Computer Vision & Machine Learning Revision Notes

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2 Disclaimer

these are my personal notes reviewing material for CV/M interviews. The notes contain text from Quora, as well as screengrabs from various sources including Andrew Ng's Machine Learning Course on Coursera, Wikipedia, The Computer Vision Course on Coursera, and other sources. Reproduced without permission. The notes are incomplete, work in progress, and may contain mistakes.

3 CV Interview Topics

Geometric vision - geometric transforms,projective geometry, homography, stereo vision, epipolar geometry, fundamental and essential matrices, geometric calibration, triangulation
Photometric vision - filtering, convolution, denoising , deblurring etc
Semantic vision - concepts of boosting, neural nets, svm, image descriptors etc

4 ToDos

- Correlation vs. Convolution
- The difference is in how the summarization iterates over the elements.
- Total Variation
- Visual Hull
- Optic Flow - Lukas-Kanade, Brox
- Cascade Detectors
- Face landmark detection

5 Computer Vision

Questions: How do you translate cameras?
graphcut
dynamic programming for stereo matching

6 Geometric Transformations

2D Scaling $S = \begin{pmatrix} s_x & 0 \\ 0 & s_y \end{pmatrix}$

2D Rotation $R = \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$
Symmetry group: SO3 vs. SE3
Reflection: Eigenvalues of the rotation matrix contain (-1)

2D Translation - only possible in homogeneous coordinates

7 Projective Geometry

7.1 Homogeneous Coordinates

$x = \begin{pmatrix} u & v & 1 \end{pmatrix}^T$ pixel space position, only 2 degrees of freedom.
Inhomgenous coordinates: $x = \begin{pmatrix} u & v \end{pmatrix}^T$
$x_w = \begin{pmatrix} x_w \\ y_w \\ w_t \\ 1 \end{pmatrix}^T$ world space position
The scale is unknown: $x \mapsto w x_w$

7.1.1 Homogenous Line/Plane Representation

$ax + by + c = 0 \rightarrow (a, b, c)^T p = 0$ for every $p = (x, y, 1)$ on the line.
The nice thing about this: for two Euclidean points $p_{1,2} = (x, y, z)$ the line connecting them is given by their cross product: $l = p_1 \times p_2$.
Similarly, for two lines, their intersection point is given by the cross product: $p = l_1 \times l_2$
From homogenous points to cartesian image points divide by z.

7.1.2 Projection

A projectivity is an invertible mapping h from to itself such that three points x_1, x_2, x_3 lie on the same line if and only if $h(x_1), h(x_2), h(x_3)$ do:

- check by fitting a line to the point and checking the third point is on the same line
- line normal coordinates in 3D

7.1.3 Projective Transformation

From world position $x \in \mathbb{R}^4 \rightarrow p \in \mathbb{R}^3$ pixel homogenous coordinates.
Projectivity : collineation : proj. transformation : homography
A point in an image projected to the world is known up to a scale factor.

7.2 Transformations Hierarchy (2D)

- Projective 8 DoF - colinearity.
- Affine 6 DoF - parallelism, ratio of areas, ratio of lengths of parallel lines.
- Similarity 4DoF - ratios of lengths, angels.
- Euclidean (3DoF).

7.2.1 Planar Homography

Relates planar images. Has eight DoFs, can be determind from four point correspondnces. (basically we're estimating the perspective transformation from one plane to another)
From point relations in Euclidean coordinates, can derive a system for the homography.
This results in a system $Ah = 0$ which is solved using SVD. The system is either determined exactly and overdetermind, in that case the smallest eigenvalue is a measure of the quality of the fit.
When working with homogenous coordinates, apply homography to homogenous coordinates and then divide by z.

Homography Line Transformation $l' = \mathbf{H}^{-T} l$
Ideal Points - intersection points of parallel lines

7.3 3D Homography

Plane normal equation: $ax_1 + bx_2 + c_x 3 + dx_4 = 0$
If $\pi^T p = 0$ the point lies on the plane / the plane passes through the point
We can fit a plane to three points by solving: $\begin{pmatrix} x_1^T \\ x_2^T \\ x_3^t \end{pmatrix} \pi = 0$

7.4 Cameras and Image Formation

The camera model relates pixels and rays in space
Optical axis - usually denoted as z, faces into the world. The optical axis passes between the camera origin and the principal point in the image.
Principal point - the point at which the principal axis passes through the image plane.
Image plane - can be visualized in front of the sensor, the formulation is equivilant.

7.5 Perspective Camera Model

The camera matrix projects from the world space to image space. Using homogenous coordinates allows for translation.
The actual image plane is actually only a few mm wide.
Central Projection Model

7.6 Camera Intrinsic K

$K = \begin{pmatrix} \alpha_x & \gamma & u_0 & 0 \\ 0 & \alpha_y & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$
Five intrinsic parameters:

- f - focal length
- image sensor format - m_x and m_y are the pixel dimensions
- γ skew coefficient between x-y
- (u_0, v_0) *principal point*, usually in the middle of the sensor
- $\alpha_x = fm_x$
- $\alpha_y = fm_y$

This model cannot represent lens distortion. Usually the model is simplified to:
$K = \begin{pmatrix} f & 0 & h/2 & 0 \\ 0 & f & w/2 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

7.7 Camera Extrinsic Parameters R, T

$\begin{bmatrix} R_{3x3} & T_{3x1} \\ 0_{1x3} & 1 \end{bmatrix}$
The extrinsic parameters define the position of the camera center and the camera's heading in world coordinates. T is the position of the origin of the world coordinate system expressed in coordinates of the camera-centered coordinate system.
The actual camera position in world coordinates is $C = -R^{-1}T = -R^T T$

7.8 Camera Intrinsic Estimation

1. For capturing a planar object:
1. set the planar object as the infinity plane: $x = (x_1 \quad x_2 \quad 0 \quad 1)^T$
2. estimate the transformation for each point using homography relations:
$x \times Hx = 0$ resolves to a series of equations that can be solved for h.
2. Normalize the points:
- Translate points s.t. centroid is at origin - Isotropic - mean distance from origin of $\sqrt{2}$
3. minimize the 2 sided reprojection error - from img1 -¿ img2 and from img2 -¿ img1
4. This relates to the maximum likelihood estimate
Given $n \geq 4$ 2D to 2D point correspondences $x_i \leftrightarrow x_i\prime$,
Determine the Maximum Likelihood Estimation of H
(this also implies computing optimal $xi' = Hxi$) Algorithm
(i) Initialization compute an initial estimate using normalized DLT or RANSAC
(ii) Geometric minimization of symmetric transfer error:
- Minimize using Levenberg-Marquardt over 9 entries of h or reprojection error:
- compute initial estimate for optimal x_i - minimize cost over $H, x1, x2, ..., xn$ - if many points, use sparse method

7.8.1 Other Considerations

- Radial lens distortion - Rolling shutter effects

8 Image Features and Matching

Local features - compact description of image regions.
Detectors are used to find salient structures in images. Common salient structures include: - corners - blobs - keypoints
A descriptor is a compact representation of the image region around that keypoint. Descriptors allow to establish matches between images by comparing descriptors. Descriptors should allow for subpixel localization.

8.1 Invariant Descriptors & Matching

Feature matching: extract features independently and match by comparing descriptors.
Feature tracking: extract at first frame, find same feature in the next frame
Image features may go through the following transformations:
Geometric transformations:

- Translation, rotation, scaling
- Perspective foreshortening

Photometric transformations:

- Non-diffuse reflections
- Illumination

Good descriptors and detectors are invariant to these transformations.
Desirable Properties

- Precise (sub pixel) localization
- Repeatable detections under rotation, tranlation, illumination, perspective distortion.
- Detect distinct/salient features

Feature Points - distinct points in image (i.e. corners)

9 Feature Detection

/TODO - all of the data in this part of the document was lost due to a bug in the text editor. Redo.

9.1 Harris Corner Detector

Stable image features should maximize the uniquness of the region, which is measured by auto-correlation.

$$\mathbf{A} = \begin{pmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{pmatrix}$$

- Cornerness depends on the eigenvalues λ_1, λ_2 of the auto-correlation matrix.
- homogenous - both small
- edge: one large, one zeroish
- corner: both large

Choosing local maxima as keypoints.
Subpixel accuracy by fitting quadratic
Variant to affine and scale transformation.

9.2 SIFT Features

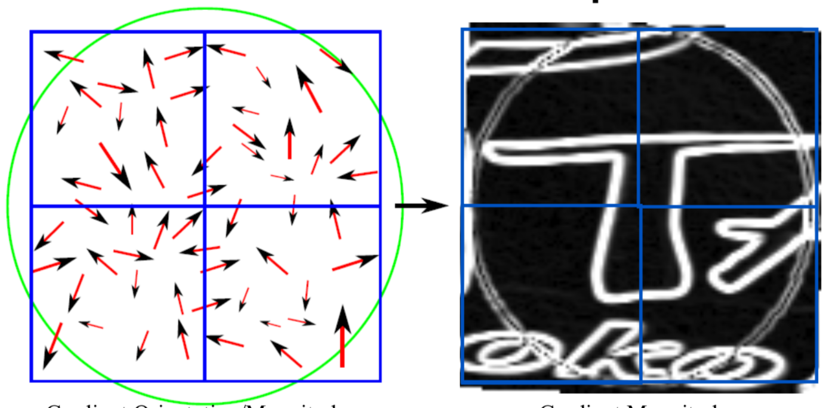
SIFT - Scale invariant image transform.
Difference of Gaussians generates candidates. Consider local extrema in scale and spatial space. Invariant to translation, rotation and scale. Quad fit for subpix accu.
Orientation Assigment - Compute gradient for each pixel in patch at selected scale - Bin gradients in histogram & smooth histogram - Select canonical orientation at peak(s) - Keypoint = 4D coordinate 0 (x, y, scale, orientation)

9.3 Affine Invariant Features

Scenario - extreme wide baseline matching
Maximally Stable Extremal Regions (MSER)
Detects extremal regions which are brighter / darker than surrounding Region is a connected component Compute its centroid + PCA Fit ellipse to canonical circle Compute orientation and re-orient in canonical space.

9.4 Lowe's SIFT Descriptors

Local descriptors based on gradient magnitude and orientation.
Ignore pixel values, use only local gradients
Gradient direction more important than positions
Partition into sectors to retain spatial information
Thresholded image gradients are sampled over 16x16 array of locations in scale space
Create array of orientation histograms
8 orientations x 4x4 histogram array = 128D
Descriptor size was chosen based on careful parameter tuning.
Hard vs. Soft Binning
Hard binning results in discontinous descriptors with small changes.
Soft binning - gradual changes to descriptor.



10 Feature Matching

Comparing image descriptors

10.1 Similarity Metrics for Patch/Line Matching

SSD - Sum of Squared Distances $SSD = \sum_x \sum_y (I(x,y) - I'(x,y))^2$
Only translation invariant.
$MSD = \frac{1}{2xy} \sum_{i,y} \left| P^{(i)}_{x,y} - P^{(j)}_{x,y} \right|^2$

10.1.1 Zero-Mean Normalized Cross Correlation

Consider two real valued functions f, g differing only by an unknown shift along the x-axis (i.e. disparity). One can use the cross-correlation to find how much g must be shifted along the x-axis to make it identical to f
The probability density of the difference $Y - X$ is formally given by the cross-correlation.
The formula essentially slides the g function along the x-axis, calculating the integral of their product at each position.

$$NCC = \frac{N(I', I)}{\sqrt{N(I,I)N(I',I')}}$$

$$N(I', I) = \sum_{xy} (I(x,y) - \bar{I})(I'(x,y) - \bar{I}')$$

This works for uniform illum. changes But even this is still fragile (i.e. non uniform changes)

10.2 Binary Descriptors

SIFT is powerful descriptor, but slow to compute

- Faster alternative: Binary Descriptors:
- Idea: Compute only sign of gradient
- Efficient test: Compare pixel intensities
- Random comparisons work already very well

Pros

- Efficient computation
- Efficient descriptor comparison via Hamming distance (1M comparison in 2ms for 64D)

Cons

- Not as good as SIFT / real-valued descriptors
- Many bits rather random = problems for efficient nearest neighbor search

10.3 Feature Matching

Spatial Search Window:

- Requires/exploits good prediction
- Can avoid far away similar-looking features
- Good for sequences

Descriptor Space:

- Initial tree setup
- Fast lookup for huge amounts of features
- More sophisticated outlier detection required
- Good for asymmetric (offline/online) problems, registration, initialization, object recognition, wide baseline matching
- Huge memory demands

10.4 Feature Tracking

Identify features and track them over video
Small difference between frames
Potential large difference overall

10.5 KLT - Kanade Lukas Tomasi

- Using the auto-correlation matrix, assumption that the motion is small.
- Linearize and solve.
- Multi scale (coarse to fine), iterate and refine over all image scales.
- Assumes brightness constancy.

Problem Affine model tries to deform sign to shape of window, tries to track this shape instead. **Solution** Perform affine alignment between first and last frame, stop tracking features with too large errors.

10.5.1 Aperture Problem

Assumption: neighbors have same displacement

10.5.2 Summary

Motivation: Exploit small motion between subsequent (video) frames
Key ideas: - Brightness constancy assumption - Linearize complex motion model and solve iteratively - Use simple model (translation) for frame-to- frame tracking - Compute affine transformation to first occurrence to avoid switching tracks

11 Stereo Vision

Two cameras, C_L and C_R
Their respective optical centers O_L and O_R
The world space point X and its projection in each one of the cameras: x_L, x_R

11.0.1 Epipolar Point

The epipolar points e_L, e_R are defined as the points where the baseline intersects each one of the camera images, or the center of each camera as projected into the other cameras image.

11.0.2 Epipolar Line

A line segment between the epipolar point and the projection of the X on the image.
A second epipolar line segment is the projection of this line onto the first camera image plane
A point in one image generates a line in the other on which its corresponding point must lie

11.0.3 Epipolar Plane

A plane that passes through both camera centers.

11.1 Essential Matrix

For two points in two images of a camera stereo pair which correspond to the same $3D$ world position, the following is true: $\mathbf{y}'^T\mathbf{E}\mathbf{y} = 0$
This relates the two calibrated cameras.
The essential matrix can be seen as a precursor to the fundamental matrix.
The essential matrix can only be used in relation to calibrated cameras, it requires known intrinsic camera parameters (for normalization).

The essential matrix can be useful for determining both the relative position and orientation between the cameras and the 3D position of corresponding image points.
E is an essential matrix iff two of its singular values are equal, third is 0.
$\mathbf{E} = \mathbf{RS}$
where
$\mathbf{S} = \begin{pmatrix} 0 & -T_z & T_y \\ T_z & 0 & -T_x \\ -T_y & T_x & 0 \end{pmatrix}$
weird transformation matrix with 3 DoF and R also has 3 DoFs
rank 2
Has both left and right nullspaces
Depends only on extrinsic parameters

11.2 Fundamental matrix

$\mathbf{F} \in \mathbb{R}^{3\times3} = \mathbf{M}_r\mathbf{RSM}_l^{-1}$
The Fundamental matrix relates corresponding points in stereo images. $\mathbf{X}'^T\mathbf{Fx} = 0$ and $\mathbf{F}_{3x3}$
Analogous to essential matrix. The fundamental matrix relates pixels (points) in each image to epipolar lines in the other image.
It is related to the essential matrix $\mathbf{E} = \mathbf{K}'^T\mathbf{FK}$ where $\mathbf{K}', \mathbf{K}$ are in the intrinsic matrices of both cameras.
$rank(\mathbf{F}) = 2$
Here a bunch notes were deleted.

11.3 Eight Point Algorithm

`https://en.wikipedia.org/wiki/Eight-point_algorithm`
From homography relation $x'Fx = 0$, F is rank 2, has seven DoF.
Eight points define a linear system, which is it contains no errors, can be solved as SVD and the smallest column of V defines F.
Otherwise this is solved in the least square sense and one computes a F' which is most similar to F and also $\|F\| = 1$. Usually compute SVD for F and drop the smallest eigenvector.
Since the whole algorithm is numerically sensitive, all points and matches need to be normalized.

11.4 Singularity Constraint - Seven Point Algorithm

From 7x9 matrix, compute eigenvalues. Solve up to $F = \alpha F_1 + (1-\alpha)F_2$ Enforce a constraint by: $\det(F) = 0$ (cubic in λ) cubic in α =¿ 1 or 3 solutions.

11.5 Five Point Algorithm

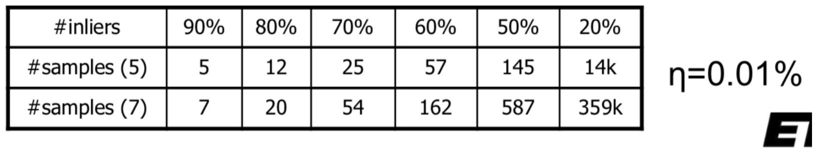
For the calibrated case, only compute **E**
Generate a 5x9 matrix for 9 enteries of **E**
$4D$ solution space, w = 1 reduces one dimension.
10 cubic polynomials generated from Perform Gauss-Jordan elimination on polynomials.

11.6 Automatic Computation of F

1. Extract features 2. Compute a set of potential matches 3. Robust estimation of **F** via RANSAC 4. Compute **F** based on all inliers 5. Look for additional matches 6. Refine **F** based on all correct matches

11.7 Robustness to False Matches - RANSAC

Number of samples requires depending on dataset size and # of inliers for high certainty that the samples contains an inlier set: $\eta = 0.01$



#inliers	90%	80%	70%	60%	50%	20%
#samples (5)	5	12	25	57	145	14k
#samples (7)	7	20	54	162	587	359k

η=0.01%

Restricted search around epipolar line (e.g. 1.5 pixels) Relax disparity restriction (along epipolar line)

12 Structure From Motion

12.1 Sequential / Incremental SfM

1. Initialize Motion
2. Initialize Structure
3. Extend Motion
Initialize:

- Compute pairwise epipolar geometry
- Find pair to initialize structure and motion
- Repeat:
- For each additional view

- Determine pose from structure
- Extend structure
- Refine structure and motion

Recap Essential matrix: $E = [t]$ • Motion for two cameras: [I—0], [R—t] • Essential Matrix decomposition: E=UVT • Recover E and t as • t=u3 or t=-u3 • R=UWVT or R=UWTVT • Four solutions, but only one meaningful
Pose Estimation from 2D-3D Matches
Generate hypothesis using 6 points (two equations per point) - planar scenes are degenerate!
Stereo Constraints and their implications:

- 1-D Epipolar Search - Arbitrary images of the same scene may be rectified based on epipolar geometry such that stereo matches lie along onedimensional scanlines. This reduces the computational complexity and also reduces the likelihood of false matches.
- Monotonic Ordering -Points along an epipolar scanline appear in the same order in both stereo images, assuming that all objects in the scene are approximately the same distance from the cameras.
- Image Brightness Constancy - Assuming Lambertian surfaces, the brightness of corresponding points in stereo images are the same. Match Uniqueness For every point in one stereo image, there is at most one corresponding point in the other image.
- Disparity Continuity - Disparities vary smoothly (i.e. disparity gradient is small) over most of the image. This assumption is violated at object boundaries.
- Disparity Limit - The search space may be reduced significantly by limiting the disparity range, reducing both computational complexity and the likelihood of false matches.
- Fronto-Parallel Surfaces - The implicit assumption made by area-based matching is that objects have fronto-parallel surfaces (i.e. depth is constant within the region of local support). This assumption is violated by sloping and creased surfaces.
- Feature Similarity - Corresponding features must be similar (e.g. edges must have roughly the same length and orientation).
- Structural Grouping - Corresponding feature groupings and their connectivity must be consistent.

Photometric issues: specularities, strongly non-Lambertian BRDF's. **Surface structure:** lack of texture, repeating texture within horopter bracket **Geometric ambiguities:** as surfaces turn away, difficult to get accurate reconstruction (affine approximate can help); at the occluding contour, likelihood of good match but incorrect reconstruction.
Motion Initialization: Cheirality Constraint

12.2 Global SfM

Initialize: • Compute pairwise epipolar geometry
Compute: • Estimate all orientations • Estimate all positions • Triangulate structure • Refine structure and motion (bundle adjustment)
Pros: More efficient, more accurate
Con: Less robust
SfM approaches often have to work on an unordered set of images often computed in the cloud with little to no time constraints and might employ different cameras
One of the challenges in SfM is to retrieve near-by images, and add images one by one to a growing graph while accounting for potential outlier images so that robust reconstruction maybe performed. (i.e. that MS/GOOG Building Rome in a Day also talked a lot about system design)
SFM might be employed on a huge set of potentional images. It's a challenge to retrieve near-by images, and add images one by one to a growing graph while accounting for potential outlier images so that robust reconstruction maybe performed.

13 Dense Correspodences and Stereo Matching

13.1 Triangulation

13.2 Disparity

Definition: difference in image location of the same 3D point between stereo images
Task: construct 3D model from 2 images of a calibrated camera.

1. Find corresponding points
2. Estimate epipolar geometry
3. Rectify images
4. Dense feature matching

5. 3D reconstruction

cv/disparity.png

Baseline - dist. between camera centers

- f - focal length
- d = disparity between the points
- z = dist from object

From triangle similarity: $\frac{B}{z} = \frac{d}{f}$
Looking at this relationship in depth:
$d = \frac{Bf}{z}$
$\frac{dd}{dz} = -\frac{Bf}{z^2}$
$dd = \frac{f}{B}dz$
$\Delta z = \frac{Z^2}{Bf}dd$
$(x', y') = (x + D(x, y), y)$
Depth resolution is better when the camera is closer to the objects.
The disparity between two points in a stereo pair is inversely proportional to their distance from the observer.

13.3 Multiple View Geometry - Single Center of Projection

Three camera views are related via a trifocal tensor
Having multiple cameras close together results in better depth resolution, less noise, etc.

13.4 Rectification

Pre-warping images such that the corresponding epipolar lines are coincident
For a rectified image pair:

- All epipolar lines are parallel to the horizontal axis of the image plane
- Corresponding points have identical vertical coordinates.

Rectification can be done for image pairs, but may prove impossible for a collection of random cameras, unless they are "parallel" of some sort
How to compute rectification?
1. Rotate both cameras s.t. they're perpendicular to the line connecting both camera centers - using the smallest rotation possible and relying on the freedom of tilt. 2. To determine the desired twist around the optical axes, make the up vector perpendicular to the camera center line -¿ the corresponding epipolar lines are horizontal and the disparity for points at infinity is 0. 3. Rescale images if necc.
Then the pixel matching can be done for a single dimension on every scanline - reduces the dimensionality of the problem to 1D search

13.4.1 How does it look in math?

Assuming one camera is $K = [I0]$
YK: TODO

13.5 Assumptions for Stereo Matching

- Small baseline - Lower precision and higher correspondences / similar appearance - Most scene points are visible in both images - Image regions are similar in appearance
Left view images will move to the left in the right image - optimization by maximizing normalized cross-correlation.
Uniqueness Constraint In an image pair each pixel has at most one corresponding pixel, if occlusion none.
Block matching has an assumption about frontal view.

13.6 Graph Cut

Treats Stereo Matching as Energy Minimization
Data term + disparity smoothness term (x) + disparity smoothness term (right)
NP-hard using graph cuts or belief propagation (2-D optimization)
Instead do dynamic programming along many directions Don't use visibility or ordering constraints Add costs of all paths
Patch Match Stereo

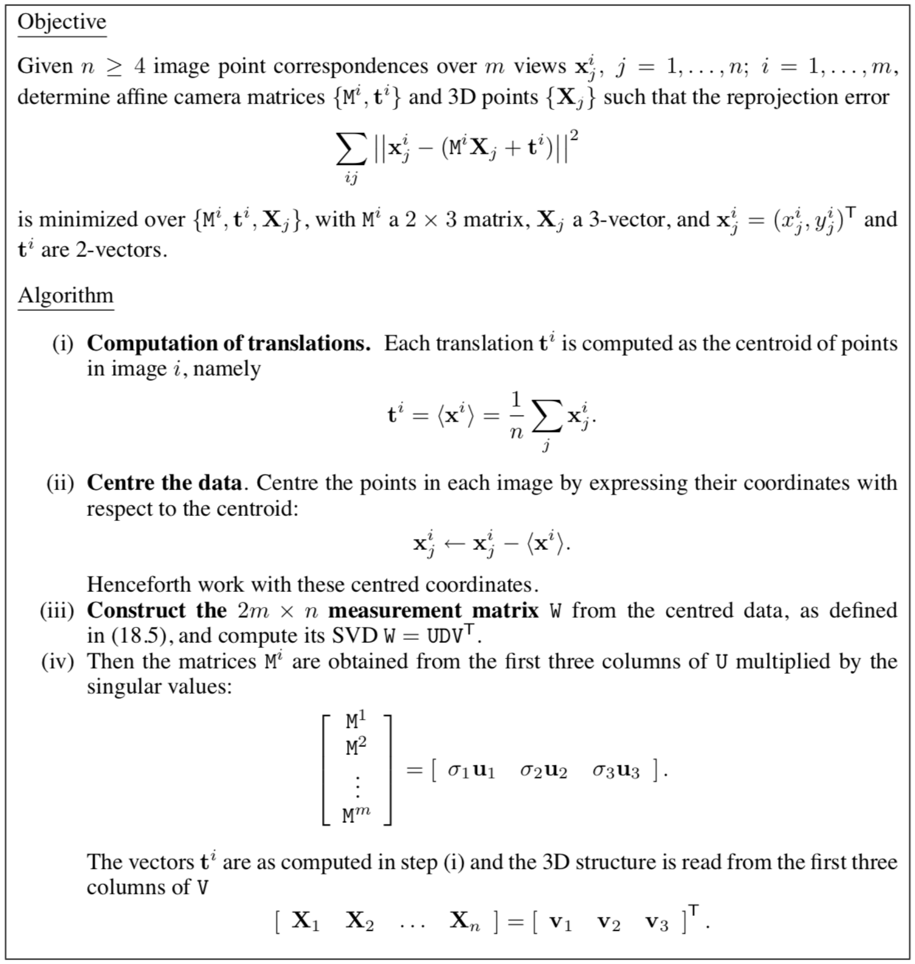
14 Stereo Reconstruction Pipeline

14.1 Stereo Photogrammetry

Small vs large baseline:
robust binocular stereo point matching adaptive point-based filtering of the merged point clouds, and efficient, high-quality mesh generation.

14.1.1 Bundle Adjustment

Bundle adjustment amounts to jointly refining a set of initial camera and structure parameter estimates for finding the set of parameters that most accurately predict the locations of the observed points in the set of available images.
The asumption is that image measurements are noisy then the equations $x_i = P_i X$ will not be satisfied exactly.
Bundle adjustment is a maximum likelihood solution (maximizes the probability of the model) under Gaussian noise assumption.
We wish to estimate projection matrices P^i and 3D points X j which project exactly to image points x^ij as x^ij = P^iXj, and also minimize the image distance between the reprojected point and detected (measured) image points xij for every view in which the 3D point appears,
Input: n $3D$ points, m views, x_{ij} is the projection of theh ith point on image j. v_{ij}
This estimation involving minimizing the reprojection error
$\min \sum_{i,j} d(P^{Xj}, x^i_j)^2$ in image space.
`https://wikimedia.org/api/rest_v1/media/math/render/svg/c3d1bb3a51f8bafc07c30a99c3f3f15e008d0259` denote the binary variables that equal 1 if point i is visible in image j. Assume also that each camera j is parameterized by a vector $\mathbf{a}_j$ and each 3D point i by a vector $\mathbf{b}_i$.
Bundle adjustment minimizes the total reprojection error with respect to all 3D point and camera parameters, specifically
It's advanatages: tolerant to missing data (sum), while providing a true ML estimate. Allows assignment of individual covariances to each measurement may also be extended to include estimates of priors and constraints on camera parameters or point positions
Cons: requires a good initialization can become an extremely large minimizition problem (solution: use last N (key)frames)
Iterative minimization:
Since each camera has 11 degrees of freedom Each 3D space point 3 degrees of freedom
n points over m views requires minimization over 3n + 11m parameters.
For Levenberg–Marquardt algorithm the matrix dimensions are (3n + 11m) (3n + 11m).
Factorizing this is on the scale of expensive to infeasible. Strategies including interleaving (block coordinate descent) camera and geometry optimization and limiting the observations.
ML Bundle Adjustment for Affine Camera Model



Objective

Given $n \geq 4$ image point correspondences over m views $\mathbf{x}^i_j$, $j = 1, \ldots, n$; $i = 1, \ldots, m$, determine affine camera matrices $\{\mathtt{M}^i, \mathbf{t}^i\}$ and 3D points $\{\mathbf{X}_j\}$ such that the reprojection error

$$\sum_{ij} \left|\left|\mathbf{x}^i_j - (\mathtt{M}^i \mathbf{X}_j + \mathbf{t}^i)\right|\right|^2$$

is minimized over $\{\mathtt{M}^i, \mathbf{t}^i, \mathbf{X}_j\}$, with $\mathtt{M}^i$ a 2×3 matrix, $\mathbf{X}_j$ a 3-vector, and $\mathbf{x}^i_j = (x^i_j, y^i_j)^\mathsf{T}$ and $\mathbf{t}^i$ are 2-vectors.

Algorithm

(i) **Computation of translations.** Each translation $\mathbf{t}^i$ is computed as the centroid of points in image i, namely

$$\mathbf{t}^i = \langle \mathbf{x}^i \rangle = \frac{1}{n} \sum_j \mathbf{x}^i_j.$$

(ii) **Centre the data**. Centre the points in each image by expressing their coordinates with respect to the centroid:

$$\mathbf{x}^i_j \leftarrow \mathbf{x}^i_j - \langle \mathbf{x}^i \rangle.$$

Henceforth work with these centred coordinates.

(iii) **Construct the** $2m \times n$ **measurement matrix** $\mathtt{W}$ from the centred data, as defined in (18.5), and compute its SVD $\mathtt{W} = \mathtt{UDV}^\mathsf{T}$.

(iv) Then the matrices $\mathtt{M}^i$ are obtained from the first three columns of $\mathtt{U}$ multiplied by the singular values:

$$\begin{bmatrix} \mathtt{M}^1 \\ \mathtt{M}^2 \\ \vdots \\ \mathtt{M}^m \end{bmatrix} = [\ \sigma_1 \mathbf{u}_1 \quad \sigma_2 \mathbf{u}_2 \quad \sigma_3 \mathbf{u}_3\ ].$$

The vectors $\mathbf{t}^i$ are as computed in step (i) and the 3D structure is read from the first three columns of $\mathtt{V}$

$$[\ \mathbf{X}_1 \quad \mathbf{X}_2 \quad \ldots \quad \mathbf{X}_n\ ] = [\ \mathbf{v}_1 \quad \mathbf{v}_2 \quad \mathbf{v}_3\ ]^\mathsf{T}.$$

Source: Multiple View Geometry
$\min_{\mathbf{a}_j, \mathbf{b}_i} \sum_{i=1}^{n} \sum_{j=1}^{m} v_{ij}\, d(\mathbf{Q}(\mathbf{a}_j,\, \mathbf{b}_i),\ \mathbf{x}_{ij})^2$
where $\mathbf{Q}(\mathbf{a}_j,\, \mathbf{b}_i)$ is the predicted projection of point i on image j and $d(\mathbf{x},\, \mathbf{y})$ denotes the Euclidean distance between the image points represented by vectors $\mathbf{x}, \mathbf{y}$

Bundle adjustment is tolerant to missing image projections
Minimizes a physically meaningful criterion
This is typically solved using Levenberg–Marquardt Algorithm
When solving the minimization problems arising in the framework of bundle adjustment, the normal equations have a sparse block structure owing to the lack of interaction among parameters for different 3D points and cameras.

LM Here the idea is that the energy terms are separable, and when structured in a matrix we have a distinct block structure with a diagonal block which depends on position only.
Computing its inverse is easy, which allows to solve for a correction in positions, which then reduce the problem of the solving the correction to the camera parameters to solving a smaller linear system.
The normal equation is then solved by Cholesky factorization, as the matrix is not necc. invertible and its inverse is not sparse.

Cost Function LM requires a cost function which is robust to outliers. LS assumes a Gaussian noise distribution and independent observations, this will lead to poor convergence in the presence of any outliers.
Using a Cauchy or Huber loss is more robust.

Reconstruction from Sequences There is an ordering on the images Small baseline - easy to identify correspondences, both in appearance
The disadvantage of a small baseline is that the 3D structure is estimated poorly. However, this disadvantage is mitigated by tracking over many views in the sequence so that the effective baseline is large.

Feature Tracking Over the sequence, have to think about how the appearance of the feature changes. Inc. updating the features can cause to drift in localization and introduce error, while affine mapping to a keyframe at every step is also not likely to work.

14.1.2 Surface Reconstruction

- Match points and compute depth field
- Approximate normals i.e. approximating the planarity from the point neighborhood
- Reconstruct surface - for example, fit planes, or Poisson surface reconstruction

15 Bundle Adjustments and SLAM

Refinement step in Structure-from-Motion.
Produce jointly optimal 3D structures P and camera poses C.
Minimize total re-projection errors z.
The cost function $\text{argmin}_X \sum_i \sum_j \Delta z_{ij}^T W_{ij} \Delta z_{ij}$
W_{ij} : Measurement error covariance $X = [P, C]$

Minimization Techniques

- GD - slow convergence near minimum
- Newtom method - second order approx, requires inverse Hessian computation - expensive
- Gauss - Newtom - estimate the Hessian $H = J^T W J$ and solve using normal equation - might get stuck and slow convergence
- Levenberg-Marquardt - Regularized Gauss-Newton with damping factor. $\lambda \to 0$: Gauss-Newton (when convergence is rapid) $\lambda \to \infty$: Gradient descent

Solving the normal equation can be computationally inefficient. Schur-Complement techinque is used to accelerate the s
(H HT H1H) HT H1 C SC S SC C C S SC S Ax b Don't solve as x=A-1b: A is sparse, but A-1 is not! • Use sparse matrix factorization to solve system 1. LU Factorization 2. QR factorization 3. Cholesky Factorization
Visual SLAM works in real-time on an ordered sequence of images acquired from a fixed camera set-up (i.e. one or two particular cameras). large scale visual SLAM is typically restricted to trajectories of a few kilometers.
SfM approaches often have to work on an unordered set of images often computed in the cloud with little to no time constraints and might employ different cameras, have been scaled to work on the "planet" level.
Bundle adjustment (BA), pose-graph optimization (BA without optimizing for 3D points, or motion-only BA), or more generally some sort of non-linear optimization is employed in many state-of-the-art SLAM systems.
BA is expensive, in SLAM there isn't enough computational budget to run BA on all frames - usually only performed on last N (key)frames.
SLAM approaches try to cut corners when it comes to feature descriptors and matching, really every stage of the pipeline, to ensure real-time performance in a budget.
In visual SLAM, this problem is just not there since you have an ordered set - you know that neighbouring images are expected to heavily overlap with each other - making the problem easier.

15.0.1 SLAM with Depth Sensors

It isn't quite correct to say you avoid the correspondence problem, but the depth information is a big advantage. One common approach is to use the iterative closest point algorithm to align the next depth map to the previous one (or to the map you are building up over time in the case of SLAM), which works when the frame-rate is high enough to expect overlap between every depth-map and for the initialization to be close enough to the correct answer to converge. This way you find correspondence between all the points at once in 3D space, without the difficult search associated with feature matching between RGB images.

15.0.2 Loop Closure / Relocalization

In Visual SLAM, the robot/camera begins from the origin and explores its environment while keeping a record of its location with respect to the origin (odometry) and creating a sparse or dense map of the environment. A perfect odometry (visual?) would solve the visual SLAM problem without ever requiring another essential component of the SLAM system, that is, visual place recognition.
Visual Place Recognition is an integral and common part of both Relocalization and Loop Closure in visual SLAM. The idea is to parse the entire database of images and find the best matching hypothesis for the current image. This implies that for both relocalization and loop closure, it is a mandatory condition that the current image is actually a revisited place and we have a matching reference image stored within our database.
So, when do we need Relocalization?
As the name suggests, it is a re-localization, that is, the robot in its current state is no more aware of its location within the map. This generally happens when visual odometry fails, that is, the robot is unable to track its pose/position due to lack of sufficient matching between the current and recent previous images. Therefore, visual place recognition is called for help and after finding a confident match from within the entire database, the robot pose is re-estimated with respect to the map.
What about Loop Closures?
The primary purpose of loop closures is to overcome the drift accumulated in the robot trajectory over the time. The odometry based on motion sensors as well as visual information is prone to errors, drifting the estimated trajectory from its actual ground truth. Therefore, intermittent searches are generally performed (using visual place recognition) to detect revisited places in order to close the loop (matched pair of places). This nullifies the drift as now we have additional information about our location within the map along with the odometry estimate.
Summary:
The main objective of Loop Closures is to correct the robot/camera trajectory, while Relocalization helps in recovering from a 'lost' state. Both rely on the fact that the current image belongs to a seen/pre-visited place and therefore both require visual place recognition to achieve their objective.

16 SLAM

SLAM uses scene matches over the previous N frames to estimate camera pose and 3D keypoint locations. There are different algorithms that can do this task:

16.1 Kalman Filters

Kalman filtering, also known as linear quadratic estimation (LQE), is an algorithm that uses a series of measurements observed over time, containing statistical noise and other inaccuracies, and produces estimates of unknown variables that tend to be more accurate than those based on a single measurement alone, by estimating a joint probability distribution over the variables for each time-frame

16.2 Particle Filters

YK: TODO
Visual SLAM is supposed to work in real-time on an ordered sequence of images acquired from a fixed camera set-up. Large scale visual SLAM is typically restricted to trajectories of a few kilometers.
FSM - structure. not necc. coherent map
SLAM - structure + map
SLAM is more complete than BA/SFM since SLAM provides 3D structures, camera localization (the L of SLAM) and mapping.
SLAM there isn't enough computational budget to run BA on all frames, and the time constraints are tough. SLAM approaches try to cut corners when it comes to feature descriptors and matching, really every stage of the pipeline, to ensure real-time performance in a budget.
The matching problem is easier, as the neighboring images are expected to heavily overlap with each other and are known (and sequential).

16.2.1 Depth Map Matching

A common approach is to use the iterative closest point algorithm to align the sequential depth maps to the previous one (or to the map), which works when the frame-rate is high enough to expect overlap between every depth-map and for the initialization to be close enough to the correct answer to converge. This way all correspondence are computed at once in 3D space, without difficult search associated with feature matching between RGB images.

16.2.2 Loop Detection/Closure

Recognizing features/structures that are already seen. This is used to correct camera's trajectory when it comes back to its starting point and minimize drift.

16.2.3 Visual SLAM a la Dai. 2017

1. SIFT features are detected and matched to the features of all previously seen frames.
We use SIFT as it accounts for the major variation encountered during hand-held RGB-D scanning, namely: image translation, scaling, and rotation. Potential matches between each pair of frames are then filtered to remove false positives and produce a list of valid pairwise correspondences as input to global pose optimization
2. Correspondence Filtering. To minimize outliers, we filter the sets of detected pairwise correspondences based on geometric and photometric consistency.
3. Key Point Correspondence Filter: For a pair of frames fi and fj with detected corresponding 3D points P from fi , and Q from fj , the key point correspondence lter nds a set of correspondences which exhibit a stable distribution and a consistent rigid transform. Use this to compute a RMSD (Kabsch Algorithm)
4. Surface Area Filtering
5. Dense Verication Finally, we perform a dense two-sided geometric and photometric verication step. For frames fi and fj , we use the computed relative transform Tij from the key point correspondence lter to align the coordinate systems of fi and fj . We measure the average depth discrepancy, normal deviation and photoconsistency of the re-projection in both directions - This is potentially sensitive to occlusion error, so we discard correspondences with - high depth discrepancy - normal deviation, - lack of photoconsistency

16.2.4 Sparse Volumetric Representation

17 Object Detection

Before deep learning, was a several step process:
1. edge detection and feature extraction using techniques like SIFT, HOG 2. Build multi-scale object representation 3. Descriptor were then compared with existing object templates to detect objects 4. Localize objects present in the image.
For example, for pedestrian detection:
SVM template + image pyramid -¿ template matching

17.0.1 Quality Metrics

Intersection over Union (IoU) :Bounding box prediction cannot be expected to be precise on the pixel level, and thus a metric needs to be defined for the extend of overlap between 2 bounding boxes.
Average Precision and Average Recall : Precision meditates how accurate are our predictions while recall accounts for whether we are able to detect all objects present in the image or not. Average Precision (AP) and Average Recall (AR) are two common metrics used for object detection.

17.1 Face Detection

Haar Cascades
Face Landmark Detection (~ 60)
3DDM Face model (identity, expression,
Basel Face Model (BFM)

17.2 QR Detection

The idea is that the feature has a distinct signature of $++--++$ so looking for signatures like this in the image, even on line by line, can quickly localize candidates for QR detection.
Another conclusion from this interview question:
Think about the function representation of the feature how the image and how it can be detected quickly. For example - as row traversal operations.
This rough estimate can be refined later.

18 Machine Learning

19 Resources

- https://github.com/afshinea/stanford-cs-229-machine-learning
- https://ml2.inf.ethz.ch/courses/aml/
- https://las.inf.ethz.ch/pai-f19
- https://www.coursera.org/learn/machine-learning/lecture/zcAuT/welcome-to-machine-learning

Regression: predict real-valued output
Classification: discrete valued outputs

20 Supervised Learning

Training set - with m number of training examples, x input variables / features, y outputs/targets
$(x^{(i)}, y^{(i)})$ is a training example

20.1 Linear Regression (Uni-variable)

Hypothesis: $h_\theta(x) = \theta_0 + \theta_1 x$
Cost function: a function that measures the performance of the hypothesis
For linear regression:

$$\min_{\theta_0,\theta_1} \frac{1}{2m} \sum_i |h_\theta(x^{(i)}) - y^{(i)}|^2$$

Squared Error Cost Function: $J = \frac{1}{2m} \sum_i (h_\theta(x^{(i)}) - y^{(i)})^2$

20.1.1 Gradient Descent for Linear Regression

For linear regression - the least squared cost function has no local minimum
GD will converge
Normal Equations can be used to perform a single step solution for linear models, but GD scales better for large training sets

20.1.2 Stochastic Gradient Descent

Computes the gradient with respect to each training example directly and aggregates it.
Can converge to a minimum much faster than batch gradient descent

20.2 Multi Variable Linear Regression

For n features, define $x \in \mathbb{R}^{n+1}$, $0th$ indexed vector, the features vector, where $x_0^{(i)} := 1 \forall i$
And $\theta = (0_0, \dots, \theta_n)$ the model
The hypothesis: $h_\theta = \sum \theta_i x_i = \theta^T x$
Update rule for linear regression:
$\theta_0 = \theta_0 - \alpha \frac{1}{m} \sum_i (h_\theta(x^{(i)}) - y^{(i)}) \cdot x_0$
and similarly for all other variables

20.2.1 Feature Scaling

If features are of very different dimensions, the cost function will have skewed contours in the energy landscape. The gradient descent has this ping-pong behavior.
It helps to scale the parameters to approx. $-1 \leq x_j^{(i)} \leq 1$

20.2.2 Mean Normalization

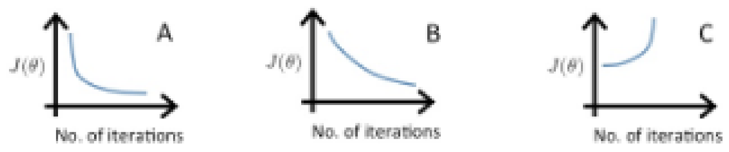
Replace x_i with $x_i - \mu_i$ to make the variable approx. 0-mean
$x_i \leftarrow \frac{x_i - \mu_i}{range}$
s = Range will be max − min

20.3 Debugging Gradient Descent

Plot the cost function when GD runs
Num of iterations depends on the algorithm / model
Automatic convergence tests:

- change in $J(\theta)$ decreases by less than 10^{-3}

If the cost function value increases, try smaller α
When visualization - either wave behavior or increase in the model
Gradient verification with FD
If α is small enough, GD should decrease for every iteration



A - good convergence
B - slow convergence
C - learning rate too high
Run GD with α with a range of values with 10-scale factor 3x from previous values
Until you find one value which is too small and one value which is too large

20.3.1 Momentum

20.3.2 Netwon

- no learning rate

For a function l with the derivative $l'(\theta)$ and second derivative, starting from an initial guess the update rule is:
$\theta := \theta - \frac{l'(\theta)}{l''\theta}$
until $l'(\theta) = 0$.
Newton method looks at the approximated tangent to $l(\theta)$ at the point θ and solves for where the line is equal to 0.

20.3.3 Newton Raphson Method

Generalization of Netwon's method to multi-variable / multi dimension settings:
$\theta = \theta - H^{-1}\nabla_{\theta} l(\theta)$
where ∇_{θ} is a vector of partial derivatives of $l(\theta)$ with respect to θ and
$H(\theta) = \frac{\partial^2 l(\theta)}{\partial\theta_i \partial\theta_j}$
Better and faster convergence than GD, but expensive, requires (careful) evaluation, Hessian needs to be invertible (full rank)
Fischer scoring - applying Newton's to logistic regression log likelihood function

20.4 Polynomial Regression

Basically, the idea here is to cheat and pre-compute the feature vector.
For example, $(x_1 := x, x_2 := x^2, x_3 := x^3)$.
The previous formulation and update rules hold: $\theta^T x$
In this case it's important to scale the variables!
Other options: sqrt, cubic, squared (which might not fit a lot of models)

20.5 Normal Equation

For a feature vector n features and m data points:
Construct a matrix $X \in \mathbb{R}^{m\times(n+1)}$ which contains all of features for all the variables + (n+1) column which contains all 1s.
$\begin{pmatrix} 1 & x_1^1 & ... & x_1^n \\ \vdots & x_2^1 & ... & x_2^n \\ 1 & x_m^1 & ... & x_m^n \end{pmatrix}$
And collect all of the observations in a vector $y \in \mathbb{R}^m$:
And we solve for a model:
$\theta = (X^T X)^{-1} X^T y$
Now, this is true only if $X^T X$ is invertible
Feature scaling is not necc. when using the normal equation.

20.6 GD vs. Normal Equation

GD

- need to choose learning rate
- need many iterations
- works well when n is large

Normal Equation

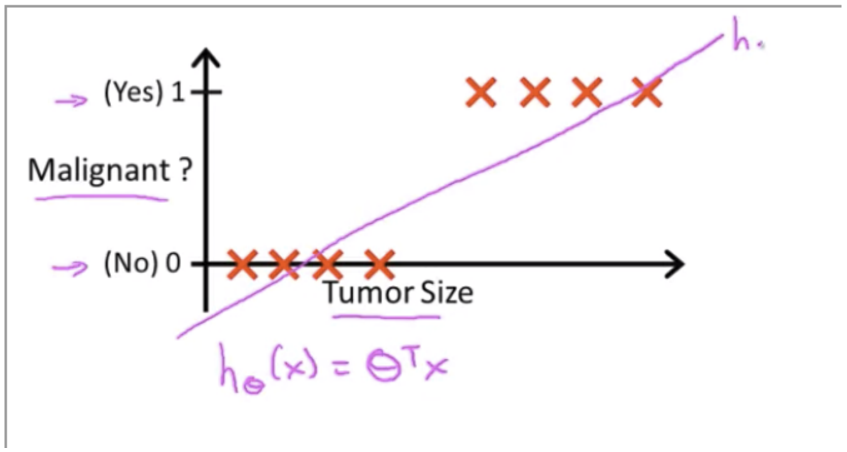
- slow for large n $O(n^3)$, n=10k is where switching over could be beneficial
- no need to choose learning rate
- direct

20.7 When is $X^T X$ non-invertible?

- linearly dependent features - i.e. size in m^2 and size in feet squared
  - remove features
- too many features $n \geq m$
  - delete features
  - use regularization

21 Classification

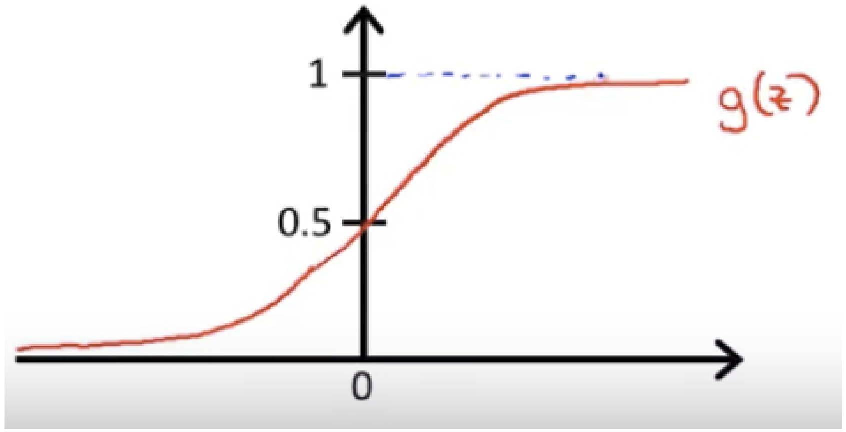
21.1 Two Class Problems



Using linear regression model + threshold:
Classification is not actually a linear function - using linear models doesn't work well.
Labels are usually 0,1 known as negative and positive classes.

21.1.1 Logistic Regression

Want a model that predict a value $0 \leq h_\theta(x) \leq 1$
Model: $h_\theta(x) = g(\theta^T x)$
Logistic/sigmoid function: $g(z) = \frac{1}{1+e^{-z}}$
Together: $h_\theta(x) = \frac{1}{1+e^{-\theta^T x}}$



Has asymptotes at 0,1

21.1.2 Interpretation of Output

$h_\theta(x)$ is the estimated probability that $y = 1$ on input x
$h_\theta(x) = P(y = 1|x;\theta)$
$P(y = 0|x;\theta) = 1 - P(y = 1|x;\theta)$

21.1.3 Decision Boundary

$g(z) \geq 0.5$ when $z > 0$
$g(\theta^T x) \geq 0.5$ when $\theta^T x \geq 0$
(basically, here we can derive this from $1 + e^{-\theta^T x} = 2$
The decision boundary is a function of the hypothesis and its parameters

21.1.4 Non Linear Decision Boundaries

Can perform a similar trick as with linear regression -¿ polynomial regression - build features such as x_1^2 etc...
So for example:

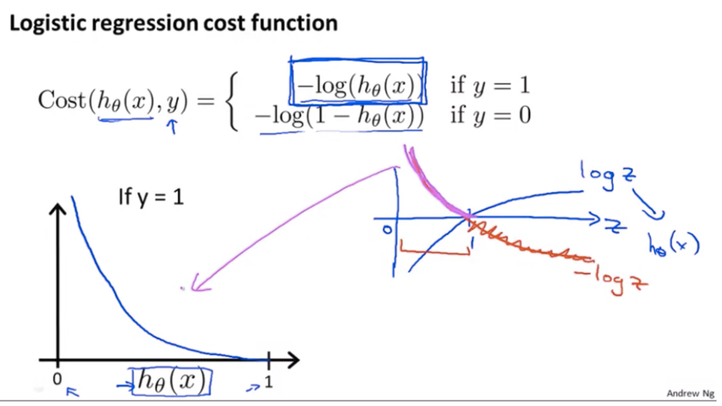
$$\theta = \begin{bmatrix} -1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$$h_\theta(x) = g(\theta^T(1, x_1, x_2, x_1^2, x_2^2))$$

The decision boundary will lie at $x_1^2 + x_2^2 = 1$

21.1.5 Cost Function

Using the linear regression cost function is non convex for the logistic regression.
$Cost(h_\theta(x),y) = \begin{cases} -\log(h_\theta(x)); \text{ if } y=1 \\ -\log(1-h_\theta(x)); \text{ if } y=0 \end{cases}$
This formulation has desirable properties:
$(h(x)=0, y=0)$ or$(h(x)=1, y=1)$ - cost = 0
Very high penalization if $(h(x)=1, y=0)$ or$(h(x)=0, y=1)$ due to the cost function going asymptotically to ∞ :



21.1.6 Simplified Cost Function

A generalized cost function is:
$Cost(h_\theta(x),y) = -y\log(h_\theta(x)) - (1-y)\log(1-h_\theta(x))$
And summarizing over all examples:
$J(\theta) = -\frac{1}{m}\sum_{i=1}^{m} y^{(i)}\log(h_\theta(x^{(i)})) + (1-y^{(i)})\log(1-h_\theta(x^{(i)}))$
To minimize, solve for parameters:
$\min_\theta J(\theta)$
Output / new prediction: $h_\theta(x) = \frac{1}{1+e^{-\theta^T x}}$
$\frac{\partial}{\partial\theta_j}J(\theta) = \frac{1}{m}\sum_i(h_\theta(x^{(i)}) - y^{(i)})x_j^{(i)}$
Exactly the same update as linear regression. Here the main difference is that h_θ went from $\theta^T x$ to $\frac{1}{1+e^{-\theta^T x}}$
And the update rules are:
$\theta_j := \theta_j - \frac{\alpha}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x_j^{(i)}$
And vectorized:
$\theta := \theta - \frac{\alpha}{m}X^T(g(X\theta) - \vec{y})$

21.2 Maximum Likelihood Estimation + Convexity

Convexity: gives us lower bounds on the first order approximation of the function (i.e. the first order approximation is guaranteed to be larger than or equal to the real function value).
Assuming that the target variables and input are related via the equation:
$y^{(i)} = \theta^T x^{(i)} + \epsilon^{(i)}$
where ϵ are IID (independently and identically distributed) error terms the captures unmodeled effects, i.e random noise.
Assuming $e^i \sim \mathcal{N}(0,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma}}\exp\left(-\frac{(\epsilon^{(i)})^2}{2\sigma^2}\right)$
That implies that: $p(y^{(i)}|x^{(i)};\theta) = \frac{1}{\sqrt{2\pi\sigma}}\exp\left(-\frac{(y^i-\theta^T x^{(i)})^2}{2\sigma^2}\right)$ - this does not depend on θ, the model is not a random variable!
For the entire model's training set X we can define this the **likelihood** function of the model : $L(\theta) = L(\theta; X; \vec{y}) = p(\vec{y}|X;\theta)$
$L(\theta) = L(\theta; X, \vec{y}) = p(\vec{y}|X;\theta)$
Since all of the observations are independent:
$L(\theta) = \Pi_i p(y^{(i)}|x^{(i)};\theta) = \Pi_i \frac{1}{\sqrt{2\pi\sigma}}\exp\left(-\frac{(y^i-\theta^T x^{(i)})^2}{2\sigma^2}\right)$
Maximum likelihood: we should choose a model θ so as maximize the probability of the data: θ should maximize $L(\theta)$.
By deriving the function that maximizes $\log L(\theta)$, product becomes a series sum and we simply need to maximize the $\frac{1}{2}\sum_i(y^{(i)} - \theta^T x^{(i)})^2$ which is the original least-squares cost function.
Note that this does not depend on σ !

21.2.1 Maximum A Posteriori

/TODO

21.3 Locally Weighted Linear Models

21.4 Optimization Techniques

There following algorithms are alternatives to GD that do not require choosing a learning rate:

- Conjugate Gradient

- BFGS
- L-BFGS

Advantages:

- No learning rate
- Faster than GD
- Line search

Disadvantages

- More complex
- Prob. don't imp. yourself

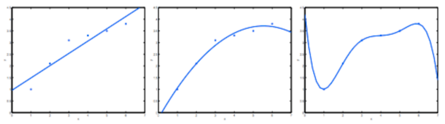
21.4.1 Multi-Class Classification Problems

21.4.2 One vs. All

For example: tagging emails according to multiple classes; weather (rainy, sunny)

For each class, train a logistic regression classifier $h_\theta^{(i)}(x)$ that predicts that probability that $y = i$.
For new input choose $\max_i h_\theta^i(x)$

22 Overfitting vs. Bias

 Underfitting -¿ high bias.
Overfitting, high variance
High variance - fitting a high order polynomial can be used to fit almost any function, not enough data to give a good hypothesis
If we have too many features, the learned hypothesis may fit the training data very well, but fail to generalize

22.0.1 Addressing Overfitting

Reduce number of features
Requires deciding which feature to keep and discard Model selection algorithms
Regularization

keep features but reduce magnitude / values of θ_j

works well when there are a lot features, each of which contributes less
Modify the cost function by penalizing the parameters:
Penalize higher order parameters: equiv to reducing the model to lower order model - simplfying the model
Penalize all parameters - trying to keep the hypothesis small, usually corresponds to smoother functions
So now the objective has a data term and a regularization term.
The regularization term: $\lambda \sum_{j=1} \theta_j^2$ keeps all of them small
If λ is very large, in linear reg., all model params will be close to 0 and $h_\theta(x) = \theta_0$

23 Measuring Model Performance

Type 1 Error - False positive - Predict an event when there was no event Type 2 Error - False negative - Predict no event when in fact there was an event.

23.0.1 Precision-Recall

Precision-Recall curves summarize the trade-off between the true positive rate and the positive predictive value for a predictive model using different probability thresholds.
Precision-recall curves are appropriate for imbalanced datasets.

24 Image Processing

24.1 Norms

L_1 norm
L_2 norm
Huber's norm
L_∞

25 Computational Photography

Bayer Pattern
Hough transform

25.1 Integral Images

The value at any point (x, y) in the summed-area table is the sum of all the pixels above and to the left of (*x*, *y*), inclusive where $i(x, y)$ is the value of the pixel at (x,y). The summed-area table can be computed efficiently in a single pass over the image:
$I(x, y) = i(x-1, y-1) + I(x, y-1) + I(x-1, y) - I(x-1, y-1)$
and similarly for any rectangular region:
$i(A, B, c, D) = I(D) - I(B) - I(C) + I(A)$
`https://upload.wikimedia.org/wikipedia/commons/thumb/5/58/Summed_area_table.png/220px-Summed_area_table.png`

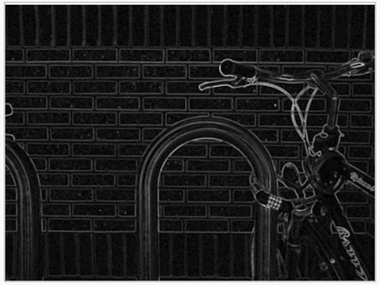
26 Filtering

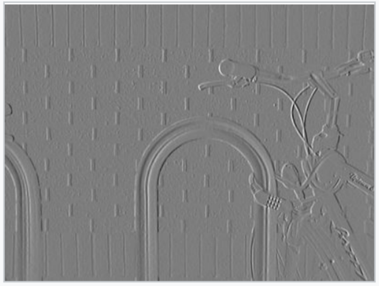
- Color Conversion
- Thresholding
- Smoothing
- Morphology
- Gradient

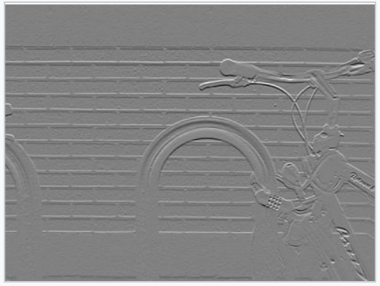
26.1 Sobel Operator

Uses 3×3 operators which are convolved with the image to compute approximate (center difference?) derivatives in x and y directions.

Grayscale test image of brick wall and bike rack


Normalized gradient magnitude from Sobel–Feldman operator


Normalized *x*-gradient from Sobel–Feldman operator


Normalized *y*-gradient from Sobel–Feldman operator

26.2 Canny Edge Detection

1. Apply Gaussian filter to smooth the image in order to remove the noise 2. Find the intensity gradients of the image The edge orientation is the atan of the intensity of the gradient in each dimension The edge magnitude is the sqrt 3. Apply non-maximum suppression to get rid of spurious response to edge detection 4. Apply double threshold to determine potential edges 5. Track edge by hysteresis: Finalize the detection of edges by suppressing all the other edges that are weak and not connected to strong edges.

- Contours
- Histograms

26.3 Convolution

27 Image Deblurring

28 Fourier Transform

29 Image Compression

30 Optic Flow

30.1 Gaussian Pyramids

Gradient consistency assumption + intensity consistency assumption
Iterative multi scale + warping
Uses an analytic formulation derived from Euler-Lagrange Equations
Results in a dense optic flow field.
Works well for small changes.

31 Interpolation

31.1 Nearest Neighbor

31.2 Bilinear Interpolation

Linear interpolation on a 2D grid.

32 Noise Models

32.0.1 Salt and Pepper / Black White

This type of noise happens due to sudden interruption in the image signal. Also known as data drop noise because statistically its drop the original data values Can be removed using median or morphological filtering.

32.0.2 Gaussian noise

Noise which has a probability density function (PDF) equal to that of the normal distribution. Can be estimated by taking a dark image and measuring the variance of the pixels. Removed by smoothing.

33 Semantic Computer Vision

Visual Odometry

34 Silhouette Segmentation / Visual Hull

35 Optic Flow

36 Image Segmentation / Pixel Labeling

37 Object Detection

38 Classification Problems

39 Event Cameras

39.0.1 Features

- Low-latency ($\sim 1\mu s$)
- No motion blur
- High dynamic range (140 dB instead of 60 dB)
- Ultra-low power (mean: 1mW vs 1W)

Traditional vision algorithms cannot be used because:

- Asynchronous pixels
- No intensity information (only binary intensity changes)

But they bring new possibilities:

- Night vision

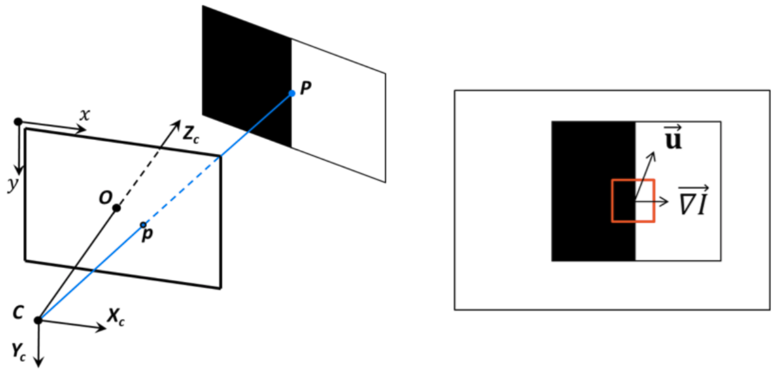
- Compact representation and data

On static scenes, they mostly produce noise
Main visible features - edges.

39.0.2 Linearized Event Generation

An event is triggered $\log I(x,t) \log I(xt - \Delta t) = \pm C$
Where C
Consider a pixel $p(x,y)$ with gradient $\nabla L(x,y)$ undergoing a motion $u \in (u,v)$ induced by a moving point $p \in \mathbb{R}^3$

$$-\nabla L \;\cdot \mathbf{u} = C$$



From brightness constancy assumption:
$L(x,y,t) = L(x+u, y+v, t+\Delta t)$ from first order approx we get the following $-\nabla L \cdot \vec{u} = C$

39.0.3 Deblurring

A blurry image can be regarded as the integral of a sequence of latent images during the exposure time, while the events indicate the changes between the latent images.
Sharp images are done by subtracting the double integral of the events

39.0.4 (Sparse) Feature Tracking In Event Space

39.0.5 Kanade–Lucas–Tomasi

Goal: extract features on frames and track them using only events in the blind time between two frames
Uses the event generation model via joint estimation of patch warping and optic flow
Disadvantages: requires GPU for real time tracking and they require knowledge of contract sensitivity, which is scene dependent and differs from pixel to pixel.

39.0.6 Image Reconstruction from Event Cameras

Recurrent neural network (main module: Unet)
Input: last reconstructed frame + sequences of event tensors (spatiotemporal 3D voxels grid: each voxel contains sum of ON and OFF events falling within the voxel)
Network processes last N events (10,000)
Trained in simulation only (without seeing a single real image) (we used our event camera simulator: `http://rpg.ifi.uzh.ch/esim.html`
Noise free simulation. We randomized the contrast sensitivity

40 Image Segmentation

Geometry Processing

41 Basics

Plane normal equation
Plane point distances
Barycentric coordinates

42 Surface Representations

Explicit:

- Mesh
- Spline surface
- Oriented planes
- Point cloud

Implicit - voxel grid:

- Signed distance fields (implicit) ¡0, 0, ¿0
- Signed distance fields (implicit)
- Occupancy grid
- Signed-distance grid
- Voxel octree
- Tetrahedral Mesh

Volumetric modeling for vision: • Flexible and robust surface representation • Handles (changes of) complex surface topologies effortlessly • Ensures watertight surface / manifold / no self- intersections • Allows to sample the entire volume of interest by storing information about space opacity • Voxel processing is often easily parallelizable
Drawbacks: Implicit surface representation

42.1 Marching Cubes

Recovers an isosurface from a volume ensures a watertight surface Can be done per voxel 15 combinations of surface intersections per cube Precise normal specification Accuracy depends on resolution
Trivial merging or overlapping of different surfaces based on the corresponding implicit functions: • minimum of the values for merging • averaging for overlapping
Limitations of Marching Cubes • Maintains 3D entries rather than a 2D surface, i.e., higher computational and memory requirements • Generates consistent topology, but not always the topology you wanted • Problems with very thin surfaces if resolution not high enough

43 ICP

Algorithm:

44 Point Cloud Merge

ICP for point cloud matching
Normal Estimation
Outlier detection / removal
Surface / mesh fitting / template fitting

44.0.1 Geometric Representations

Bezier curves

45 Laplacian Deformation

46 Math

46.1 Algebra

Matrix $A \in \mathbb{R}^{m \times n}$ is a matrix with m rows and n columns
$AA^{-1} = \mathbb{I}$
A matrix that does not have inverse is $_singular_or_degenerate$
Transpose: $A_{ij} = A^T_{ji}$
Taylor Expansion:
Gradient:
Chain rule:
Finite Difference Approximation: