Time-varying Signals Recovery via Graph Neural Networks

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Motivation

- The recovery of time-varying graph signals is an important problem in sensor networks and time series forecasting.
- Spatial-temporal graph signal reconstruction typically relies on optimization-based methods that require prior assumptions about the signals, which can be inflexible for real-world applications.
- We propose Time Graph Neural Network (**TimeGNN**) as a solution, composed of an encoder-decoder architecture with a specialized loss function.
- TimeGNN relaxes the smoothness assumption using a learning module and shows competitive performance in real datasets.

Methodology

The Time-varying Graph Signal Reconstruction (TGSR) is for a time varying graph signal $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_M]$ with $\mathbf{x}_s \in \mathbb{R}^N$ is performed by solving the following optimization problem

$$\min_{\tilde{\mathbf{X}}} \frac{1}{2}\|\mathbf{J} \circ \tilde{\mathbf{X}} - \mathbf{Y}\|_{\mathcal{F}}^2 + \frac{v}{2}\operatorname{tr}\left((\tilde{\mathbf{X}}\mathbf{D}_h)^{\top}\mathbf{L}\tilde{\mathbf{X}}\mathbf{D}_h\right), \quad (1)$$

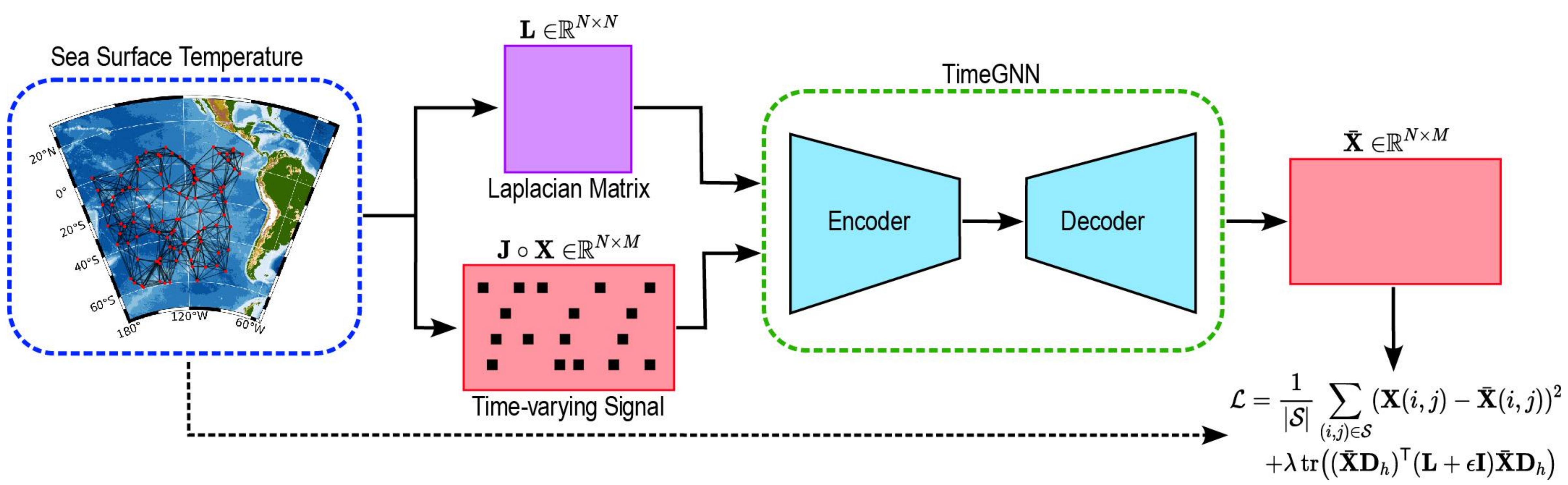
Where $\mathbf{J} \in \{0,1\}^{N\times M}$ is a sampling matrix, v is a regularization term, $\mathbf{Y}$ is the matrix of observed values, and $\mathbf{D}_h$ is a difference temporal operator yielding a temporal difference signal

$$\mathbf{X}\mathbf{D}_h = [\mathbf{x}_2 - \mathbf{x}_1, \ldots, \mathbf{x}_M - \mathbf{x}_{M-1}]$$

Equation (1) has been proven to exhibit slow convergence due to the ill-conditioned nature of the global smoothness $S_2(\tilde{\mathbf{X}}\mathbf{D}_h)$. Our algorithm addresses this issue by introducing a learnable module, which is regularized by the Sobolev smoothness term

$$\operatorname{tr}\left((\tilde{\mathbf{X}}\mathbf{D}_h)^{\top}(\mathbf{L} + \epsilon\mathbf{I})\mathbf{L}\tilde{\mathbf{X}}\mathbf{D}_h\right)$$

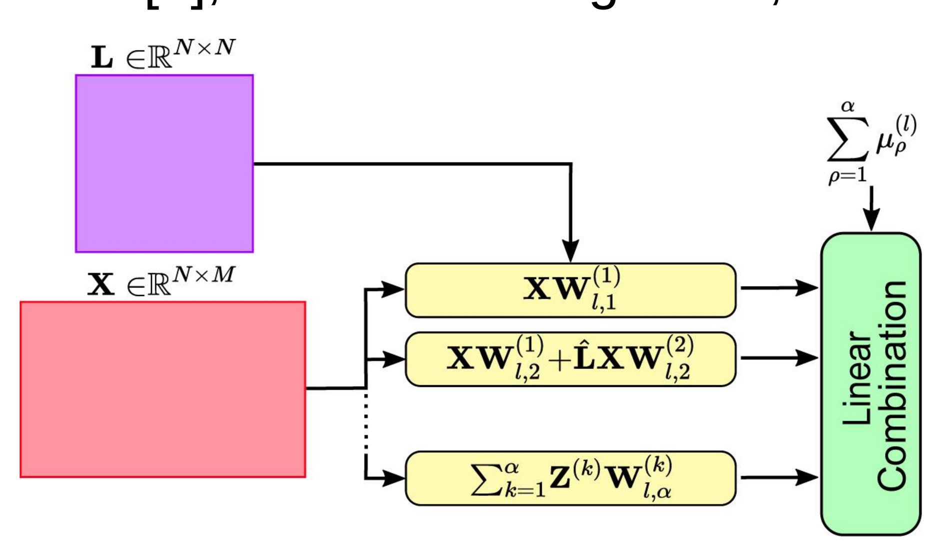
TimeGNN captures the spatio-temporal information of the data utilizing an encoding process to transform the term $(\mathbf{J} \circ \mathbf{X})\mathbf{D}_h$ into an H-dimensional latent vector, which is then decoded with the final layer. The pipeline of our architecture is as follows



We utilize a spectral filtering operation and introduce a novel convolutional layer that consists of two components: 1) a series of Chebyshev graph filters [1], with increasing order, and 2) a linear combination layer.

$$\mathrm{H}^{(l+1)} = \sum_{p=1}^{\alpha} \mu_{\rho}^{(l)} \sum_{k=1}^{\rho} \mathbf{Z}^{(k)}\mathbf{W}_{l,p}^{(k)}$$

Propagation rule of each layer of TimeGNN



Performance of TimeGNN using real-world time-varying data

We compared TimeGNN with several existing methods:

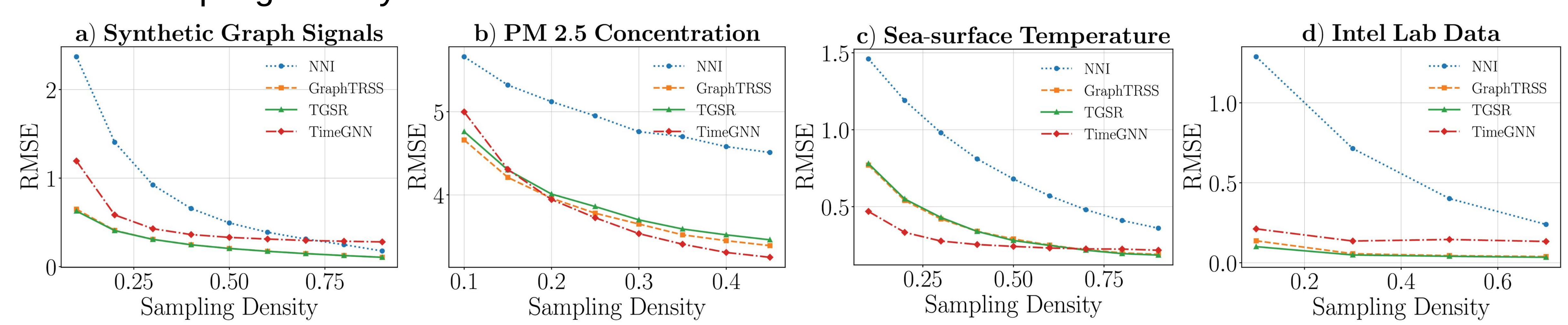
- Graph Convolutional Networks (GCN) [2]
- Natural Neighbor Interpolation (NNI) [3]
- Time-varying Graph signal Reconstruction via Sobolev Smoothness (GraphTRSS) [4]
- Conventional TGSR [5]

We used four distinct time-varying datasets for comparison:

a) Synthetic graph signals
b) PM 2.5 concentration
c) Sea-surface temperature
d) Intel Lab data.

TimeGNN shows competitive performance against SOTA methods

After constructing the graphs for each dataset using k-NN with the node's coordinate locations and following a random sampling strategy with varying densities, we performed hyperparameter optimization. We then evaluated all the methods using Monte Carlo cross-validation with 50 repetitions for each sampling density m.



Method	Synthetic Graph Signals			PM2.5 Concentration			Sea-surface Temperature			Intel Lab Data		
	RMSE	MAE	MAPE	RMSE	MAE	MAPE	RMSE	MAE	MAPE	RMSE	MAE	MAPE
GCN (Kipf and Welling [2])	11.296	8.446	1.123	4.657	2.959	0.550	3.766	2.922	0.548	2.998	2.327	0.120
NNI (Kiani et al. [3])	0.775	0.436	0.255	4.944	2.956	0.593	0.772	0.561	0.067	0.661	0.291	0.015
GraphTRSS (Giraldo et al. [4])	**0.260**	0.256	0.178	3.824	2.204	0.377	0.357	0.260	0.029	**0.056**	**0.023**	**0.037**
TGSR (Qiu et al. [5])	0.263	**0.193**	**0.144**	3.898	2.279	0.394	0.360	0.263	0.030	0.069	0.037	0.002
TimeGNN (ours)	0.452	0.323	0.226	**3.809**	**2.172**	**0.362**	**0.275**	**0.203**	**0.023**	0.156	0.095	0.005

The best and second-best performing methods on each dataset are shown in **red** and blue, respectively.

Conclusions

- ✓ We introduced a GNN architecture for the recovery of time-varying graph signals.
- ✓ We proposed a new convolutional layer composed of a cascade of Chebyshev graph filters.
- ✓ Our framework showed competitive performance against several approaches for reconstructing graph signals.

References

[1] M. Defferrard, X. Bresson, and P. Vandergheynst, "Convolutional neural networks on graphs with fast localized spectral filtering," in NeurIPS, 2016.

[2] T. N. Kipf and M.Welling, "Semi-supervised classification with graph convolutional networks," in ICLR, 2017.

[3] K. Kiani and K. Saleem, "K-nearest temperature trends: A method for weather temperature data imputation," in ICISDM, 2017.

[4] J. H. Giraldo et al., "Reconstruction of time-varying graph signals via Sobolev smoothness," IEEE T-SIPN, vol. 8, pp. 201–214, 2022.

[5] K. Qiu et al., "Time-varying graph signal reconstruction," IEEE Journal of Selected Topics in Signal Processing, vol. 11, no. 6, pp. 870–883, 2017.

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