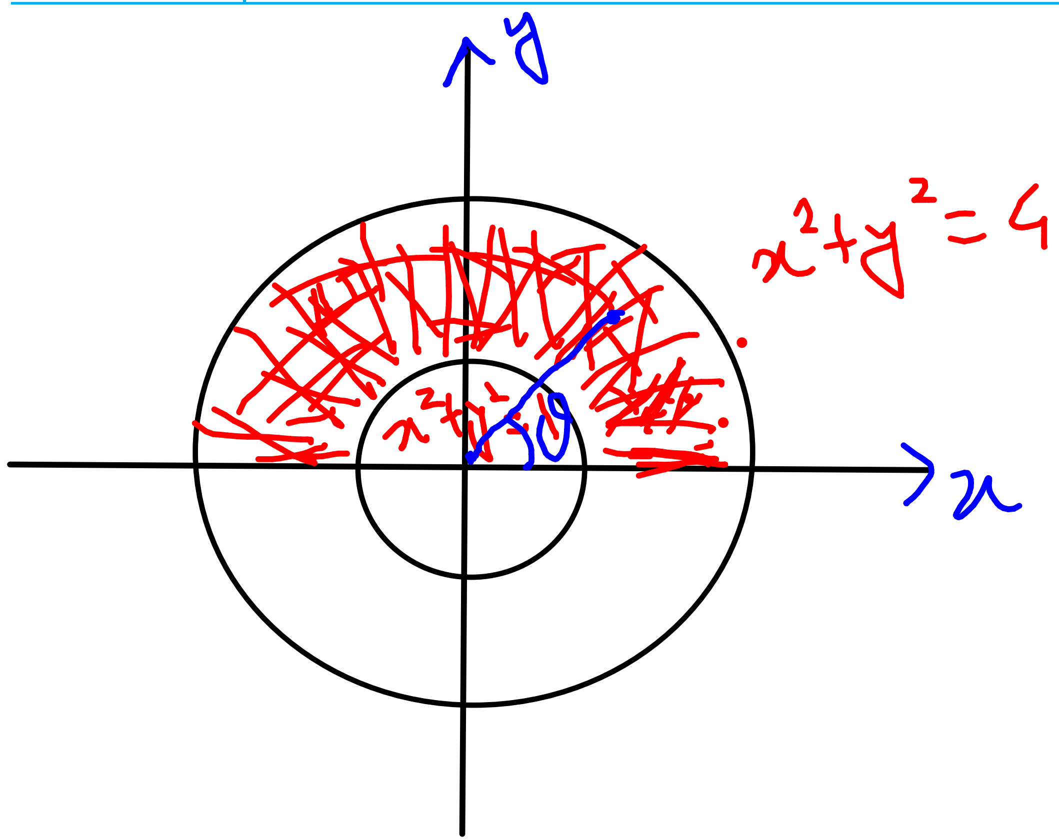
12.3 DOUBLE INTEGRALS IN POLAR COORDINATES



range of r & θ for the shaded region

$0 \le \theta \le \pi$

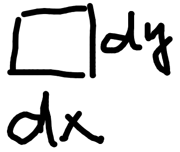
$1 \le r \le 2$

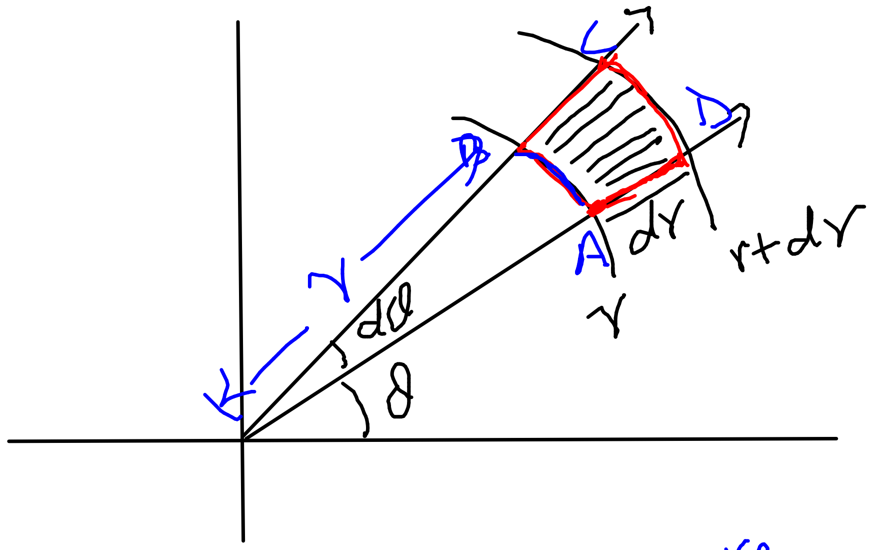
$x = r\cos\theta$

$y = r\sin\theta$

$$J = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix} = r$$

$dx\,dy \approx r\,dr\,d\theta$



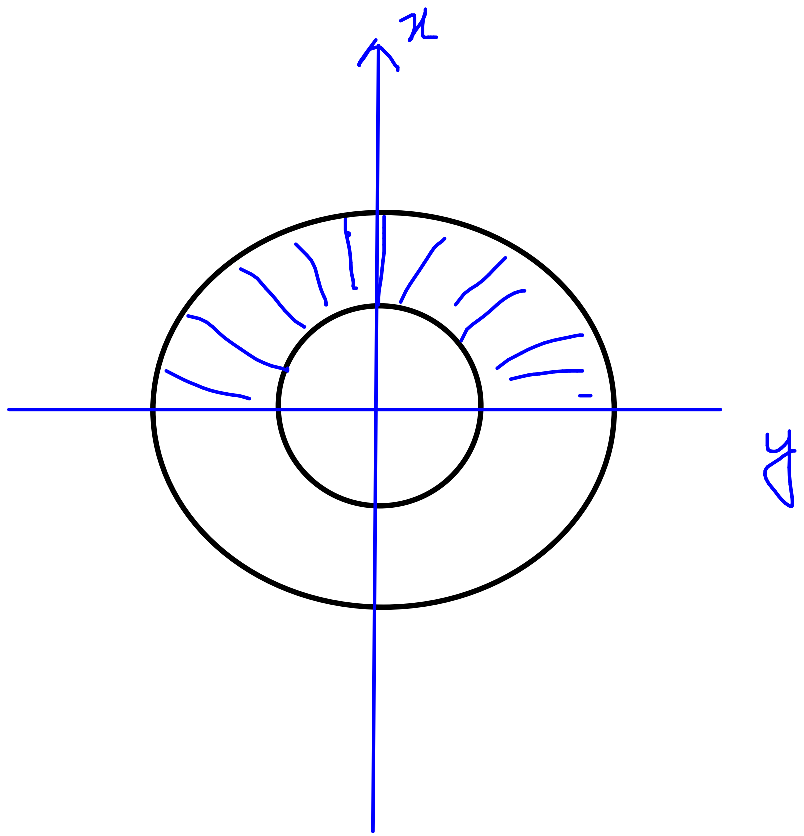


$AB \approx r\,d\theta$

$AD = dr$

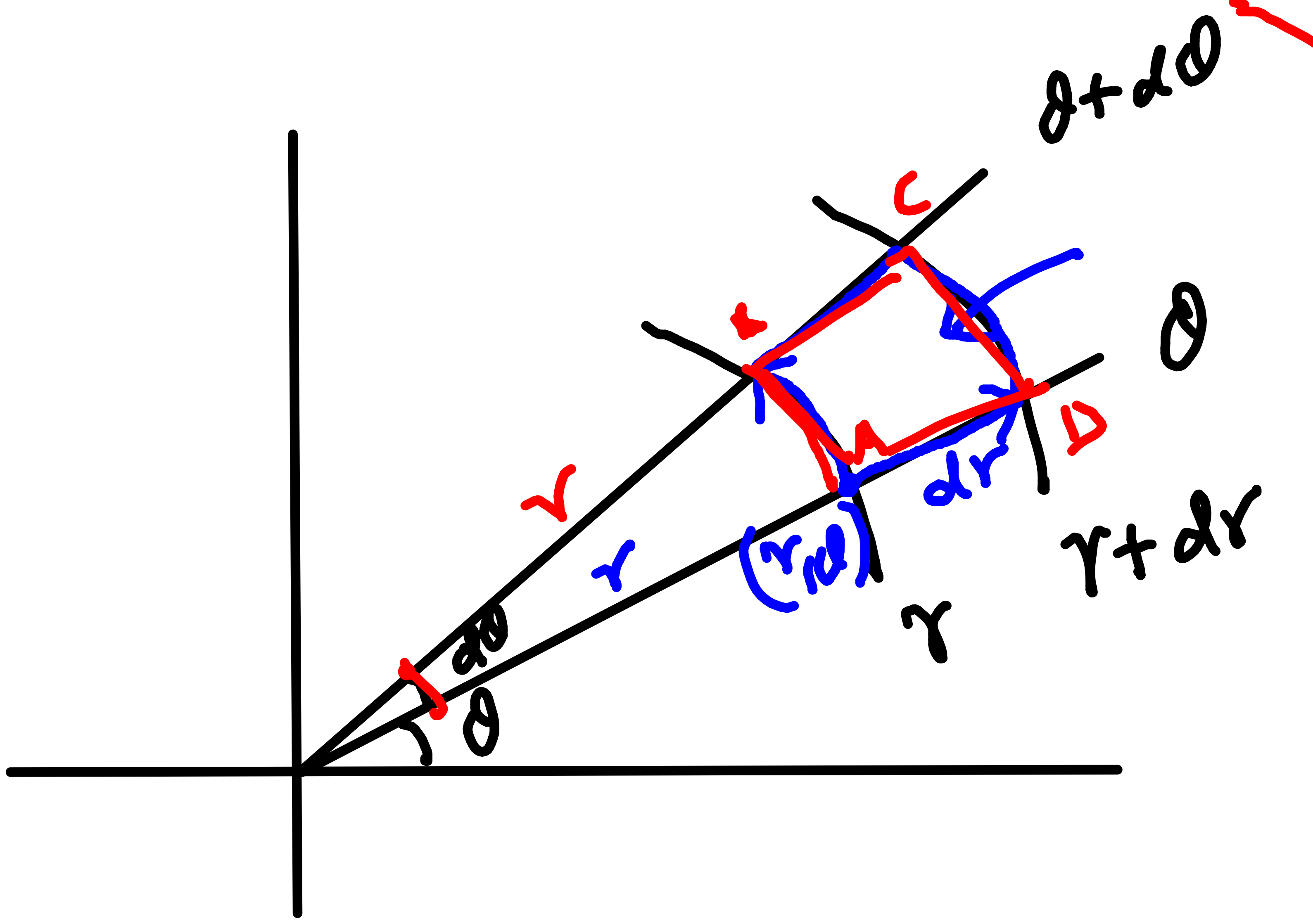
area $(ABCD) = (r\,d\theta)\,dr = r\,dr\,d\theta$

EXAMPLE 1 Evaluate $\iint_R (3x + 4y^2)\, dA$, where R is the region in the upper half-plane bounded by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.



$$\int_1^2 \int_0^{\pi} \left(3r\cos\theta + 4r^2\sin^2\theta\right) r \, d\theta \, dr$$

$$= \frac{15\pi}{2}$$



$\approx r\,dr\,d\theta$

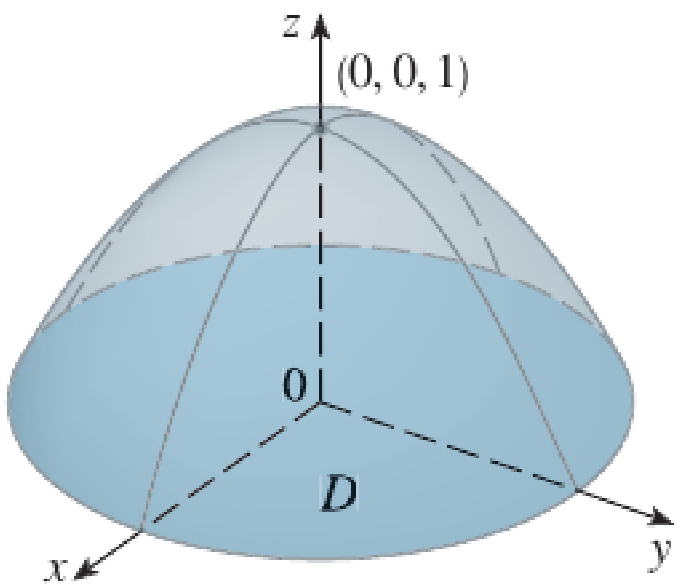
$\widehat{AB} = r\,d\theta$

$AD = dr$

$\text{area}(ABCD) \approx (r\,d\theta)(dr)$

$= \underline{r}\,dr\,d\theta$

V EXAMPLE 2 Find the volume of the solid bounded by the plane $z = 0$ and the paraboloid $z = 1 - x^2 - y^2$.



V EXAMPLE 3 Find the volume of the solid that lies under the paraboloid $z = x^2 + y^2$, above the xy-plane, and inside the cylinder $x^2 + y^2 = 2x$.