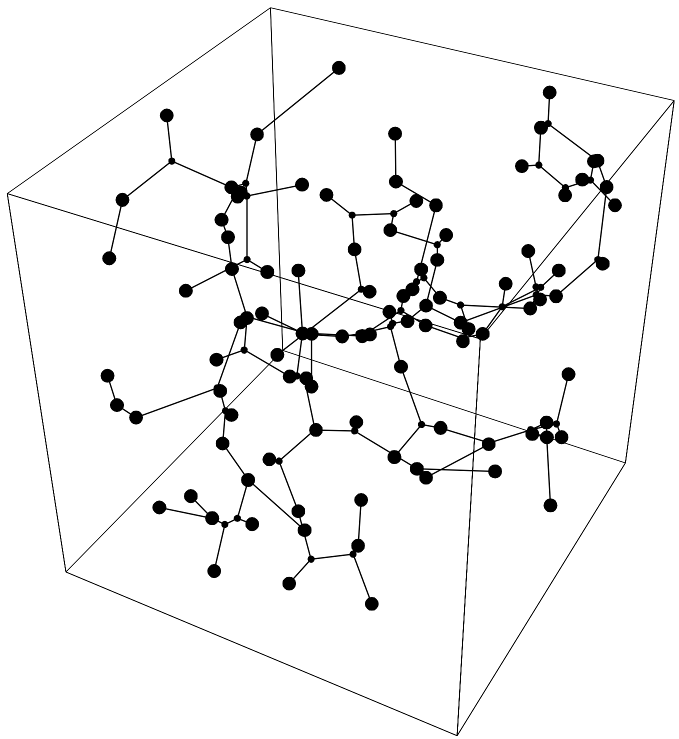
Steiner Tree and Minimum Spanning Tree for
a small random point set in unit cube

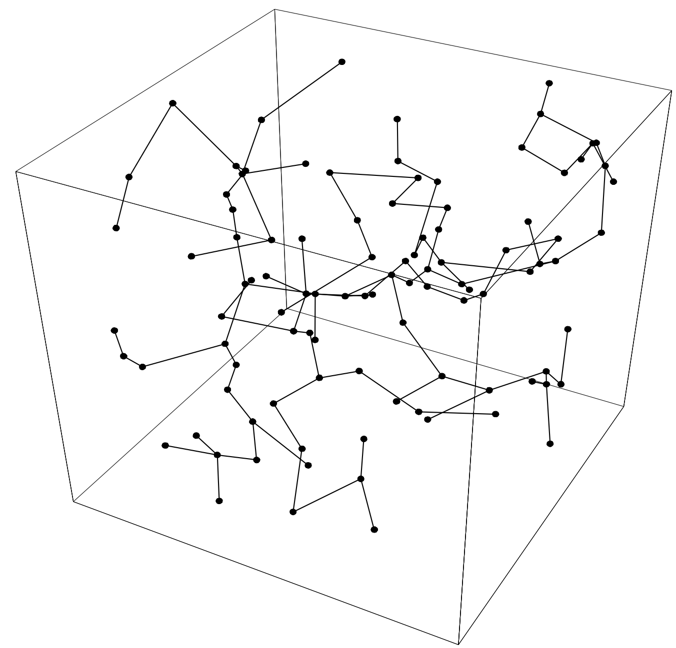
Minimal Interconnecting Tree for data without any structure



Steiner Tree and Minimum Spanning Tree for
a medium sized random point set in unit cube

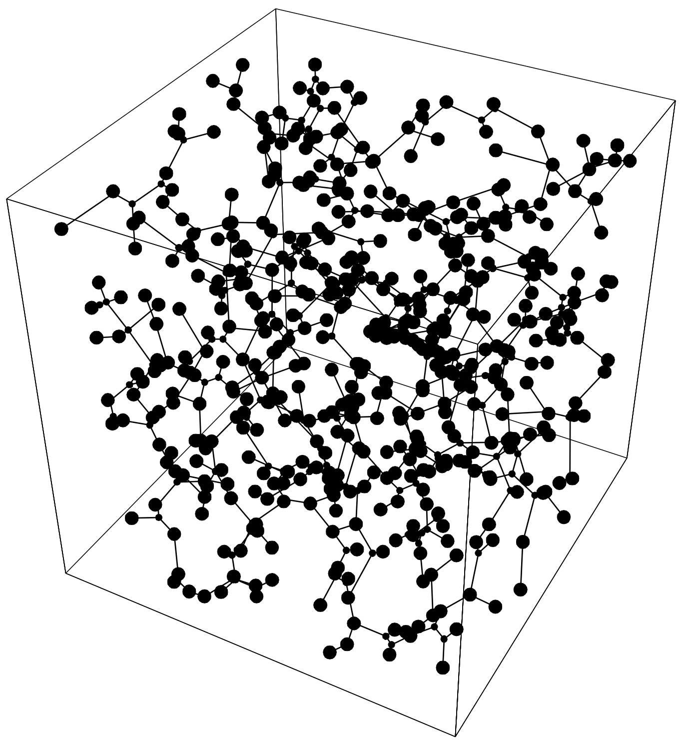
Minimal Interconnecting Tree for data without any structure

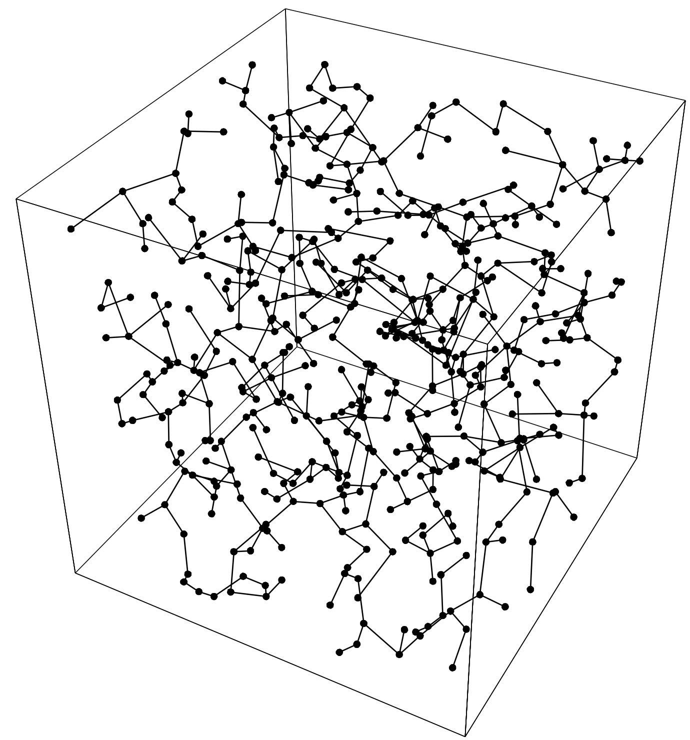




Steiner Tree and Minimum Spanning Tree for
Hundred random points in unit cube

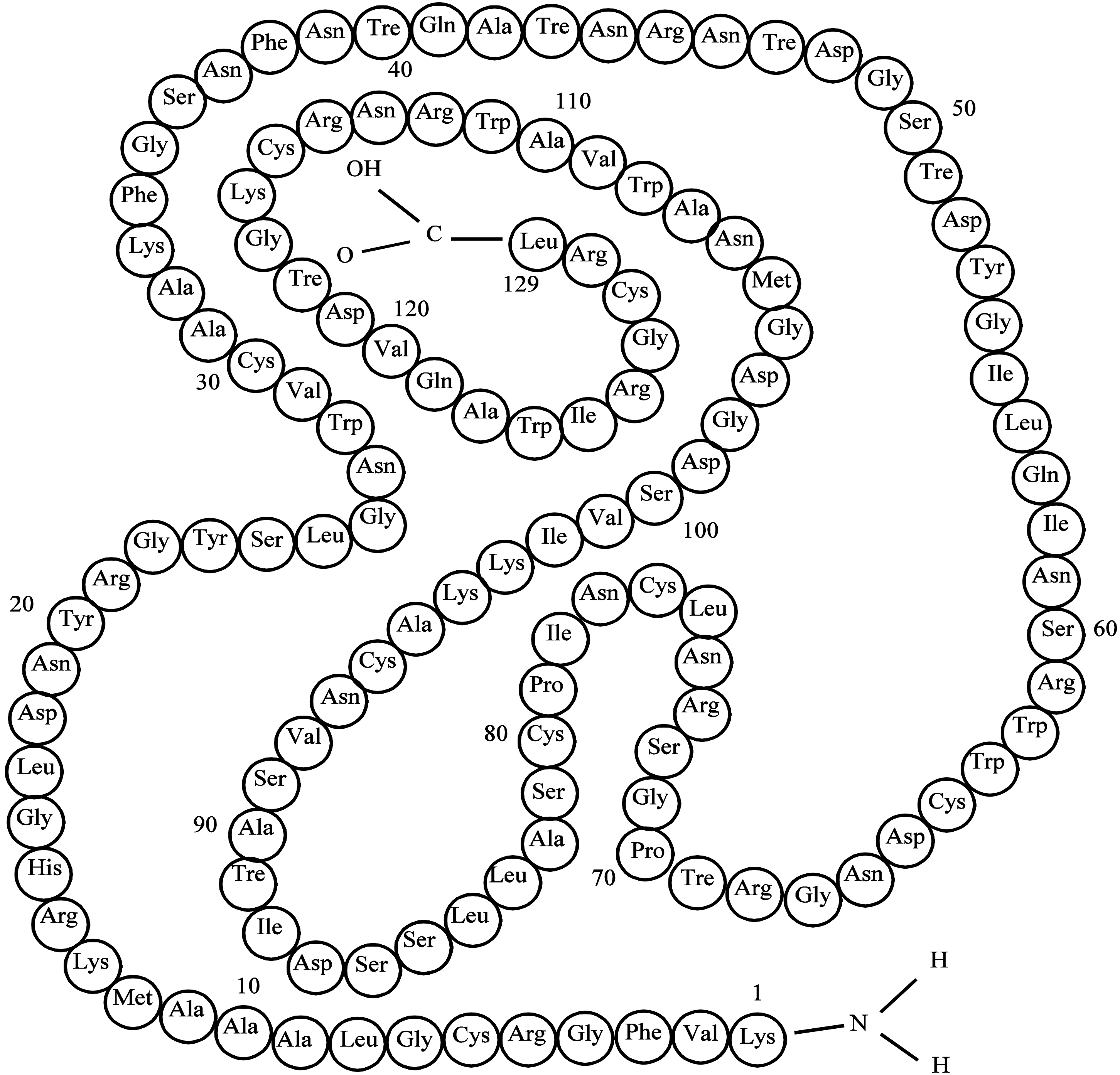
Minimal Interconnecting Tree for data without any structure

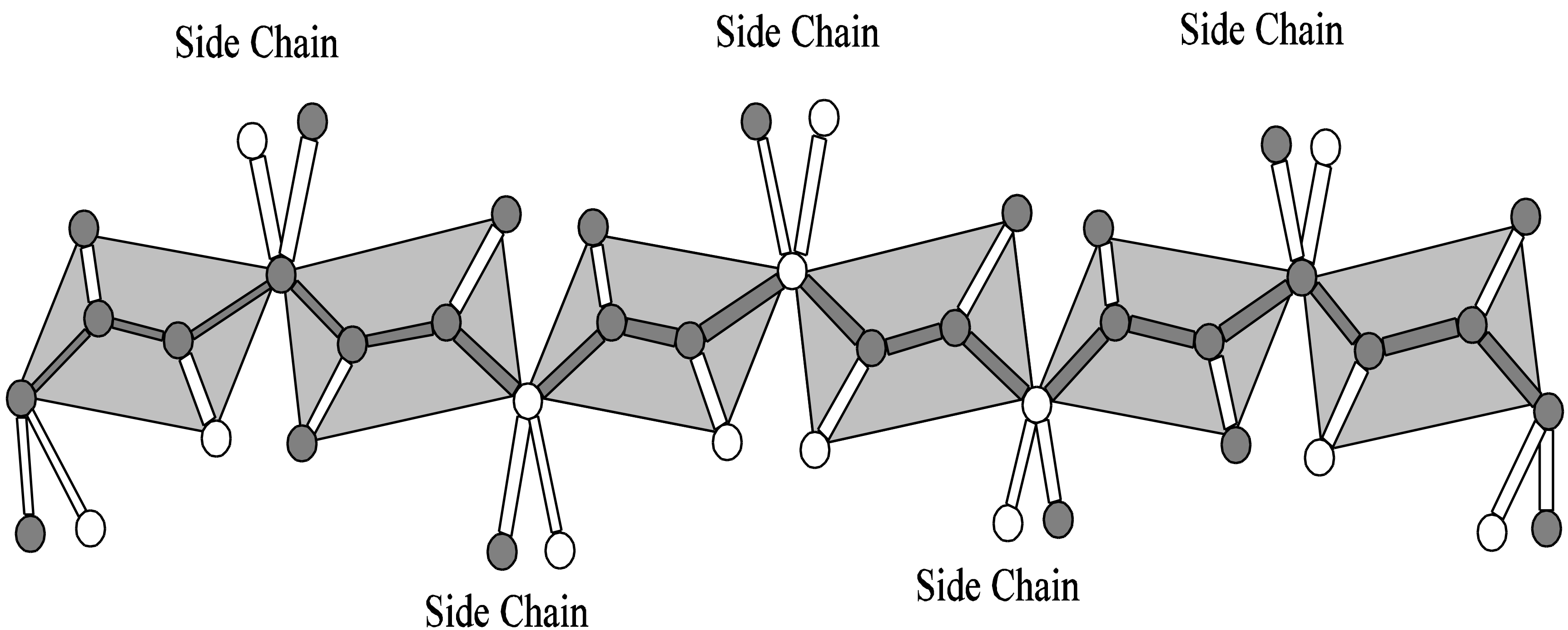




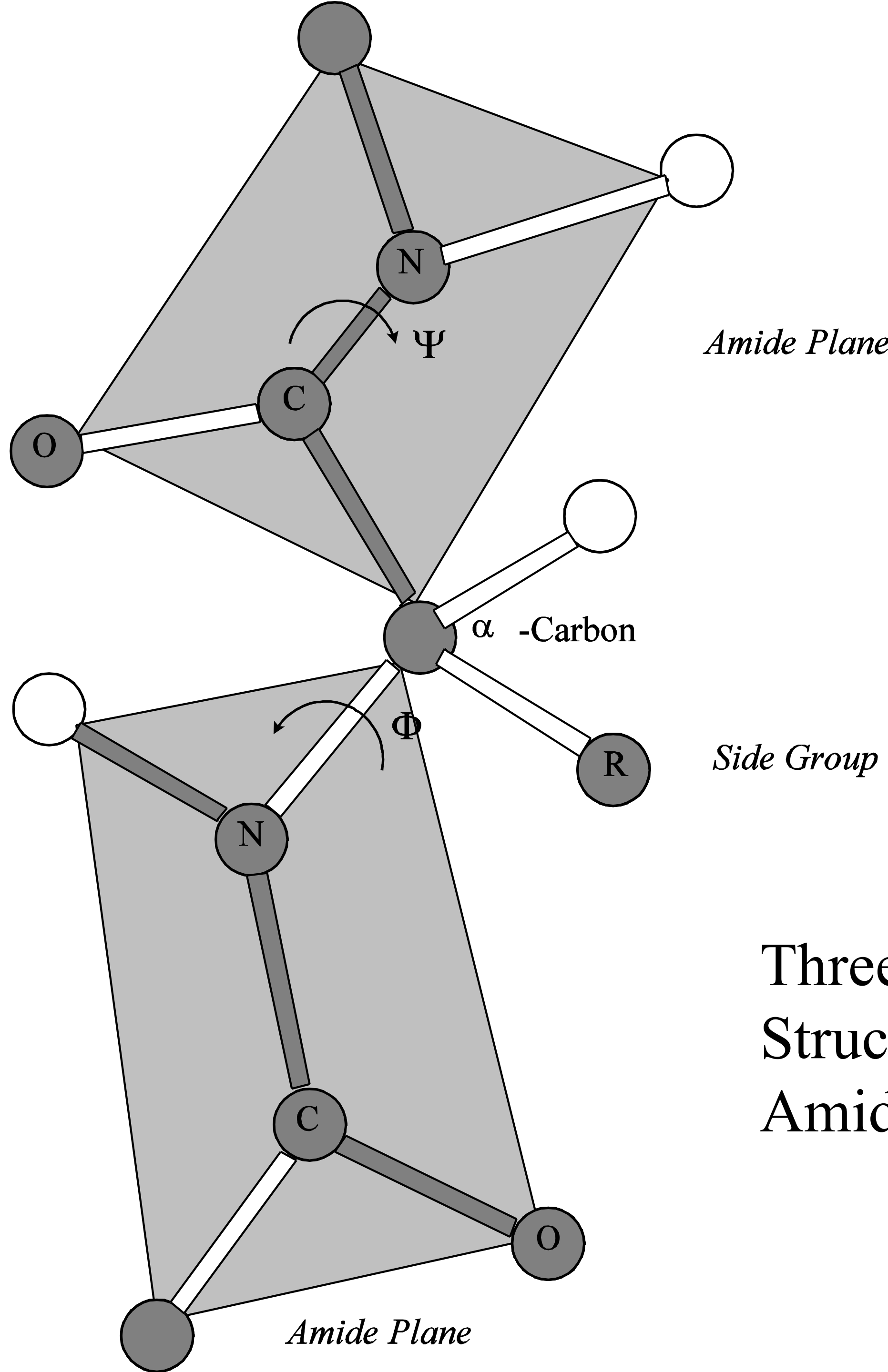
Steiner Tree and Minimum Spanning Tree for
500 random points in unit cube

Amino Acid Sequence of a Protein (Lysozyme)

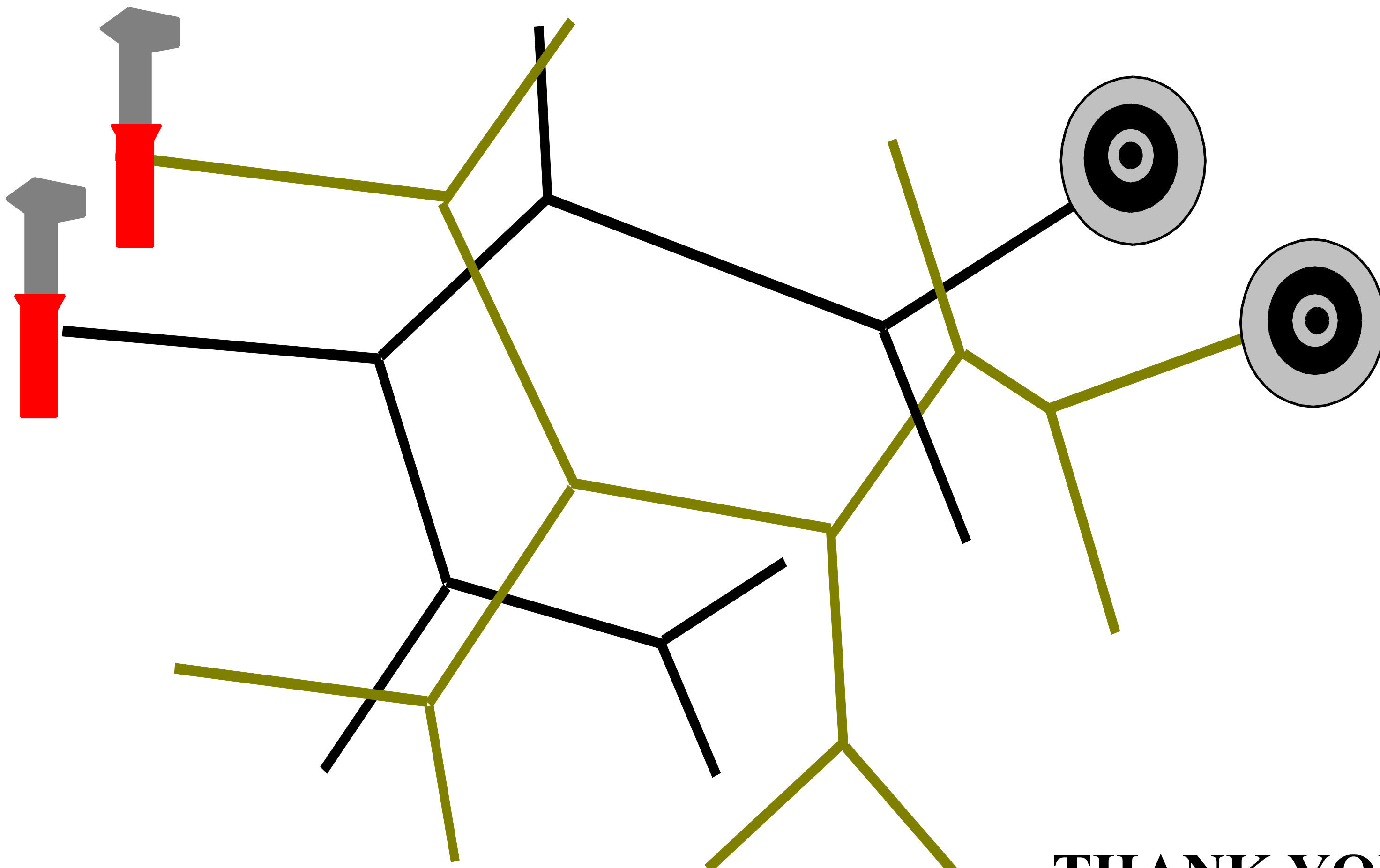




Network of Amide Planes in a Protein



Three Dimensional
Structure of Two
Amide Planes



THANK YOU