MAT 104

mathematical methods II

→ Matrix Algebra (Linear Algebra)
(geometry behind matrices)

→ Ordinary Differential Equation

$$\frac{dy}{dx} + Py = Q$$

But we will finish

Calculus first

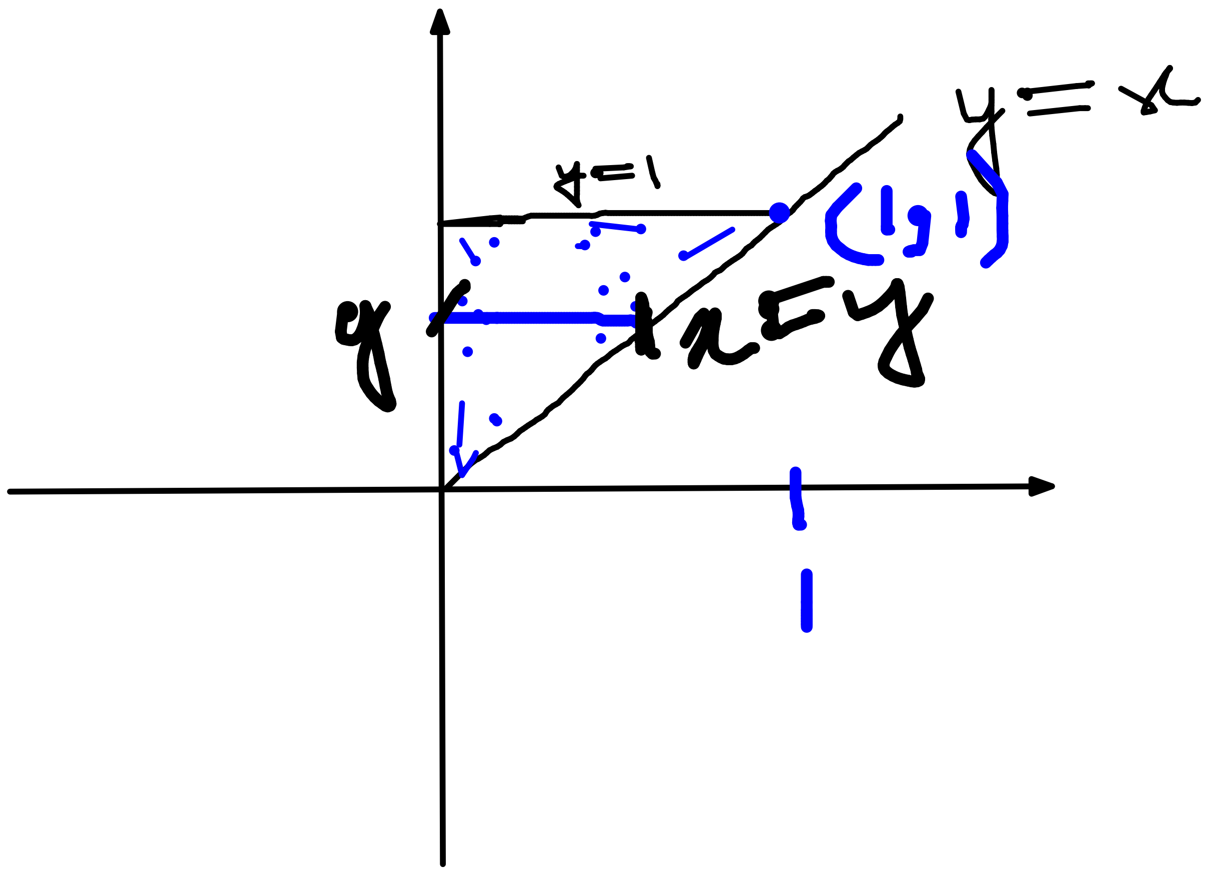
→ Essential Calculus

12.5 & later

Calculate the iterated integral by first reversing the order of integration.

$$\int_0^1 \int_x^1 \cos(y^2)\, dy\, dx \quad = \quad \int_a^b \int_c^d \cos(y^2)\, dx\, dy$$

$a, b, c, d = ??$



$$= \int_0^1 \int_0^y \cos(y^2)\, dx\, dy$$

$$\int_0^y \cos(y^2)\, dx = \cos(y^2) \int_0^y dx$$

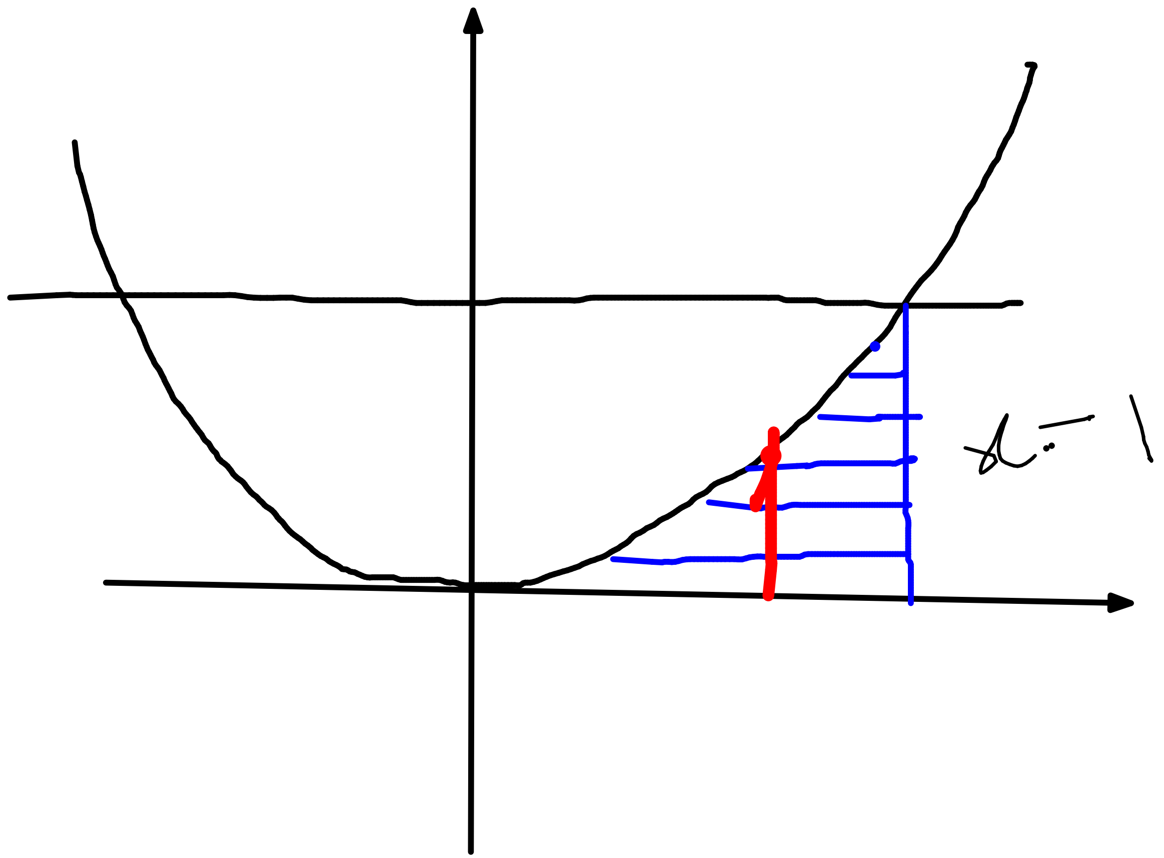
$$= \cos(y^2)\, y$$

$$\int_0^1 y\cos(y^2)\, dy = \frac{\sin(1)}{2}$$

Calculate the iterated integral by first reversing the order of integration.

$$\int_0^1 \int_{\sqrt{y}}^1 \frac{ye^{x^2}}{x^3}\,dx\,dy$$

sketch the region of integration



$x = \sqrt{y}$

$y = x^2$

$$\int_0^1 \int_0^{x^2} \frac{ye^{x^2}}{x^3}\,dy\,dx$$

$$= \int_0^1 \frac{e^{x^2}}{x^3} \int_0^{x^2} y\,dy\,dx$$

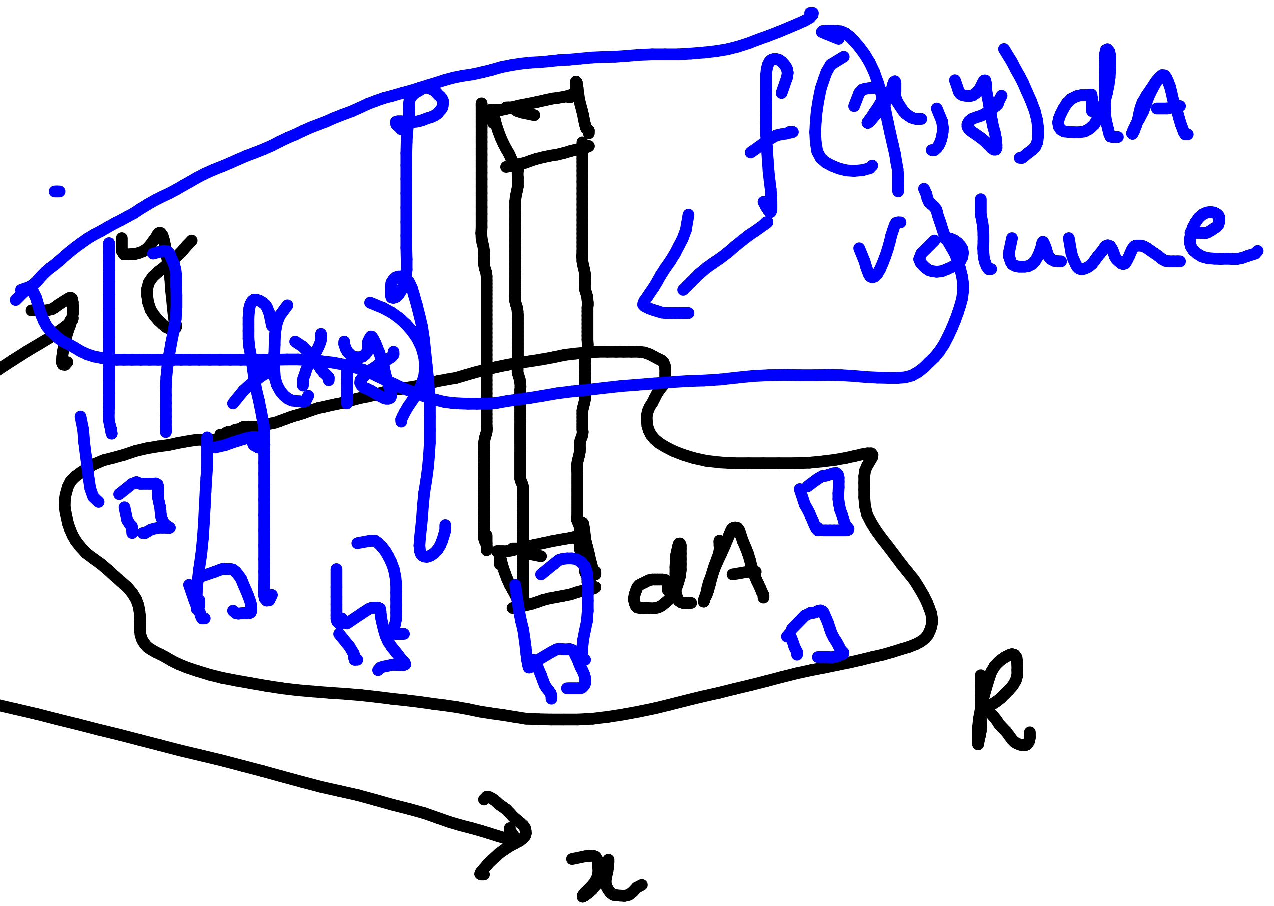
$$= \int_0^1 \frac{e^{x^2}}{x^3} \frac{x^4}{2} \, dx$$

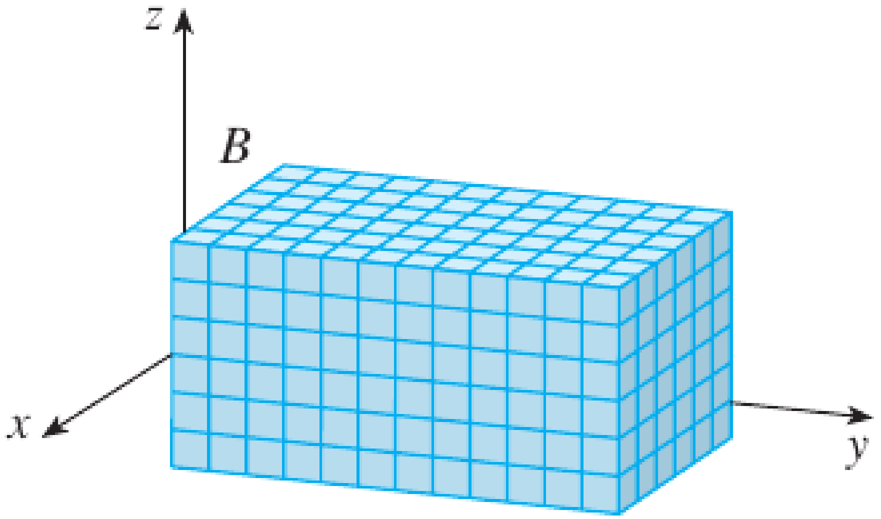
$$= \frac{e-1}{4}$$

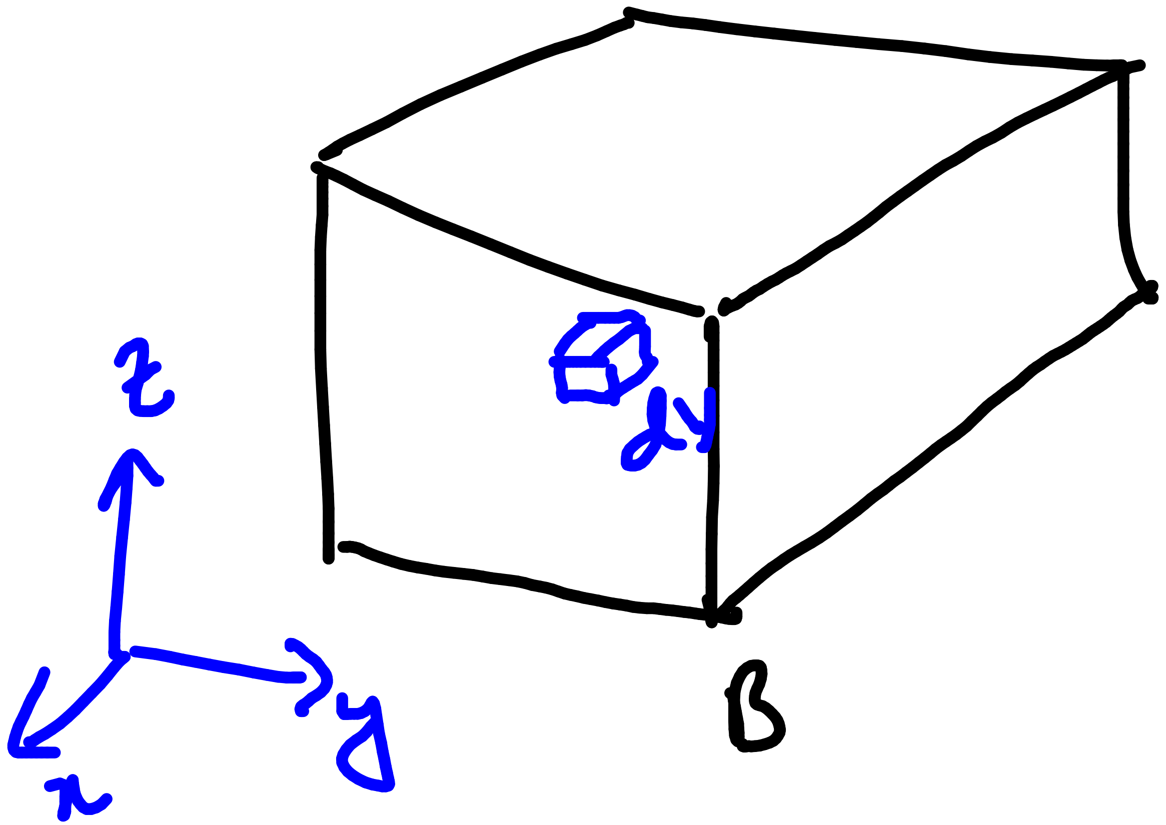
12.5 TRIPLE INTEGRALS

$$\iint_{R} \underbrace{f(x,y)\,dA}_{??}$$

area gets replaced with volume







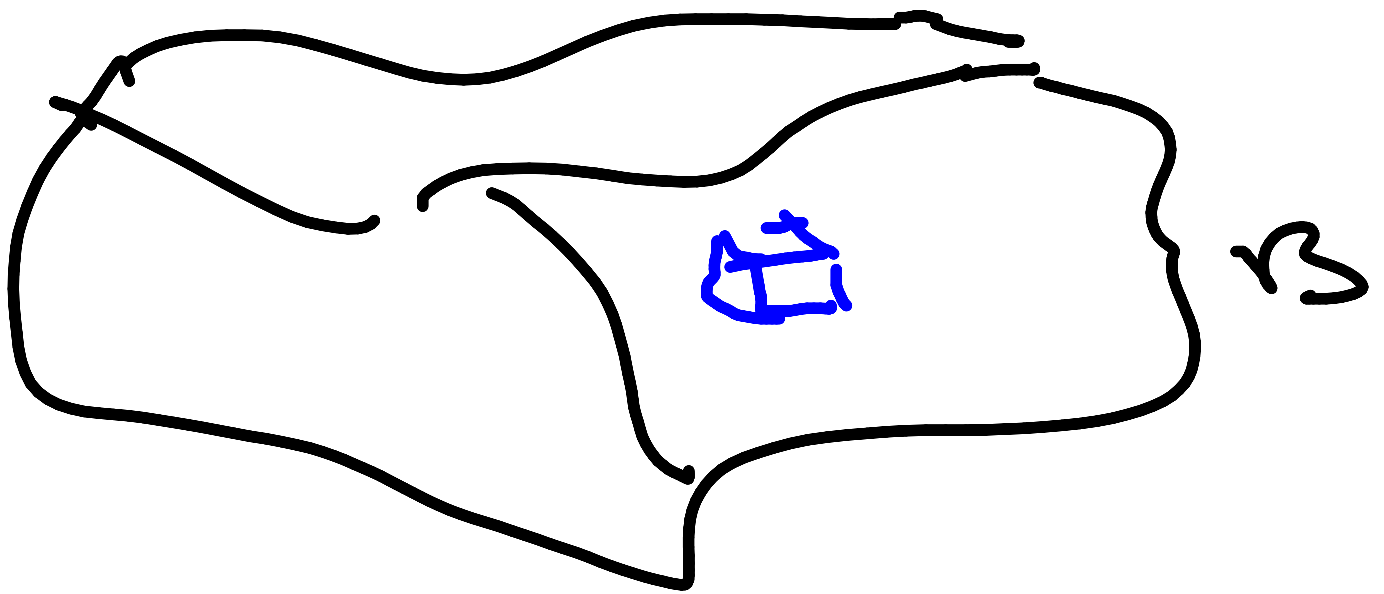
density

$$\rho(x,y,z) = \frac{1}{z}$$

Q: find the mass of the box

$$dm = \rho(x,y,z)\,dV$$

$$\iiint_B \rho(x,y,z)\,dV = \iiint_B dm = \text{total mass}$$

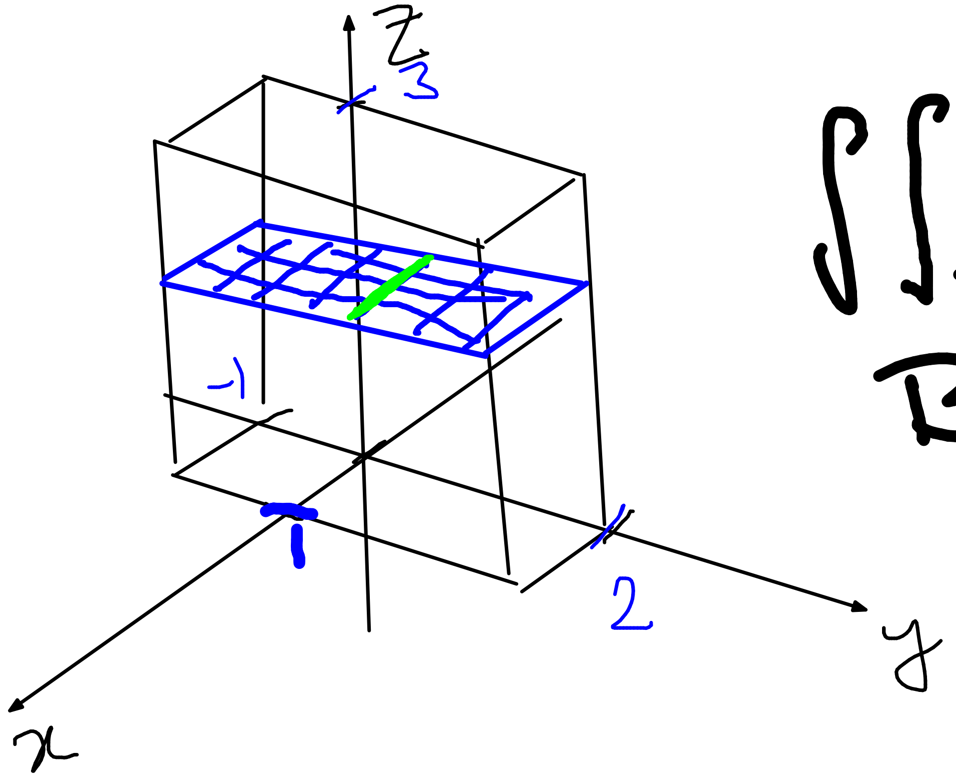


$$\iiint_B f(x,y,z)\,dV$$

→ $f(x,y,z)$ will always be some kind of density
e.g. mass/volume, charge/volume

EXAMPLE 1 Evaluate the triple integral $\iiint_B xyz^2\,dV$, where B is the rectangular box given by

$$B = \{(x, y, z) \mid 0 \leqslant x \leqslant 1,\ -1 \leqslant y \leqslant 2,\ 0 \leqslant z \leqslant 3\}$$



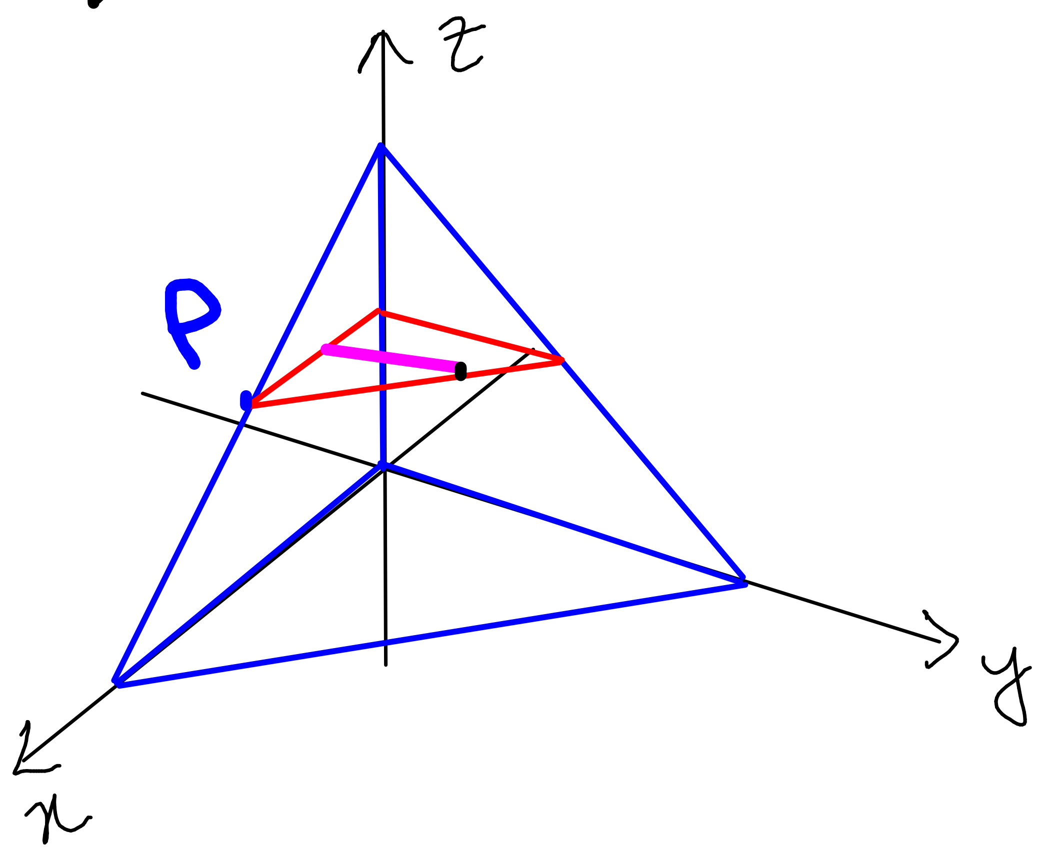
$$\iiint_B xyz^2\,dV = \int_0^3 \int_{-1}^2 \int_0^1 xyz^2\,dx\,dy\,dz$$

$$= \frac{1}{2}\int_0^3 \int_{-1}^2 yz^2\,dy\,dz = \text{do the work}$$

$$= 27/4$$

EXAMPLE 2 Evaluate $\iiint_E z\,dV$, where E is the solid tetrahedron bounded by the four planes $x = 0$, $y = 0$, $z = 0$, and $x + y + z = 1$.

→ sketch E



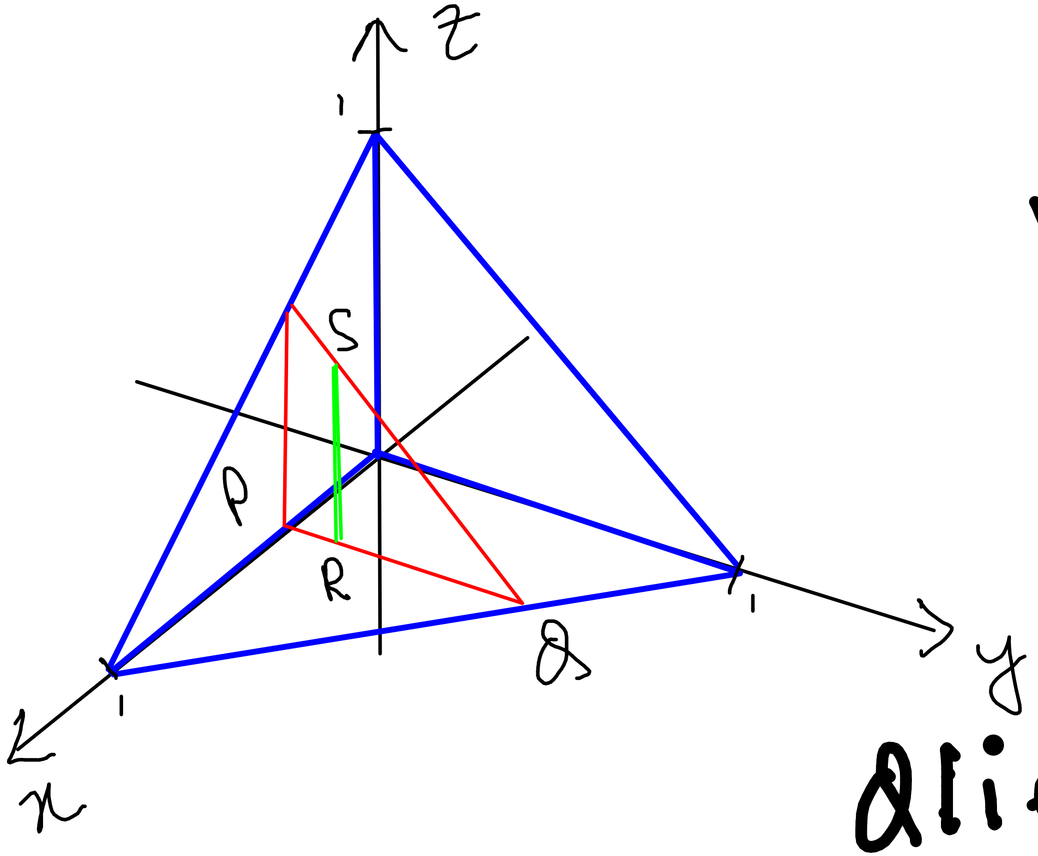
↳ how does this plane look like??

$$\int_0^1 \int_0^{1-z} \int_0^{1-x-z} z \; dy\,dx\,dz$$

$x+y+z=1$

$y = 1-x-z$

EXAMPLE 2 Evaluate $\iiint_E z\, dV$, where E is the solid tetrahedron bounded by the four planes $x = 0$, $y = 0$, $z = 0$, and $x + y + z = 1$.



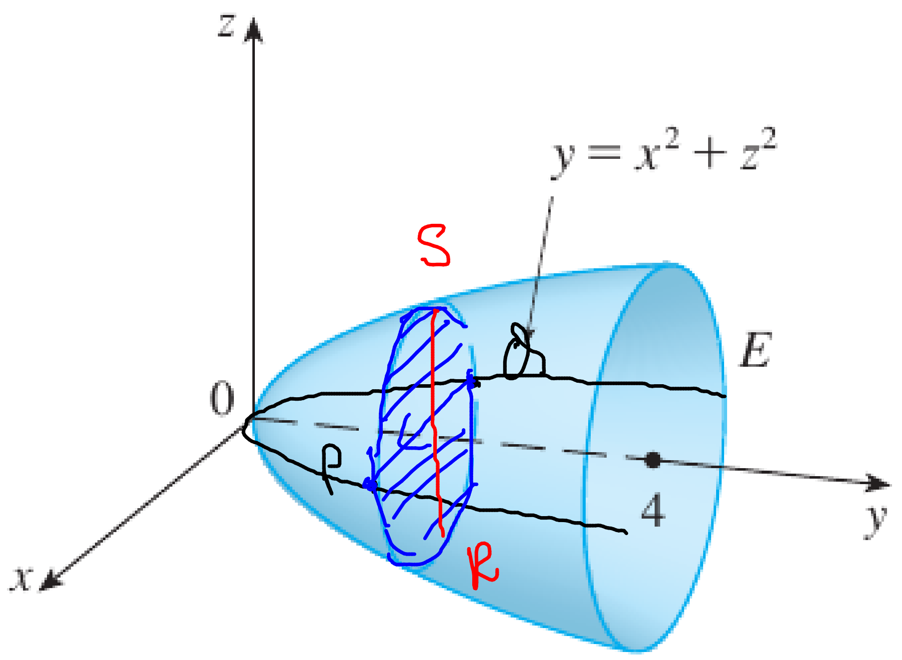
$$\int_0^1 \int_0^{1-x} \int_0^{1-x-y} z \quad dz\, dy\, dx$$

$$= \frac{1}{24}$$

Q lies on $x+y=1$
$y = 1-x$

S lies on
$x+y+z=1$
$z = 1-x-y$

V EXAMPLE 3 Evaluate $\iiint_E \sqrt{x^2 + z^2}\, dV$, where E is the region bounded by the paraboloid $y = x^2 + z^2$ and the plane $y = 4$.



$$\int_0^4 \int_{-\sqrt{y}}^{\sqrt{y}} \int_{-\sqrt{y-x^2}}^{\sqrt{y-x^2}} \sqrt{x^2+z^2}\, dz\, dx\, dy$$

H.W. rewrite this integration in some other order

Q: P, Q are on x-y plane] $z = 0$
also on the paraboloid

$y = x^2 + z^2$

$y = x^2 \mid x = \pm\sqrt{y}$

Q: $y = x^2 + z^2 \mid z^2 = y - x^2$

$z = \pm\sqrt{y - x^2}$

EXAMPLE 4 Use a triple integral to find the volume of the tetrahedron T bounded by the planes $x + 2y + z = 2$, $x = 2y$, $x = 0$, and $z = 0$.

Use a triple integral to find the volume of the given solid.

The solid bounded by the cylinder $y = x^2$ and the planes $z = 0$, $z = 4$, and $y = 9$

Sketch the solid whose volume is given by the iterated integral.

$$\int_0^2 \int_0^{2-y} \int_0^{4-y^2} dx\, dz\, dy$$

47. Find the region E for which the triple integral

$$\iiint_E (1 - x^2 - 2y^2 - 3z^2)\, dV$$

is a maximum.