MAT 104

mathematical methods II

→ Matrix Algebra (Linear Algebra)

(geometry behind matrices)

→ Ordinary Differential Equation

$$\frac{dy}{dx} + Py = Q$$

But we will finish

Calculus first

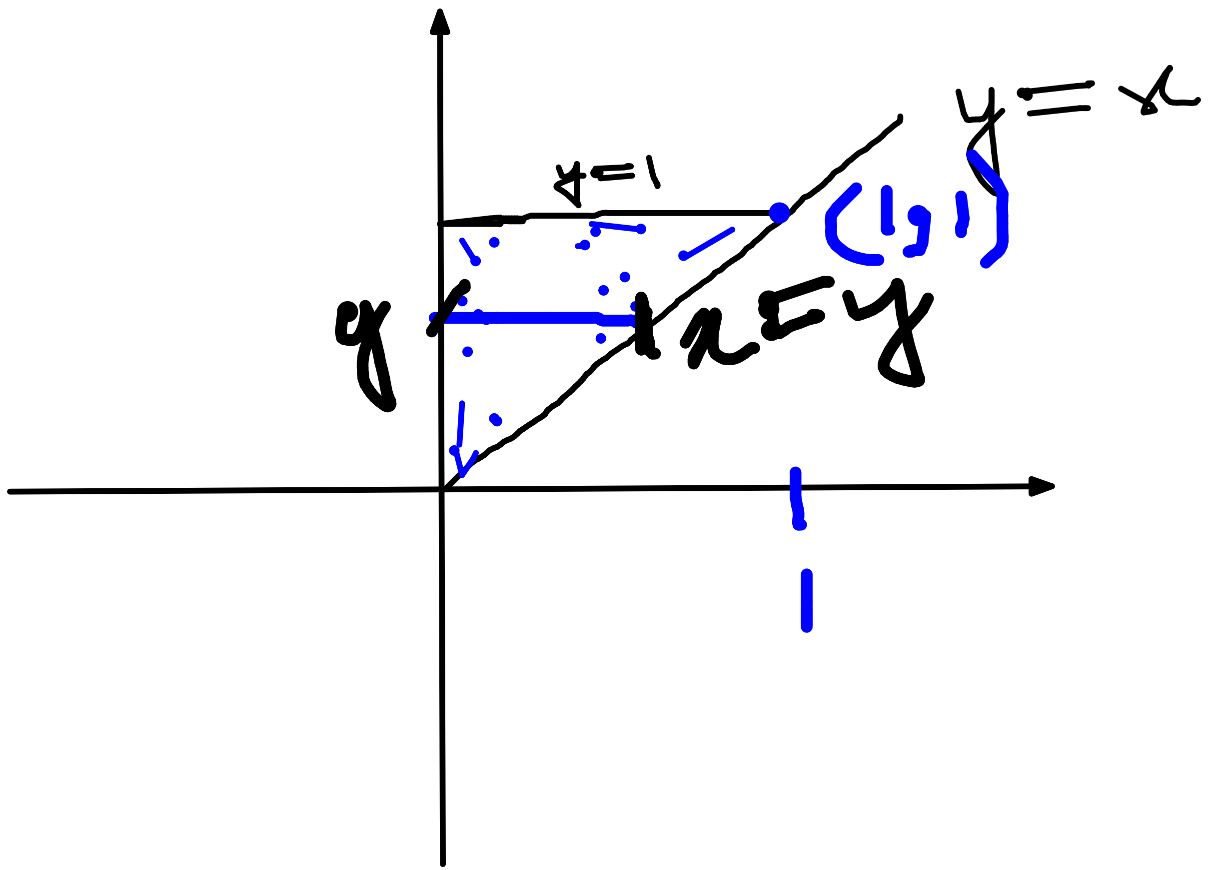
$\rightarrow$ Essential Calculus

12.5 & later

Calculate the iterated integral by first reversing the order of integration.

$$\int_0^1 \int_x^1 \cos(y^2)\, dy\, dx \quad = \quad \int_a^b \int_c^d \cos(y^2)\, dx\, dy$$

$a, b, c, d = ??$



$$= \int_0^1 \int_0^y \cos(y^2)\, dx\, dy$$

$$\int_0^y \cos(y^2)\, dx = \cos(y^2) \int_0^y dx$$

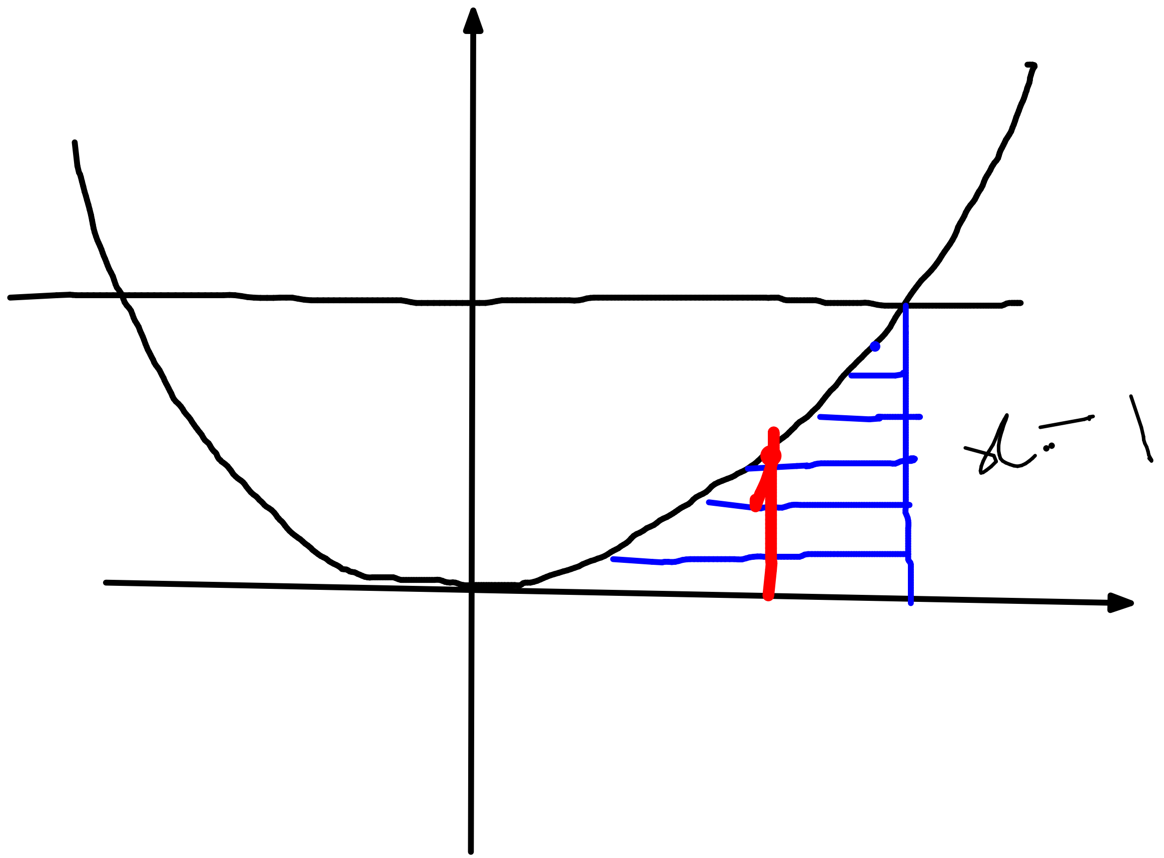
$$= \cos(y^2)\, y$$

$$\int_0^1 y\cos(y^2)\,dy = \frac{\sin(1)}{2}$$

Calculate the iterated integral by first reversing the order of integration.

$$\int_0^1 \int_{\sqrt{y}}^1 \frac{ye^{x^2}}{x^3}\,dx\,dy$$

sketch the region of integration



$x = \sqrt{y}$

$y = x^2$

$$\int_0^1 \int_0^{x^2} \frac{ye^{x^2}}{x^3}\,dy\,dx$$

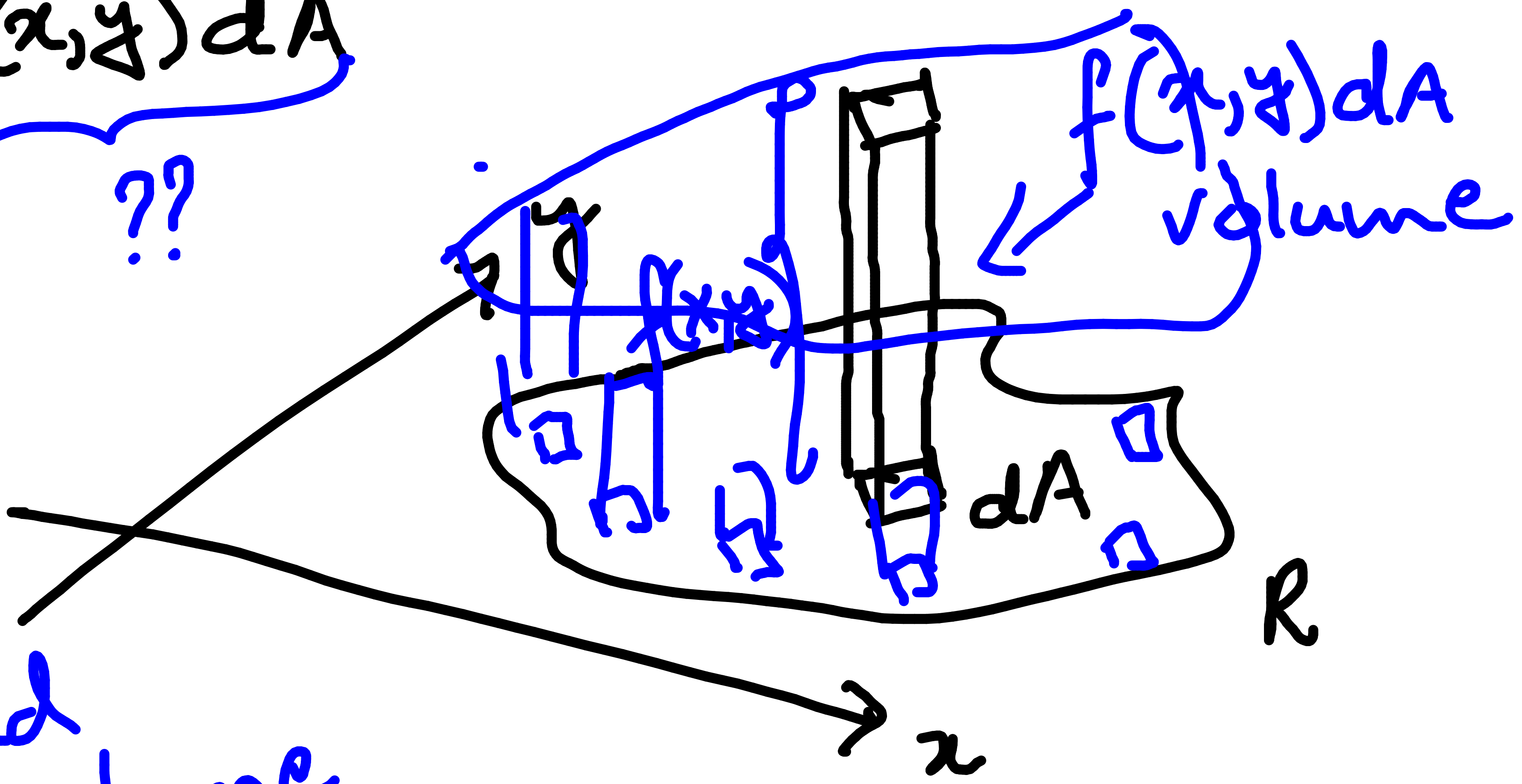
$$= \int_0^1 \frac{e^{x^2}}{x^3} \int_0^{x^2} y\,dy\,dx$$

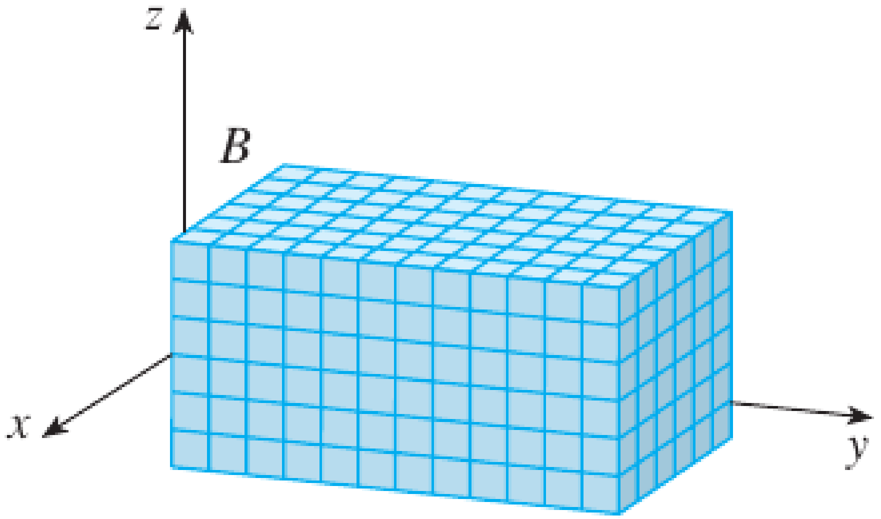
$$= \int_0^1 \frac{e^{x^2}}{x^3} \frac{x^4}{2} \, dx$$

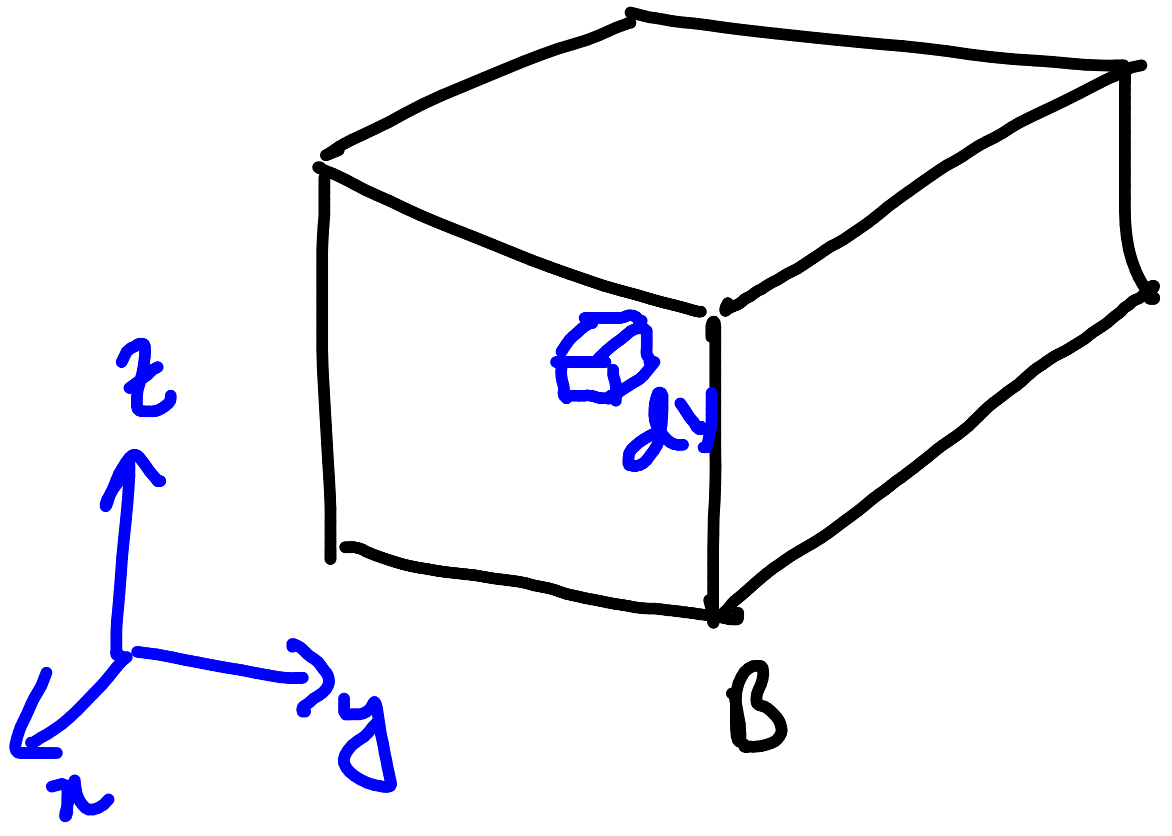
$$= \frac{e-1}{4}$$

$$\iint_{R} \underbrace{f(x,y)\,dA}_{??}$$

area gets replaced with volume







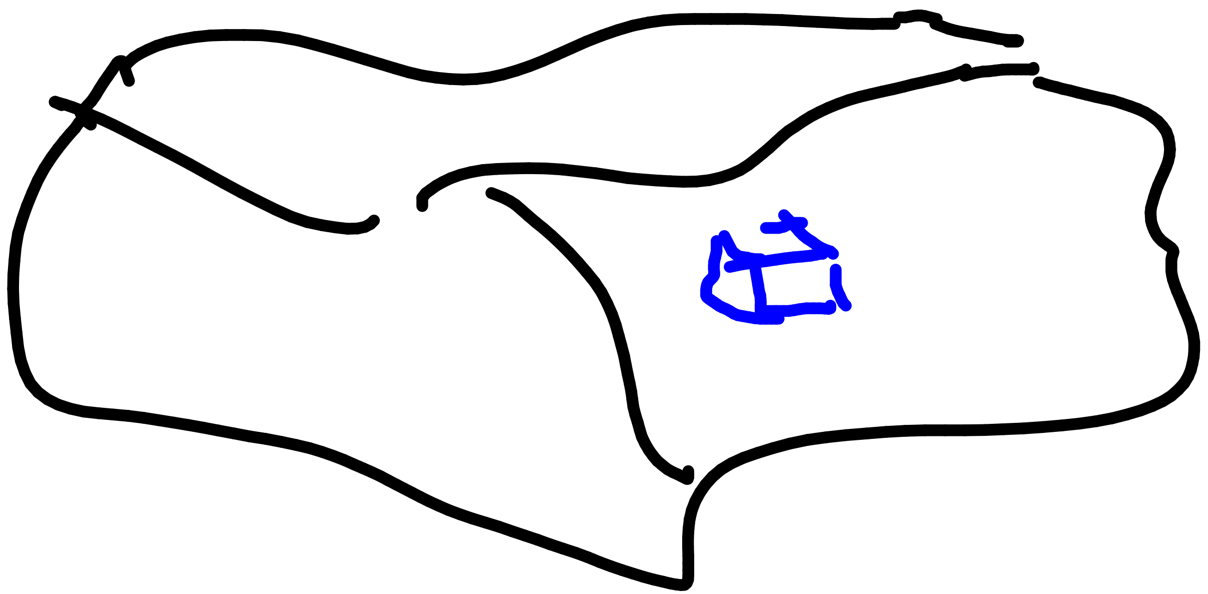
density

$$\rho(x,y,z) = \frac{1}{z}$$

Q: find the mass of the box

$$dm = \rho(x,y,z)\,dV$$

$$\iiint_B \rho(x,y,z)\,dV = \iiint_B dm = \text{total mass}$$



B

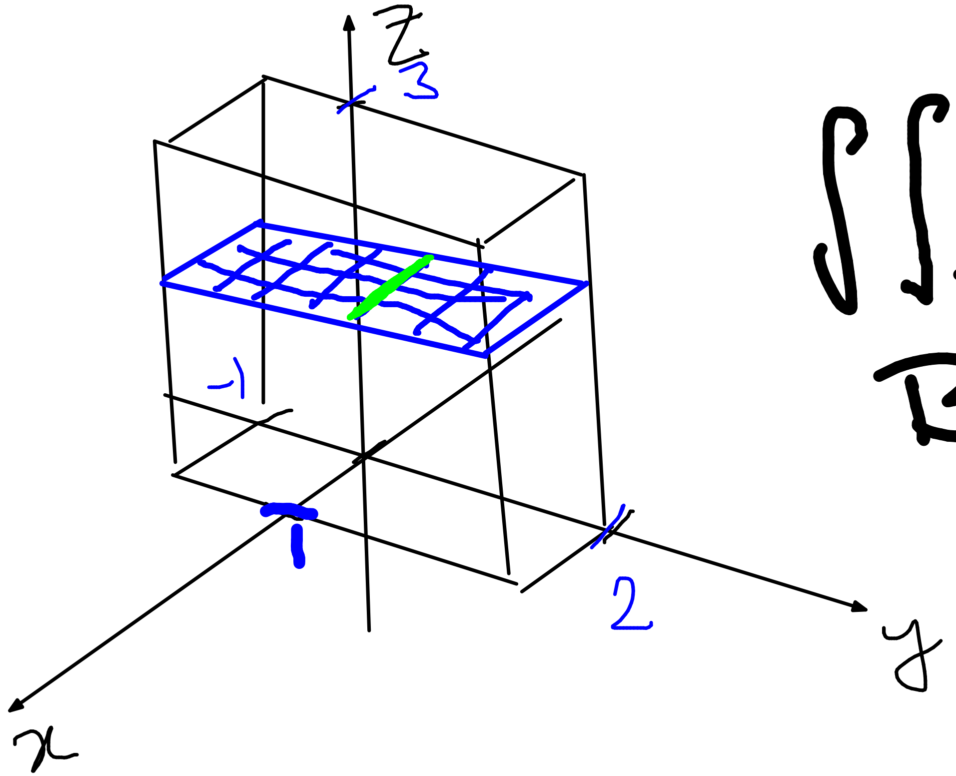
$$\iiint_B f(x,y,z)\,dV$$

→ $f(x,y,z)$ will always be some kind of density

e.g. mass/volume, charge/volume

EXAMPLE 1 Evaluate the triple integral $\iiint_B xyz^2\,dV$, where B is the rectangular box given by

$$B = \{(x, y, z) \mid 0 \leqslant x \leqslant 1,\ -1 \leqslant y \leqslant 2,\ 0 \leqslant z \leqslant 3\}$$



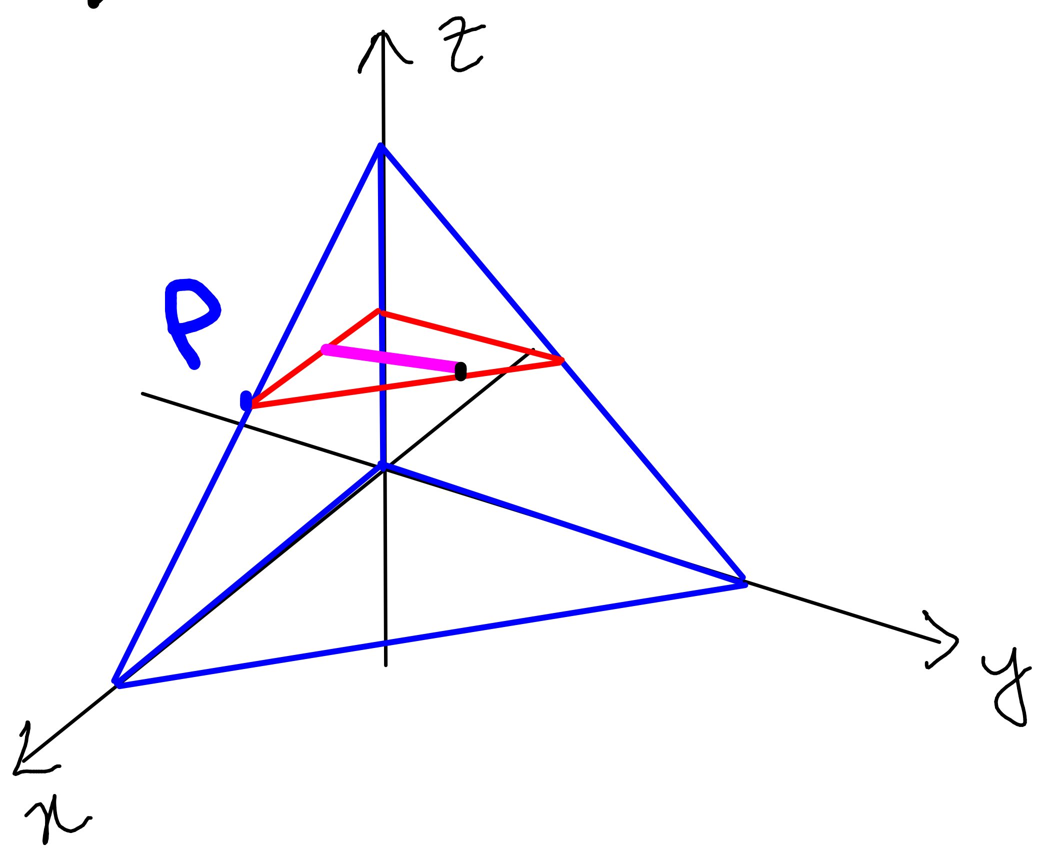
$$\iiint_B xyz^2\,dV = \int_0^3 \int_{-1}^2 \int_0^1 xyz^2\,dx\,dy\,dz$$

$$= \frac{1}{2}\int_0^3 \int_{-1}^2 yz^2\,dy\,dz = \text{do the work}$$

$$= 27/4$$

EXAMPLE 2 Evaluate $\iiint_E z\,dV$, where E is the solid tetrahedron bounded by the four planes $x = 0$, $y = 0$, $z = 0$, and $x + y + z = 1$.

→ sketch E



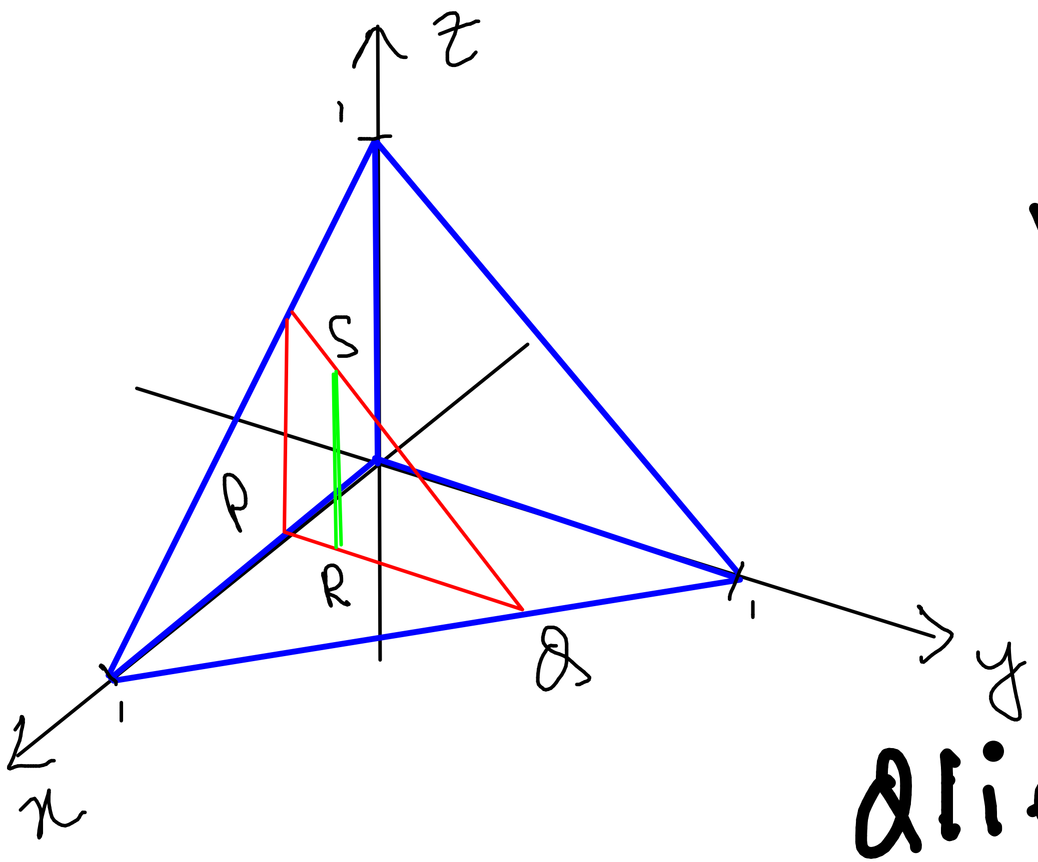
↳ how does this plane look like??

$$\int_0^1 \int_0^{1-z} \int_0^{1-x-z} z \; dy\, dx\, dz$$

$x+y+z=1$

$y = 1-x-z$

EXAMPLE 2 Evaluate $\iiint_E z\,dV$, where E is the solid tetrahedron bounded by the four planes $x = 0$, $y = 0$, $z = 0$, and $x + y + z = 1$.

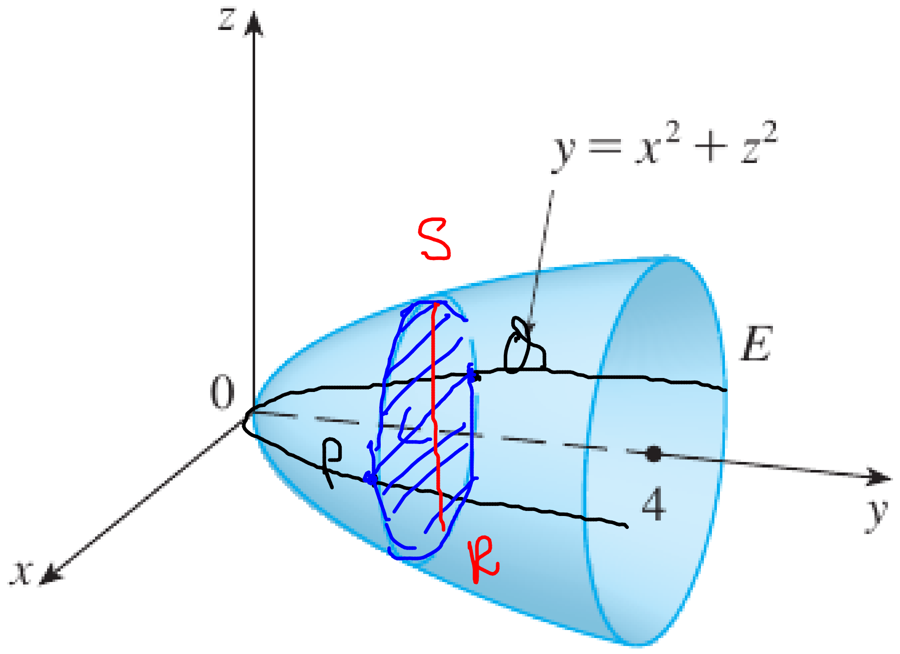


$$\int_0^1 \int_0^{1-x} \int_0^{1-x-y} z \quad dz\,dy\,dx = \frac{1}{24}$$

Q lies on $x+y=1$
$y = 1-x$

S lies on $x+y+z = 1$
$z = 1-x-y$

EXAMPLE 3 Evaluate $\iiint_E \sqrt{x^2 + z^2}\, dV$, where E is the region bounded by the paraboloid $y = x^2 + z^2$ and the plane $y = 4$.



$$\int_0^4 \int_{-\sqrt{y}}^{\sqrt{y}} \int_{-\sqrt{y-x^2}}^{\sqrt{y-x^2}} \sqrt{x^2+z^2}\; \underbrace{dz\, dx\, dy}$$

H.W. rewrite this integration in some other order

Q: P, Q are one x-y plane] $z = 0$
also on the paraboloid

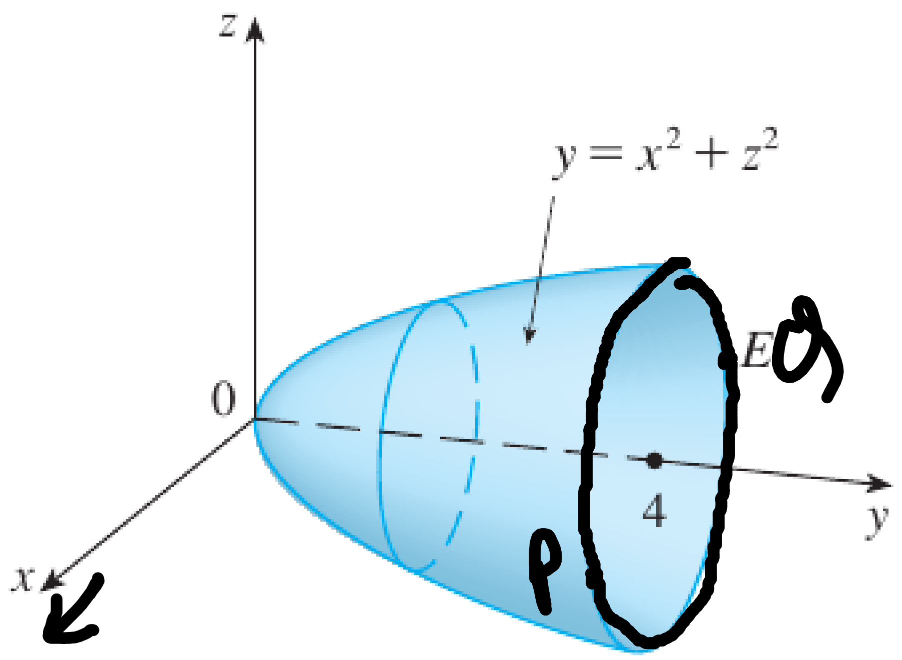
$y = x^2 + z^2$

$y = x^2 \mid x = \pm\sqrt{y}$

Q: $y = x^2 + z^2 \mid z^2 = y - x^2$

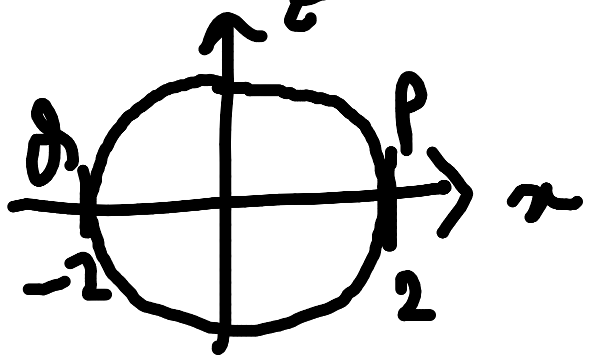
$z = \pm\sqrt{y - x^2}$

EXAMPLE 3 Evaluate $\iiint_E \sqrt{x^2 + z^2}\, dV$, where E is the region bounded by the paraboloid $y = x^2 + z^2$ and the plane $y = 4$.



$$\int_{-2}^{2} \int \int \quad dy \; dz \; dx$$

$4 = x^2 + z^2$



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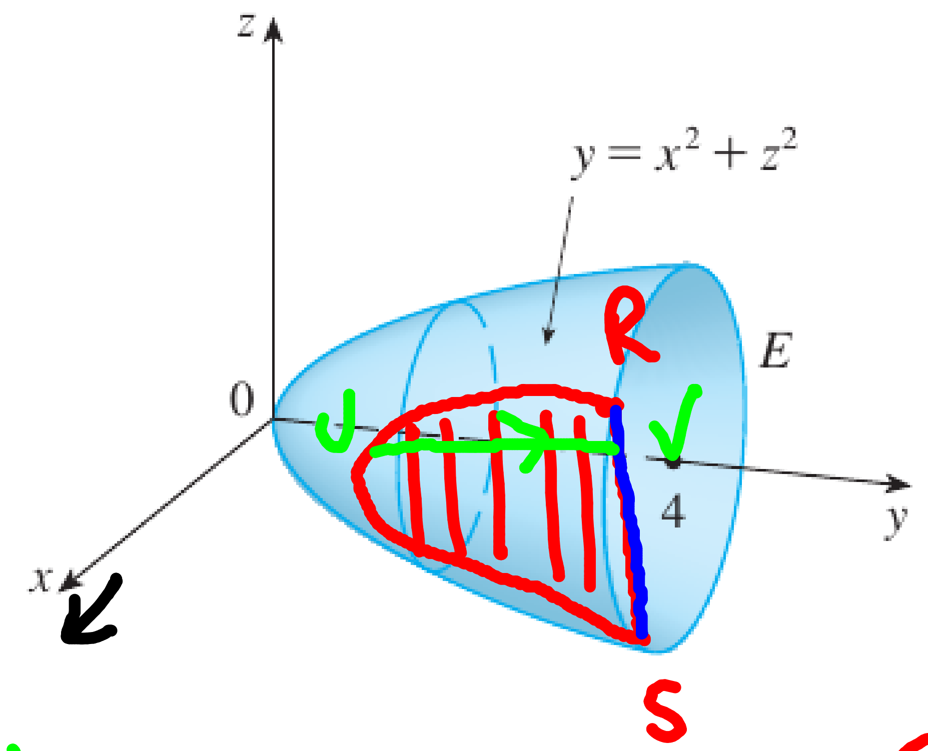
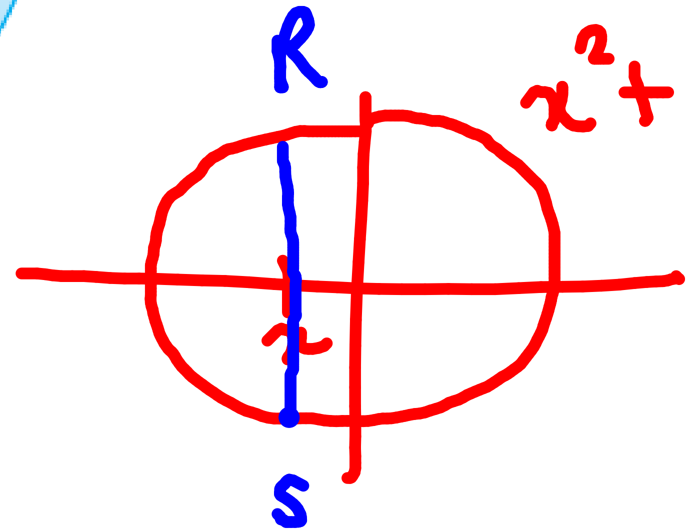


$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{x^2+z^2}^{4} \qquad dy\;dz\;dx$$

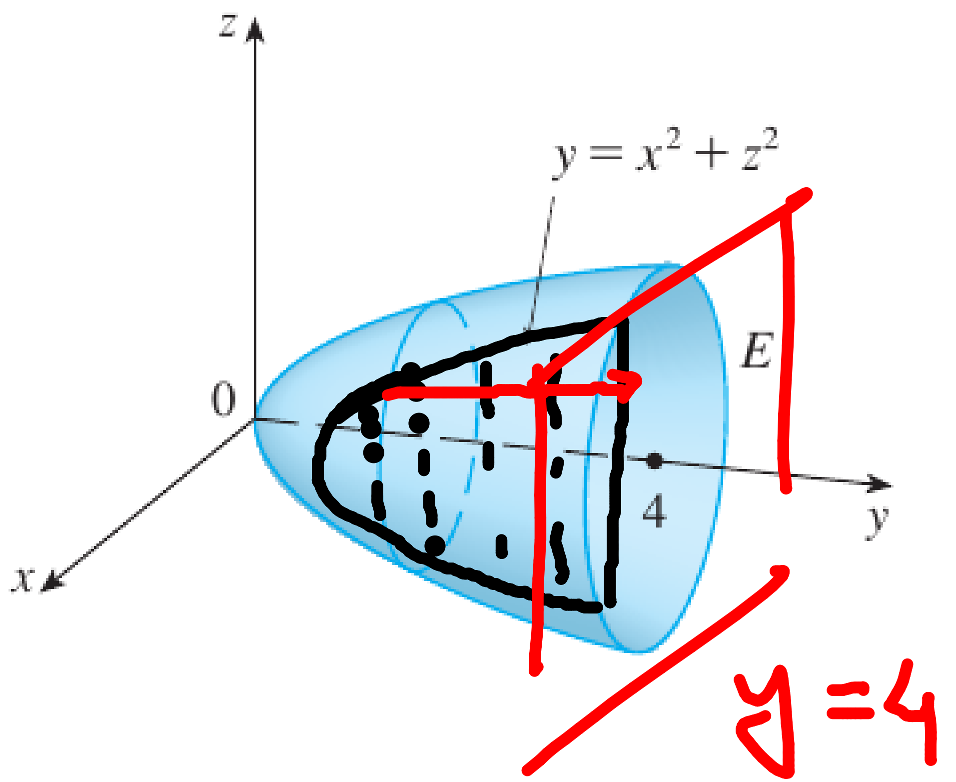
U

$y = x^2 + z^2$

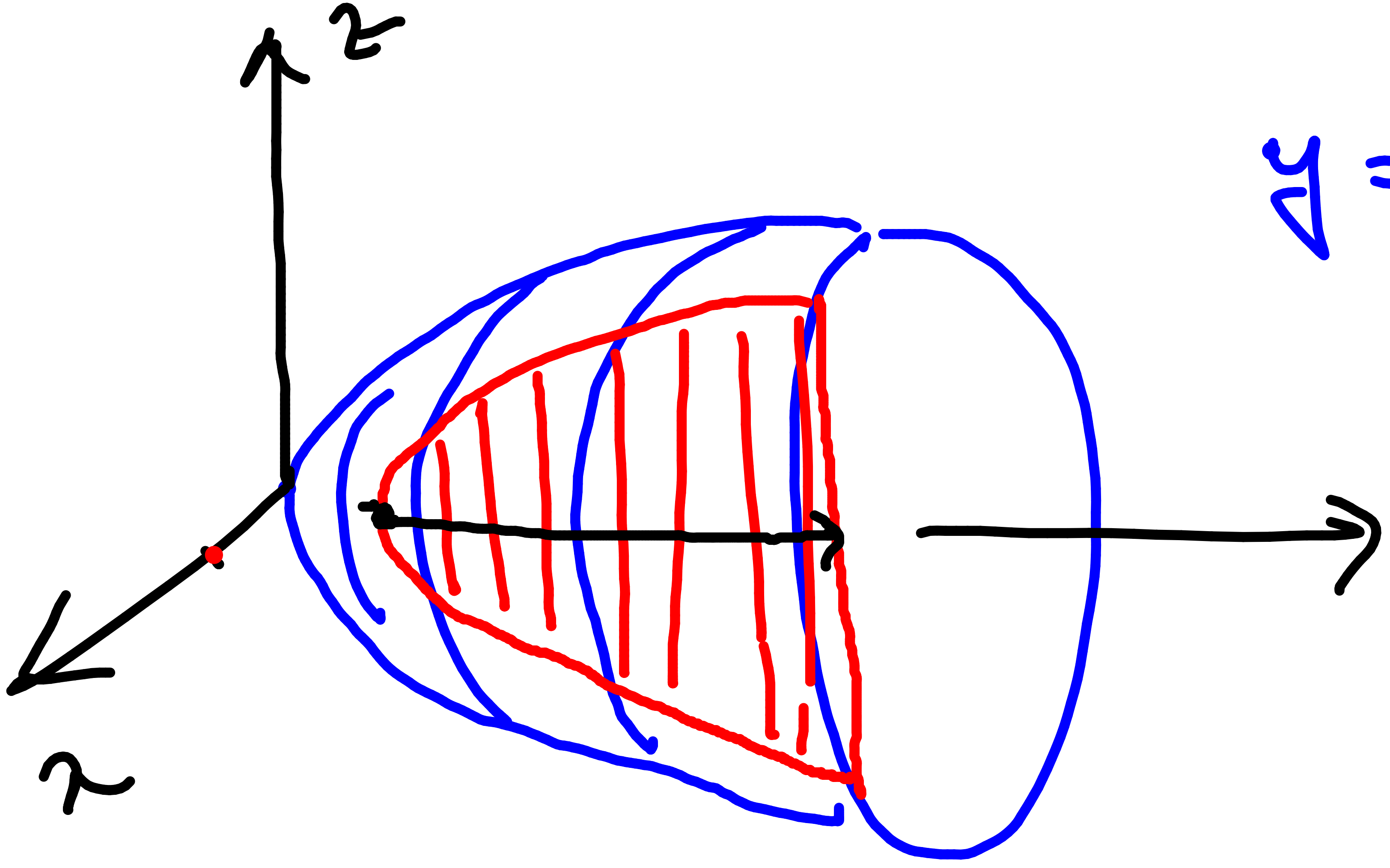


$x^2 + z^2 = 4$

V EXAMPLE 3 Evaluate $\iiint_E \sqrt{x^2 + z^2}\, dV$, where E is the region bounded by the paraboloid $y = x^2 + z^2$ and the plane $y = 4$.



$$\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{x^2+z^2}^{4} \sqrt{x^2+z^2}\; dy\; dz\; dx$$



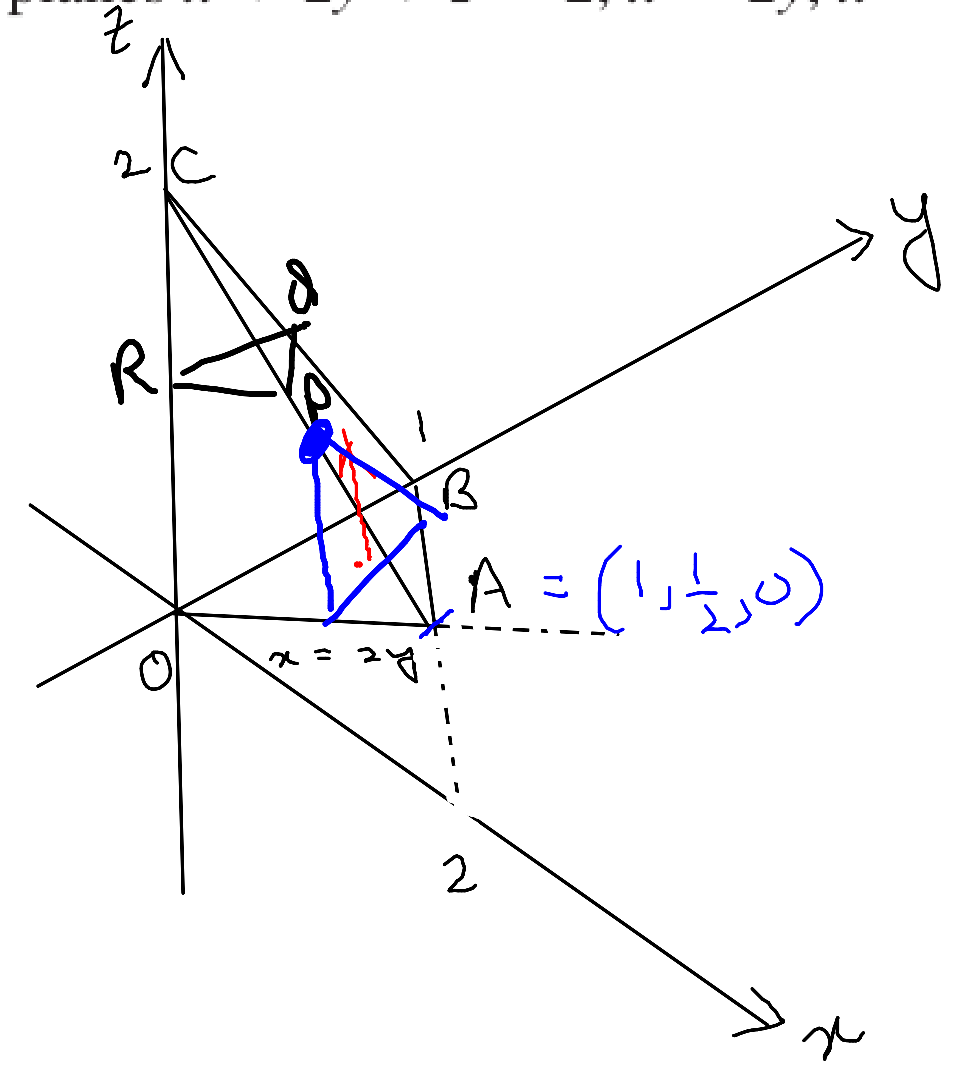
$y = x^2 + z^2$

Q: intersect the paraboloid with the plane $x = 1$

$x = 1$

$$\boxed{y = x^2 + z^2}$$

EXAMPLE 4 Use a triple integral to find the volume of the tetrahedron T bounded by the planes $x + 2y + z = 2$, $x = 2y$, $x = 0$, and $z = 0$.



$$\int_0^1 \int_0^{2-2x} \qquad dy\, dz\, dx$$

$x = 2y$

$x + 2y + z = 2$

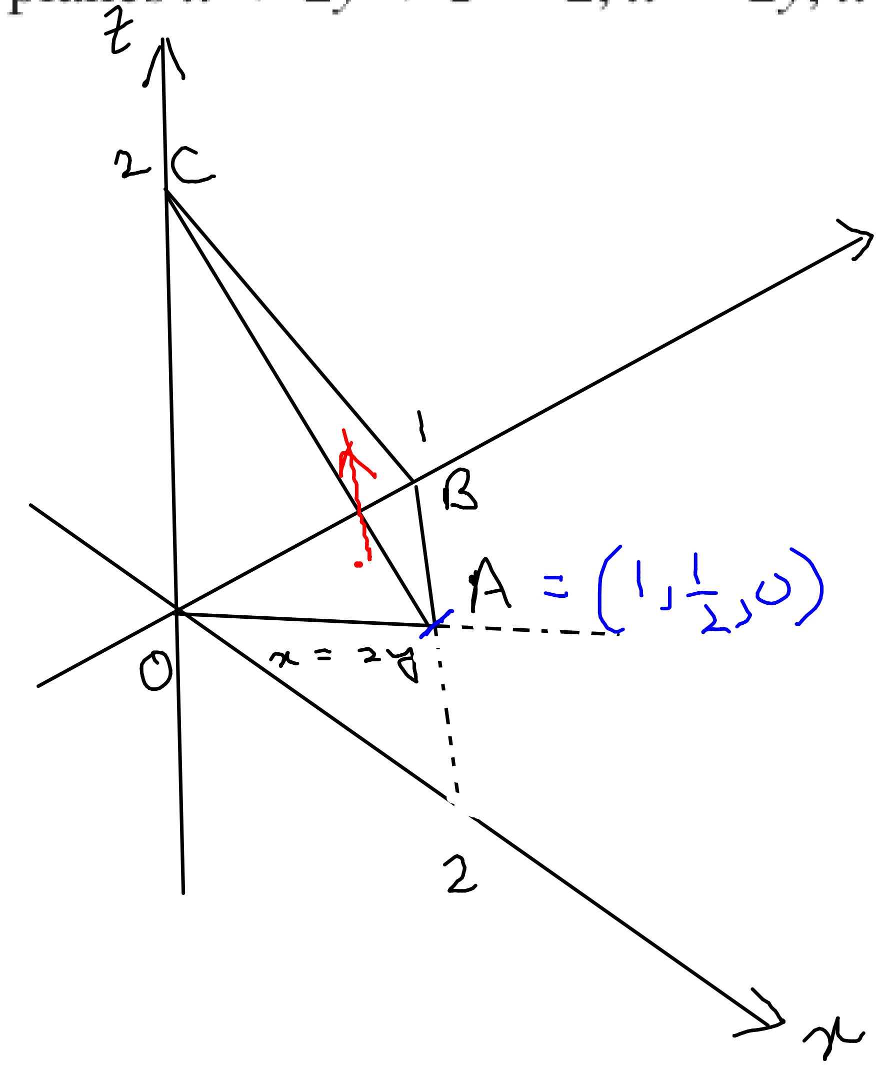
$x + x + z = 2$

$z = 2 - 2x$

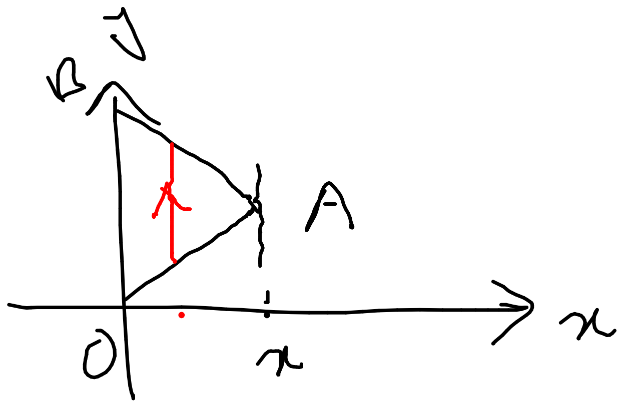
EXAMPLE 4 Use a triple integral to find the volume of the tetrahedron T bounded by the planes $x + 2y + z = 2$, $x = 2y$, $x = 0$, and $z = 0$.

solve for A

$$\left.\begin{aligned} x+2y &= 2 \\ x &= 2y \end{aligned}\right\} \quad x = ? \quad y = ?$$



$$BA: \left.\begin{aligned} x+2y+z &= 2 \\ z &= 0 \end{aligned}\right\} \quad x+2y = 2$$

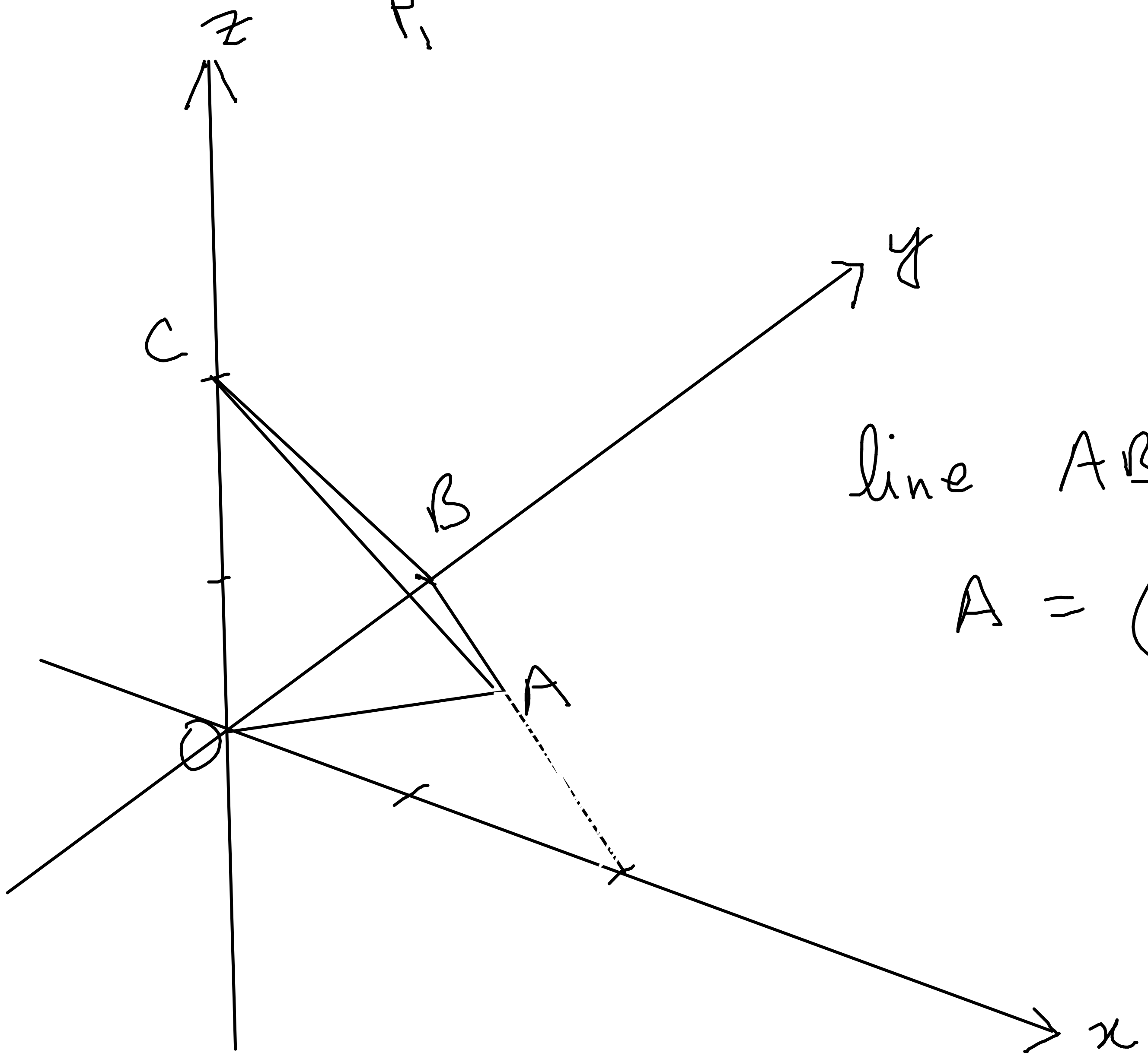


$$\int_0^1 \int_{x/2}^{(2-x)/2} \int_0^{(2-x-2y)} 1 \; dz\, dy\, dx$$

EXAMPLE 4 Use a triple integral to find the volume of the tetrahedron T bounded by the planes $\underbrace{x + 2y + z = 2}_{P_1}$, $\underbrace{x = 2y}$, $x = 0$, and $z = 0$.

→ sketch this tetrahedron T

→ set up the limits of $\iiint_{?}^{?} dV$



line AB: $x + 2y = 2$

$A = \left(1, \frac{1}{2}, 0\right)$

$$\int_0^1 \qquad dx$$

EXAMPLE 4 Use a triple integral to find the volume of the tetrahedron T bounded by the planes $\underbrace{x + 2y + z = 2}_{P_1}$, $\underbrace{x = 2y}$, $x = 0$, and $z = 0$.

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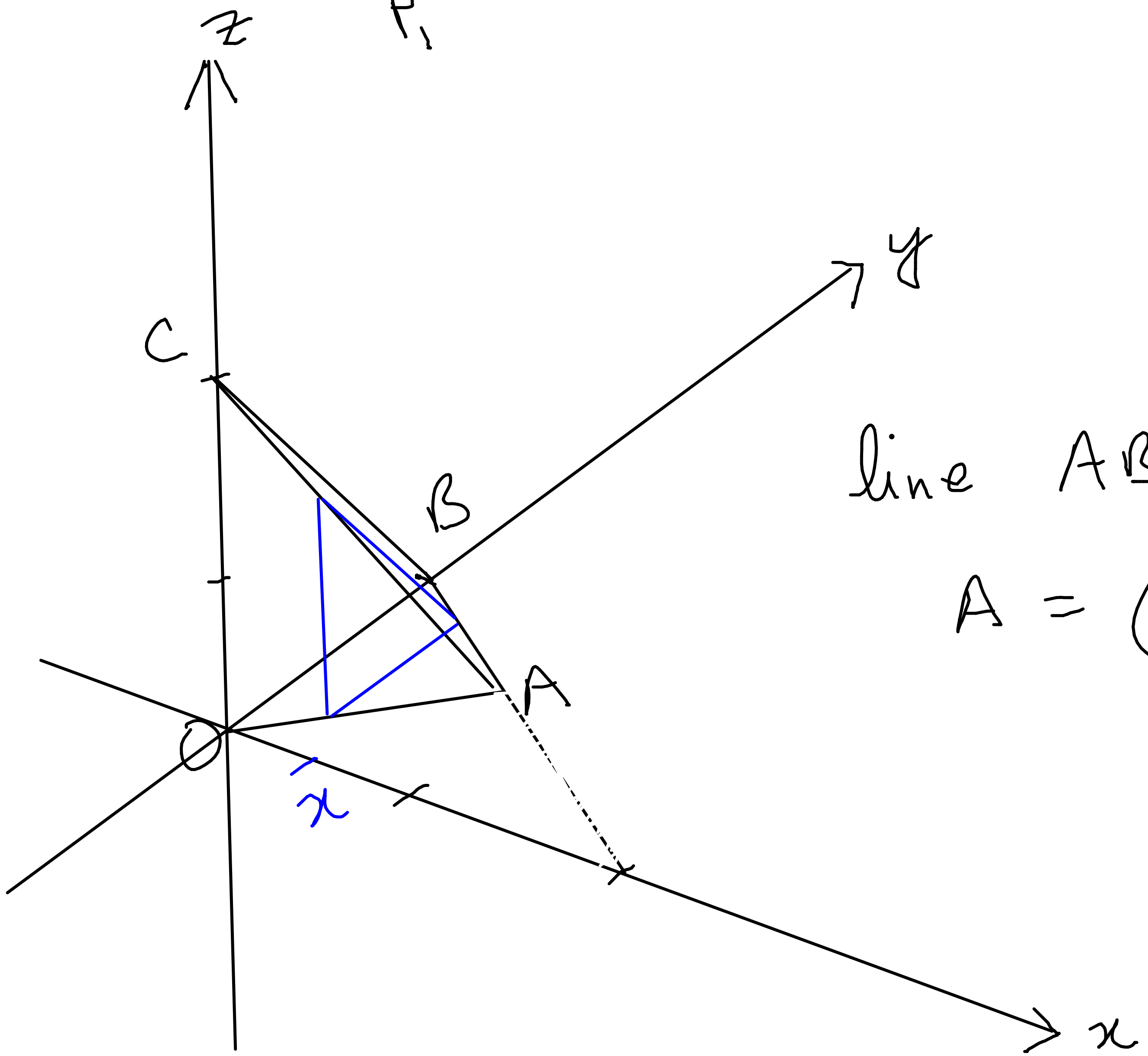

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$$\int_0^1 \int_{x/2}^{(2-x)/2} \int \; dz \, dy \, dx$$

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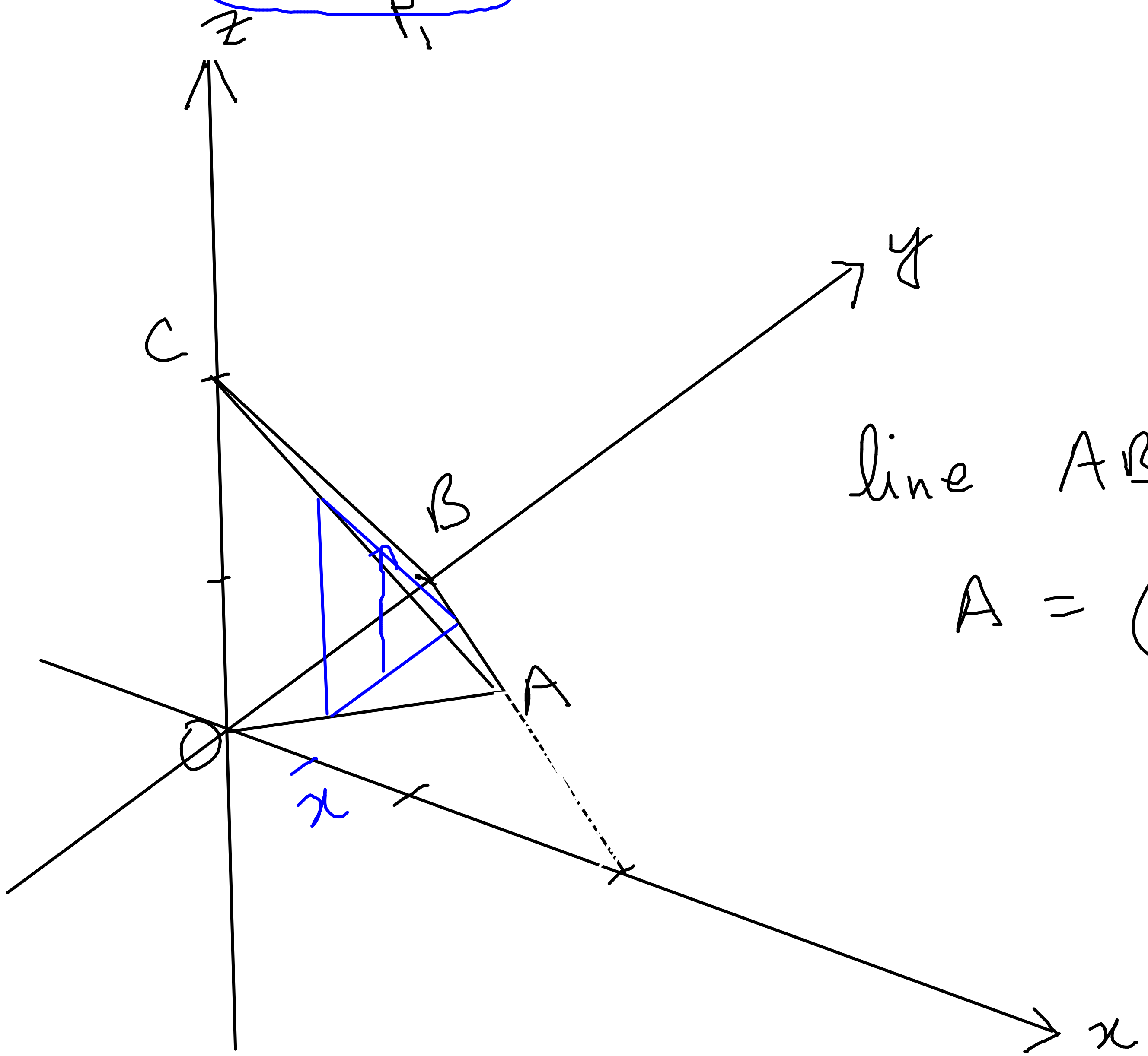


line AB: $x + 2y = 2$

$A = \left(1, \frac{1}{2}, 0\right)$

$$\int_0^1 \int_{x/2}^{(2-x)/2} \int_0^{2-x-2y} dz\, dy\, dx$$

$$\int_0^1 \int_{\frac{x}{2}}^{\frac{(2-x)}{2}} \int_0^{2-x-2y} dzdydx$$

Go

Examples »

Solution

Keep Practicing >

Show Steps

$\int_0^1 \int_{\frac{x}{2}}^{\frac{2-x}{2}} \int_0^{2-x-2y} 1dzdydx = \frac{1}{3}$ (Decimal: $0.33333\ldots$)

Steps

$$\int_0^1 \int_{\frac{x}{2}}^{\frac{2-x}{2}} \int_0^{2-x-2y} 1dzdydx$$

Show Steps

$$\int_0^{2-x-2y} 1dz = 2-x-2y$$

$$= \int_0^1 \int_{\frac{x}{2}}^{\frac{2-x}{2}} (2-x-2y)dydx$$

Show Steps

$$\int_{\frac{x}{2}}^{\frac{2-x}{2}} (2-x-2y)dy = 2-2x-x\frac{2-x}{2}+\frac{x^2}{2}-\frac{(2-x)^2}{4}+\frac{x^2}{4}$$

$$= \int_0^1 \left(2-2x-x\frac{2-x}{2}+\frac{x^2}{2}-\frac{(2-x)^2}{4}+\frac{x^2}{4}\right)dx$$

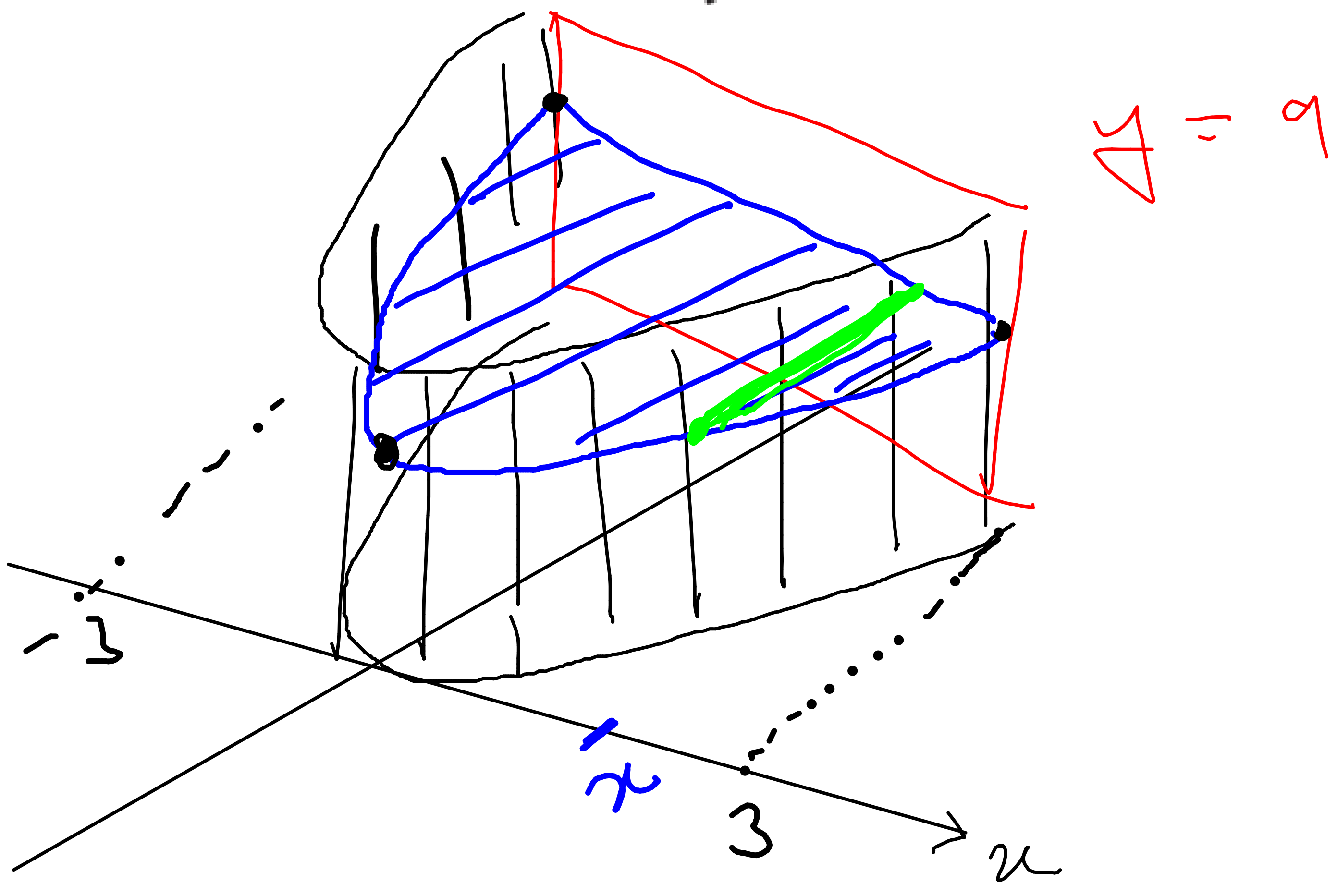
Show Steps

$$\int_0^1 \left(2-2x-x\frac{2-x}{2}+\frac{x^2}{2}-\frac{(2-x)^2}{4}+\frac{x^2}{4}\right)dx = \frac{1}{3}$$

Q.

Use a triple integral to find the volume of solid. en

The solid bounded by the cylinder $y = x^2$ and the planes $z = 0$, $z = 4$, and $y = 9$



$$\int_0^4 \int_0^9 \int_{-\sqrt{y}}^{\sqrt{y}} 1 \; dx\,dy\,dz$$

$$= \int_0^4 \int_{-3}^3 \int_{x^2}^9 1 \; dy\,dx\,dz$$

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Use a triple integral to find the volume of solid. en

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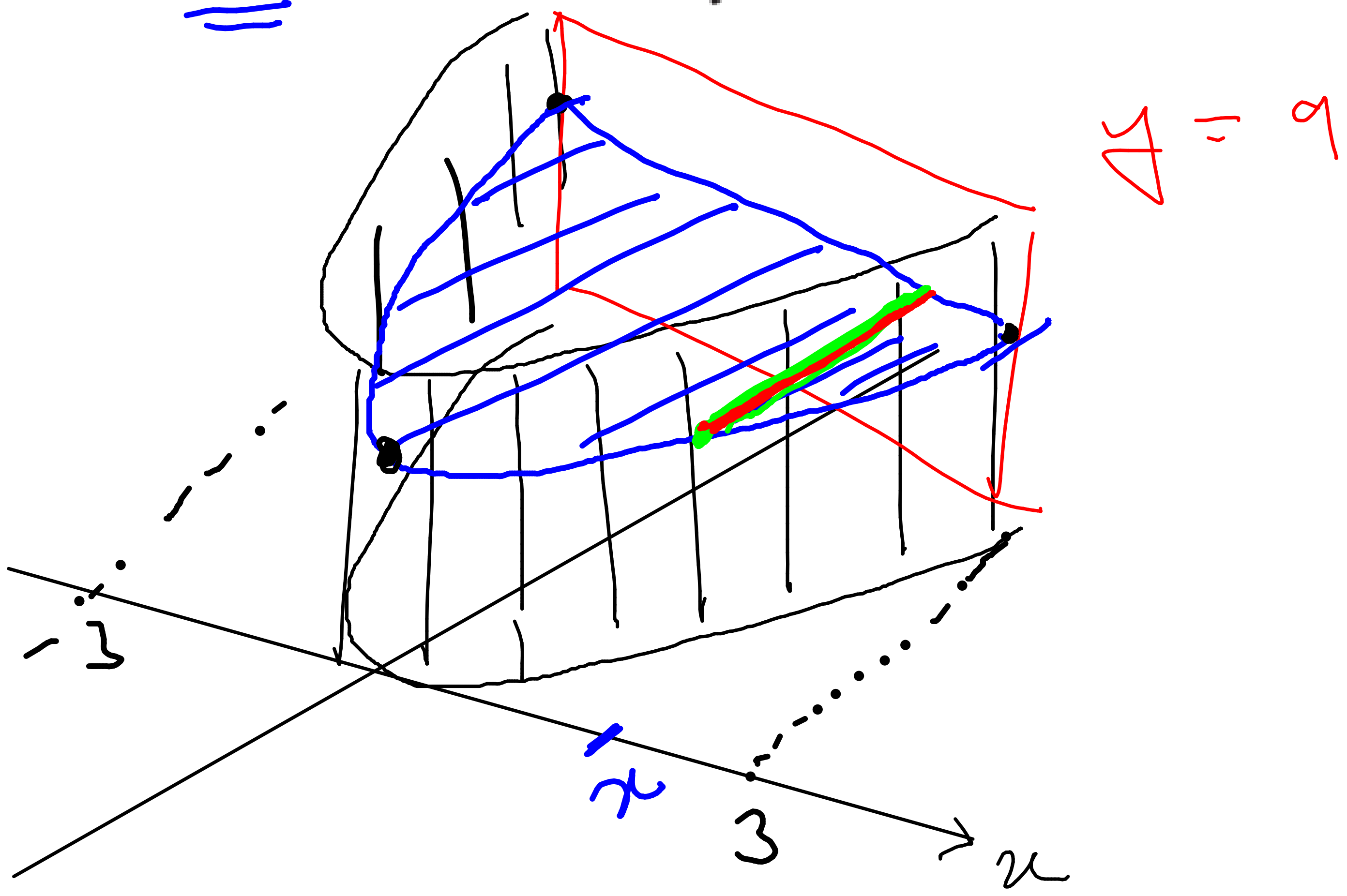

(x, y, z)

$0 \le z \le 4$

& for each z in this range what are the range or x&y

$-3 \le x \le 3$

& for each of x above

$x^2 \le y \le 9$

Sketch the solid whose volume is given by the iterated integral.

$$\int_0^2 \int_0^{2-y} \int_0^{4-y^2} dx\, dz\, dy$$

47. Find the region E for which the triple integral

$$\iiint_E (1 - x^2 - 2y^2 - 3z^2)\, dV$$

is a maximum.