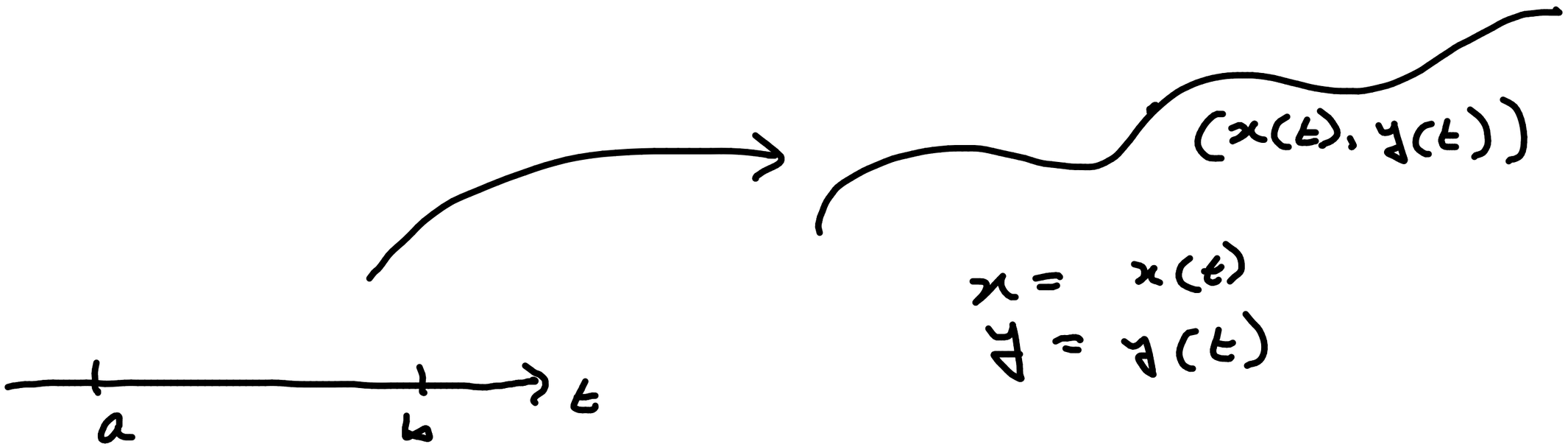
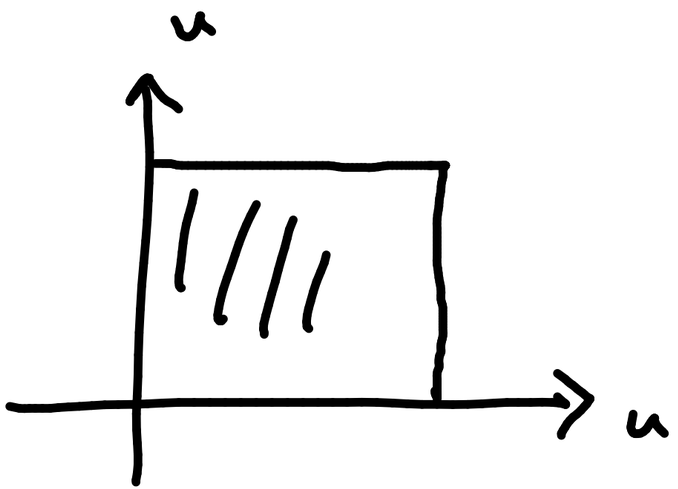
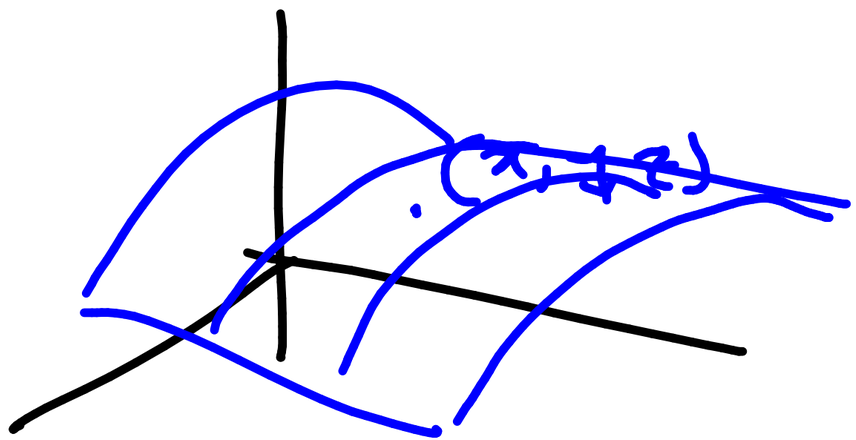
Recall parametric curves:



Parametric Surfaces



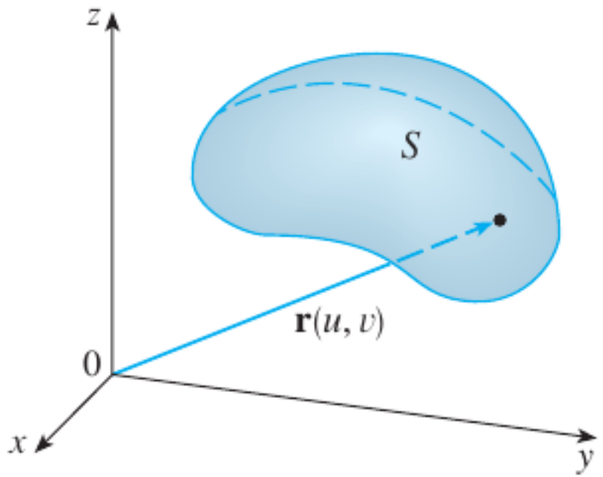


$$x = x(u,v)$$

$$y = y(u,v)$$

$$z = z(u,v)$$

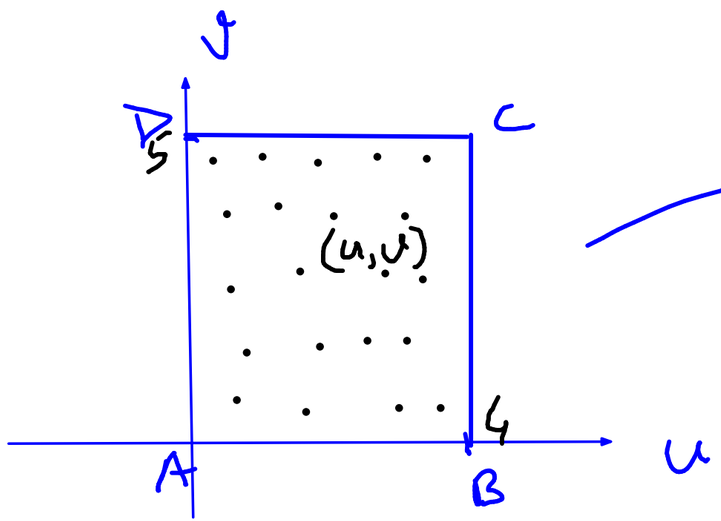
13.6 PARAMETRIC SURFACES AND THEIR AREAS



$$\mathbf{r}(u, v) = x(u, v)\,\mathbf{i} + y(u, v)\,\mathbf{j} + z(u, v)\,\mathbf{k}$$

Q. $0 \le u \le 4$

$0 \le v \le 5$

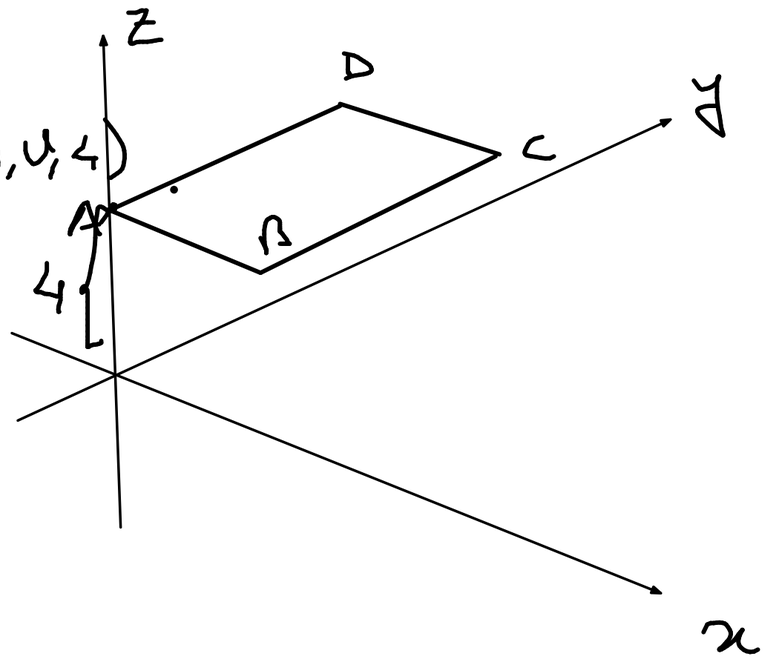


$x = u$

$y = v$

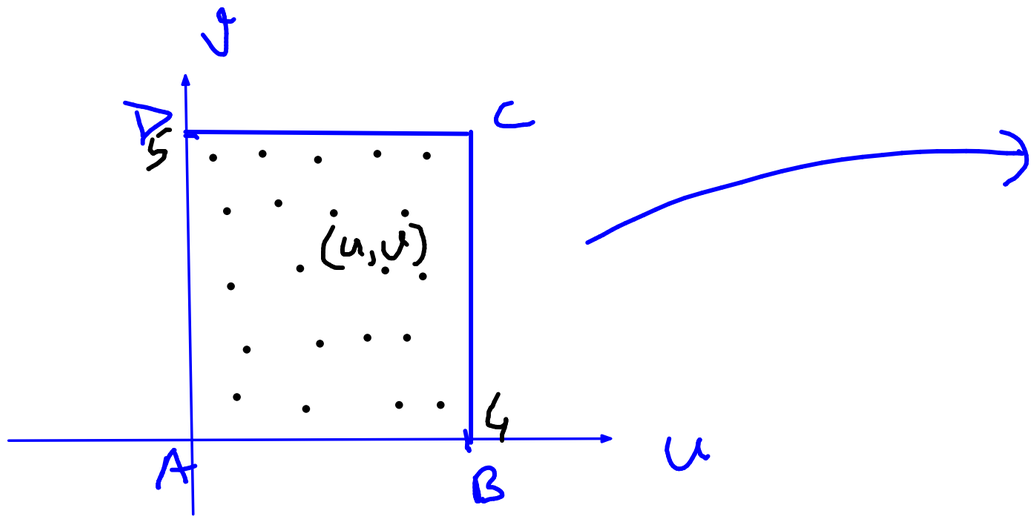
$z = 4$

$(x, y, z) = (u, v, 4)$



Q. $0 \le u \le 4$

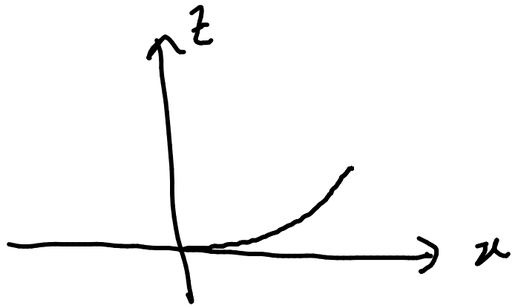
$0 \le v \le 5$

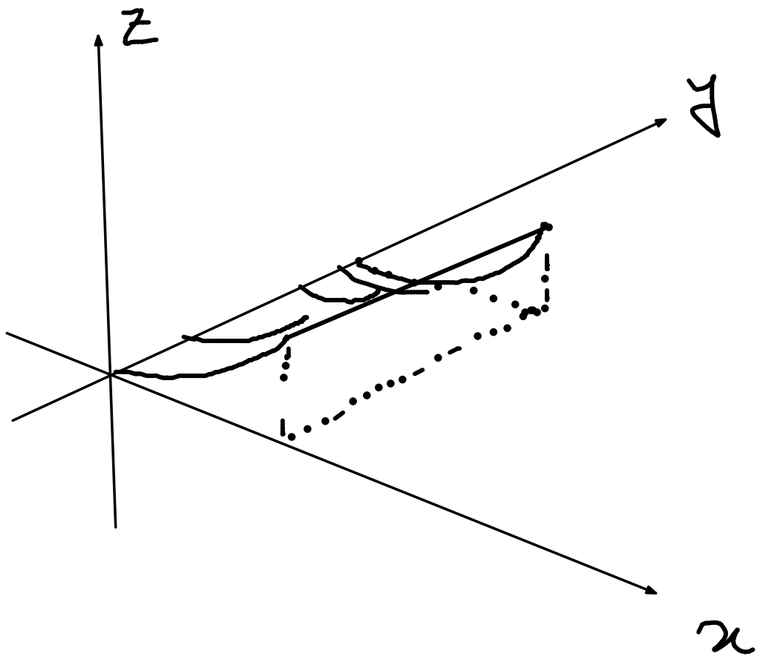


$x = u$

$y = v$

$z = u^2$

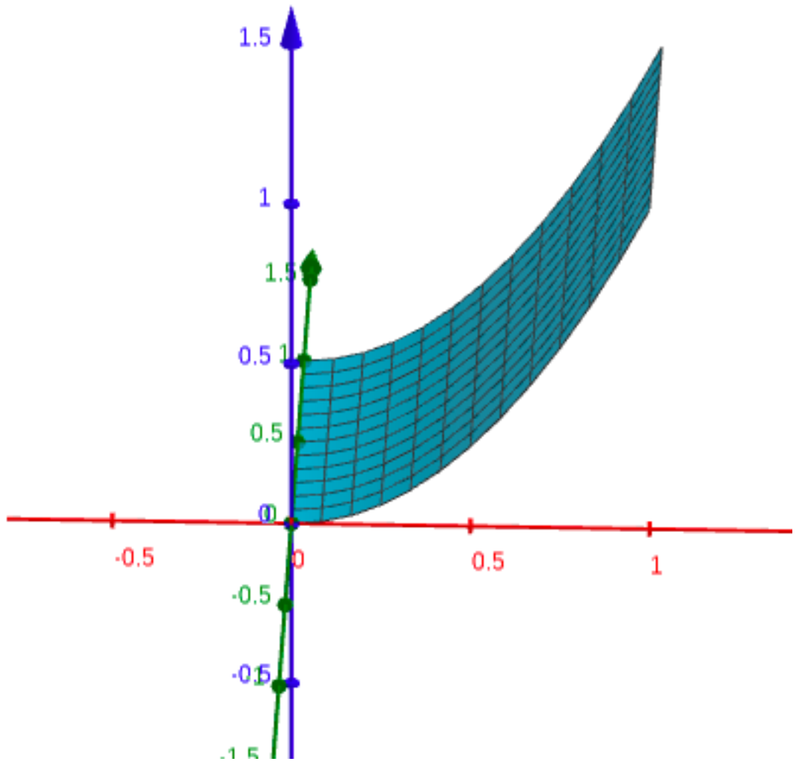




$$a = \text{Surface}(u, v, u^2, u, 0, 1, v, 0, 1)$$

$$\rightarrow \begin{pmatrix} u \\ v \\ u^2 \end{pmatrix}$$

Input...



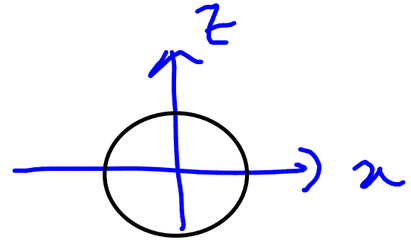
EXAMPLE 1 Identify and sketch the surface with vector equation

$$\mathbf{r}(u, v) = 2\cos u\,\mathbf{i} + v\,\mathbf{j} + 2\sin u\,\mathbf{k}$$

$x = 2\cos u$

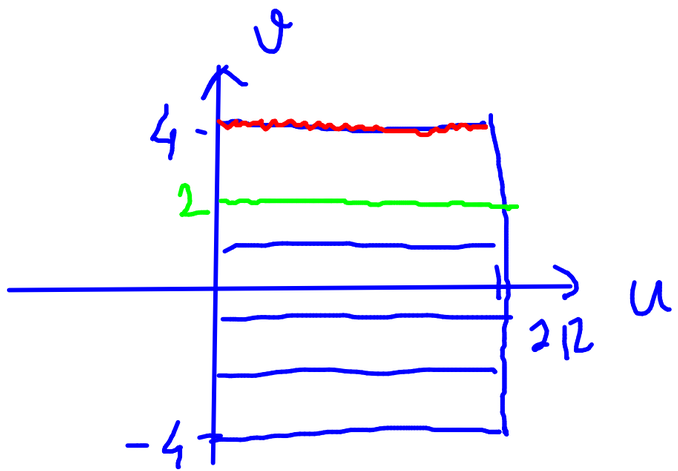
$y = v$

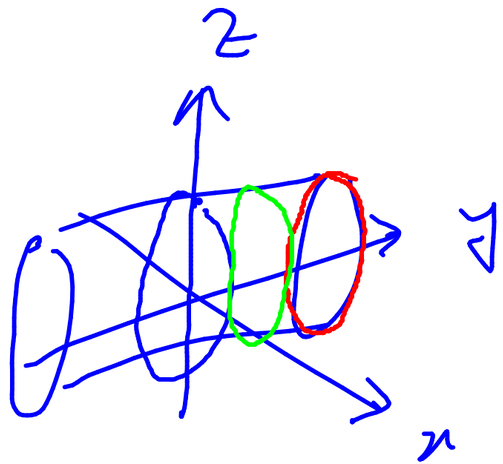
$z = 2\sin u$



$0 \le u \le 2\pi$

$-4 \le v \le 4$

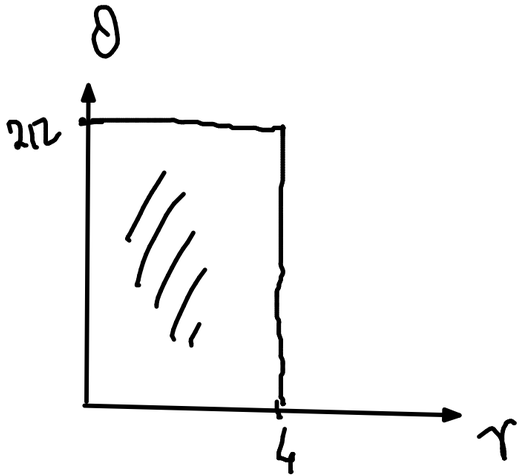




EXAMPLE 2 Use a computer algebra system to graph the surface

$$\mathbf{r}(u, v) = \langle (2 + \sin v) \cos u, (2 + \sin v) \sin u, u + \cos v \rangle$$

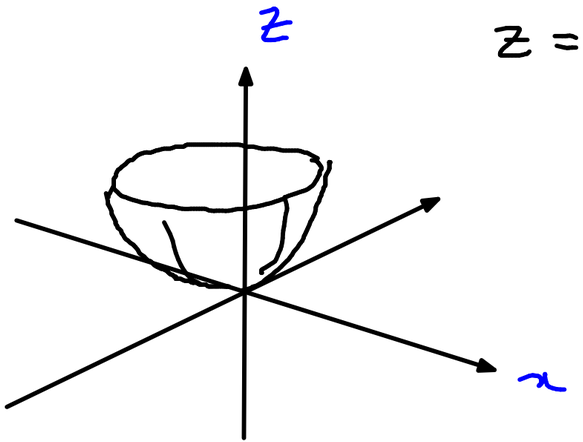
d.



$x = r\cos\theta$

$y = r\sin\theta$

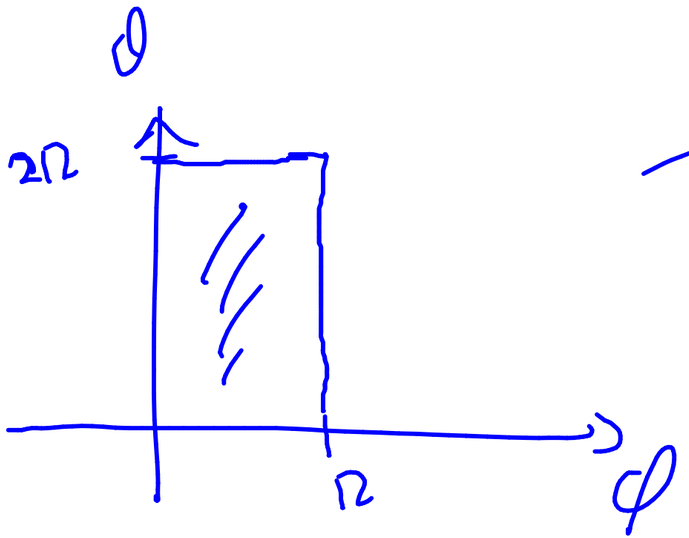
$z = r^2$



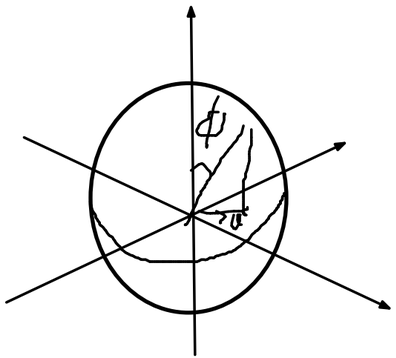
$z = x^2 + y^2$

EXAMPLE 3 Find a vector function that represents the plane that passes through the point P_0 with position vector $\mathbf{r}_0$ and that contains two nonparallel vectors $\mathbf{a}$ and $\mathbf{b}$.

EXAMPLE 4 Find a parametric representation of the sphere $x^2 + y^2 + z^2 = 3^2$



$$x = 3 \sin\phi \cos\theta$$
$$y = 3 \sin\phi \sin\theta$$
$$z = 3 \cos\phi$$



EXAMPLE 5 Find a parametric representation for the cylinder

$$x^2 + y^2 = 4 \qquad 0 \leqslant z \leqslant 1$$

$x = 2\cos\theta$

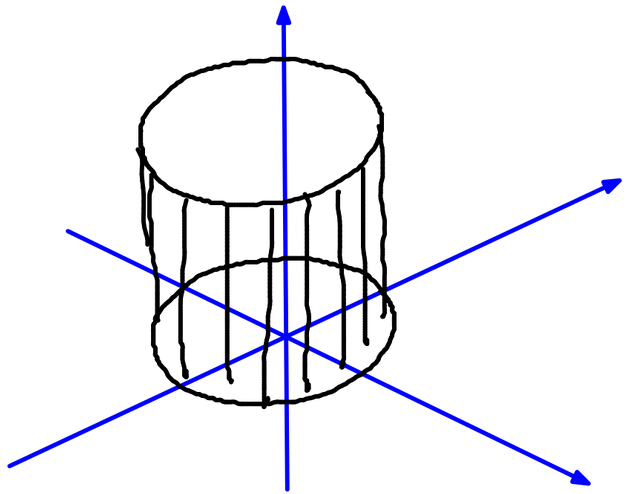
$y = 2\sin\theta$

$z = z$

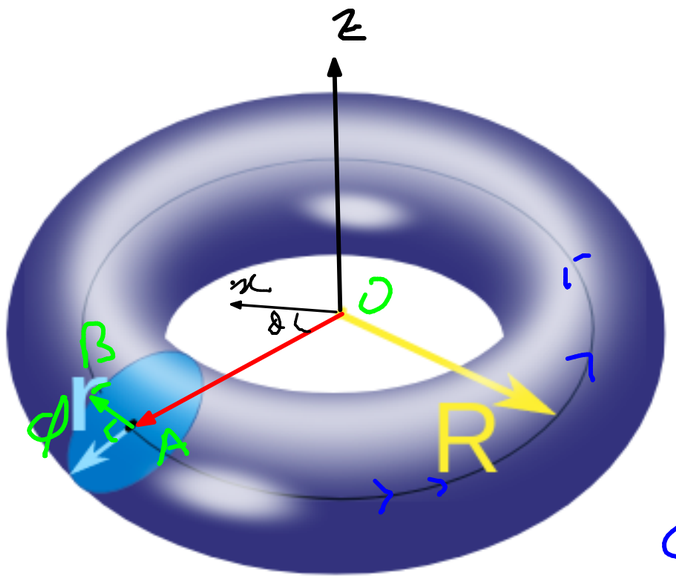
range for θ & z

$0 \leq \theta \leq 2\pi$

$0 \leq z \leq 1$



Q. find a parametric representation for a torus



ϕ : angle between the vector $\overrightarrow{AB}$ & $\overrightarrow{OA}$

$0 \le \phi \le 2\pi$

$0 \le \theta \le 2\pi$

Q: find coordinates of the point B: in terms of r, R, θ, ϕ

$x = ?$

$y = ?$

$z = ?$

H. W.