
Salt Reconstruction in Full-Waveform Inversion

A parametric level-set approach

Ajinkya Kadu
Utrecht University, Netherlands

in Collaboration with:

Tristan van Leeuwen (UU) and Wim Mulder (Shell/TUD)

SIAM GS-17, Erlangen, Germany

September 14, 2017

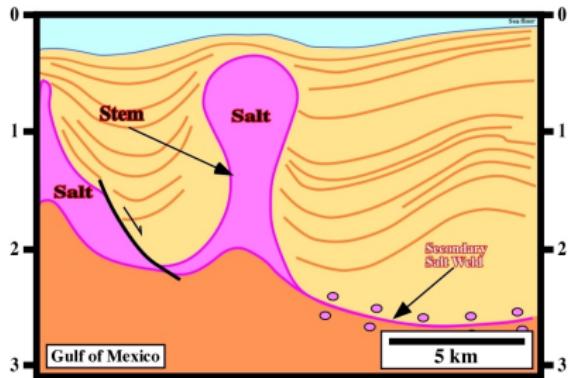


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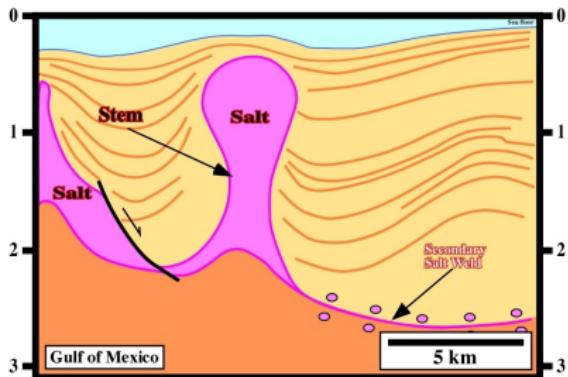
Motivation



	velocity [km/s]	density [g/cm ³]
water	1.5	1
layers	1.8-3.5	1.5-2
Salt	4.6	2.1-2.2
Anhydrite	4-4.2	2.9-3
Basalt	4.5	2.7-3.1

salt-bodies are important for hydrocarbon exploration!

Motivation

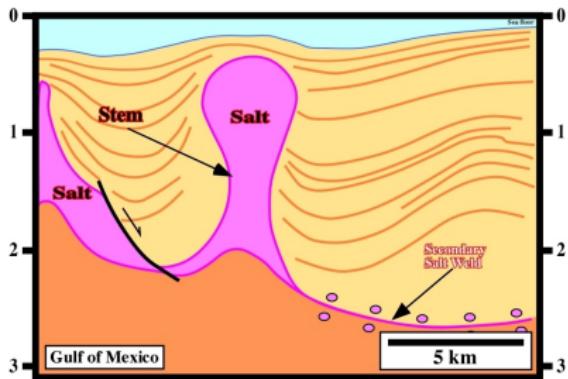


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salt-bodies are important for hydrocarbon exploration!

- ▶ cost of drilling **2 to 12M USD** per well [DoE, 2016]
- ▶ significant **dry hole** costs (30% upwards failure chance)

Outline

Seismic Inversion

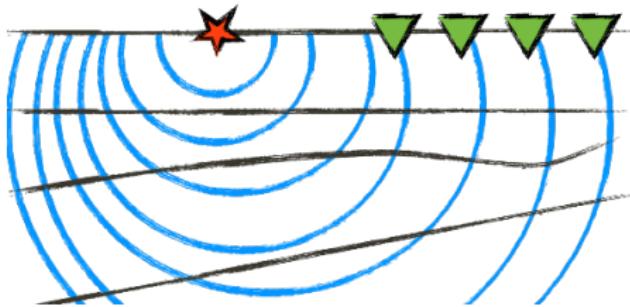
Level-Set Method

Joint Reconstruction Approach

Results & Conclusions

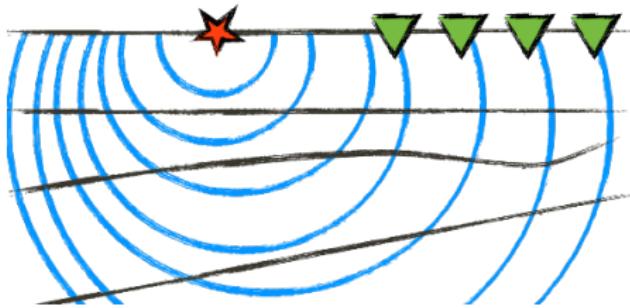
Seismic experiment

A controlled explosion (dynamite or airgun)



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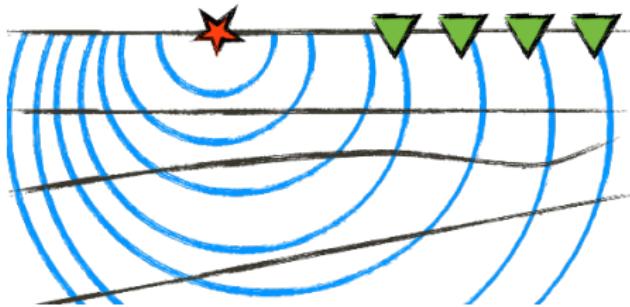


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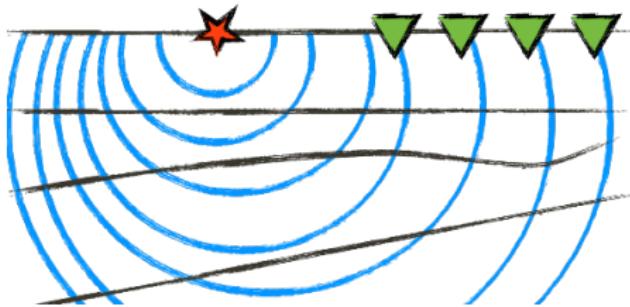
$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u + q$$

- Frequency space - a scalar Helmholtz equation:

$$\underbrace{(\omega^2 m(\mathbf{x}) + \nabla^2)}_{A(\mathbf{m})} u(\omega, \mathbf{x}) = q(\omega, \mathbf{x})$$

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Full waveform inversion

$$\begin{aligned} & \text{minimize} && \frac{1}{2} \|P\mathbf{u} - \mathbf{d}\|_2^2 \\ & \text{subject to} && A(\mathbf{m})\mathbf{u} = \mathbf{q} \end{aligned}$$

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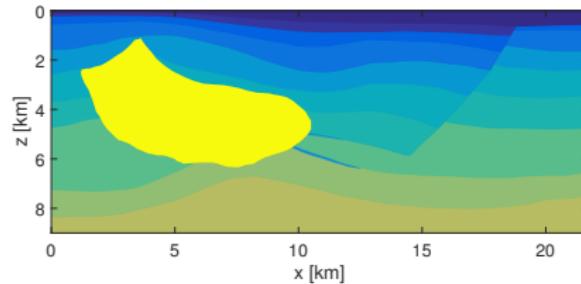
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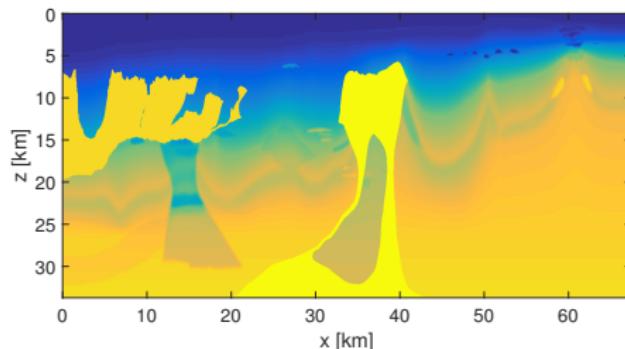
$$\mathbf{m}^{(k+1)} = \mathbf{m}^{(k)} - \gamma_k H_k^{-1} \nabla f(\mathbf{m}^{(k)})$$

- ▶ ill-posed problem, no unique solution, heavily depends on \mathbf{m}^0
[Santosa and Symes, 1989, Bunks et al., 1995,
van Leeuwen and Herrmann, 2015, Bharadwaj et al., 2016]

Earth models



2D Hess VTI Model



BP 2004 Model

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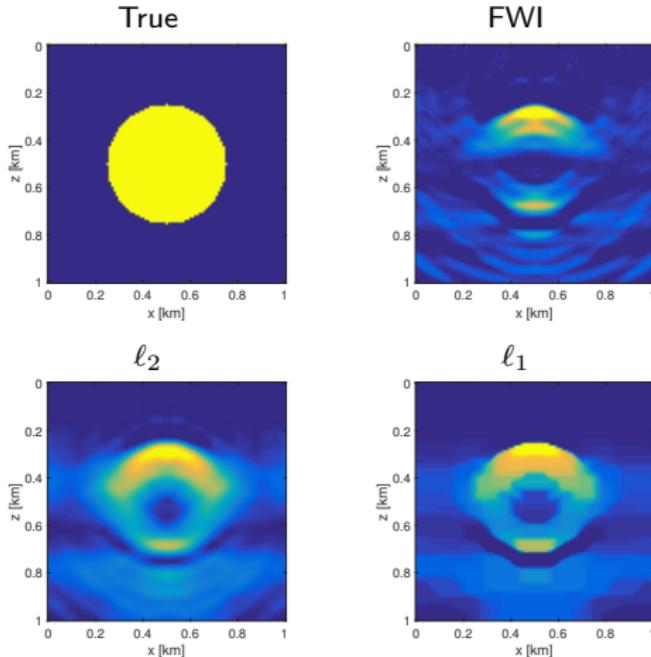
Seismic Inversion

- Total-variation(ℓ_1) [Rudin et al., 1992]

$$g(\mathbf{m}) = \|\nabla \mathbf{m}\|_1$$

An example

- salt($v=4000\text{m/s}$) in a constant sediment($v=2500\text{m/s}$) - Camembert model
- Data generated for [5, 8, 10, 15] Hz, Gaussian noise with 50 dB SNR



Outline

Seismic Inversion

Level-Set Method

Joint Reconstruction Approach

Results & Conclusions

Level-Set Method

Classical approach

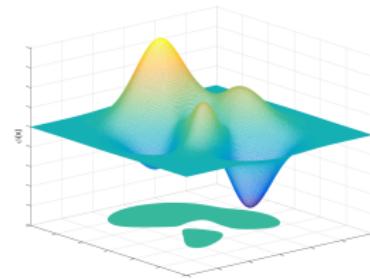
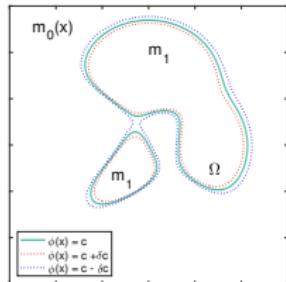
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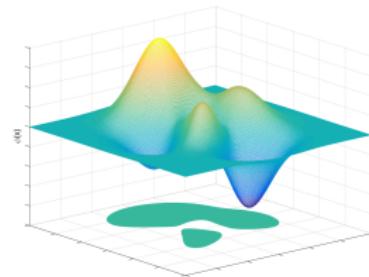
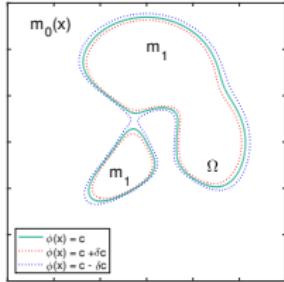
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- ▶ Hamilton-Jacobi equation [Osher and Sethian, 1988]

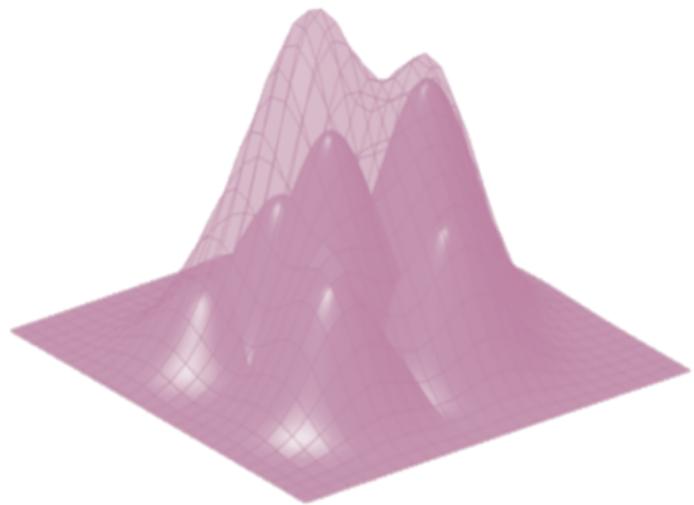
$$\dot{\phi} = v |\nabla \phi|$$

choice for v : gradient - ∇f [Burger, 2001, Dorn and Lesselier, 2006]

Parametric level-set method

- ▶ Represent level-set in lower-dimensional space [Aghasi et al., 2011]:

$$\phi(\mathbf{x}) = \sum_{i=1}^m \alpha_i \Psi(\|\mathbf{x} - \boldsymbol{\chi}_i\|)$$



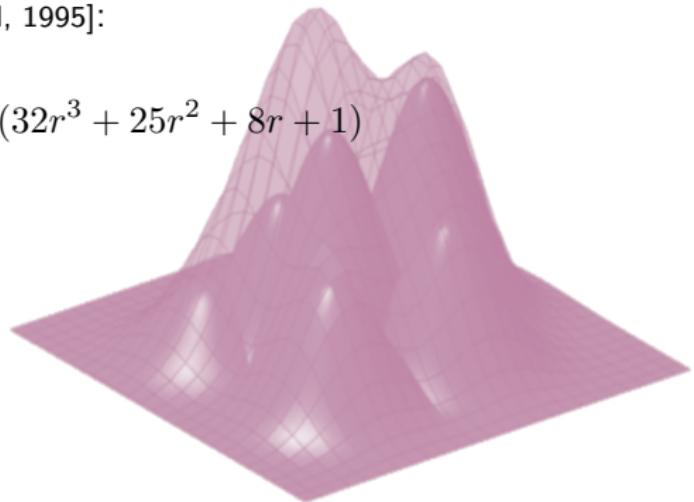
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- ▶ Open Question: Is this problem well-posed?

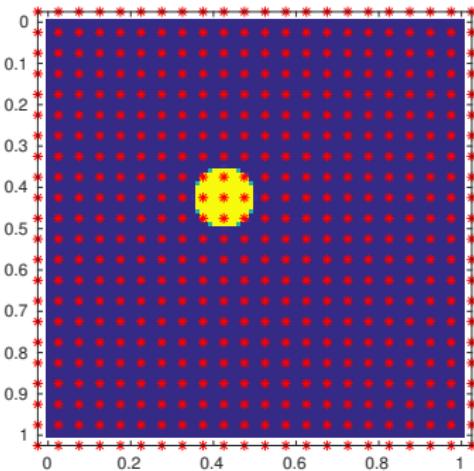
Revisiting an example

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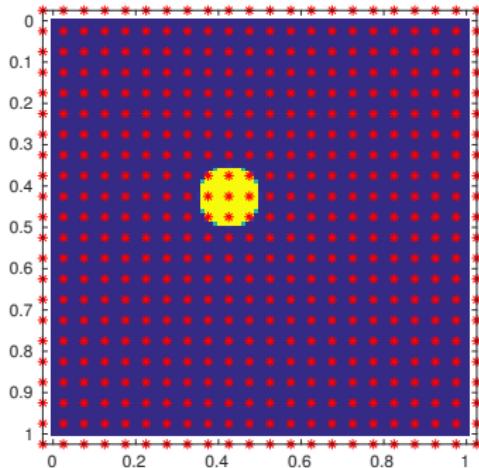
RBF placements



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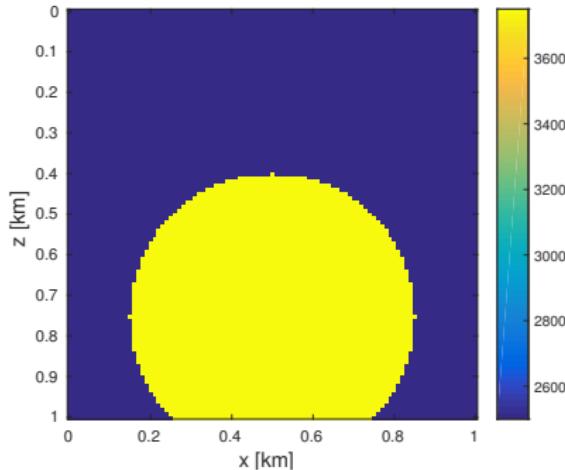
RBF placements



Level-set Evolution

A more challenging example

Shifted camembert model (Resolution: 101×101 pixels)



- ▶ data generated for $[5, 6, 7, 8]$ Hz. (Gaussian noise of 50 dB SNR)
- ▶ RBF grid: 7×7 (dimensions reduced by ≈ 200 times)

A more challenging example

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An inverse problem

$$\begin{aligned} & \text{minimize}_{\boldsymbol{\alpha}, \mathbf{m}_0} \quad \frac{1}{2} \|F(B(\boldsymbol{\alpha}, \mathbf{m}_0)) - \mathbf{d}\|_2^2 \\ & \text{subject to} \quad l \leq \mathbf{m}_0 \leq u \end{aligned}$$

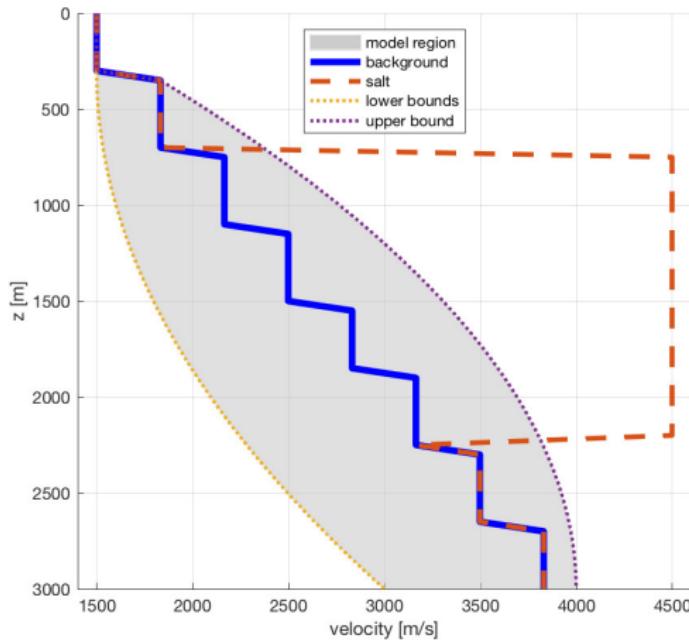
where, $B(\boldsymbol{\alpha}, \mathbf{m}_0) = \mathbf{m}_0 \odot (\mathbf{1} - h_\epsilon(A\boldsymbol{\alpha})) + m_1 h_\epsilon(A\boldsymbol{\alpha})$

Simultaneous updates

Gauss-Newton updates:

$$\begin{bmatrix} \boldsymbol{\alpha}^{k+1} \\ \mathbf{m}_0^{k+1} \end{bmatrix} \leftarrow \begin{bmatrix} \boldsymbol{\alpha}^k \\ \mathbf{m}_0^k \end{bmatrix} - \gamma_k \begin{bmatrix} \tilde{H}_{\boldsymbol{\alpha}} & 0 \\ 0 & \tilde{H}_{\mathbf{m}_0} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{g}_{\boldsymbol{\alpha}} \\ \mathbf{g}_{\mathbf{m}_0} \end{bmatrix}$$

constraining sediment



box constraints are handled using proximal operator [Parikh et al., 2014]

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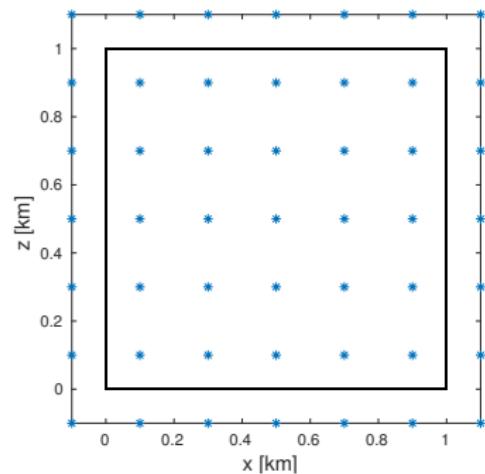
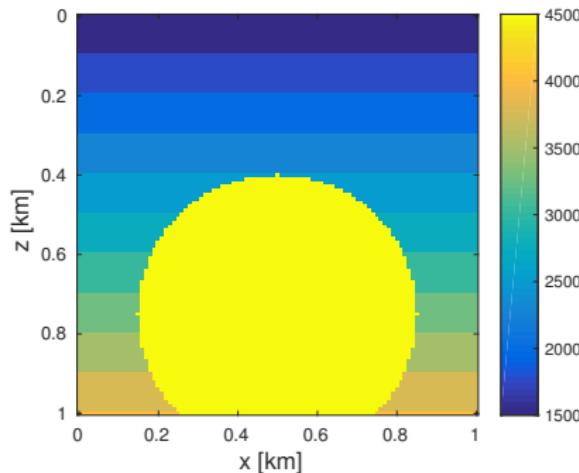
Joint Reconstruction Approach

Results & Conclusions

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18

Experiment details



Experiment details

- ▶ shifted Camembert (101×101 pixels) model with sediment layers
- ▶ Reflection Expt: 6 sources (200m apart) and 21 receivers (50m apart)
- ▶ frequencies: [3, 4.67, 6.33, 8] Hz
- ▶ surface multiples off! absorbing boundary conditions (all four sides)
- ▶ Source: Ricker wavelet with peak freq of 10 Hz and 0 phase shift

- ▶ No **inverse crime**! Data generated using different code.
- ▶ Source Estimation at every iterate (only scalar).

- ▶ RBF grid: 7×7

- ▶ WAVEFORM toolbox from SLIM [Da Silva and Herrmann, 2016].

Preliminary results

Concluding remarks

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Future Roadmap:

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- ▶ testing the framework on **real 2D marine data**
(data provided by Shell/NAM)

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Thanks!

Read:

- ▶ A. Kadu, T. van Leeuwen and W. A. Mulder Salt Reconstruction in Full-Waveform Inversion With a Parametric Level-Set Method. *IEEE Transactions on Computational Imaging* 3.2 (2017): 305-315.
- ▶ A. Kadu, T. van Leeuwen and W. A. Mulder Parametric level-set full-waveform inversion in the presence of salt bodies. In *SEG Technical Program Expanded Abstracts* 2017 (pp. 1518-1522).

Email:

- ▶ a.a.kadu@uu.nl

Web:

- ▶ <https://ajinkyakadu125.github.io>
- ▶ <https://github.com/ajinkyakadu125/>

Special Thanks to:

Eldad Haber, Felix Herrmann, Bas Peters, Rajiv Kumar and SLIM group at UBC