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Indian Institute of Technology Kanpur
CS637 Embedded and Cyber-Physical Systems
Homework Assignment 1
Deadline: September 27, 2020

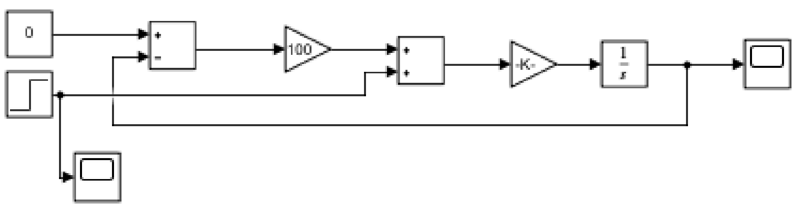
Instructions: *Total:* **40 marks**

1. This question paper contains a total of 9 pages (9 sides of paper). Please verify.
2. Write your name, roll number, department, section on **every side of every sheet** of this booklet
3. Write final answers **neatly** in the given boxes.

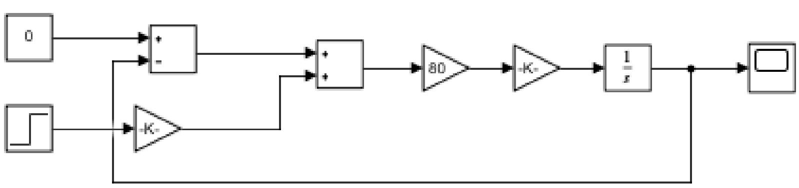
Problem 1. (20 points) Problem 7 in the Exercises of Chapter 2 in [LS15].
[LS15] Edward A. Lee and Sanjit A. Seshia, Introduction to Embedded Systems, A Cyber-Physical Systems Approach, Second Edition, http://LeeSeshia.org, ISBN 978-1-312-42740-2, 2015.

Answer a)

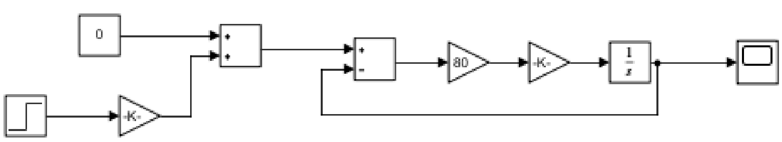
Below is the model of helicopter model with a control system with the following parameters:
Desired angular velocity: ψ(t) = 0
Moment of Inertia: $I_{yy} = 100$
Gain parameter: K = (1, 10, 50, 100)
Top-rotor torque: T_t(t) = bu(t); (used step signal of step size 5)



Helicopter System



Equivalent Helicopter System



Equivalent Helicopter System

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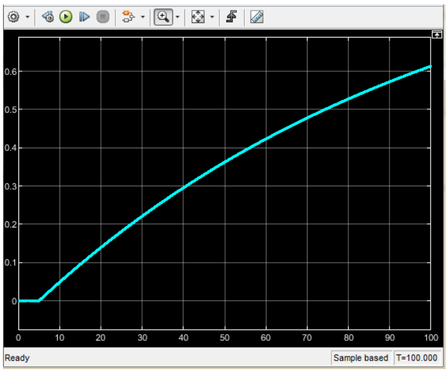
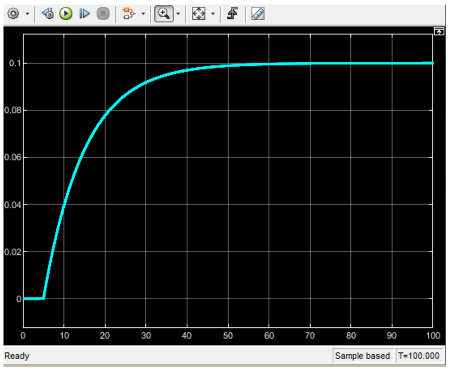
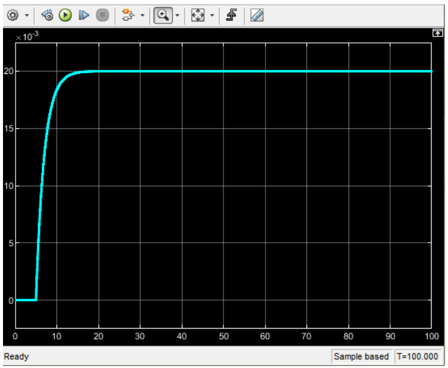


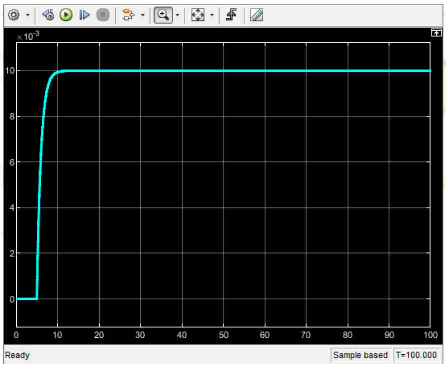
K = 1: Unstable System



K = 10



K = 50



K = 100

Explanation:

At K = 1, the system is unstable as the desired angular velocity is undoubtedly increasing with increase in time. At K = 10, the system attains close to the desired angular velocity 0.1 after 60 time units. As we further increase the values of K to 50 or 100, the desired angular velocity attained will be 0.002 and 0.001 and the system will become stable at 5-10 time units. Hence, it can be concluded that at K >= 50, the system is able to maintain a stationary state with no angular velocity.

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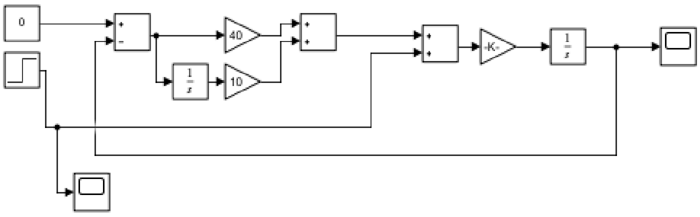
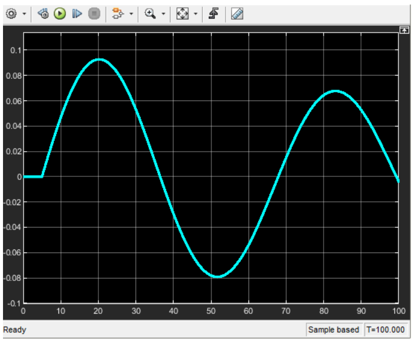
Answer b)

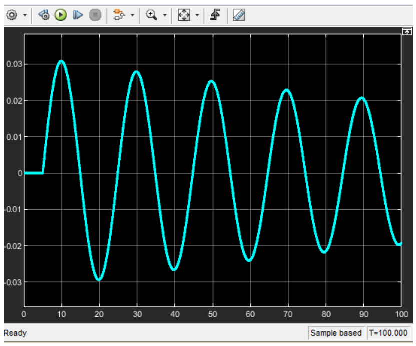
Below are the results using the propositional-integrator controller in place of simple scale by K actor. K1, K2 = {{1,1}, {1,10}, {10,1}, {10,10}, {50,10}, {50,50}, {100, 10}, {40, 10}}



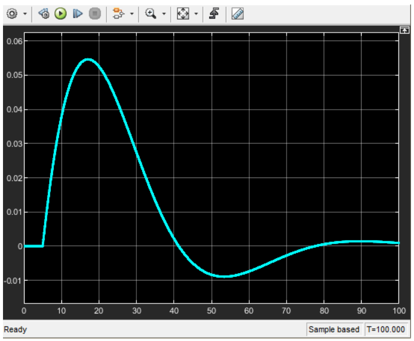
Helicopter System with PI Controller



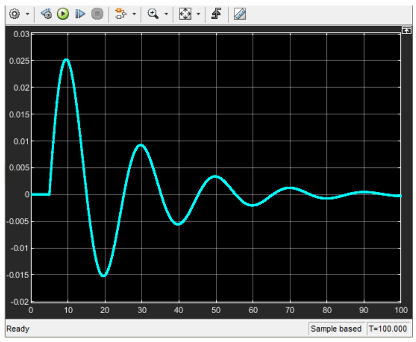
K1 = 1, K2 = 1



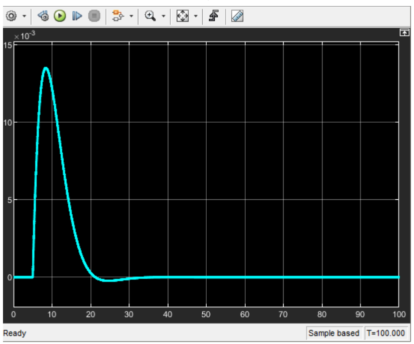
K1 = 1, K2 = 10



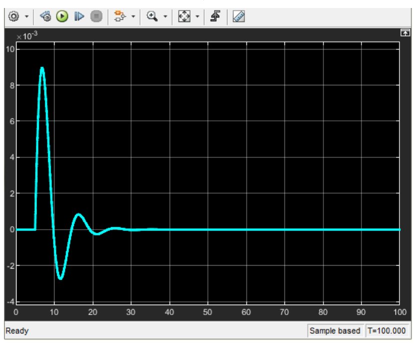
K1 = 10, K2 = 1



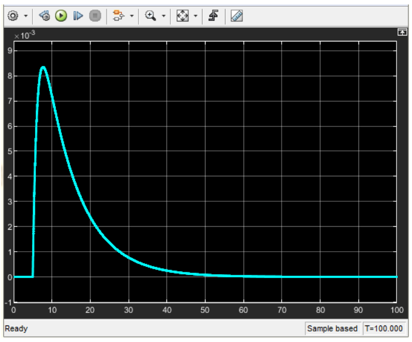
K1 = 10, K2 = 10



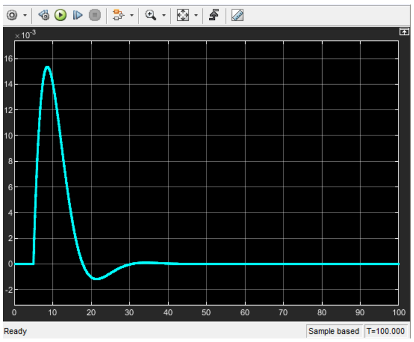
K1 = 50, K2 = 10



K1 = 50, K2 = 50



K1 = 100, K2 = 10



K1 = 40, K2 = 10

Explanation:

- **K1 = 1, K2 = 1**: Oscillations are present and the desired angular velocity is not attained
- **K1 = 1, K2 = 10**: Increase in Oscillations with increase in Integrator parameter K2 relative to K1
- **K1 = 10, K2 = 1**: No Oscillations when proportion parameter K1 is approximately 5 - 10 times the integrator parameter K2 (this is also seen in K1 = 100, 50; K2 = 10 example)
- **K1 = 10, K2 = 10**: Oscillations will be present if K1==K2 but with an increase in K1, K2 value, the desired angular velocity is attained relatively early.
- **K1 = 50, K2 = 10**: The desired angular velocity is attained very early at 20 time-steps with no oscillations in the beginning of the curve.
- **K1 = 50, K2 = 50**: One dip of oscillation is observed, hence K2 can be maintained at 10 value.
- **K1 = 100, K2 = 10**: No Oscillations at all It can be concluded that K2 = 10 will be suited to avoid oscillations.
- **K1 = 40, K2 = 10**: Tried to explore right value of K1, conclusion could be that if K1 is maintained around 50, 40, the desired velocity will be attained around 25 time units with out any oscillations.

Conclusion:

At K1 = 50, K2 = 10, the system remains to be the most stable i.e. the desired angular velocity is attained at the earliest 25 time units without any oscillations.

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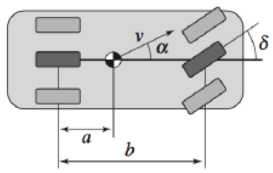
Problem 2. (20 points) The states of the linearized model of the vehicle steering system represent the lateral deviation of the vehicle from the x-axis and the angle between the vehicle axis and the x-axis. The output of the linearized model is only the first state. Construct a Simulink model for the vehicle steering system with its controller that includes an observer. The dynamics are available in Example 6.4 and Example 7.3 in [AM09]. Apply a sinusoidal signal as the reference trajectory that specifies the desired deviation of the vehicle from the x-axis with time. Plot the output (lateral deviation of the vehicle from the x-axis) with time. [AM09] K. J. Astrom and R. M. Murray. Feedback Systems: An Introduction for Scientists and Engineers. Princeton University Press, 2009. http://www.cds.caltech.edu/~murray/books/AM05/pdf/am08-complete_22Feb09.pdf.

Answer

Below diagram show the simulink implementatin of vehical steering system. The desired deviation of the vehicle from the x-axis with time is represented by a sine wave of frequency 0.5. The system dynamics can be represented by below A, B matrix and the observer can be defined by the following differential equation. The simulation w.r.t. different K, Kr values has been explained through plots (using L = [2,1]).



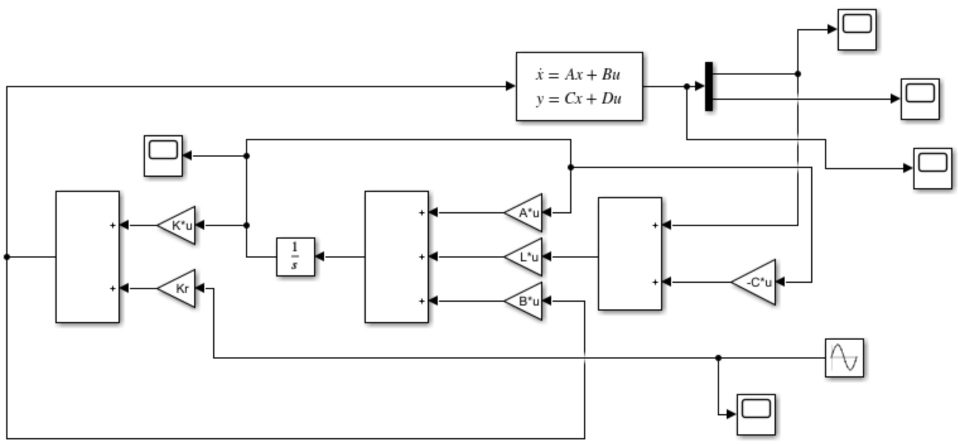
Vehicle Dynamics

$$\frac{d\hat{x}}{dt} = A\hat{x} + Bu + L(y - C\hat{x})$$

Observer

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \; B = \begin{bmatrix} \gamma \\ 1 \end{bmatrix}, C = [1 \;\; 0], \; D = 0$$

Vehicle Steering System: $\gamma = a/b$; γ is taken as 0.4 for this example



Vehical Steering system with an observer

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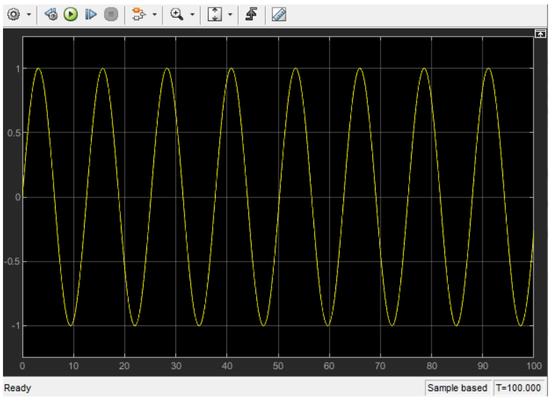
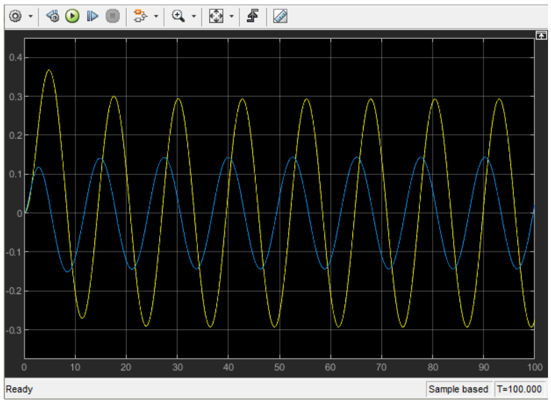
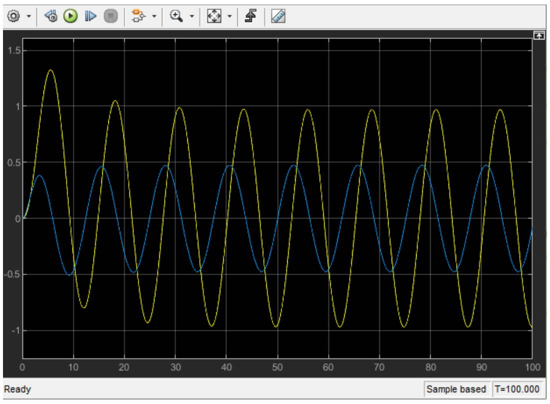


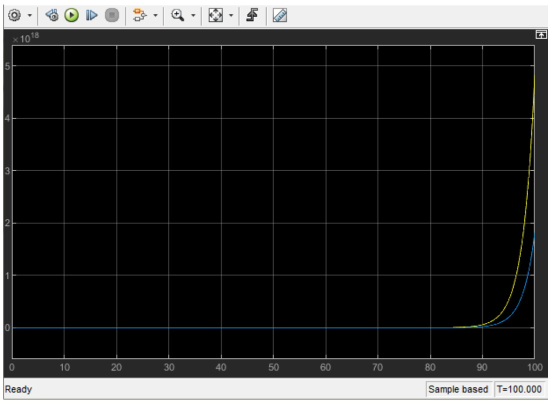
Input Signal: Sine wave with frequency 0.5



State of the system for K, Kr = [0.25, 1.3, 0.25]



State of the System for K, Kr = [0.1, 0.85, 0.5]



State of the System: Unstable when eigen values of A matrix is positive

Observations:

- **Plot 1:** Reference signal given to model the deviation of vehicle from the x-axis
- **Plot 2:** The state of the system at K, Kr = [0.25, 1.3, 0.25]; it can be observed that at these parameter values, system is not able to attain value 1, the range of sine wave is [0.3, -0.3]
- **Plot 3:** The state of the system at K, Kr = [0.1, 0.85, 0.5]; it can be observed that sine wave range is stabilised to [+1, -1] after 1st cycle and it is similar to reference single as shown in plot 1.
- **Plot 4:** The state of the system is not stable. This plot is generated based on K value that resulted in positive eigen values for A matrix (using place function).