

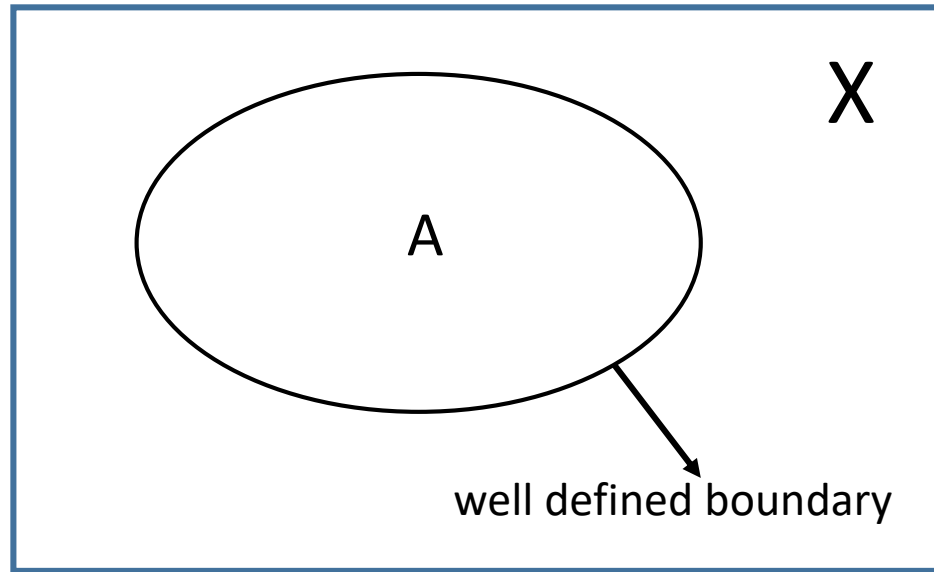
# UNIT-I

Fuzzy logic, Fuzzy Set Theory, Crisp Sets, Fuzzy sets

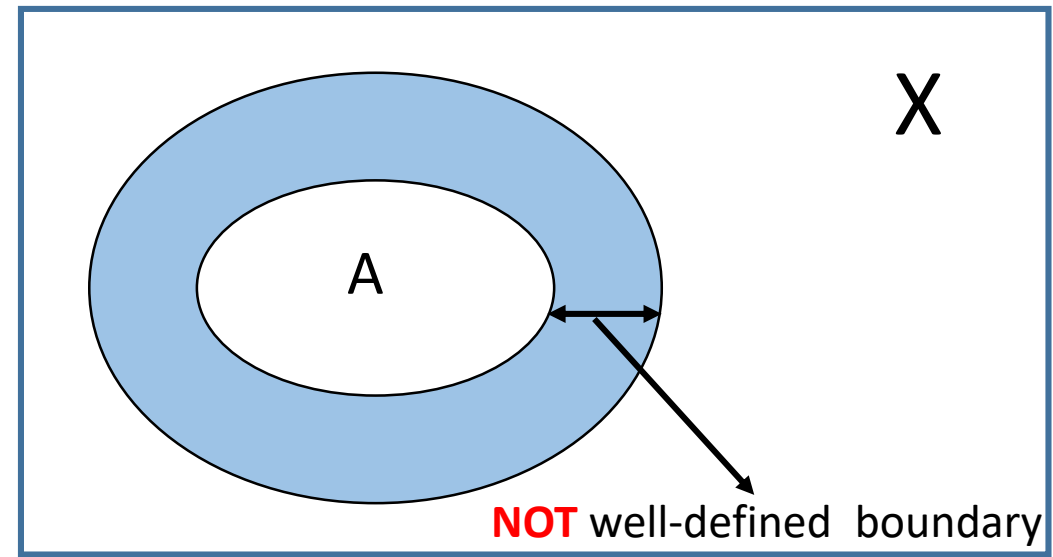
# Contents

- Operation on Classical Sets
- Properties of Classical (Crisp) Sets
- Mapping of Classical Sets to Functions Fuzzy Sets
- Notation Convention for Fuzzy Sets
- Fuzzy Set Operations

# Classical Set vs Fuzzy Set



**Classical (*crisp*) set theory**



**Fuzzy set theory**

- A classical set is defined by **crisp boundaries (or clearly defined boundaries, or sharp boundaries, or well-defined boundaries)**. In classical set theory, a set possesses a **well-defined boundary**, This means that an element in a classical set is either a member(1) or a non-member(0).
- A fuzzy set can exhibit an arbitrary membership degree between **zero and one** due to its **vague or ambiguous set boundaries or NOT well-defined boundaries**.

# Classical (Crisp) Set

- A **set** is a collection of objects with a common property.
- A **universe of discourse**,  $X$ , as a collection of objects all having the same characteristics.
- $X$  could be the  $N, Z, Q, R$  depending on the context.
- For the set of the vowels of the alphabet,  $X$  would be all the letters of the alphabet.
- The individual elements in the universe  $X$  will be denoted as  $x_i$ .
- The features of the elements in  $X$  can be **discrete**, countable integers or **continuous** valued quantities on the real line. The total number of elements in a set  $A$  is called its cardinal number, denoted by  $|A|$
- Collections of elements within a universe are called **sets**.
- Collections of elements within sets are called **subsets**
- The collection of all possible sets in the universe is called the whole set (**power set**). The **power set of set  $A$  is  $|P(A)| = 2^{|A|}$**

# Representation of Classical/Crisp Sets:

- Enumeration of elements:  $A = \{x_1, x_2, \dots, x_n\}$
- Definition by property:  $A = \{x \in X \mid x \text{ has property } P\}$
- Characteristic function/Membership function:  $\mu_A(x) : X \rightarrow \{0, 1\}$

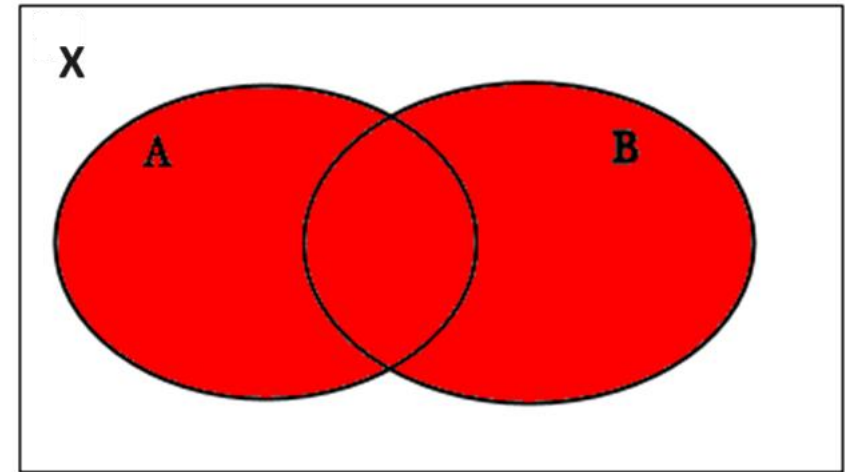
$$\mu_A(x) = \begin{cases} 1, & x \text{ is member of } A \\ 0, & x \text{ is not member of } A \end{cases}$$

# Classical (Crisp) Set: Operations

## Union:

$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$ , universe of discourse is  $X$

- Represents all those elements in the universe that reside in (or belong to) the set  $A$ , the set  $B$ , or both sets  $A$  and  $B$ .
- This operation is also called the **logical or**



$A \cup B$

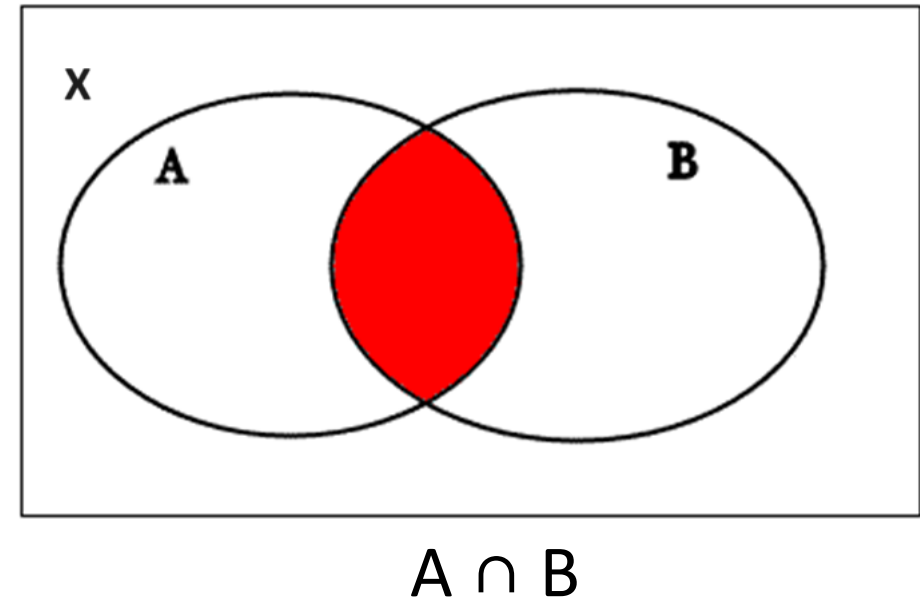
# Classical (Crisp) Set: Operations

## Intersection:

$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$ , universe of discourse is  $X$

Represents all those elements in the universe  $X$  that simultaneously reside in (or belong to) both sets  $A$  and  $B$ .

- This operation is also called the **logical and**

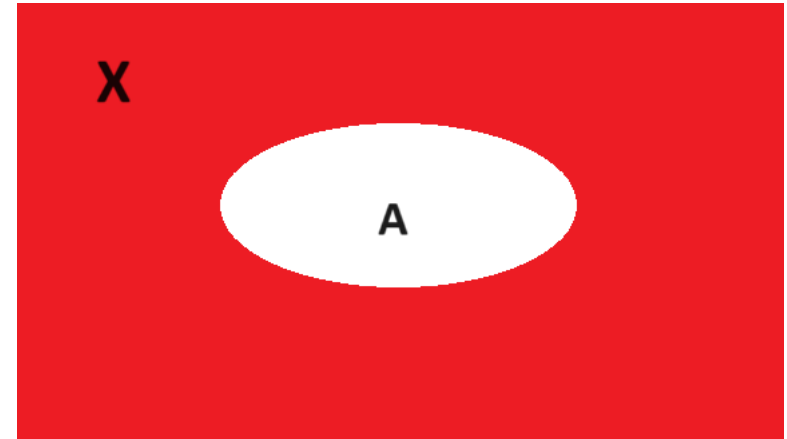


# Classical (Crisp) Set: Operations

## Complement:

$A' = \{x \mid x \in X \text{ but } x \notin A\}$ , universe of discourse is  $X$

Represents collection of all elements in the universe that do not reside in the set  $A$ .





# Classical (Crisp) Set: Operations

## Difference:

$$A - B = \{x \mid x \in A \text{ but } x \notin B\}$$

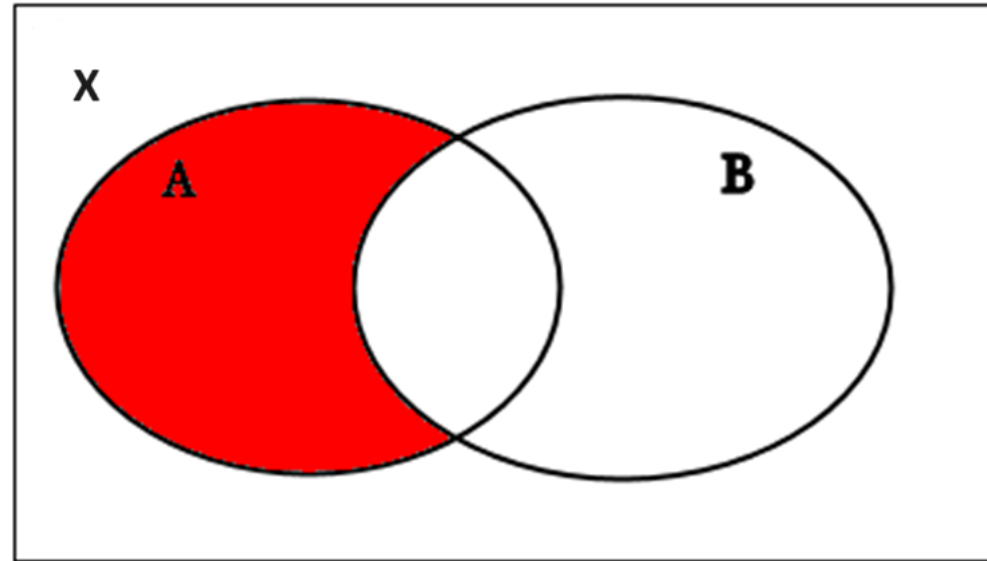
Represents the collection of all elements in the universe that reside in A and that do not reside in B simultaneously.

Similarly,

$$B - A = \{x \mid x \in B \text{ but } x \notin A\}$$

$$A - B = A \cap \overline{B}$$

$$B - A = B \cap \overline{A}$$

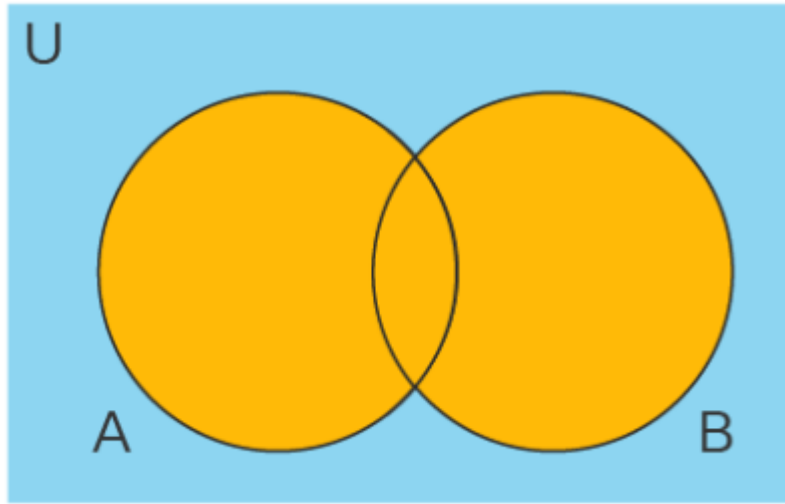


A - B

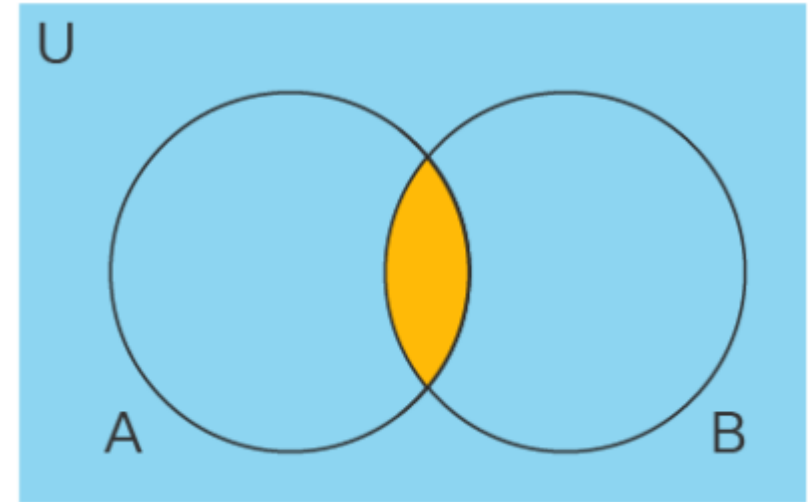
# Classical (Crisp) Set: De Morgan's law

(i)  $(A \cup B)' = A' \cap B'$

(ii)  $(A \cap B)' = A' \cup B'$



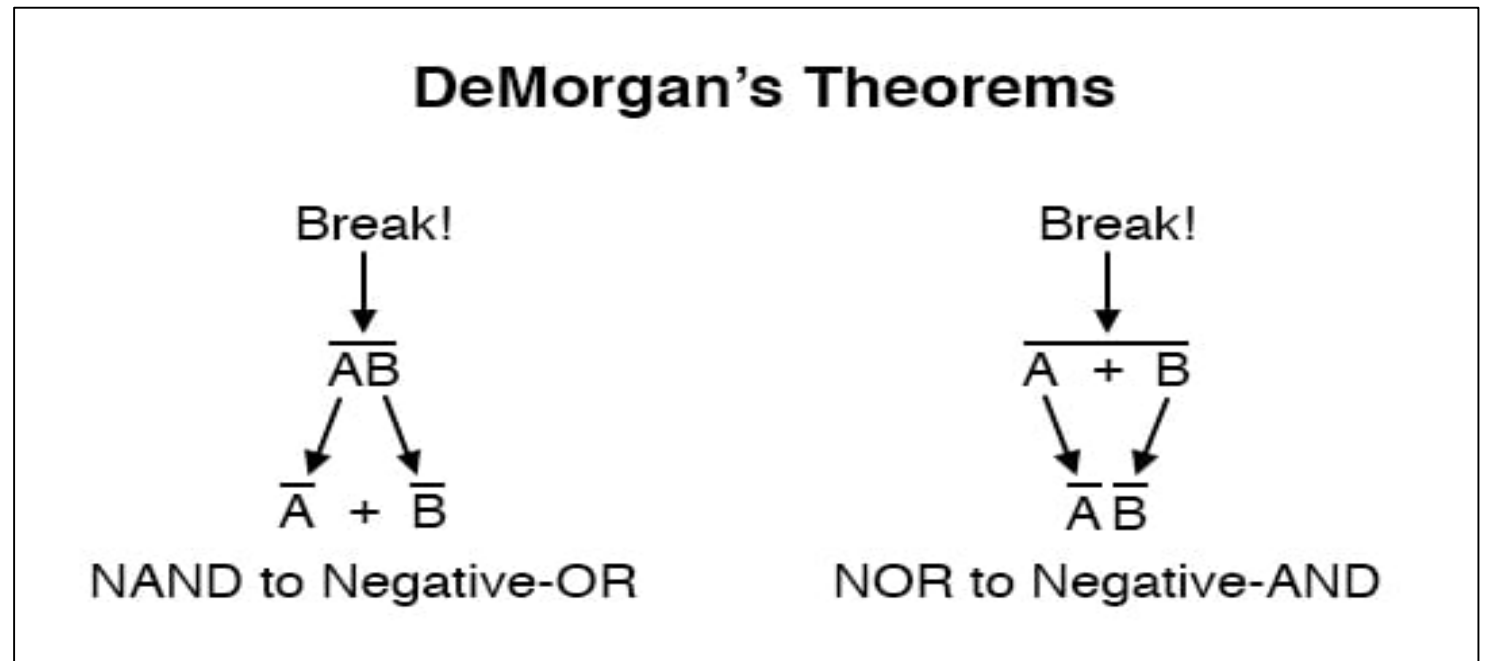
$$(A \cup B)' = A' \cap B'$$



$$(A \cap B)' = A' \cup B'$$

# Classical (Crisp) Set: De Morgan's law

- De Morgan's theorem may be thought of in terms of *breaking* a long bar symbol.
- When a long bar is broken, the operation directly underneath the break changes *from addition to multiplication, or vice versa*, and the broken bar pieces remain over the individual variables.



# Classical (Crisp) Set: Properties

Law of contradiction	$A \cap \bar{A} = \emptyset$
Law of the excluded middle	$A \cup \bar{A} = X$
Idempotency	$A \cap A = A, A \cup A = A$
Involution	$\overline{\bar{A}} = A$
Commutativity	$A \cap B = B \cap A, A \cup B = B \cup A$
Associativity	$(A \cup B) \cup C = A \cup (B \cup C)$ $(A \cap B) \cap C = A \cap (B \cap C)$
Distributivity	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Absorption	$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$
Absorption of complement	$A \cup (\bar{A} \cap B) = A \cup B$ $A \cap (\bar{A} \cup B) = A \cap B$
DeMorgan's laws	$\overline{A \cup B} = \bar{A} \cap \bar{B}$ $\overline{A \cap B} = \bar{A} \cup \bar{B}$

← NOT hold in  
← Fuzzy set

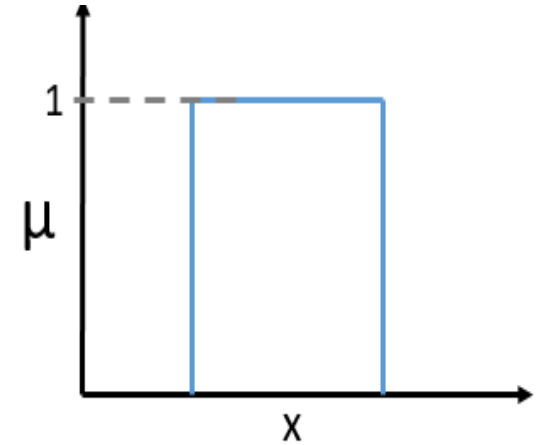
# Mapping of Classical Sets to Functions

- The **characteristic function** or **membership function**  $\mu_A$  is defined by:

$$\mu_A(x) = \begin{cases} 1, & \text{if } x \in A \\ 0, & \text{if } x \notin A \end{cases}$$

- *The characteristic function  $\mu_A$  of a crisp set A is analogous to the membership function of a fuzzy set.*
- *There are only two values which represents whether an element **belongs to a set (i.e. member of the set, whose membership value is 1 (one)) or not belongs to the set (i.e. not a member of the set, whose value is zero (0))***

$$\mu_A(x) : X \rightarrow \{0, 1\}$$



# Why fuzzy?

- The word “fuzzy” means “vagueness (ambiguity)”.

Fuzzy → (vague, uncertain, inexact etc.)

In real world, human thinking and reasoning (analysis, logic, interpretation) frequently involved fuzzy information.

*“ Fuzzy system have been provide solution”*

- Statistical models deal with *random events and outcomes*; fuzzy models attempt to capture *and quantify nonrandom imprecision*.
- *Randomness refers to an event that may or may not occur. Fuzziness refers to the boundary of a set that is not precise.*
- *Example : Randomness: frequency of car accidents, Fuzziness: the seriousness of a car accident.*

# Fuzzy logic and Fuzzy set:

- A logic based on the two truth values TRUE and FALSE is sometimes inadequate when *describing human reasoning*.
- Fuzzy logic uses the whole interval between *0 (FALSE)* and *1 (TRUE)* to describe human reasoning.
- Fuzzy logic deals with Fuzzy set.

*Definition:* Let *X* be the universal set, The fuzzy set *A* in *X* is a set of ordered pairs;  
$$A := \{(x; \mu_A(x)) : x \in X\}, \quad \mu_A(x): X \rightarrow [0, 1]$$

where,  $\mu_A(x): X \rightarrow [0, 1]$  is called the membership function. The value of  $\mu_A(x)$  is called the grade of membership of *x* in *A*

*Definition:* Let  $x \in X$ , then *x* is called

- (i) *Not include /Not a member* in the fuzzy set if  $\mu_A(x) = 0$
- (ii) *Partial include/Partial Membership* if  $0 < \mu_A(x) < 1$
- (iii) *Full include/Full membership* if  $\mu_A(x) = 1$

# Formal definitions of a fuzzy set:

- For any fuzzy set,  $A$ , the function  $\mu_A$  represents the membership function for which  $\mu_A(x)$  indicates the degree of membership that  $x$ , of the universal set  $X$ , belongs to set  $A$  and is, usually, expressed as a number between 0 and 1.

$$\mu_A(x) : X \rightarrow [0,1]$$

- Fuzzy sets can be either **discrete** or **continuous**



# Representation Methods Of Fuzzy Set

If elements are **discrete**, then the fuzzy set A on X can be represented by

$$(i) \quad A = \{(x, \mu_A(x)), x \in X\}$$

$$(ii) \quad A = \sum_i^n \mu_A(x_i) / x_i$$

$$(iii) \quad A = \alpha_1 / x_1 + \alpha_2 / x_2 + \alpha_3 / x_3 + \dots + \alpha_n / x_n, \text{ where } x_i \in X, \alpha_i = \mu_A(x_i)$$

(v) Graphical method

- Where the symbol “/” is not a division sign but indicates that the top number  $\mu_A(x)$  is the membership value of the element x in the bottom.
- Summation symbol used only for notations. This NOT actual summation

# Discrete Fuzzy Set:

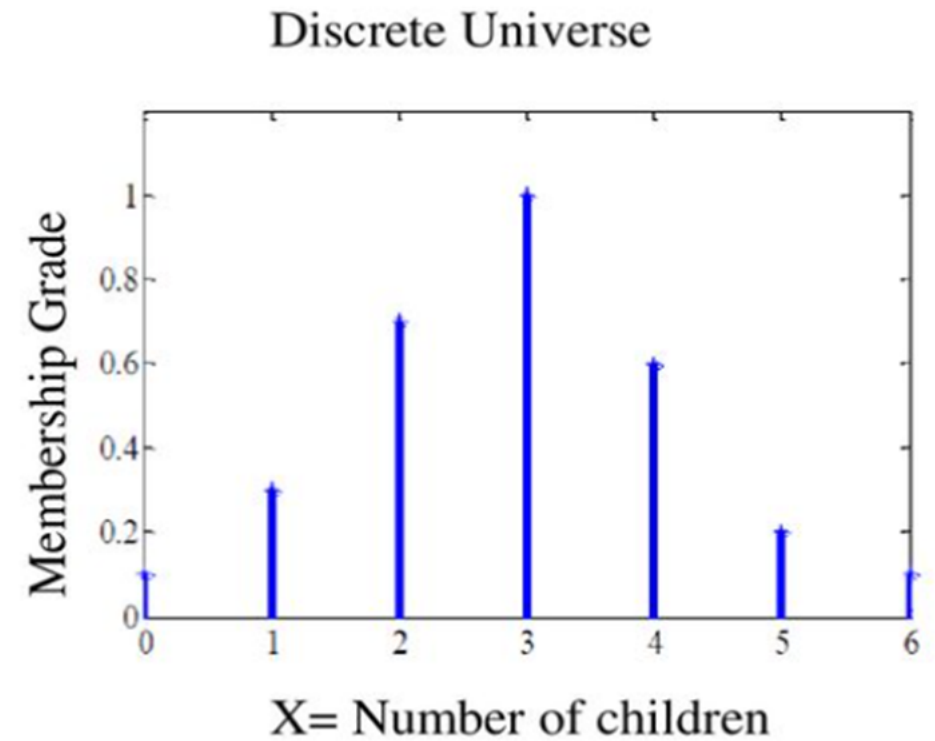
$$A = \{\mu_A(x_1)/x_1, \dots, \mu_A(x_n)/x_n\} = \{\mu_A(x_i)/x_i \mid x_i \in X\}$$

$$A = \sum_{x_i \in A} \mu_A(x_i) / x_i$$

Let  $X = \{0, 1, 2, 3, 4, 5, 6\}$  be a set of numbers of children a family may possibly have.

Fuzzy set  $A$  with “sensible number of children in a family” may be described by

$$A = \{(0, 0.1), (1, 0.3), (2, 0.7), (3, 1), (4, 0.6), (5, 0.2), (6, 0.1)\}$$



# Representation Methods Of Fuzzy Set

If elements are **continuous**, then the fuzzy set A on X can be represented by

1)  $\int_X \mu_A(x)/x$

2)

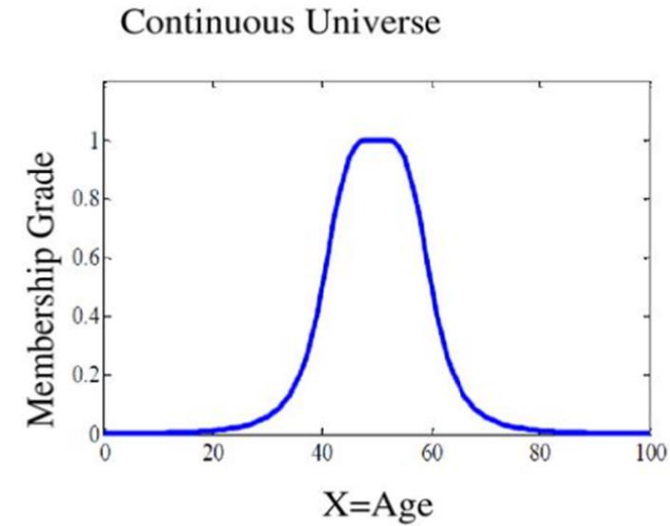
$x_1$	$x_2$	$x_3$	$x_4$	$x_5$
0.2	0.5	0.4	0.7	0.6

## 3) Graphical method

- Where the symbol “/” is not a division sign but indicates that the top number  $\mu_A(x)$  is the membership value of the element x in the bottom.
- **Integration** symbol used only for notations. This NOT actual integration

# Continuous Fuzzy Set:

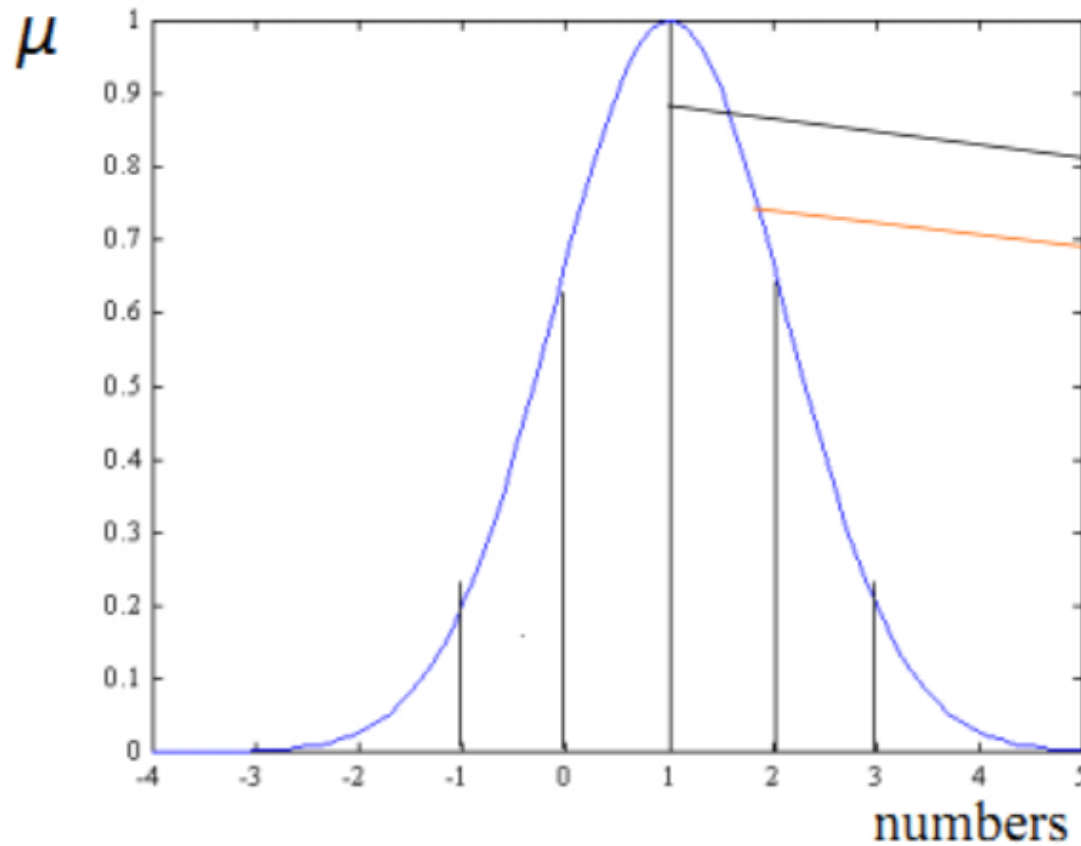
$$A = \int_X \mu_A(x)/x$$



$X = \mathbb{R}^+$  be the set of possible ages for human beings. fuzzy set  $B =$  “about 50 years old” may be expressed as  $B = \{(x, \mu_A(x) | x \in X\}$ , where  $\mu_A(x) = 1/(1 + ((x-50)/10)^4$

Question: draw the graph for the continuous functions (i)  $\mu_A(x) = \frac{1}{1+x^2}$ ,  
(ii)  $\mu_A(x) = \left(\frac{1}{1+x^2}\right)^2$

# Fuzzy Sets( Discrete and continuous)



**Example:**  
Discrete and  
Continuous  
fuzzy sets to  
represent the set  
of numbers  
“close to 1”

# Crisp set vs Fuzzy set

- Crisp set :

- Membership function
- Membership degree:  $\{0,1\}$

*A crisp set/classical set,  $A$ , the **characteristic function** for each element  $x$  of  $X$  can be represented by **ordered pairs**  $(x, 0)$  or  $(x, 1)$ , which indicates  $x \notin A$  and  $x \in A$  respectively.*

- Fuzzy set :

- Membership function: user specify
- Membership degree:  $[0,1]$

*A fuzzy set  $A$  in the universe  $X$  is a set of **ordered pairs**  $A = \{(x, \mu_A(x)), x \in X\}$  where  $\mu_A(x)$  is the grade of membership of  $x$  in  $A$*

# Fuzzy set:

- A **classical set** is defined by crisp(exact) boundaries, i.e., there is **no uncertainty** about the set boundaries.
- A fuzzy set is defined by its **ambiguous boundaries**, i.e., there exists **uncertainty about the set boundaries**.
- A set without crisp boundary is known as fuzzy set. The transition from “belong to a set” to “not-belong to the set” is **gradual** and this smooth *transition is characterized by the membership function*.
- *Fuzzy sets allows a **grading** of to what extent an element of a set belongs to that specific set.*
- The value 0 means that  $x$  is not a member of the fuzzy set; the value 1 means that  $x$  is fully a member of the fuzzy set.
- The values between 0 and 1 characterize fuzzy members, which belong to the fuzzy set only partially.

# Fuzzy Set

- Weather is **Hot**?

Crisp: **Yes** (1) ! **No**(0)!

Fuzzy: Extremely Hot (1) Very Hot(0.80) warm(0.40)

little warm(0.0)

- Words like young, tall, good are fuzzy.

There is **no single** quantitative value which defines the term **young**.

For some people, age **25** is young, and for others, age **35** is young.

The concept **young has no clear boundary**.

Age **35** has some possibility of being young and usually depends on the context in which it is being considered.

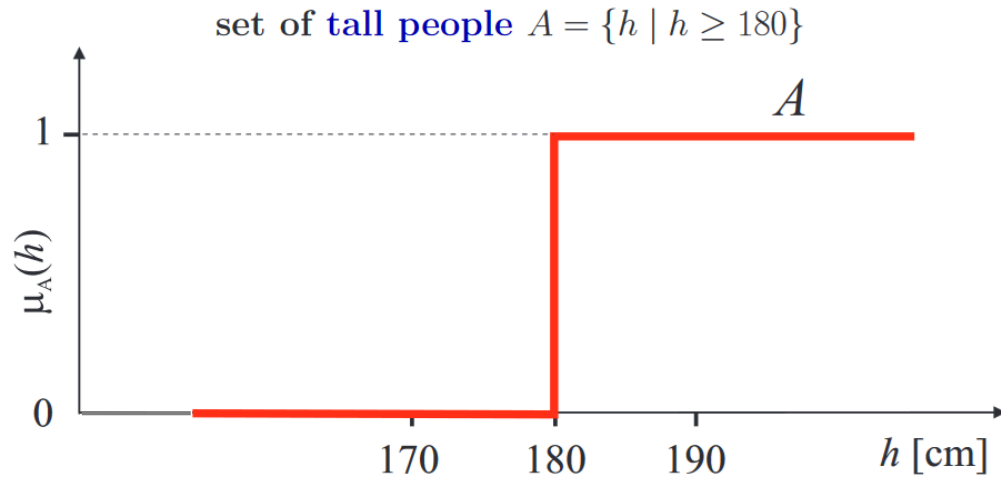
*Fuzzy set theory is an extension of classical set theory where elements have degree of membership/grade of membership.*



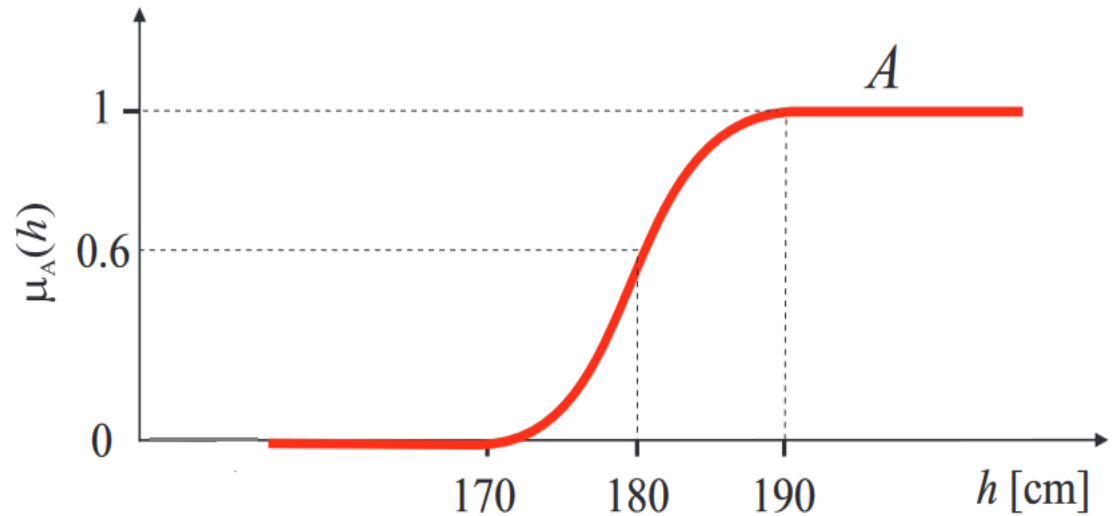
# Fuzzy Set

- Set of all **tall** peoples representation in classical and fuzzy set

:Classical Set Approach:



:Fuzzy Set Approach:



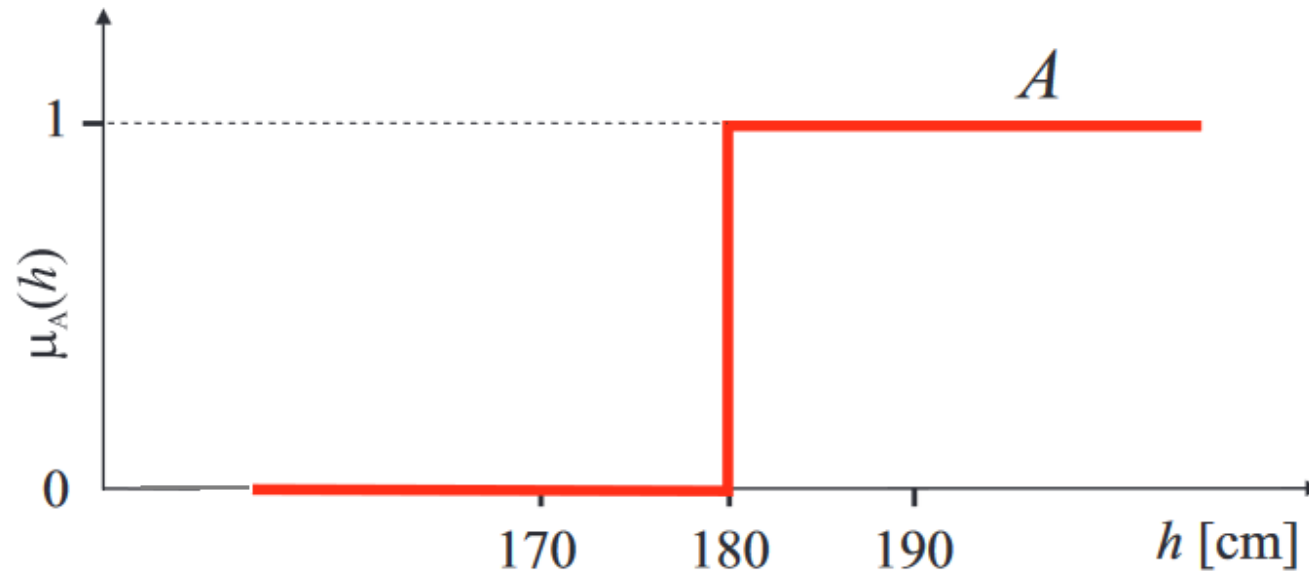
$$\mu_A(h) = \begin{cases} 1 & h \text{ is full member of } A \quad (h \geq 190) \\ (0, 1) & h \text{ is partial member of } A \quad (170 < h < 190) \\ 0 & h \text{ is not member of } A \quad (h \leq 170) \end{cases}$$

# Logical Propositions

“John is tall” ... true or false

John's height:  $h_{John} = 180.0$        $\mu_A(180.0) = 1$  (true)

$h_{John} = 179.5$        $\mu_A(179.5) = 0$  (false)



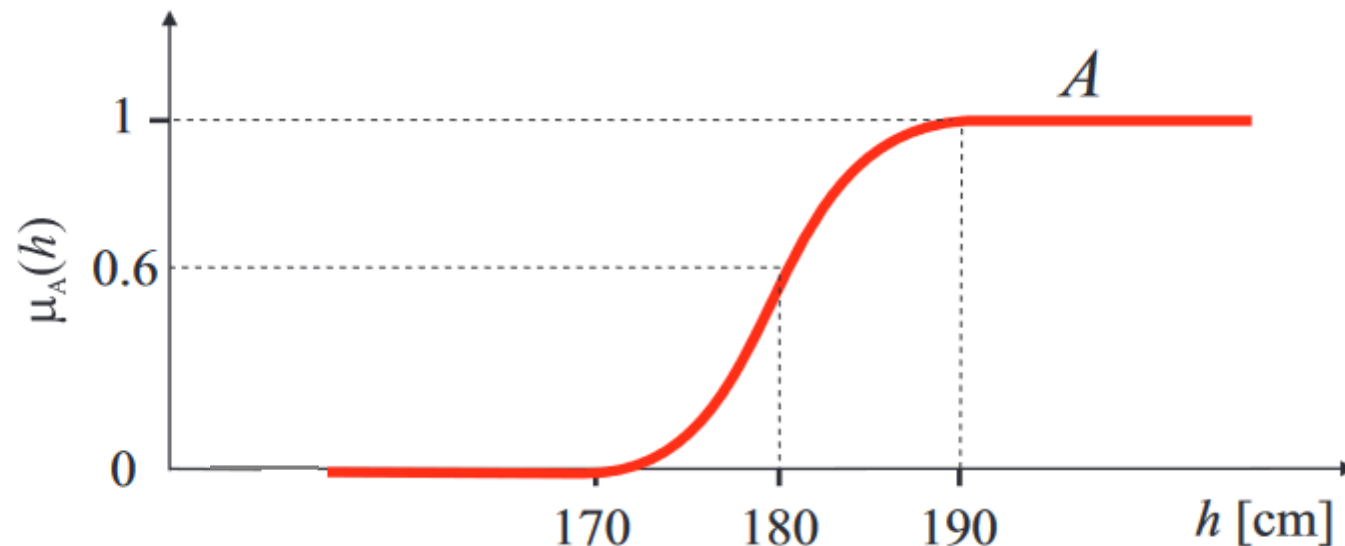
# Fuzzy Logic Propositions

“John is tall” ... degree of truth

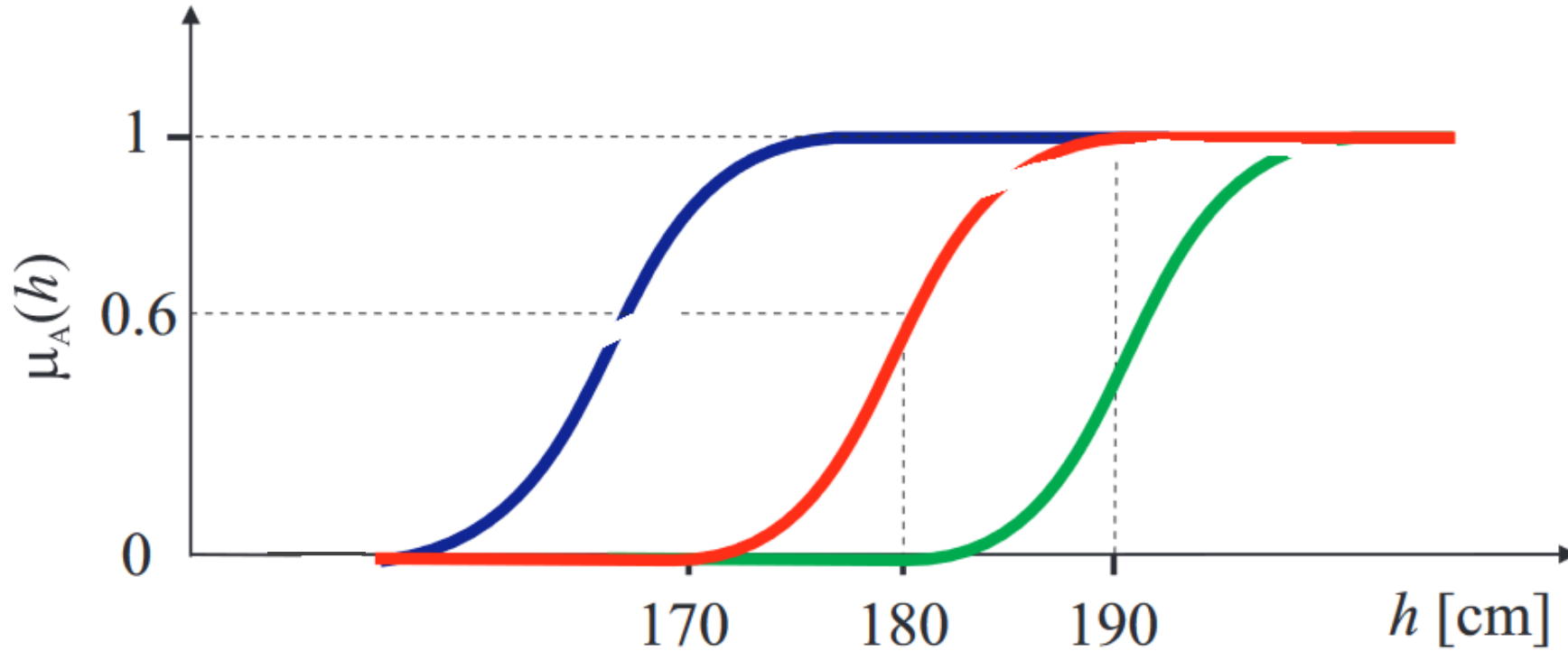
John's height:  $h_{John} = 180.0$        $\mu_A(180.0) = 0.6$

$h_{John} = 179.5$        $\mu_A(179.5) = 0.56$

$h_{Paul} = 201.0$        $\mu_A(201.0) = 1$



# Based on Subjective and Context Dependent



tall in China

tall in Europe

tall in NBA

# Fuzzy Set

Lets take an example the set of **YOUNG PEOPLE**.

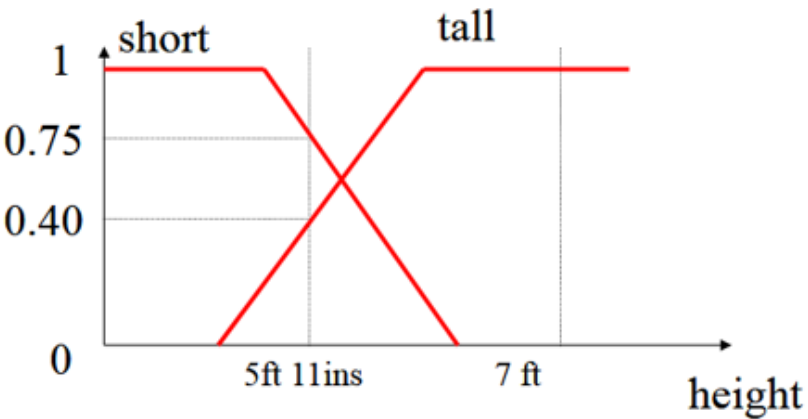
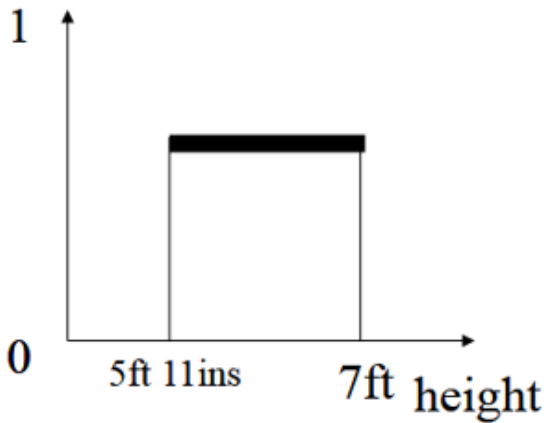
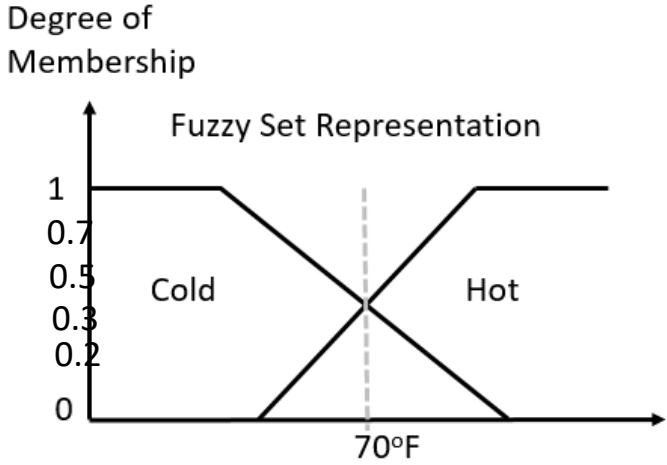
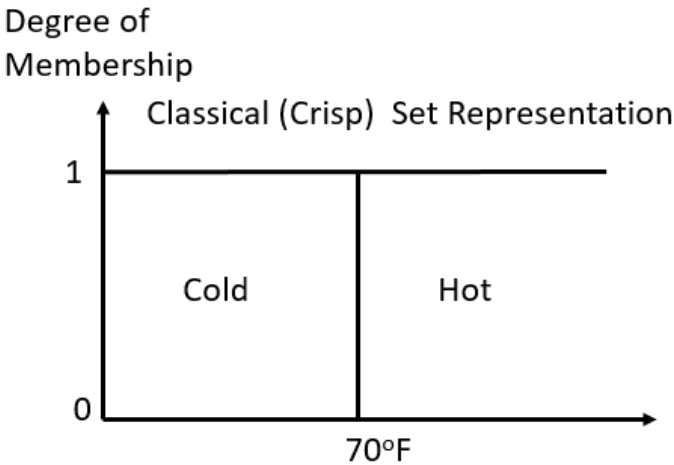
- A **one year old baby** will clearly be **a member of the set**, and **a 100 years old person will not be a member** of this set, but what about people at the age of 20, 30, or 40 years?
- Zadeh proposed a **degree of membership/grade of membership**, such that the transition from membership to non-membership is **gradual rather than abrupt**.
- The **grade of membership** for all its members thus describes a fuzzy set. An item's **grade of membership** is normally a real number between 0 and 1, often denoted by the Greek letter  $\mu$ .
- **There is no formal basis for how to determine the grade of membership. It depends on the user and on the context.**

# Fuzzy Set:

## Classical Set:

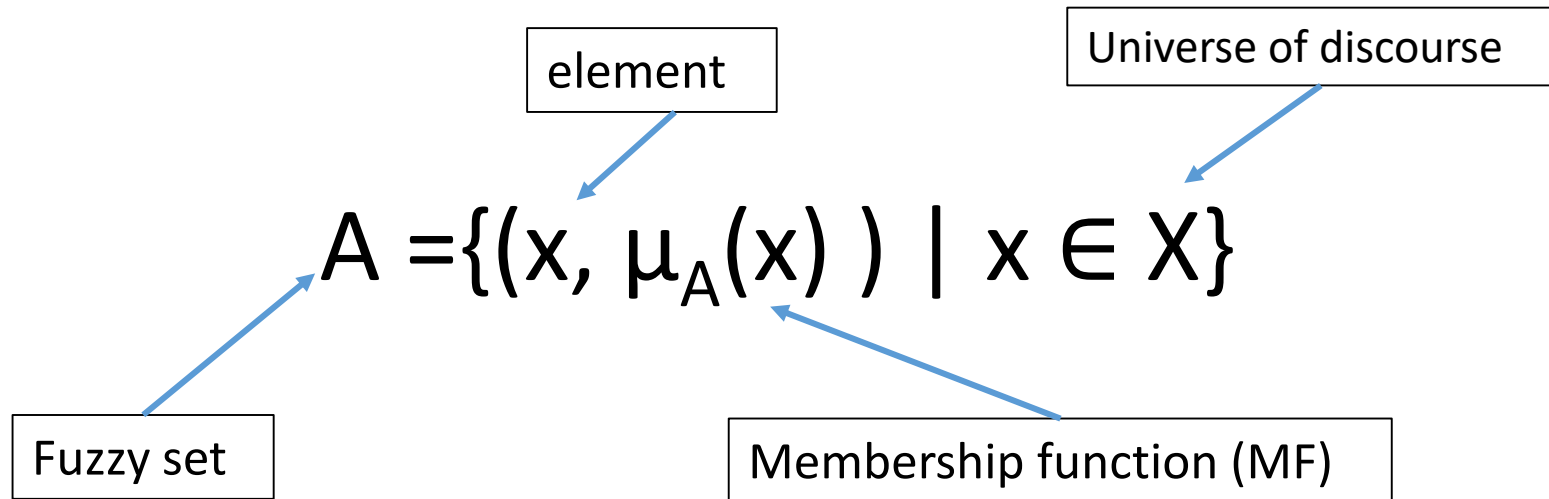
If the temperature is higher than 70°F, it is hot; otherwise, it is not hot.

- Cases:
- |                        |         |
|------------------------|---------|
| – Temperature = 100°F  | Hot     |
| – Temperature = 70.1°F | Hot     |
| – Temperature = 69.9°F | Not hot |
| – Temperature = 50°F   | Not hot |



# Formal definitions of a fuzzy set:

- A fuzzy set is uniquely defined by its membership function
- A fuzzy set **A** in **X** is defined by a set of ordered pairs.



- The **MF** maps every element of **X** to a membership value membership grade between **0** and **1**.

# Fuzzy set operations:

- The **Union (Disjunction)** of these two sets in terms of function-theoretic terms is given as follows (the symbol  $\vee$  is the maximum):

$$\text{Union } A \cup B : \mu_{A \cup B}(x) = \mu_A(x) \vee \mu_B(x) = \max(\mu_A(x), \mu_B(x))$$

- The **Intersection (Conjunction)** of these two sets in function-theoretic terms is given by (the symbol  $\wedge$  is the minimum operator):

$$\text{Intersection } A \cap B : \mu_{A \cap B}(x) = \mu_A(x) \wedge \mu_B(x) = \min(\mu_A(x), \mu_B(x))$$

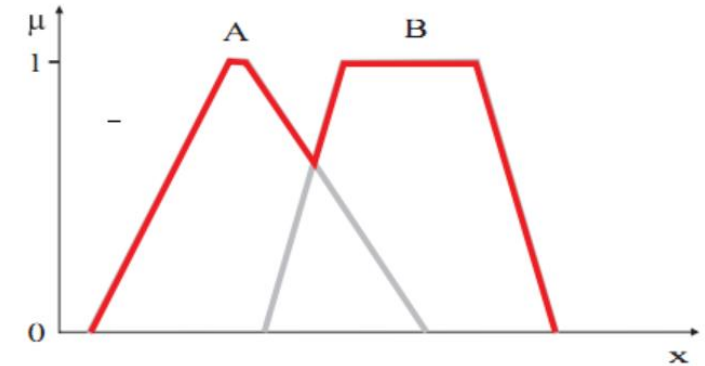
- The **Complement (negation)** of a single set on universe  $X$ , say  $A$ , is given by

$$\text{complement}(\bar{A}): \quad \mu_{\bar{A}}(x) = 1 - \mu_A(x)$$

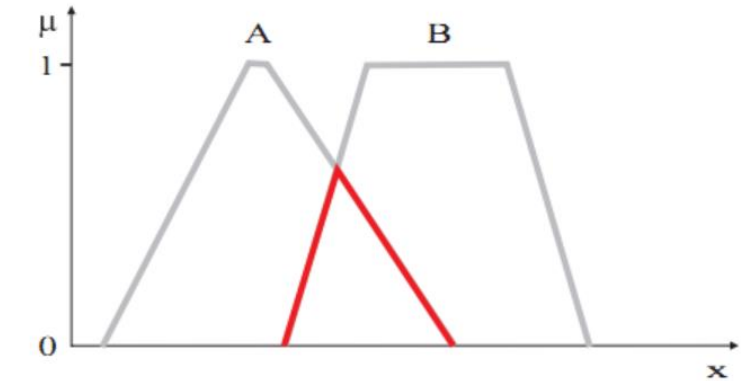


# Fuzzy Set operations

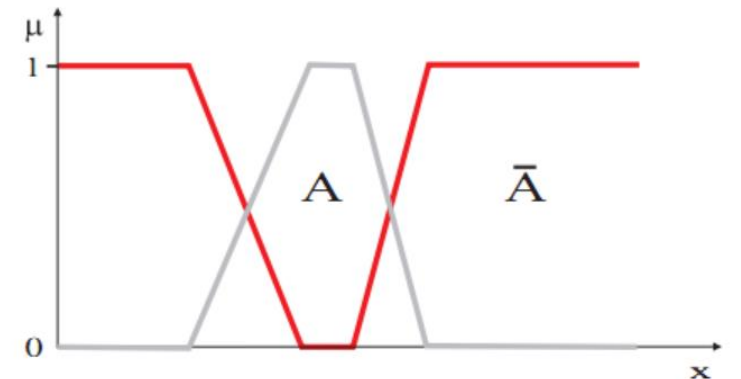
$$\mu_{A \cup B}(x) = \mu_A(x) \vee \mu_B(x) = \max(\mu_A(x), \mu_B(x))$$



$$\mu_{A \cap B}(x) = \mu_A(x) \wedge \mu_B(x) = \min(\mu_A(x), \mu_B(x))$$



$$\mu_{\bar{A}}(x) = 1 - \mu_A(x)$$



# Fuzzy set operations

Example:

$$A = \{(x_1, 0.5), (x_2, 0.7), (x_3, 0)\} \quad B = \{(x_1, 0.8), (x_2, 0.2), (x_3, 1)\}$$

**Union:**

$$A \cup B = \{(x_1, 0.8), (x_2, 0.7), (x_3, 1)\}$$

Because

$$\begin{aligned}\mu_{A \cup B}(x_1) &= \max(\mu_A(x_1), \mu_B(x_1)) \\ &= \max(0.5, 0.8) \\ &= 0.8\end{aligned}$$

$$\mu_{A \cup B}(x_2) = 0.7 \quad \text{and} \quad \mu_{A \cup B}(x_3) = 1$$

# Fuzzy Set Operations

Example:

$$A = \{(x_1, 0.5), (x_2, 0.7), (x_3, 0)\} \quad B = \{(x_1, 0.8), (x_2, 0.2), (x_3, 1)\}$$

**Intersection:**

$$A \cap B = \{(x_1, 0.5), (x_2, 0.2), (x_3, 0)\}$$

Because

$$\begin{aligned}\mu_{A \cap B}(x_1) &= \min(\mu_A(x_1), \mu_B(x_1)) \\ &= \min(0.5, 0.8) \\ &= 0.5\end{aligned}$$

$$\mu_{A \cap B}(x_2) = 0.2 \quad \text{and} \quad \mu_{A \cap B}(x_3) = 0$$

# Fuzzy Set Operations

Example:

$$A = \{(x_1, 0.5), (x_2, 0.7), (x_3, 0)\}$$

**Complement:**

$$A^c = \{(x_1, 0.5), (x_2, 0.3), (x_3, 1)\}$$

Because

$$\mu_{\bar{A}}(x_1) = 1 - \mu_A(x_1)$$

$$= 1 - 0.5$$

$$= 0.5$$

$$\mu_{\bar{A}}(x_2) = 0.3 \text{ and } \mu_{\bar{A}}(x_3) = 1$$

# Fuzzy set

**Containment/Subset:** The fuzzy set **A** is contained in the fuzzy set **B** (or A is a subset of B) if and only if  $\mu_A \leq \mu_B$ .

We will denote **subset** between two fuzzy sets as  $A \subseteq B$ .

From this definition of containment, it can easily be seen that for two fuzzy sets **A** and **B** that  $A = B$  if and only if  $A \subseteq B$  and  $B \subseteq A$ .

The notion of a **proper subset** will be the same as the definition of subset but with  $\mu_A < \mu_B$  and be denoted as  $A \subset B$ .

**Empty Fuzzy Set :** A fuzzy set **A** is **empty** if and only if its membership function is identically zero on  $X$ ,  $\mu_A \equiv 0$ .

**Fuzzy Set Equality :** Two fuzzy sets A and B are equal if and only if  $\mu_A(x) = \mu_B(x)$  for all  $x \in X$ . We will denote equality as  $A = B$ .

# Support, core, boundary

## *The Support of fuzzy set:*

The support of a fuzzy set (denoted  $\text{supp}$ ) is the crisp set of all  $x \in X$  for which  $\mu_A(x) > 0$

## *The (core) of a fuzzy set:*

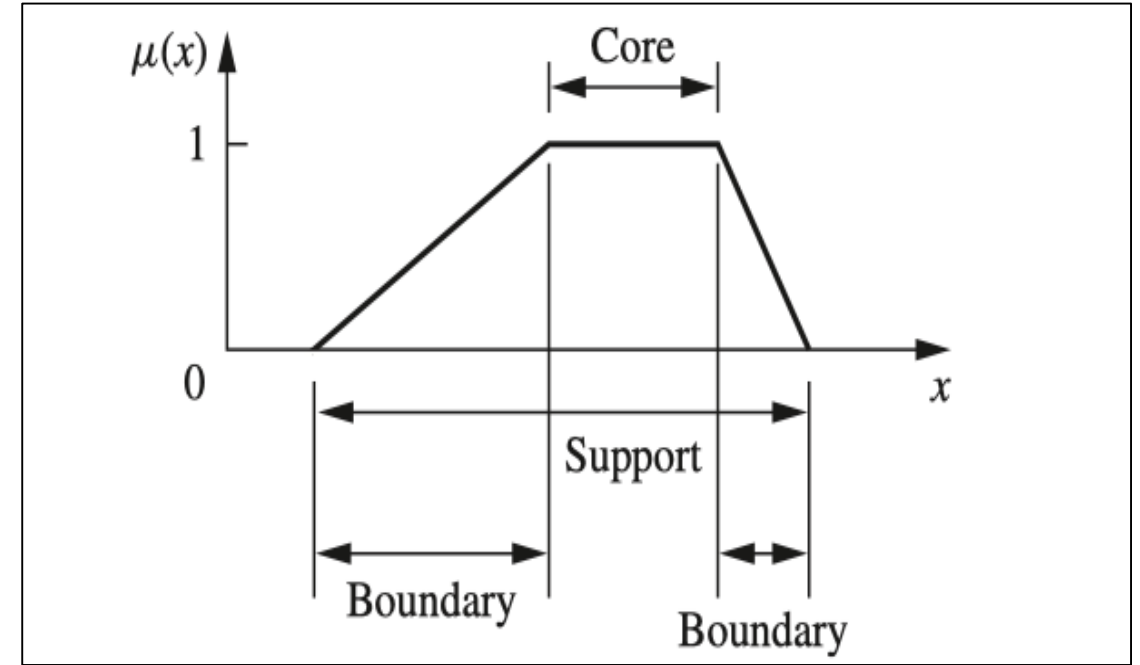
is the crisp set for which  $\mu_A(x) = 1$

## *The (boundary) of a fuzzy set:*

is the crisp set for which  $0 < \mu_A(x) < 1$

## *Normal fuzzy set:*

A fuzzy subset  $A$  of the universal set  $X$  is called **normal** if there exists an  $x \in X$  such that  $\mu_A(x) = 1$ . Otherwise  $A$  is **subnormal**.



# Fuzzy sets:

## *Fuzzy singleton set:*

Fuzzy set whose support is a single point in  $X$  with  $\mu_A(x) = 1$  is called fuzzy singleton

## *Fuzzy crossover point:*

Crossover point of a fuzzy set  $A$  is a point  $x$  in  $X$  such that  $\{(x \mid \mu_A(x) = 0.5)\}$

## *$\alpha$ -cut :*

$\alpha$ -cut of a fuzzy set  $A$  is set of all points  $x$  in  $X$  such that  $\{(x \mid \mu_A(x) \geq \alpha)\}$

## *Convexity:*

Convexity  $\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_A(x_1), \mu_A(x_2))$ . Then  $A$  is convex, where  $\lambda \in [0,1]$

## *Bandwidth:*

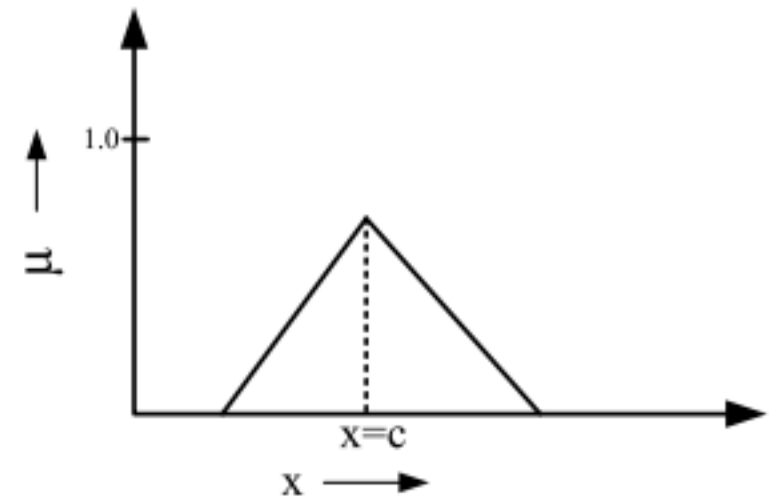
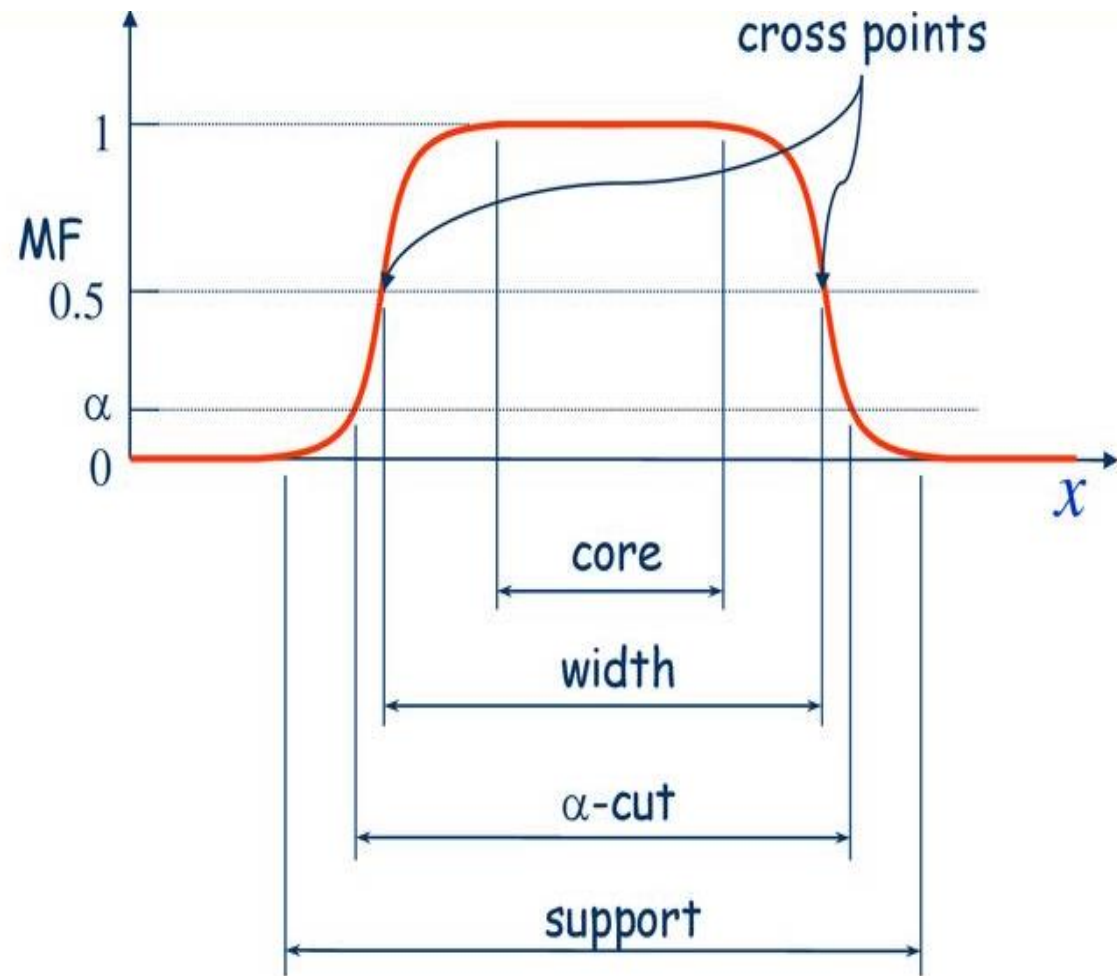
Bandwidth( $A$ ) =  $|x_2 - x_1|$ , where  $x_2 = 0.5$  and  $x_1 = 0.5$  are crossover points.

## *Symmetry:*

Symmetry  $\mu_A(c+x) = \mu_A(c-x)$  for all  $x \in X$ . Then  $A$  is symmetric.

## *Fuzzy numbers:*

A fuzzy number  $A$  is a fuzzy set in the real line  $R$  that satisfies the conditions for *normality* and *convexity*.



*Symmetry:*



Q ) Let  $X = \{a, b, c, d\}$  and fuzzy set  $A = 0.8/a + 1.0/b + 0.3/c + 0.1/d$

(i)  $\alpha$  -cut sets for  $\alpha = 0.1, 0.3, 0.8, 1.0$

(ii) Strong  $\alpha$  -cut sets for  $\alpha = 0.1, 0.3, 0.8, 1.0$

**Example (Normality) :** Let  $A = 0.2/1 + 0.5/2 + 0.8/3 + 1/4 + 0.7/5 + 0.3/6$ , Since  $\mu_A(4) = 1$ , then this fuzzy set is **normal**.

**While the fuzzy set**  $A = 0.2/1 + 0.5/2 + 0.8/3 + 0.7/5 + 0.3/6$  **is subnormal.**

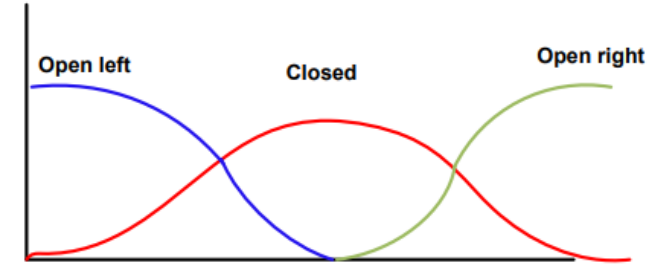
# Fuzzy sets:

- Open left, open right, closed: A fuzzy set is open left if

**Open left :** If  $\lim_{x \rightarrow -\infty} \mu_A(x) = 1$  and  $\lim_{x \rightarrow +\infty} \mu_A(x) = 0$

**Open right :** If  $\lim_{x \rightarrow -\infty} \mu_A(x) = 0$  and  $\lim_{x \rightarrow +\infty} \mu_A(x) = 1$

**Closed If :**  $\lim_{x \rightarrow -\infty} \mu_A(x) = \lim_{x \rightarrow +\infty} \mu_A(x) = 0$



- **Height :** The maximum value of the membership is called *height*.

Example: Let  $A = 0.8/1 + 0.5/2 + 0.9/3 + 0.1/4$  . the height of this fuzzy set is 0.9

- **Containment or Subset:** Fuzzy set A is contained in fuzzy set B (or A is a subset of B) if  $\mu_A(x) \leq \mu_B(x)$  for all x.

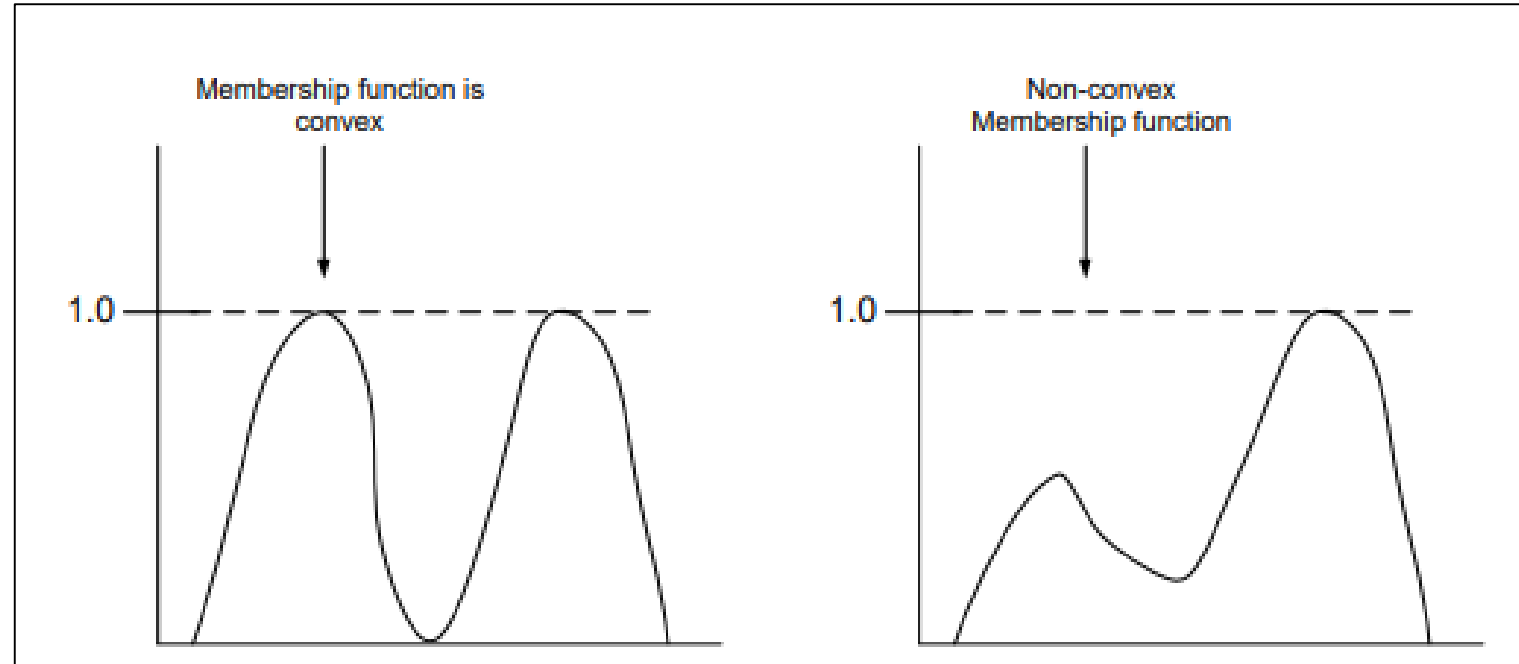
# Convexity:

## Classical Set:

A subset  $S$  of  $R$  is said to be convex if and only if, for all  $x_1, x_2 \in S$  and for every real number  $\lambda$  satisfying  $0 \leq \lambda \leq 1$ , we have  $\lambda x_1 + (1 - \lambda)x_2 \in S$ : For example, any interval  $S = [a_1, a_2]$  is a convex set since the above condition is satisfied;  $[0, 1]$  and  $[3, 4]$  are convex, but  $[0, 1] \cup [3, 4]$  is not.

## Fuzzy Convexity:

$$\mu_A(\lambda x_1 + (1 - \lambda)x_2) \geq \min(\mu_A(x_1), \mu_A(x_2))$$

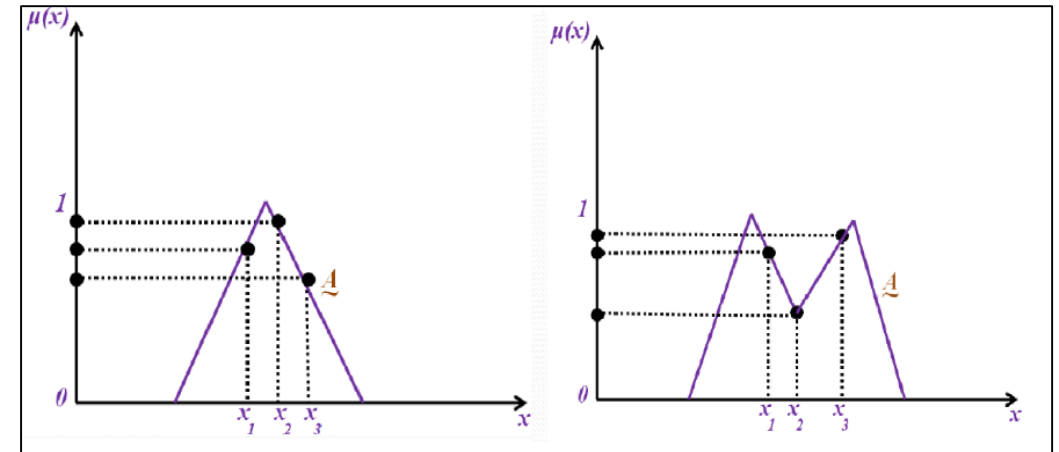


**Question:** Let  $X = [1, 5]$  and  $A := \int_X^{1/x}/x$ , is  $A$  convex?

# Fuzzy convexity

- A convex fuzzy set is described by a membership function whose membership values are **strictly monotonically increasing**, or **whose membership values are strictly monotonically decreasing**, or whose membership values are strictly monotonically increasing then strictly monotonically decreasing with increasing values for elements in the universe.
- Another way, if, for any element  $t$  in between  $x_1$  and  $x_2$  in a fuzzy set  $A$ , the relation  $x_1 < t < x_2$  implies that

$$\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_A(x_1), \mu_A(x_2))$$



- How do you distinguish a fuzzy set from a crisp set?

Answer:

- Do the Laws of Contradiction and Excluded Middle hold?

Answer: However, if A is a non-crisp set, then neither law will hold. Indeed, note that for a non-crisp set, there exists some  $x \in A$  such that  $\mu_A(x) \in (0, 1)$ , i.e.  $\mu_A(x) \neq 0, 1$ . Thus, we have

- $\mu_{A \cap A'}(x) = \min\{\mu_A(x), 1 - \mu_A(x)\} \neq 0$
- $\mu_{A \cup A'}(x) = \max\{\mu_A(x), 1 - \mu_A(x)\} \neq 1$

# Cardinality

- Cardinality of fuzzy set A, is  $|A| = \sum_i^n \mu_A(x)$
- Relative cardinality of set A is :  $||A|| = |A|/|X|$

Example : Let  $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$$A = 0.2/1 + 0.5/2 + 0.8/3 + 1/4 + 0.7/5 + 0.3/6$$

$$B = 0.3/1 + 0.9/3 + 0.8/6 + 0.1/7 + 0.4/8 + 0.6/9 + 1/10$$

Then find the fuzzy cardinality?

# Cartesian product and co-product

- Let  $A$  and  $B$  be fuzzy sets in  $X$  and  $Y$ , respectively. The Cartesian product of  $A$  and  $B$ , denoted by  $A \times B$ , is a fuzzy set in the product space  $X \times Y$  with membership function

$$\mu_{A \times B}(x, y) = \min(\mu_A(x), \mu_B(y)).$$

- Cartesian co-product  $A+B$  is fuzzy set with the membership function

$$\mu_{A+B}(x, y) = \max(\mu_A(x), \mu_B(y)).$$

# Difference between Fuzzy and Crisp set

## Fuzzy Set

- Fuzzy set have gradual transition from membership to non-membership or vice versa with many membership degrees/values.
- Infinite number of values:  $[0,1]$
- Fuzzy controller
- Gradual membership
- Ambiguous boundary or not well defined boundary

## Classical/ Crisp set

- In classical/crisp set the transition is abrupt/sudden not gradual.
- Bi-valued :  $\{0,1\}$
- Digital system design
- Total membership
- Precise or well defined boundary



# Assignment-1

Consider a universal set X which is defined on the age domain.

$X := \{5, 15, 25, 35, 45, 55, 65, 75, 85\}$

Answer the following:

1. Find fuzzy sets such as *infant*, *young*, *adult* and *senior* in X
2. Find the *support* set of each fuzzy set.
3. Find the  *$\alpha$ -cut set* is derived from fuzzy set *young*.
4. Is the fuzzy set *adult* is *normal* or *subnormal*?
5. What is the height of fuzzy set *senior*

age(element)	infant	young	adult	senior
5	0	0	0	0
15	0	0.2	0.1	0
25	0	1	0.9	0
35	0	0.8	1	0
45	0	0.4	1	0.1
55	0	0.1	1	0.2
65	0	0	1	0.6
75	0	0	1	1
85	0	0	1	1

# Assignment-2

Consider of the fuzzy sets A, B and C defined on  $X = [0, 10]$  of real numbers by the membership grade functions

$$(i) \mu_A(x) = \frac{1}{1+x^2}$$

$$(ii) \mu_B(x) = 2^x$$

$$(iii) \mu_C(x) = \frac{x}{x+2}$$

Q1) Calculate: (i)  $A', B', C'$  ; (ii)  $A \cup B, A \cup C, B \cup C$ ; (iii)  $A \cup B \cup C$ ; (iv)  $A \cap B \cap C$ ; (v)  $A \cap C'$  ; (vi)  $\overline{A \cap B}$ ;

Q2) Calculate the  **$\alpha$ -cuts** and **strong  $\alpha$ -cuts** of the three fuzzy in the sets for some values of  $\alpha$ , for example  $\alpha = 0.2, 0.1, 0.4, 1.0$ .

# Assignment-3

- (i) How do you distinguish a fuzzy set from a crisp set?
- (ii) What is the difference between Fuzzy set and Probability ?

# Difference between Fuzzy set and Probability ?

- The key difference between fuzzy logic and probability is how they handle uncertainty
    - Fuzzy Sets: Deal with **vagueness** and **imprecision**.
    - **Probability: Deals with randomness and unpredictability**
  - Fuzziness describes event ambiguity. It measures the degree to which an event occurs, NOT whether it occurs.
  - **Randomness describes the uncertainty of event occurrence, an event occurs or not.**
  - Fuzzy Sets: Use a membership function to assign degrees of membership.
  - **Probability: Use a probability function to assign likelihoods to events.**
  - Fuzzy Sets: The degrees of membership of elements in a set do not need to sum to 1.
  - **Probability: The probabilities of all possible outcomes in a probability space must sum to 1**
  - Fuzzy Sets: Used in scenarios requiring human-like reasoning and dealing with imprecise information.
  - **Probability: Used in scenarios involving statistical analysis and prediction of future events based on known data.**
- ( Fuzziness describes the lack of distinction of an event, whereas chance describes the uncertainty in the occurrence of the event. The event will occur or not occur; but is the description of the event clear enough to measure its occurrence or nonoccurrence? )

**\*\*\* Fuzziness is a type of deterministic uncertainty**