Computer Vision & Machine Learning Revision Notes

Yeara Kozlov

Contents

1. Frontmatter

1.1 Disclaimer

these are my personal notes reviewing material for CV/M interviews. The notes contain text from Quora, as well as screengrabs from various sources including Andrew Ng's Machine Learning Course on Coursera, Wikipedia, The Computer Vision Course on Coursera, and other sources. Reproduced without permission. The notes are incomplete, work in progress, and may contain mistakes.

1.2 CV Interview Topics

Geometric Vision - geometric transforms, projective geometry, homography, stereo vision, epipolar geometry, fundamental and essential matrices, geometric calibration, triangulation.

Photometric Vision - filtering, convolution, denoising, deblurring.

Semantic Vision - object detection, segmentation, tracking, pose estimation, visual hull, optic flow.

Machine Learning - linear regression, logistic regression, generative models, svm, gaussian mixture models, boosting, neural nets.

1.3 ToDos

- Total Variation
- Visual Hull
- Cascade Detectors
- Face landmark detection
- Graphcut
- Dynamic programming for stereo matching
- Pose Estimation from 2D-3D Matches
- Camera translation

2. Machine Learning

These notes are based on the Andrew Ng's Coursera course. After finishing supervised learning I switched to the lecture notes from Standford Machine Learning course, which are much more thorough.

2.1 Resources

- https://github.com/afshinea/stanford-cs-229-machine-learning
- https://ml2.inf.ethz.ch/courses/aml/
- https://las.inf.ethz.ch/pai-f19
- https://www.coursera.org/learn/machine-learning/lecture/zcAuT/welcome-to-machine-learning
- https://web.stanford.edu/class/archive/cs/cs109/cs109.1166/pdfs/37

Regression: predict real-valued output
Classification: discrete valued outputs

2.2 Supervised Learning

Training set - with m number of training examples, x input variables / features, y outputs/targets
$(x^{(i)}, y^{(i)})$ is a training example

2.2.1 Linear Regression (Uni-variable)

Hypothesis: $h_\theta(x) = \theta_0 + \theta_1 x$
Cost function: a function that measures the performance of the hypothesis
For linear regression:

$$\min_{\theta_0,\theta_1} \frac{1}{2m} \sum_i |h_\theta(x^{(i)}) - y^{(i)}|^2$$

Squared Error Cost Function: $J = \frac{1}{2m} \sum_i (h_\theta(x^{(i)}) - y^{(i)})^2$

Gradient Descent for Linear Regression

For linear regression - the least squared cost function has no local minimum
GD will converge
Normal Equations can be used to perform a single step solution for linear models, but GD scales better for large training sets

Stochastic Gradient Descent

Computes the gradient with respect to each training example directly and aggregates it.
Can converge to a minimum much faster than batch gradient descent

2.2.2 Multi Variable Linear Regression

For n features, define $x \in \mathbb{R}^{n+1}$, $0th$ indexed vector, the features vector, where $x_0^{(i)} := 1 \forall i$
And $\theta = (0_0, \ldots, \theta_n)$ the model
The hypothesis: $h_\theta = \sum \theta_i x_i = \theta^T x$
Update rule for linear regression:
$\theta_0 = \theta_0 - \alpha \frac{1}{m} \sum_i (h_\theta(x^{(i)}) - y^{(i)}) \cdot x_0$
and similarly for all other variables

Feature Scaling

If features are of very different dimensions, the cost function will have skewed contours in the energy landscape. The gradient descent has this ping-pong behavior.
It helps to scale the parameters to approx. $-1 \leq x_j^{(i)} \leq 1$

Mean Normalization

Replace x_i with $x_i - \mu_i$ to make the variable approx. 0-mean
$x_i \leftarrow \frac{x_i - \mu_i}{range}$
s = Range will be max $-$ min

2.2.3 Debugging Gradient Descent

Plot the cost function when GD runs
Num of iterations depends on the algorithm / model
Automatic convergence tests:

- change in $J(\theta)$ decreases by less than 10^{-3}

If the cost function value increases, try smaller α
When visualization - either wave behavior or increase in the model
Gradient verification with FD
If α is small enough, GD should decrease for every iteration

A - good convergence
B - slow convergence
C - learning rate too high

Run GD with α with a range of values with 10-scale factor 3x from previous values
Until you find one value which is too small and one value which is too large

Momentum

Netwon

- no learning rate

For a function l with the derivative $l'(\theta)$ and second derivative, starting from an initial guess the update rule is:
$\theta := \theta - \frac{l'(\theta)}{l''\theta}$
until $l'(\theta) = 0$.
Newton method looks at the approximated tangent to $l(\theta)$ at the point θ and solves for where the line is equal to 0.

Newton Raphson Method

Generalization of Netwon's method to multi-variable / multi dimension settings:
$\theta = \theta - H^{-1} \nabla_\theta l(\theta)$
where ∇_θ is a vector of partial derivatives of $l(\theta)$ with respect to θ and
$H(\theta) = \frac{\partial^2 l(\theta)}{\partial \theta_i \partial \theta_j}$
Better and faster convergence than GD, but expensive, requires (careful) evaluation, Hessian needs to be invertible (full rank)
Fischer scoring - applying Newton's to logistic regression log likelihood function

2.2.4 Polynomial Regression

Basically, the idea here is to cheat and pre-compute the feature vector.
For example, $(x_1 := x, x_2 := x^2, x_3 := x^3)$.
The previous formulation and update rules hold: $\theta^T x$
In this case it's important to scale the variables!
Other options: sqrt, cubic, squared (which might not fit a lot of models)

2.2.5 Normal Equation

For a feature vector n features and m data points:

Construct a matrix $X \in \mathbb{R}^{m \times (n+1)}$ which contains all of features for all the variables + (n+1) column which contains all 1s.

$$\begin{pmatrix} 1 & x_1^1 & ... & x_1^n \\ \vdots & x_2^1 & ... & x_2^n \\ 1 & x_m^1 & ... & x_m^n \end{pmatrix}$$

And collect all of the observations in a vector $y \in \mathbb{R}^m$:

And we solve for a model:

$\theta = (X^T X)^{-1} X^T y$

Now, this is true only if $X^T X$ is invertible

Feature scaling is not necessary when using the normal equation.

2.2.6 GD vs. Normal Equation

GD

- need to choose learning rate
- need many iterations
- works well when n is large

Normal Equation

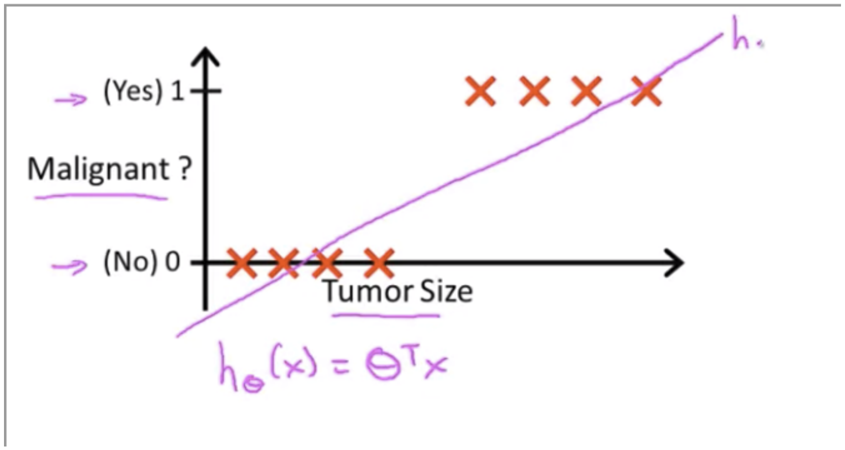
- slow for large n $O(n^3)$, $n = 10k$ is where switching over could be beneficial
- no need to choose learning rate
- direct

2.2.7 When is $X^T X$ non-invertible?

- linearly dependent features - i.e. size in m^2 and size in feet squared
  - remove features
- too many features $n \geq m$
  - delete features
  - use regularization

2.3 Classification

2.3.1 Two Class Problems



Using linear regression model + threshold:

Classification is not actually a linear function - using linear models doesn't work well.

Labels are usually 0,1 known as negative and positive classes.

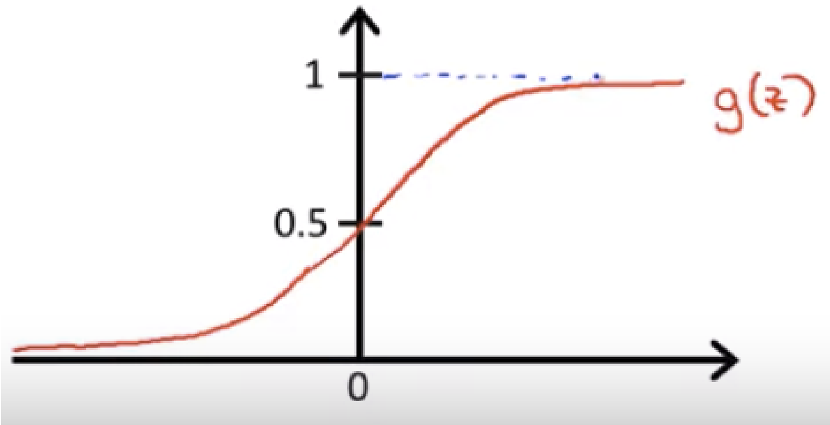
Logistic Regression

Want a model that predict a value $0 \leq h_\theta(x) \leq 1$

Model: $h_\theta(x) = g(\theta^T x)$

Logistic/sigmoid function: $g(z) = \frac{1}{1+e^{-z}}$

Together: $h_\theta(x) = \frac{1}{1+e^{-\theta^T x}}$



Has asymptotes at 0,1

Interpretation of Output

$h_\theta(x)$ is the estimated probability that $y = 1$ on input x

$h_\theta(x) = P(y = 1|x;\theta)$

$P(y = 0|x;\theta) = 1 - P(y = 1|x;\theta)$

Decision Boundary

$g(z) \geq 0.5$ when $z > 0$

$g(\theta^T x) \geq 0.5$ when $\theta^T x \geq 0$

(basically, here we can derive this from $1 + e^{-\theta^T x} = 2$

The decision boundary is a function of the hypothesis and its parameters

Non Linear Decision Boundaries

Can perform a similar trick as with linear regression -¿ polynomial regression - build features such as x_1^2 etc...

So for example:

$$\theta = \begin{bmatrix} -1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$$h_\theta(x) = g(\theta^T(1, x_1, x_2, x_1^2, x_2^2))$$

The decision boundary will lie at $x_1^2 + x_2^2 = 1$

Cost Function

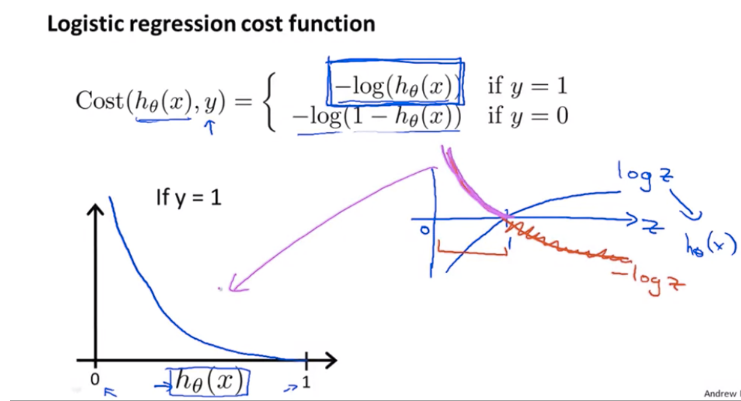
Using the linear regression cost function is non convex for the logistic regression.

$Cost(h_\theta(x), y) = \begin{cases} -\log(h_\theta(x));\ \text{if } y = 1 \\ -\log(1 - h_\theta(x));\ \text{if } y = 0 \end{cases}$

This formulation has desirable properties:

$(h(x) = 0, y = 0)$ or$(h(x) = 1, y = 1)$ - cost = 0

Very high penalization if $(h(x) = 1, y = 0)$ or$(h(x) = 0, y = 1)$ due to the cost function going asymptotically to ∞ :



Simplified Cost Function

A generalized cost function is:

$Cost(h_\theta(x), y) = -y\log(h_\theta(x)) - (1 - y)\log(1 - h_\theta(x))$

And summarizing over all examples:

$J(\theta) = -\frac{1}{m}\sum_{i=1}^{m} y^{(i)}\log(h_\theta(x^{(i)})) + (1 - y^{(i)})\log(1 - h_\theta(x^{(i)}))$

To minimize, solve for parameters:

$\min_\theta J(\theta)$

Output / new prediction: $h_\theta(x) = \frac{1}{1+e^{-\theta^T x}}$

$\frac{\partial}{\partial_{\theta_j}} J(\theta) = \frac{1}{m} \sum_i (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)}$

Exactly the same update as linear regression. Here the main difference is that h_θ went from $\theta^T x$ to $\frac{1}{1+e^{-\theta^T x}}$

And the update rules are:

$\theta_j := \theta_j - \frac{\alpha}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)}$

And vectorized:

$\theta := \theta - \frac{\alpha}{m} X^T (g(X\theta) - \vec{y})$

2.3.2 Maximum Likelihood Estimation + Convexity

Convexity: gives us lower bounds on the first order approximation of the function (i.e. the first order approximation is guaranteed to be larger than or equal to the real function value).

Assuming that the target variables and input are related via the equation:

$y^{(i)} = \theta^T x^{(i)} + \epsilon^{(i)}$

where ϵ are IID (independently and identically distributed) error terms the captures unmodeled effects, i.e random noise.

Assuming $e^i \sim \mathcal{N}(0, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(\epsilon^{(i)})^2}{2\sigma^2}\right)$

That implies that: $p(y^{(i)}|x^{(i)}; \theta) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(y^i - \theta^T x^{(i)})^2}{2\sigma^2}\right)$ - this does not depend on θ, the model is not a random variable!

For the entire model's training set X we can define this the likelihood function of the model : $L(\theta) = L(\theta; X; \vec{y}) = p(\vec{y}|X; \theta)$

$L(\theta) = L(\theta; X, \vec{y}) = p(\vec{y}|X; \theta)$

Since all of the observations are independent:

$L(\theta) = \Pi_i p(y^{(i)}|x^{(i)}; \theta) = \Pi_i \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(y^i - \theta^T x^{(i)})^2}{2\sigma^2}\right)$

Maximum likelihood: we should choose a model θ so as maximize the probability of the data: θ should maximize $L(\theta)$.

By deriving the function that maximizes $\log L(\theta)$, product becomes a series sum and we simply need to maximize the $\frac{1}{2} \sum_i (y^{(i)} - \theta^T x^{(i)})^2$ which is the original least-squares cost function.

Note that this does not depend on σ !

Maximum A Posteriori

YK: TODO

2.3.3 Locally Weighted Linear Models

2.3.4 Optimization Techniques

There following algorithms are alternatives to GD that do not require choosing a learning rate:

- Conjugate Gradient
- BFGS
- L-BFGS

Advantages:

- No learning rate
- Faster than GD
- Line search

Disadvantages

- More complex
- Prob. don't imp. yourself

Multi-Class Classification Problems

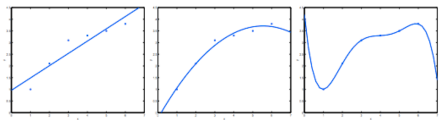
One vs. All

For example: tagging emails according to multiple classes; weather (rainy, sunny)

For each class, train a logistic regression classifier $h_\theta^{(i)}(x)$ that predicts that probability that $y = i$.

For new input choose $\max_i h_\theta^i(x)$

2.4 Bias vs. Variance

 Underfitting -¿ high bias.

Overfitting, high variance

High variance - fitting a high order polynomial can be used to fit almost any function, not enough data to give a good hypothesis

If we have too many features, the learned hypothesis may fit the training data very well, but fail to generalize

2.4.1 Model Selection and Train/Validation/Test Sets

Just because a learning algorithm fits a training set well, that does not mean it is a good hypothesis. It could over fit and as a result your predictions on the test set would be poor. The error of your hypothesis as measured on the data set with which you trained the parameters will be lower than the error on any other data set.

Given many models with different polynomial degrees, we can use a systematic approach to identify the 'best' function. In order to choose the model of your hypothesis, you can test each degree of polynomial and look at the error result.

One way to break down our dataset into the three sets is:

Training set: 60Cross validation set: 20Test set: 20We can now calculate three separate error values for the three different sets using the following method:

Optimize the parameters in θ using the training set for each polynomial degree. Find the polynomial degree d with the least error using the cross validation set. Estimate the generalization error using the test set with $J_{test}(\theta(d))$, ($d = \theta$ from polynomial with lower error). This way, the degree of the polynomial d has not been trained using the test set.

2.4.2 Regularization

Penalizing the θ too much leads to high bias in the model, i.e. strong underfitting.

When adding regularization, the error should be computed without the regularization term - i.e. now the error function and the cost function are different.

How to choose regularization weight:

- Create a list of lambdas (i.e. $\lambda \in 0, 0.01, 0.1, 1, 10, 100$);
- Create a set of models with different degrees or any other variants.
- Iterate through the λs and for each λ go through all the models to learn some θ.
- Compute the cross validation error using the learned θ (computed with λ) on the $J_{CV}(\theta) without regularization. Select the best combo that produces the lowest error on the cross$
- Using the best combo θ, λ apply it to the test set to see if it generalizes.

Addressing Overfitting

Reduce number of features

Requires deciding which feature to keep and discard Model selection algorithms

Regularization

- keep features but reduce magnitude / values of θ_j
- works well when there are a lot features, each of which contributes less

Modify the cost function by penalizing the parameters:

Penalize higher order parameters: equiv to reducing the model to lower order model - simplfying the model

Penalize all parameters - trying to keep the hypothesis small, usually corresponds to smoother functions

So now the objective has a data term and a regularization term.

The regularization term: $\lambda \sum_{j=1} \theta_j^2$ keeps all of them small

If λ is very large, in linear reg., all model parameters will be close to 0 and $h_\theta(x) = \theta_0$

2.4.3 Polynomial Degree and Bias vs. Overfitting

The relationship between the degree of the polynomial d and the underfitting or overfitting of our hypothesis.

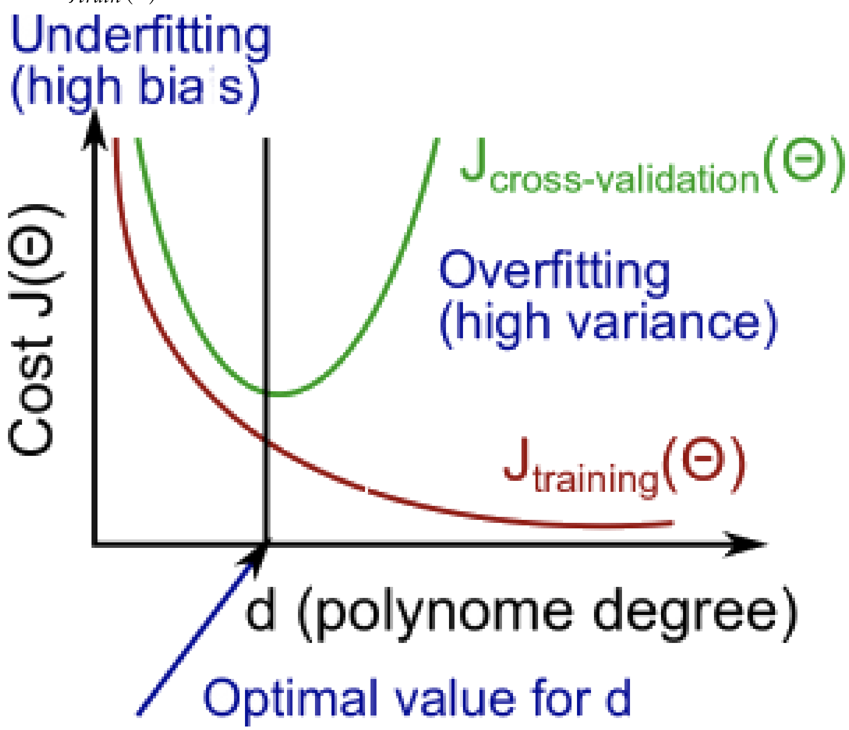
We need to distinguish whether bias or variance is the problem contributing to bad predictions. High bias is underfitting and high variance is overfitting.

Ideally, we need to find a golden mean between these two. The *training* error will tend to decrease as we increase the degree d of the polynomial.

At the same time, the *cross validation error* will tend to decrease as we increase d up to a point, and then it will increase as d is increased, forming a convex curve.

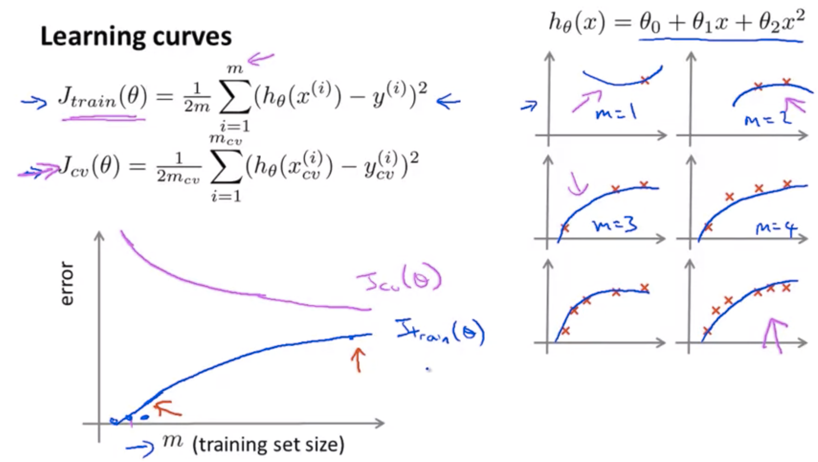
High bias (underfitting): both $J_{train}(\theta)$ and $J_{CV}(\theta)$ will be high. Also, $J_{CV}(\theta) \approx J_{train}(\theta)$.

High variance (overfitting): $J_{train}(\theta)$ will be low and $J_{CV}(\theta)$ will be much greater than $J_{train}(\theta)$.



2.4.4 Learning Curve

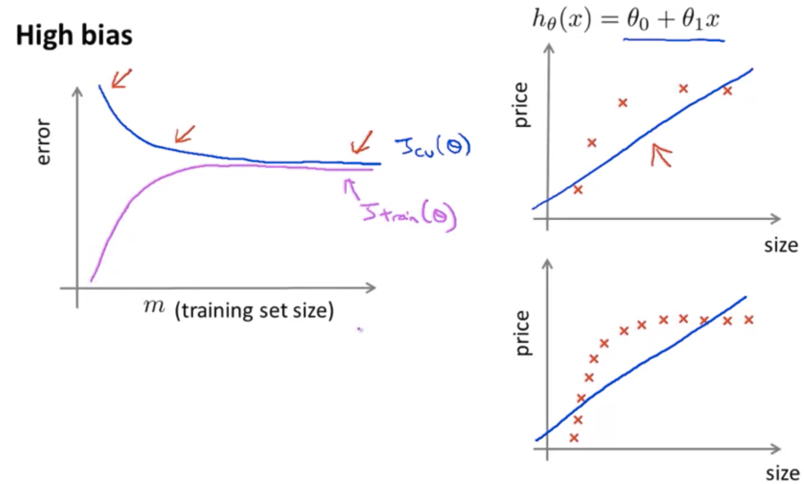
With a smaller number of training examples, the error cost on the training set or cross validation set should remain relatively low. As the number of training examples grows, the error will increase (but approximately asymptotically), as it become harder to fit all examples perfectly. At the same time, the cross-validation error should decrease.



High Bias Case

In this case the model does not have enough degrees of freedom to fit the data. Adding more data points will not help the performance of the model.

One will observe that the cross validation error will decrease initially, and then stagnate. The training error will increase until it comes close to the cross-validation error. Both of these errors will be high.



High Variance Case

The training error will remain low, although increase slightly as it becomes harder and harder to fit more training examples.

The cross validation error will remain high, maybe will decrease.

We will observe a large gap between the training error and the cross validation error. In the high variance case, adding more training examples can help.

2.4.5 Evaluating a Hypothesis

Regression error = $h_\theta(x^(i)) - y^{(i)}$

Misclassification error = $\sum err(h_\theta(x^{(i)}), y^{(i)})$

Trouble shooting options:

Getting more training examples $\leftarrow$ Trying smaller sets of features Trying additional features Trying polynomial features Increasing or decreasing λ

What to do when a hypothesis is not performing well on, for example, new data?

2.5 Measuring Model Performance

Type 1 Error - False positive - Predict an event when there was no event Type 2 Error - False negative - Predict no event when in fact there was an event.

Precision-Recall

Precision-Recall curves summarize the trade-off between the true positive rate and the positive predictive value for a predictive model using different probability thresholds.

Precision-recall curves are appropriate for imbalanced datasets.

ROC - Receiver Operating Characteristic curve

Summarize the trade-off between the true positive rate and false positive rate for a predictive model using different probability thresholds.

ROC curves are appropriate when the observations are balanced between each class

Convolution is a mathematical operation on two functions that produces a third function expressing how the shape of one is modified by the other.

$(f \star g)(t) = \int_{-\infty}^{\infty} f(\tau) \cdot g(t-\tau) d\tau = \int_{-\infty}^{\infty} f(t-\tau) \cdot g(\tau) d\tau$

Commutative.

For functions which only have limited support the integration is only done on the valid domain.

2.5.1 L1 vs L2 Norm

L2 norm strongly penalizes outliers. For good data with some very far outlier it might not generate the "best" fit as judged by a human observer.

L1 favors sparse coefficients.

2.6 Support Vector Machines

2.7 Decision Trees

Recursive repartition of the data.

2.7.1 Random Forest Regression

An ensemble of decision trees. During learning tree nodes are split using random variable subset of data features.

All trees vote to produce final result.

For best results trees should be as independent as possible. Splitting using a random subset of features achieves this.

Averaging the product of the trees reduces overfitting to noise

5-100 Trees.

2.7.2 Random Fern Regressors

2.8 Boosting

Learning strong classifiers from weak classifiers.

2.9 Naive Bayes Classifier

YK: TODO

2.10 RANSAC

A method for dealing with noisy data.

Partition the method

Is not determinant, depends on the subset selection, and is not guaranteed to converge.

1. Select a random subset of the original data. Call this subset the hypothetical inliers. 2. A model is fitted to the set of hypothetical inliers. 3. All other data are then tested against the fitted model. Those points that fit the estimated model well, according to some model-specific loss function, are considered as part of the consensus set. 4. The estimated model is reasonably good if sufficiently many points have been classified as part of the consensus set. 5. Afterwards, the model may be improved by reestimating it using all members of the consensus set.

"‘ Given: data – a set of observations model – a model to explain observed data points n – minimum number of data points required to estimate model parameters k – maximum number of iterations allowed in the algorithm t – threshold value to determine data points that are fit well by model d – number of close data points required to assert that a model fits well to data

Return: bestFit – model parameters which best fit the data (or nul if no good model is found)

iterations = 0 bestFit = nul bestErr = something really large while iterations ¡ k maybeInliers = n randomly selected values from data maybeModel = model parameters fitted to maybeInliers alsoInliers = empty set for every point in data not in maybeInliers if point fits maybeModel with an error smaller than t add point to alsoInliers if the number of elements in alsoInliers is ¿ d betterModel = model parameters fitted to all points in maybeInliers and alsoInliers thisErr = a measure of how well betterModel fits these points if thisErr ¡ bestErr bestFit = betterModel bestErr = thisErr increment iterations return bestFit "‘

2.11 Bagging/Boosting

Collaborative filtering

2.12 Generative Models

2.13 Dimension Reduction

2.13.1 PCA

2.14 Unsupervised Learning

Algorithms for finding structure in data.

2.15 Clustering

The clustering problem: given an unlabeled data set, group the data into coherent subsets or into coherent clusters for us.

2.15.1 K Means

- K number of clusters + initialization
- Training set $x^{(1)}, x^{(2)} \ldots, x^{(m)}$
- $x \in \mathbb{R}^n$
- By convention, drop $x_0 = 1$

Randomly initialize K cluster centers While not converged: 1. iterate over data and assign a cluster for each data point based on distance to center 2. recompute the cluster mean

If a cluster becomes empty - remove the cluster

Or randomly re-initialize the cluster

K Means for Non Separated Clusters

K Means Cost Function

Assuming:

$c^{(i)}$ index of cluster to which the example $x^{(i)}$ belongs to.

$\mu_k \in \mathbb{R}^n$ cluster centroid

$\mu_{c^{(i)}} \in \mathbb{R}^n$ location of the cluster centroid to which example $x^{(i)}$ has been assigned

Example cost for point $Cost(x^{(i)}) = \|x^{(i)} - \mu_{c^{(i)}}\|^2$

$J(c^{(1)}, \ldots, c^{(k)}) = \frac{1}{m}\sum_i \|x^{(i)} - \mu_{c^{(i)}}\|^2$

The objective is to minimize the cost function *distortion* with respect to the clusters (both labelling and centers).

So what k-means algorithm is actually doing is:

1. minimize the cost function with respect to cluster assignments $c^{(i)}$ 2. minimize the cost function with respect to cluster centroids μ_k

(so basically block coordinate descent?)

Random Initialization

- $K < m$
- Randomly pick K training examples and set the cluster means to these examples

K-mean can get stuck in a local optima - to avoid this a good option is to run k-mean multiple times and get as good global optimum

For multiple initializations - run K-means loads of times, pick the clustering which results in the lowest cost function

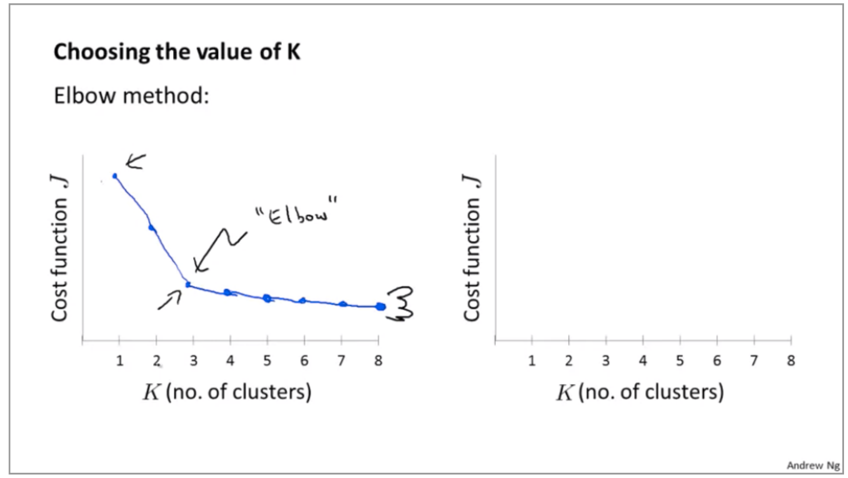
This works well for small $K < 10$.

For large Ks it is not as effective.

Number of Clusters - Elbow Method

Choosing the right K

Plot the cost function with respect to the number of clusters:



In practice it is usually a bit harder, and it is not clear that there is such a transition where the distortion stops.

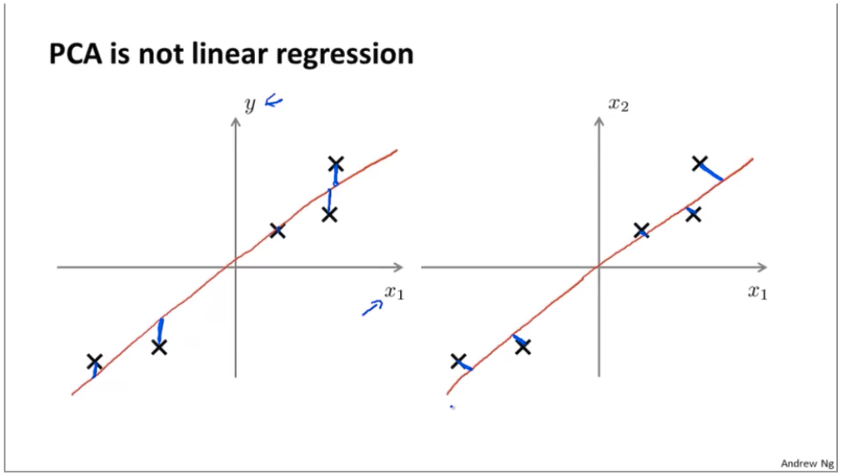
2.16 Dimensionality Reduction

2.16.1 PCA

PCA is trying to find a lower dimension representation of that data which minimizes the squared distance error of the data from the representation.

Before PCA it is standard practice to perform mean normalization and feature scaling.

PCA vs Linear Regression



We do not treat y as a special variable
Minimized projected error vs. minimize distance from line

2.17 GMM and EM

2.18 Data Generation Using Simulation

Generating good synthetic data: realism, diverse,
Want to render images which are as different as possible from each other
Parametric model of humans - procedural generation

2.19 Neural Networks

http://karpathy.github.io/neuralnets/

2.20 ML Algorithm Design

General process of building a ML product:

1. What is the objective? prediction, recommendation, clustering, search, etc.
2. Pick the right algorithm: supervised vs unsupervised, classification vs regression, generalized linear model / decision tree / neural network / etc.
3. Pick / engineer relevant features based on available data.
4. Pick metrics for model performance.
5. Optionally, comment on how to optimize the model for production.