

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

HyperArt – Generating Hyperbolic patterns for Regular and Non-Regular p-gons

Ajit Datar

Department of Computer Science
University of Minnesota, Duluth

Graduate Thesis Colloquium, 2005

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

A HyperArt rendition of Circle Limit III

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

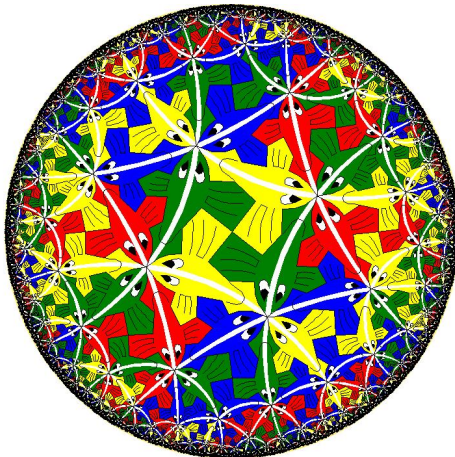
Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary



Outline

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

- 1 Motivation
 - Hyperbolic Patterns eh?
- 2 Theory
 - Lets Learn Geometry
 - Definitions Galore
- 3 Algorithms
 - Concepts
 - Regular tessellations
 - Non-regular tessellations
- 4 HyperArt
 - Features
 - Design

Not *Just* Pretty Pictures

- Tessellation is a covering of the plane with (symmetrical) patterns.
- Types of tessellations :
 - Euclidean – Penrose tiling.
 - Spherical – Temari balls
 - Hyperbolic – Escher's Circle Limit Patterns.
- Challenging and aesthetically rewarding!



HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

From Desk to Desktop

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

- First hyperbolic patterns - M. C. Escher – *by hand!*.
- H. S. M. Coxeter – well known geometer – non-Euclidean geometry
- Dr. Douglas Dunham – computer algorithms and programs.

Goals

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

- Implement and refine *Regular* and *Non-regular* hyperbolic tessellation algorithms.
- To provide a unifying extensible programming framework for these algorithms.

Absolute Geometry

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry

Definitions Galore

Algorithms

Concepts

Regular tessellations

Non-regular
tessellations

HyperArt

Features

Design

Summary

- A straight line segment can be drawn joining any two points.
- Any straight line segment can be extended indefinitely in a straight line.
- Given any straight line segment, a circle can be drawn having the segment as radius and one endpoint as center.
- All right angle are congruent.

Absolute Geometry

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry

Definitions Galore

Algorithms

Concepts

Regular tessellations

Non-regular
tessellations

HyperArt

Features

Design

Summary

- A straight line segment can be drawn joining any two points.
- Any straight line segment can be extended indefinitely in a straight line.
- Given any straight line segment, a circle can be drawn having the segment as radius and one endpoint as center.
- All right angle are congruent.

Absolute Geometry

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry

Definitions Galore

Algorithms

Concepts

Regular tessellations

Non-regular
tessellations

HyperArt

Features

Design

Summary

- A straight line segment can be drawn joining any two points.
- Any straight line segment can be extended indefinitely in a straight line.
- Given any straight line segment, a circle can be drawn having the segment as radius and one endpoint as center.
- All right angle are congruent.

Absolute Geometry

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

- A straight line segment can be drawn joining any two points.
- Any straight line segment can be extended indefinitely in a straight line.
- Given any straight line segment, a circle can be drawn having the segment as radius and one endpoint as center.
- All right angle are congruent.

Absolute Geometry

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

- A straight line segment can be drawn joining any two points.
- Any straight line segment can be extended indefinitely in a straight line.
- Given any straight line segment, a circle can be drawn having the segment as radius and one endpoint as center.
- All right angle are congruent.

The parallel postulate

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

Euclidean Parallel postulate

If two lines are drawn which intersect a third in such a way that the sum of the inner angles on one side is less than two right angles, then the two lines inevitably must intersect each other on that side if extended far enough.

Types of geometries

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

Reinterpret the Euclidean parallel postulate as ...

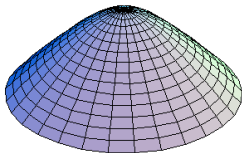
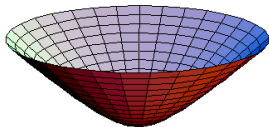
Elliptical/Spherical Parallel postulate

Through any point in the plane, there exists no lines parallel to the given line.

Hyperbolic parallel postulate

For any infinite straight line L and a point P not on it, there are many infinitely extending straight lines that pass through P and do not intersect L .

Weierstrass model



- Upper sheet of the hyperboloid of revolution $z = \sqrt{1 + x^2 + y^2}$.
- Hyperbolic lines are the arcs lying in this plane

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry

Definitions Galore

Algorithms

Concepts

Regular tessellations

Non-regular
tessellations

HyperArt

Features

Design

Summary

Poincaré model

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry

Definitions Galore

Algorithms

Concepts

Regular tessellations

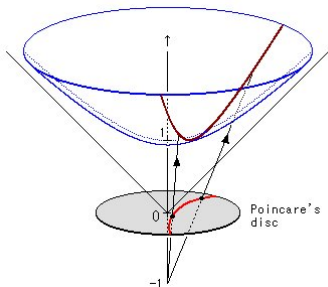
Non-regular
tessellations

HyperArt

Features

Design

Summary



- Projection of Weierstrass on a unit circle at origin towards $(0, -1, 0)$
- Hyperbolic lines are
 - arcs orthogonal to boundary
 - diameters of the disk
- “boundaryless” model

Parts of a Diagram

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

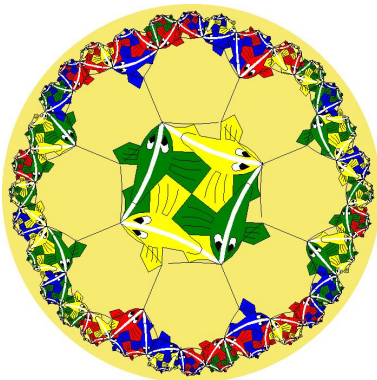
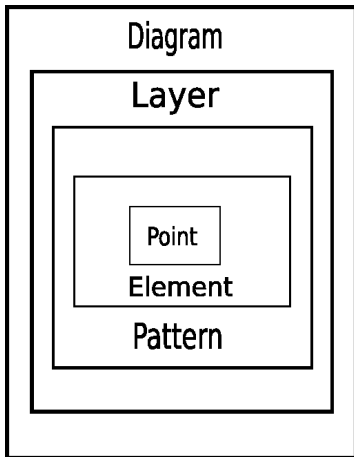
Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary



Central P-gon and Fundamental region

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

Definition

Central p-gon pattern is a pattern which remains invariant under certain transformations of the hyperbolic plane.

Definition

The fundamental region is a region in the hyperbolic plane which when transformed by all the transformations in the symmetry group, will cover the hyperbolic plane. A *Fundamental Pattern* is a pattern in the Fundamental region.

Central P-gon and Fundamental region

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

Definition

Central p-gon pattern is a pattern which remains invariant under certain transformations of the hyperbolic plane.

Definition

The fundamental region is a region in the hyperbolic plane which when transformed by all the transformations in the symmetry group, will cover the hyperbolic plane. A *Fundamental Pattern* is a pattern in the Fundamental region.

Symmetry groups

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

p is the number of sides/vertexes of the p-gon.

q is the number of p-gons meeting at a vertex.

- *Symmetry group* is a group of transformations of the hyperbolic plane that preserve a pattern.
- Infinitely many symmetry groups.
- eg Symmetry groups of Regular tessellations p, q
 - $[p, q] - 3$ reflections
 - $[p, q] + - 3$ rotations
 - $[p+, q] - p$ rotations about origin and a reflection across p-gon edge
 - $[p, q+] - q$ rotations about vertex and a reflection across p-gon edge

Symmetry groups

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

p is the number of sides/vertexes of the p-gon.

q is the number of p-gons meeting at a vertex.

- *Symmetry group* is a group of transformations of the hyperbolic plane that preserve a pattern.
- Infinitely many symmetry groups.
- eg Symmetry groups of Regular tessellations p, q
 - $[p, q] -$ 3 reflections
 - $[p, q] + -$ 3 rotations
 - $[p+, q] -$ p rotations about origin and a reflection across p-gon edge
 - $[p, q+] -$ q rotations about vertex and a reflection across p-gon edge

Symmetry groups

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

p is the number of sides/vertexes of the p-gon.

q is the number of p-gons meeting at a vertex.

- *Symmetry group* is a group of transformations of the hyperbolic plane that preserve a pattern.
- Infinitely many symmetry groups.
- eg Symmetry groups of Regular tessellations p, q
 - $[p, q] -$ 3 reflections
 - $[p, q] +$ - 3 rotations
 - $[p+, q] -$ p rotations about origin and a reflection across p-gon edge
 - $[p, q+] -$ q rotations about vertex and a reflection across p-gon edge

Classification of Algorithms

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts

Regular tessellations

Non-regular
tessellations

HyperArt

Features

Design

Summary

- According to p-gon type
 - Regular p-gon algorithms
 - Non-regular p-gon algorithms
- According to replication order
 - Hamiltonian methods.
 - Spanning-tree methods
- According to *central* p-gon
 - P-gon center at the origin
 - P-gon vertex at the origin

Classification of Algorithms

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts

Regular tessellations

Non-regular
tessellations

HyperArt

Features

Design

Summary

- According to p-gon type
 - Regular p-gon algorithms
 - Non-regular p-gon algorithms
- According to replication order
 - Hamiltonian methods.
 - Spanning-tree methods
- According to *central* p-gon
 - P-gon center at the origin
 - P-gon vertex at the origin

Classification of Algorithms

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts

Regular tessellations

Non-regular
tessellations

HyperArt

Features

Design

Summary

- According to p-gon type
 - Regular p-gon algorithms
 - Non-regular p-gon algorithms
- According to replication order
 - Hamiltonian methods.
 - Spanning-tree methods
- According to *central* p-gon
 - P-gon center at the origin
 - P-gon vertex at the origin

Classification of Algorithms

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts

Regular tessellations

Non-regular
tessellations

HyperArt

Features

Design

Summary

- According to p-gon type
 - Regular p-gon algorithms
 - Non-regular p-gon algorithms
- According to replication order
 - Hamiltonian methods.
 - Spanning-tree methods
- According to *central* p-gon
 - P-gon center at the origin
 - P-gon vertex at the origin

Edge Adjacency and Orientation

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

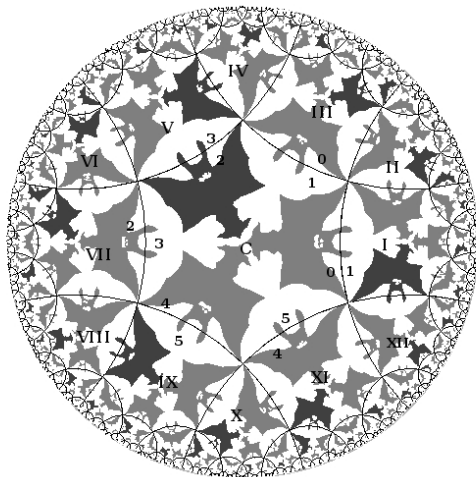
Concepts

Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary



Exposure

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

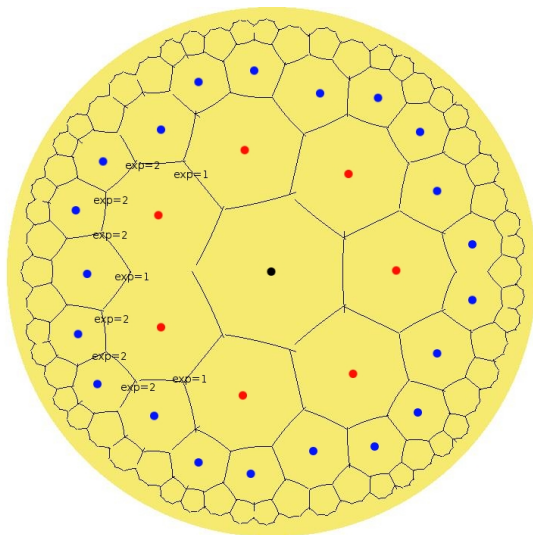
Concepts

Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary



Exposure of a
vertex in the
 k^{th} layer : the
number of
p-gons in the
 $(k + 1)^{st}$ layer
sharing that
vertex.

Generating the central p-gon

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

- Needed for Regular p-gon algorithms.
- Fundamental pattern needs to be replicated to fill up central p-gon
- which is then replicated to fill up the disk
- Transformations used :
 - Reflection across p-gon radius
 - Reflection across edge bisector
 - Rotation around p-gon center

Generating the central p-gon

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

- Needed for Regular p-gon algorithms.
- Fundamental pattern needs to be replicated to fill up central p-gon
- which is then replicated to fill up the disk
- Transformations used :
 - Reflection across p-gon radius
 - Reflection across edge bisector
 - Rotation around p-gon center

Generating the central p-gon

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

- Needed for Regular p-gon algorithms.
- Fundamental pattern needs to be replicated to fill up central p-gon
- which is then replicated to fill up the disk
- Transformations used :
 - Reflection across p-gon radius
 - Reflection across edge bisector
 - Rotation around p-gon center

Generating the central p-gon

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

- Needed for Regular p-gon algorithms.
- Fundamental pattern needs to be replicated to fill up central p-gon
- which is then replicated to fill up the disk
- Transformations used :
 - Reflection across p-gon radius
 - Reflection across edge bisector
 - Rotation around p-gon center

Generating the central p-gon

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

- Needed for Regular p-gon algorithms.
- Fundamental pattern needs to be replicated to fill up central p-gon
- which is then replicated to fill up the disk
- Transformations used :
 - Reflection across p-gon radius
 - Reflection across edge bisector
 - Rotation around p-gon center

Generating the central p-gon

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

- Needed for Regular p-gon algorithms.
- Fundamental pattern needs to be replicated to fill up central p-gon
- which is then replicated to fill up the disk
- Transformations used :
 - Reflection across p-gon radius
 - Reflection across edge bisector
 - Rotation around p-gon center

Generating the central p-gon

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

- Needed for Regular p-gon algorithms.
- Fundamental pattern needs to be replicated to fill up central p-gon
- which is then replicated to fill up the disk
- Transformations used :
 - Reflection across p-gon radius
 - Reflection across edge bisector
 - Rotation around p-gon center

Regular P-gon Pattern Replication

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts

Regular tessellations

Non-regular
tessellations

HyperArt

Features
Design

Summary

- Spanning tree method of replication.
- The idea is to
 - Start with the central p-gon
 - Visit each shared vertex.
 - And **recursively** process the **optimal** number of p-gons around it.

Lets look at an example in HyperArt

Regular P-gon Pattern Replication

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts

Regular tessellations

Non-regular
tessellations

HyperArt

Features
Design

Summary

- Spanning tree method of replication.
- The idea is to
 - Start with the central p-gon
 - Visit each shared vertex.
 - And **recursively** process the **optimal** number of p-gons around it.

Lets look at an example in HyperArt

Regular P-gon Pattern Replication

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts

Regular tessellations

Non-regular
tessellations

HyperArt

Features
Design

Summary

- Spanning tree method of replication.
- The idea is to
 - Start with the central p-gon
 - Visit each shared vertex.
 - And **recursively** process the **optimal** number of p-gons around it.

Lets look at an example in HyperArt

Regular P-gon Pattern Replication

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts

Regular tessellations

Non-regular
tessellations

HyperArt

Features
Design

Summary

- Spanning tree method of replication.
- The idea is to
 - Start with the central p-gon
 - Visit each shared vertex.
 - And **recursively** process the **optimal** number of p-gons around it.

Lets look at an example in HyperArt

Regular P-gon Pattern Replication

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts

Regular tessellations

Non-regular
tessellations

HyperArt

Features
Design

Summary

- Spanning tree method of replication.
- The idea is to
 - Start with the central p-gon
 - Visit each shared vertex.
 - And **recursively** process the **optimal** number of p-gons around it.

Lets look at an example in HyperArt

Regular P-gon Pattern Replication

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts

Regular tessellations

Non-regular
tessellations

HyperArt

Features
Design

Summary

- Spanning tree method of replication.
- The idea is to
 - Start with the central p-gon
 - Visit each shared vertex.
 - And **recursively** process the **optimal** number of p-gons around it.

Lets look at an example in HyperArt

Non-Regular P-gon Pattern Replication

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations

Non-regular
tessellations

HyperArt

Features
Design

Summary

- P-gon *is* the Fundamental region.
- But now we have different number of p-gons meeting at each vertex.
- Fundamental region moved to put vertex at center.
- \therefore the concept of layers is slightly changed.
- Except for some implementation differences, replication is then same as the Regular P-gon case.

Lets look at an example in HyperArt

Non-Regular P-gon Pattern Replication

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations

Non-regular
tessellations

HyperArt

Features
Design

Summary

- P-gon *is* the Fundamental region.
- But now we have different number of p-gons meeting at each vertex.
- Fundamental region moved to put vertex at center.
- \therefore the concept of layers is slightly changed.
- Except for some implementation differences, replication is then same as the Regular P-gon case.

Lets look at an example in HyperArt

Non-Regular P-gon Pattern Replication

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations

Non-regular
tessellations

HyperArt

Features
Design

Summary

- P-gon *is* the Fundamental region.
- But now we have different number of p-gons meeting at each vertex.
- Fundamental region moved to put vertex at center.
- \therefore the concept of layers is slightly changed.
- Except for some implementation differences, replication is then same as the Regular P-gon case.

Lets look at an example in HyperArt

Non-Regular P-gon Pattern Replication

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations

Non-regular
tessellations

HyperArt

Features
Design

Summary

- P-gon *is* the Fundamental region.
- But now we have different number of p-gons meeting at each vertex.
- Fundamental region moved to put vertex at center.
- \therefore the concept of layers is slightly changed.
- Except for some implementation differences, replication is then same as the Regular P-gon case.

Lets look at an example in HyperArt

Non-Regular P-gon Pattern Replication

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations

Non-regular
tessellations

HyperArt

Features
Design

Summary

- P-gon *is* the Fundamental region.
- But now we have different number of p-gons meeting at each vertex.
- Fundamental region moved to put vertex at center.
- \therefore the concept of layers is slightly changed.
- Except for some implementation differences, replication is then same as the Regular P-gon case.

Lets look at an example in HyperArt

Non-Regular P-gon Pattern Replication

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

- P-gon *is* the Fundamental region.
- But now we have different number of p-gons meeting at each vertex.
- Fundamental region moved to put vertex at center.
- \therefore the concept of layers is slightly changed.
- Except for some implementation differences, replication is then same as the Regular P-gon case.

Lets look at an example in HyperArt

Non-Regular P-gon Pattern Replication

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations

Non-regular
tessellations

HyperArt

Features
Design

Summary

- P-gon *is* the Fundamental region.
- But now we have different number of p-gons meeting at each vertex.
- Fundamental region moved to put vertex at center.
- \therefore the concept of layers is slightly changed.
- Except for some implementation differences, replication is then same as the Regular P-gon case.

Lets look at an example in HyperArt

HyperArt Features From User's Perspective

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

- XML data file format
- Crossplatform – Written in Qt from trolltech
- Single unified interface for different types of algorithms
- Layer and Frame toggling
- Algorithm animation and stepping
- Modern UI with zooming, panning and printing support.
- Diagram export to popular image formats.

HyperArt Features From User's Perspective

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

- XML data file format
- Crossplatform – Written in Qt from trolltech
- Single unified interface for different types of algorithms
- Layer and Frame toggling
- Algorithm animation and stepping
- Modern UI with zooming, panning and printing support.
- Diagram export to popular image formats.

HyperArt Features From User's Perspective

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

- XML data file format
- Crossplatform – Written in Qt from trolltech
- Single unified interface for different types of algorithms
- Layer and Frame toggling
- Algorithm animation and stepping
- Modern UI with zooming, panning and printing support.
- Diagram export to popular image formats.

HyperArt Features From User's Perspective

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

- XML data file format
- Crossplatform – Written in Qt from trolltech
- Single unified interface for different types of algorithms
- Layer and Frame toggling
- Algorithm animation and stepping
- Modern UI with zooming, panning and printing support.
- Diagram export to popular image formats.

HyperArt Features From User's Perspective

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

- XML data file format
- Crossplatform – Written in Qt from trolltech
- Single unified interface for different types of algorithms
- Layer and Frame toggling
- Algorithm animation and stepping
- Modern UI with zooming, panning and printing support.
- Diagram export to popular image formats.

HyperArt Features From User's Perspective

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

- XML data file format
- Crossplatform – Written in Qt from trolltech
- Single unified interface for different types of algorithms
- Layer and Frame toggling
- Algorithm animation and stepping
- Modern UI with zooming, panning and printing support.
- Diagram export to popular image formats.

HyperArt Features From User's Perspective

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

- XML data file format
- Crossplatform – Written in Qt from trolltech
- Single unified interface for different types of algorithms
- Layer and Frame toggling
- Algorithm animation and stepping
- Modern UI with zooming, panning and printing support.
- Diagram export to popular image formats.

HyperArt Features From User's Perspective

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

- XML data file format
- Crossplatform – Written in Qt from trolltech
- Single unified interface for different types of algorithms
- Layer and Frame toggling
- Algorithm animation and stepping
- Modern UI with zooming, panning and printing support.
- Diagram export to popular image formats.

HyperArt Features From Researcher's Perspective

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

- Document view architecture.
- Abstraction of Diagram and View classes.
- Importing old dat files to the new XML format.
- Element based patterns instead of pen based.
- Source-code available under GPL license on sourceforge.net.

HyperArt Features From Researcher's Perspective

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

- Document view architecture.
- Abstraction of Diagram and View classes.
- Importing old dat files to the new XML format.
- Element based patterns instead of pen based.
- Source-code available under GPL license on sourceforge.net.

HyperArt Features From Researcher's Perspective

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

- Document view architecture.
- Abstraction of Diagram and View classes.
- Importing old dat files to the new XML format.
- Element based patterns instead of pen based.
- Source-code available under GPL license on sourceforge.net.

HyperArt Features From Researcher's Perspective

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

- Document view architecture.
- Abstraction of Diagram and View classes.
- Importing old dat files to the new XML format.
- Element based patterns instead of pen based.
- Source-code available under GPL license on sourceforge.net.

HyperArt Features From Researcher's Perspective

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

- Document view architecture.
- Abstraction of Diagram and View classes.
- Importing old dat files to the new XML format.
- Element based patterns instead of pen based.
- Source-code available under GPL license on sourceforge.net.

Design of HyperArt

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

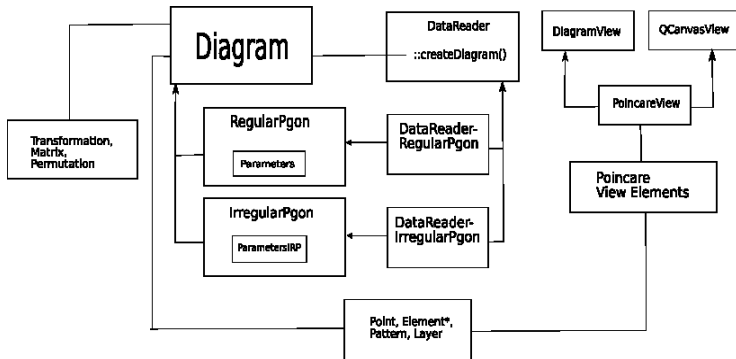


Diagram classes

- **Diagram** – abstract base class
 - **RegularPgon**
 - **IrregularPgon**
- **Parameters** – For RegularPgon
 - **ParametersIRP** – For IrregularPgon
- **Element**
 - **Circle**
 - **EuclidPoly, EuclidPolyLine**
 - **HyperPoly, HyperPolyLine**
- Other utility classes such as **Matrix, Permutation, Transformation**

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

Diagram classes

- **Diagram** – abstract base class
 - **RegularPgon**
 - **IrregularPgon**
- **Parameters** – For RegularPgon
 - **ParametersIRP** – For IrregularPgon
- **Element**
 - **Circle**
 - **EuclidPoly, EuclidPolyLine**
 - **HyperPoly, HyperPolyLine**
- Other utility classes such as **Matrix, Permutation, Transformation**

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

Diagram classes

- **Diagram** – abstract base class
 - **RegularPgon**
 - **IrregularPgon**
- **Parameters** – For RegularPgon
 - **ParametersIRP** – For IrregularPgon
- **Element**
 - **Circle**
 - **EuclidPoly, EuclidPolyLine**
 - **HyperPoly, HyperPolyLine**
- Other utility classes such as **Matrix, Permutation, Transformation**

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

Diagram classes

- **Diagram** – abstract base class
 - **RegularPgon**
 - **IrregularPgon**
- **Parameters** – For RegularPgon
 - **ParametersIRP** – For IrregularPgon
- **Element**
 - **Circle**
 - **EuclidPoly, EuclidPolyLine**
 - **HyperPoly, HyperPolyLine**
- Other utility classes such as **Matrix, Permutation, Transformation**

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

Presentation classes

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

- **DiagramView** – abstract base view class
 - **PoincareView** – derives from DiagramView and QCanvasView
- Canvas counterparts of Element classes such as **CanvasHyperPoly**

Presentation classes

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

- **DiagramView** – abstract base view class
 - **PoincareView** – derives from DiagramView and QCanvasView
- Canvas counterparts of Element classes such as **CanvasHyperPoly**

Reader classes

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

- **DataReader** – abstract base class for readers, also a Diagram Factory
 - **DataReaderRegularPgon**
 - **DataReaderIrregularPgon**

Summary

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

- Provided the unifying and extensible **HyperArt framework**
- Implemented and refined Regular and Non-Regular P-gon algorithms
- Future Work
 - Diagram Designer for HyperArt
 - Implement other algorithms.

Summary

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

- Provided the unifying and extensible **HyperArt framework**
- Implemented and refined Regular and Non-Regular P-gon algorithms
- Future Work
 - Diagram Designer for HyperArt
 - Implement other algorithms.

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Motivation

Hyperbolic Patterns
eh?

Theory

Lets Learn Geometry
Definitions Galore

Algorithms

Concepts
Regular tessellations
Non-regular
tessellations

HyperArt

Features
Design

Summary

Lets see some nice designs ...

For Further Reading I

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Appendix
For Further Reading



HSM. Coxeter

Non-Euclidean Geometry.

Mathematical Association of America, 1988, ISBN
0-88385- 522-4.



Goldennumber.net

Penrose Tiling

<http://goldennumber.net/penrose.htm>, 1997-2005.



Diana Vandervoort

Temari balls exhibition.

<http://temari.com>, 2003.

For Further Reading II

HyperArt –
Generating
Hyperbolic
patterns for
Regular and
Non-Regular
p-gons

Ajit Datar

Appendix
For Further Reading



Wikipedia

Hyperboloid of 2 sheets

<http://en.wikipedia.org/wiki/Image:HyperboloidOfTwoSheets>
2004.



D. Dunham

Creating Repeating Hyperbolic Patterns

Computer Graphics, 1981, Volume 15, Number 3.



A. Datar

Hyperart webpage.

<http://hyperart.sourceforge.net>, 2005.