



ERWIN KREYSZIG

ADVANCED ENGINEERING MATHEMATICS

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GUIDES AND MANUALS	
Maple Computer Guide Mathematica Computer Guide	
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same
topics
next
sem

PART A Ordinary Differential Equations (ODEs)

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What's ODE?

Ordinary Differential Equation

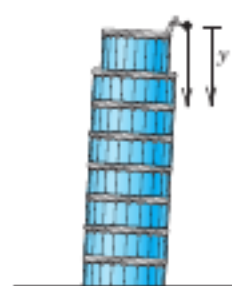
Idea: we know something about $\frac{df}{dx}$ or $\frac{d^2f}{dx^2}$

& we wish to find out about f .

$$\left. \begin{array}{l} \frac{df}{dx} = 2, \text{ find } f(x) \\ \frac{df}{dx} + f = 5 \end{array} \right\}$$

$$\left. \begin{array}{l} \frac{d^2f}{dx^2} + \frac{df}{dx} + f = e^x \\ \text{find } f(x) \end{array} \right\}$$

Why care about knowing to solve ODEs:



Falling stone

$$y'' = g = \text{const.}$$

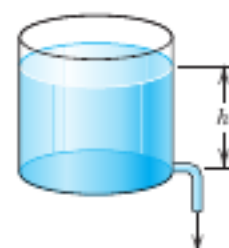
(Sec. 1.1)



Parachutist

$$mv' = mg - bv^2$$

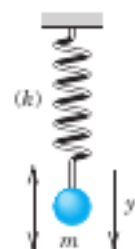
(Sec. 1.2)



Water level h

$$h' = -k\sqrt{h}$$

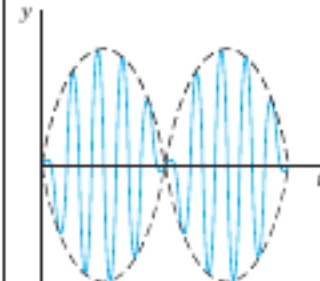
(Sec. 1.3)



Displacement y

$$my'' + ky = 0$$

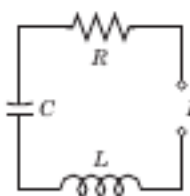
(Secs. 2.4, 2.8)



Beats of a vibrating system

$$y'' + \omega_0^2 y = \cos \omega t, \quad \omega_0 \approx \omega$$

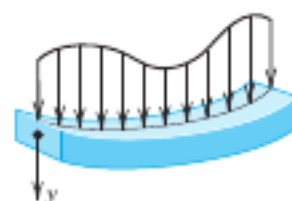
(Sec. 2.8)



Current I in an RLC circuit

$$LI'' + RI' + \frac{1}{C}I = E'$$

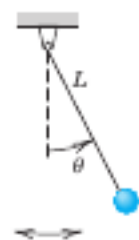
(Sec. 2.9)



Deformation of a beam

$$EIy'''' = f(x)$$

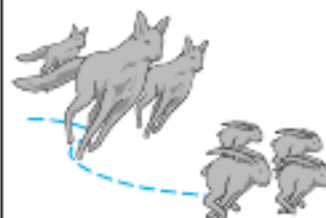
(Secs. 3.2, 3.3)



Pendulum

$$L\theta'' + g \sin \theta = 0$$

(Secs. 4.1, 4.2)



Lotka-Volterra predator-prey model

$$\begin{aligned} y_1' &= ay_1 - by_1y_2 \\ y_2' &= ky_1y_2 - ly_2 \end{aligned}$$

(Secs. 5.1, 5.2)

H.W. → Just read section 1.1 & 1.2.

→ Don't solve exercise problems.

mathematical modelling: converting real world scenarios
into mathematical equations.
↳ in particular into ODE eqns.

1.3 Separable ODEs

1st order
& separable ODEs

$$\left[\frac{dy}{dx} + y^2 = \sin(x) \right], \text{ solve for } y$$

$$\left[\frac{d^2 y}{dx^2} + y^2 = \sin(x) \right] \text{ 2nd order ODE}$$

$$\left[\frac{dy}{dx} + y = \frac{d^2}{dx^2}(e^x) \right] \text{ 1st order ODE}$$

Q. //

$$y' = (x+1)e^{-x}y^2$$

$$\frac{dy}{dx} = (1+x)e^{-x}y^2$$

$$\frac{1}{y^2} dy = (1+x)e^{-x} dx$$

$$\int \frac{1}{y^2} dy = \int (1+x)e^{-x} dx$$

$$\frac{1}{y} = -(x+2)e^{-x} + C$$

&

$$y = \frac{1}{(x+2)e^{-x} + C}$$

Use separation of variables
 \hookrightarrow try to move all x in one side
 & all y in other side
 & integrate

C is an arbitrary constant

d.

Solve $y' = -2xy$, $y(0) = 1.8$.

initial condition

$$\frac{dy}{dx} = -2xy$$

$$\frac{1}{y} dy = -2x dx$$

$$\ln y = -x^2 + C$$

$y(0) = 1.8$

$$\ln 1.8 = -0^2 + C$$

$$C = \ln 1.8$$

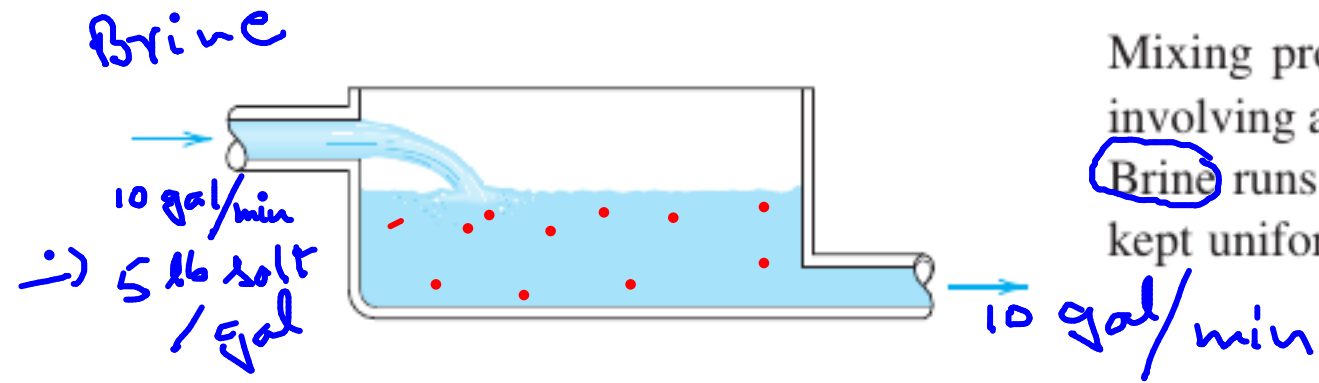
Solve using separation of variables
use the extra condition to eliminate the constant C

$$\ln y = -x^2 + \ln 1.8$$

$$\ln\left(\frac{y}{1.8}\right) = -x^2$$
$$\frac{y}{1.8} = e^{-x^2}$$

$$y = 1.8 e^{-x^2}$$

EXAMPLE 5 Mixing Problem



Mixing problems occur quite frequently in chemical industry. We explain here how to solve the basic model involving a single tank. The tank in Fig. 11 contains 1000 gal of water in which initially 100 lb of salt is dissolved. Brine runs in at a rate of 10 gal/min, and each gallon contains 5 lb of dissolved salt. The mixture in the tank is kept uniform by stirring. Brine runs out at 10 gal/min. Find the amount of salt in the tank at any time t .

$y(t)$: amount of salt in the tank at time t

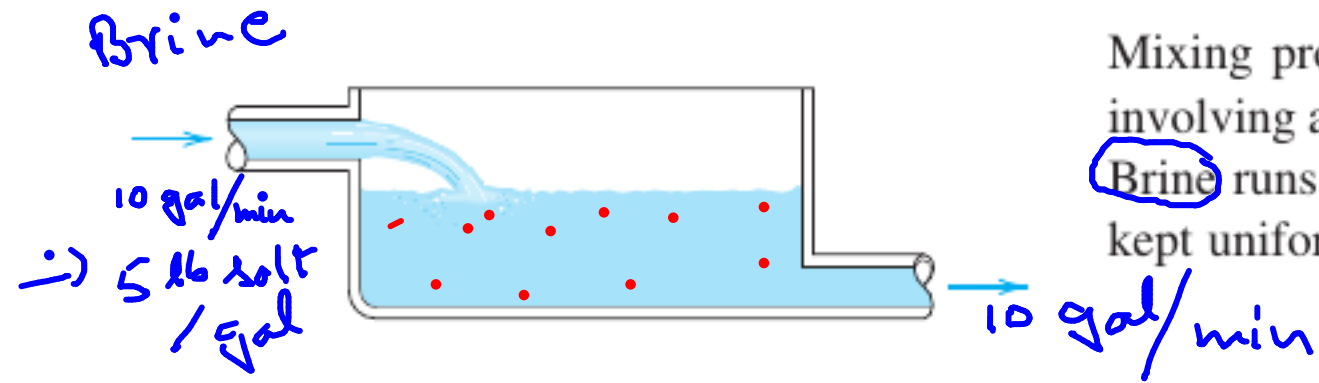
$$y(t) = ??$$

$$y(0) = 100$$

initial salt density ??
0.1 lb / gal

$$\lim_{t \rightarrow \infty} y(t) = \text{guess} = 5000 \text{ lb}$$

EXAMPLE 5 Mixing Problem



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$y(t)$: amount of salt in the tank at time t

$$y(t) = ??$$

$$y(0) = 100$$

$$\begin{aligned} \frac{dy}{dt} &= \text{rate of change of salt mass} \\ &= \text{inflow rate} - \text{outflow rate} \\ &= 50 - \frac{10}{1000} y \end{aligned}$$

Brine \leftrightarrow salt + water

initial salt density ??

$$0.1 \text{ lb / gal}$$

Solve y from

$$\frac{dy}{dt} = 50 - \frac{y}{100}, \quad y(0) = 100$$

Solve $\frac{dy}{dt} = 50 - \frac{y}{100}$ using separation of variables.

$$\frac{dy}{dt} = \frac{5000 - y}{100}$$

$$\frac{1}{5000 - y} dy = \frac{1}{100} dt$$

$$\int \frac{1}{5000 - y} dy = \int \frac{1}{100} dt$$

$$-\ln(5000 - y) = \frac{t}{100} + C$$

$$t = 0$$

$$y = 100$$

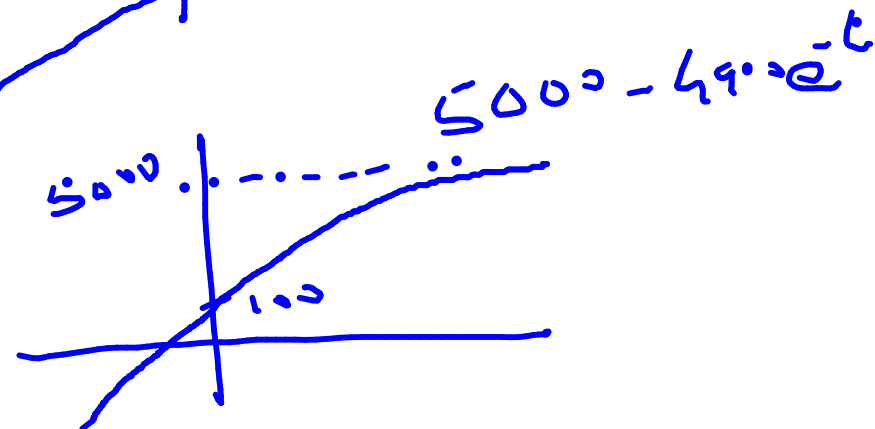
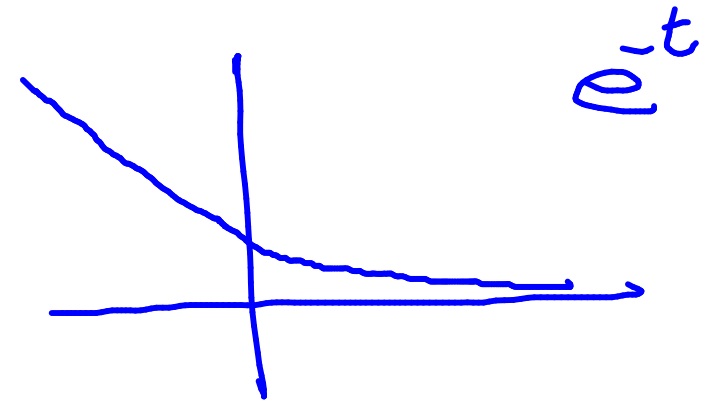
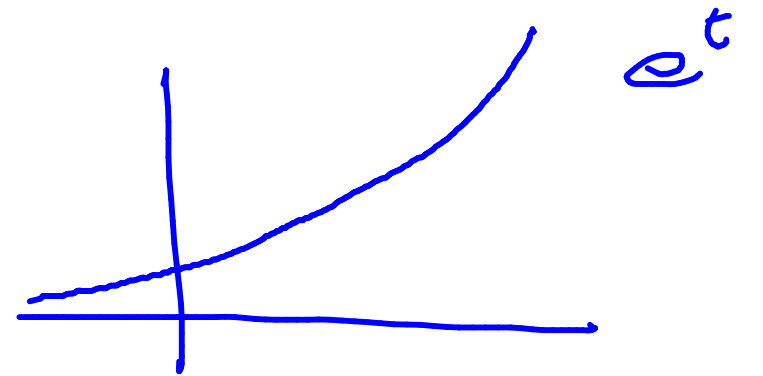
$$C = -\ln 4900$$

$$\ln\left(\frac{5000 - y}{4900}\right) = -\frac{t}{100}$$

$$y = ??$$

$$5000 - y = 4900 e^{-t/100}$$

$$y = 5000 - 4900 e^{-t/100}$$



Extended Method: Reduction to Separable Form

$$y' = f\left(\frac{y}{x}\right)$$

$$2xyy' = y^2 - x^2.$$

32. Friction. If a body slides on a surface, it experiences friction F (a force against the direction of motion). Experiments show that $|F| = \mu|N|$ (*Coulomb's⁶ law of kinetic friction without lubrication*), where N is the normal force (force that holds the two surfaces together; see Fig. 15) and the constant of proportionality μ is called the *coefficient of kinetic friction*. In Fig. 15 assume that the body weighs 45 nt (about 10 lb; see front cover for conversion). $\mu = 0.20$ (corresponding to steel on steel), $\alpha = 30^\circ$, the slide is 10 m long, the initial velocity is zero, and air resistance is negligible. Find the velocity of the body at the end of the slide.

