ERWIN KREYSZIG
ADVANCED ENGINEERING MATHEMATICS

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GUIDES AND MANUALS
Maple Computer Guide
Mathematica Computer Guide

Student Solutions Manual
and Study Guide

Instructor's Manual

Whats ODE ?

$\underbrace{\text{Ordinary } \underbrace{\text{Differential } \underbrace{\text{Equation}}}}$.

Idea: we know something about $\frac{df}{dx}$ or $\frac{d^2f}{dx^2}$

& we wish to find out about f.

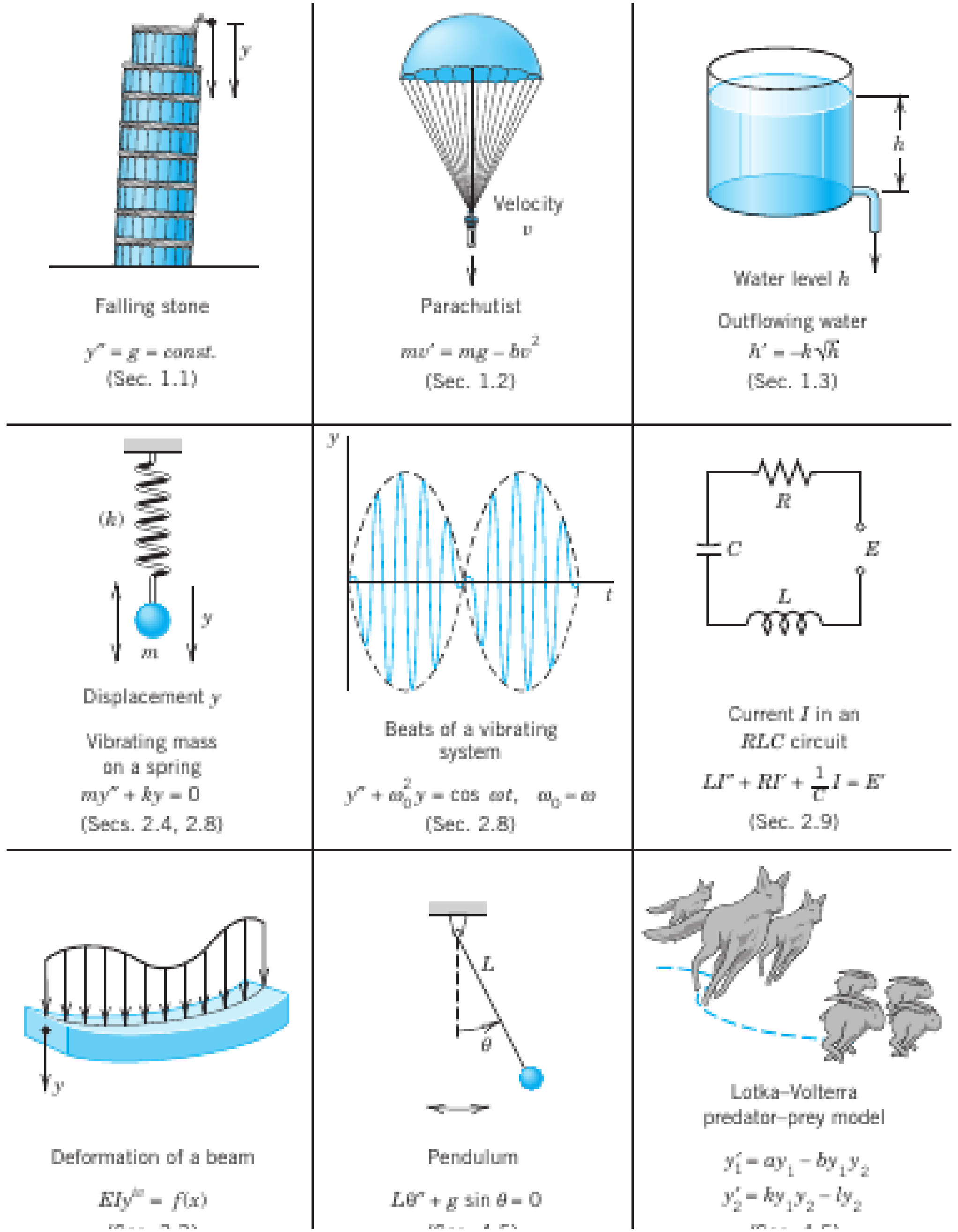
$$\frac{df}{dx} = 2, \quad \text{find } f(x)$$

$$\frac{df}{dx} + f = 5$$

$$\frac{d^2f}{dx^2} + \frac{df}{dx} + f = e^x$$

find $f(x)$

Why care about knowing to solve ODEs :



H.W. → Just read section 1.1 & 1.2.

→ Don't solve exercise problems.

mathematical modelling: converting real world scenarios into mathematical equations.

↳ in particular into ODE eqns.

1.3 Separable ODEs

1st order ODEs

& separable

$$\left[\frac{dy}{dx} + y^2 = \sin(x)\right]$$

, solve for y

$$\left[\frac{d^2y}{dx^2} + y^2 = \sin(x)\right]$$

2nd order ODE

$$\frac{dy}{dx} + y = \frac{d^2}{dx^2}(e^x)$$

1st order ODE

Q.

$$y' = (x + 1)e^{-x}y^2$$

Use separation of variables
↳ try to move all x in one side
& all y in other side
& integrate

$$\frac{dy}{dx} = (1+x)e^{-x}y^2$$

$$\frac{1}{y^2}dy = (1+x)e^{-x}dx$$

$$\int \frac{1}{y^2}dy = \int (1+x)e^{-x}dx$$

$$-\frac{1}{y} = -(x+2)e^{-x} + c \, ,$$

c is an arbitrary constant

&

$$\boxed{y = \frac{1}{(x+2)e^{-x} + c}}$$

d.

Solve $y' = -2xy$, $y(0) = 1.8$.

initial condition

Solve using separation of variables

use the extra condition to eliminate the arbitrary constant C

$$\frac{dy}{dx} = -2xy$$

$$\frac{1}{y}\,dy = -2x\,dx$$

$$\ln y = -x^2 + C$$

$y(0) = 1.8$

$$\ln 1.8 = -0^2 + C$$

$$C = \ln 1.8$$

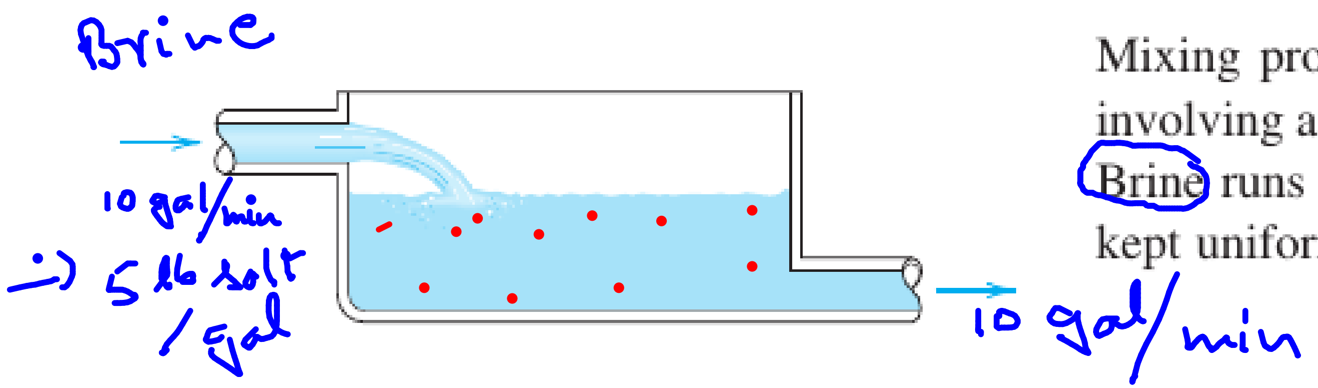
$$\ln y = -x^2 + \ln 1.8$$

$$\ln\left(\frac{y}{1.8}\right) = -x^2$$

$$\frac{y}{1.8} = e^{-x^2}$$

$$\boxed{y = 1.8e^{-x^2}}$$

EXAMPLE 5 Mixing Problem



Mixing problems occur quite frequently in chemical industry. We explain here how to solve the basic model involving a single tank. The tank in Fig. 11 contains 1000 gal of water in which initially 100 lb of salt is dissolved. Brine runs in at a rate of 10 gal/min, and each gallon contains 5 lb of dissoved salt. The mixture in the tank is kept uniform by stirring. Brine runs out at 10 gal/min. Find the amount of salt in the tank at any time t.

Brine $\Leftrightarrow$ salt + water

$y(t)$: amount of salt in the tank at time t

initial salt density ??

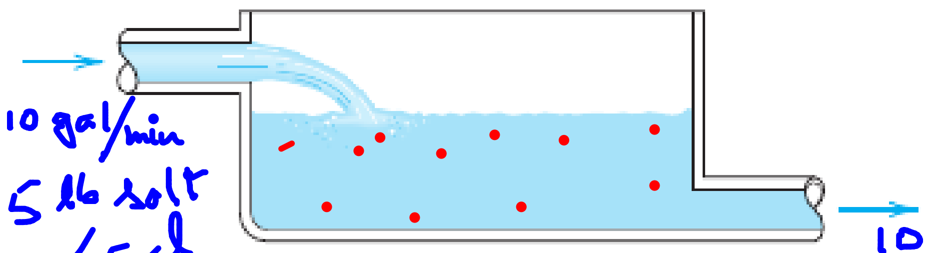
0.1 lb/gal

$y(t) = ??$

$y(0) = 100$

$\lim_{t \to \infty} y(t) =$ guess $= 5000$ lb

EXAMPLE 5 Mixing Problem



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Brine ⇔ salt + water

$y(t)$: amount of salt in the tank at time t

$y(t) = ??$

$y(0) = 100$

$$\frac{dy}{dt} = \text{rate of change of salt mass}$$
$$= \text{inflow rate} - \text{outflow rate}$$
$$= 50 - \frac{10}{1000} y$$

initial salt density ??

0.1 lb/gal

Solve y from

$$\frac{dy}{dt} = 50 - \frac{y}{100}, \quad y(0) = 100$$

Solve $\frac{dy}{dt} = 50 - \frac{y}{100}$ using separation of variables.

$$\frac{dy}{dt} = \frac{5000 - y}{100}$$

$$\frac{1}{5000 - y}\, dy = \frac{1}{100}\, dt$$

$$\int \frac{1}{5000 - y}\, dy = \int \frac{1}{100}\, dt$$

$$- \ln(5000 - y) = \frac{t}{100} + C$$

$t = 0$

$y = 100$

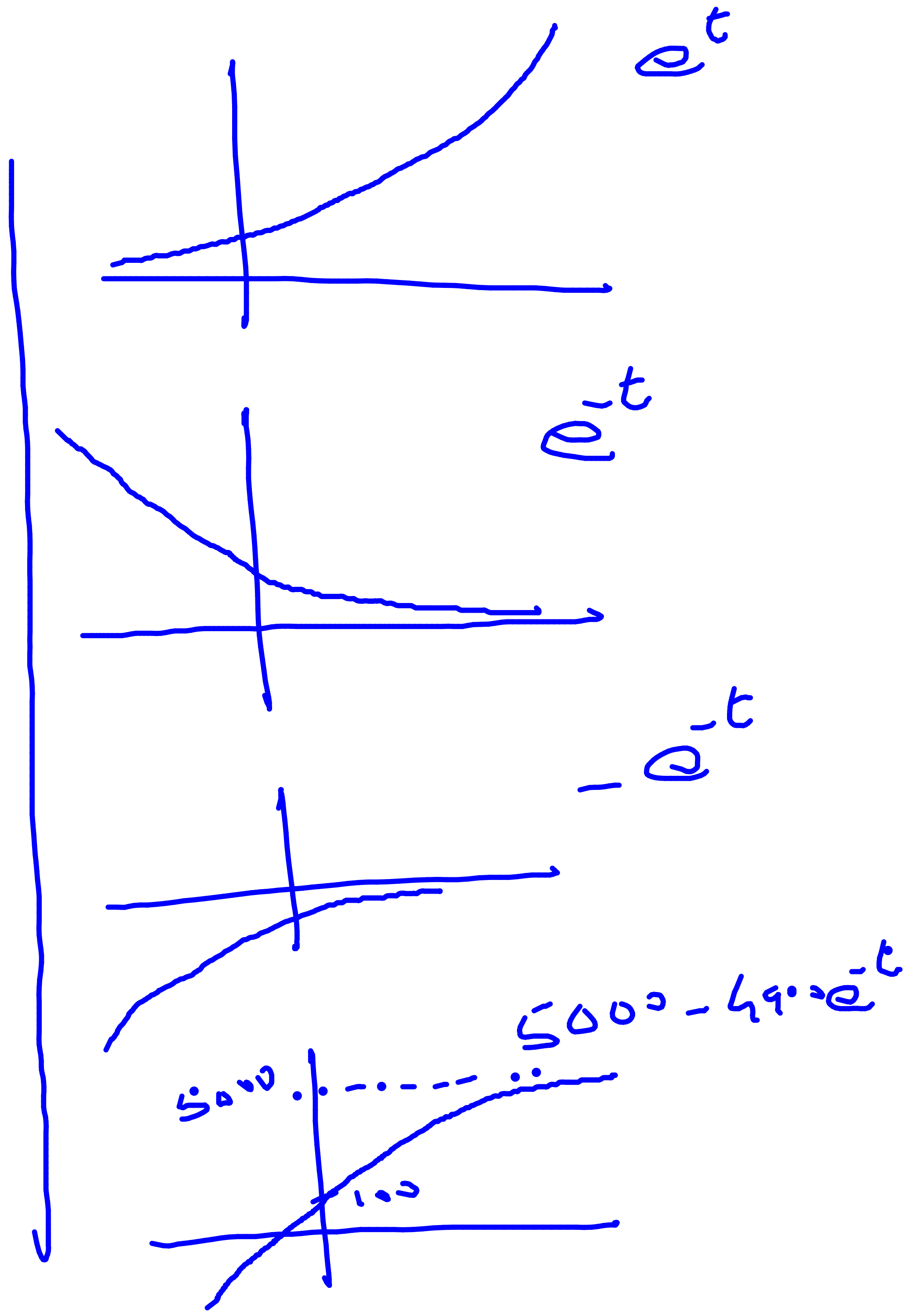
$C = - \ln 4900$

$$\ln\left(\frac{5000 - y}{4900}\right) = - \frac{t}{100}$$

$y = ??$

$$5000 - y = 4900\, e^{-t/100}$$

$$\boxed{y = 5000 - 4900\, e^{-t/100}}$$



Recall : what did we start ?? ODE

In differential equations, we solve for a function :

↪ we have some information about derivatives

e.g. $\frac{dx}{dt} = 2$, find $x(t)$.

$2\frac{d^2x}{dt^2} = 7$, find $x(t)$

→ We learnt last time : the first thing we should try when solving an ODE

Variable separable

$$\boxed{x\frac{dy}{dx} = 5x^2y}$$

is this an ODE ??

$$\frac{1}{y}\,dy = \frac{5x^2}{x}\,dx$$

$$\int \frac{1}{y}\,dy = \int 5x\,dx$$

$$\sin\left(\frac{dy}{dx}\right) = x$$

$$\frac{dy}{dx} = \sin^{-1}(x)$$

Extended Method: Reduction to Separable Form

$$\boxed{y' = f\left(\frac{y}{x}\right)} \quad [\qquad]$$

$$\boxed{y/x = v}$$

$$2xyy' = y^2 - x^2.$$

$$2xy\frac{dy}{dx} = y^2 - x^2$$

try to move x & y in two sides

↓ ??

$$\left(\begin{smallmatrix}\text{only}\\ y\text{ terms}\end{smallmatrix}\right) dy = \left(\begin{smallmatrix}\text{only}\\ x\text{ terms}\end{smallmatrix}\right) dx$$

$$\boxed{v = \frac{y}{x}}$$

& eliminate y

$$2\frac{dy}{dx} = \frac{y^2 - x^2}{xy}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\left(\frac{y}{x}\right) - \left(\frac{x}{y}\right)\right]$$

$$\frac{dy}{dx} = \frac{1}{2}\left[v - \frac{1}{v}\right]$$

$$y = vx$$

$$\frac{dy}{dx} = \frac{d}{dx}(vx)$$

$$= v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{1}{2}\left[v - \frac{1}{v}\right]$$

↳ is this separable ??

$$x\frac{dv}{dx} = -\frac{1}{2}\left[\frac{v^2+1}{v}\right]$$

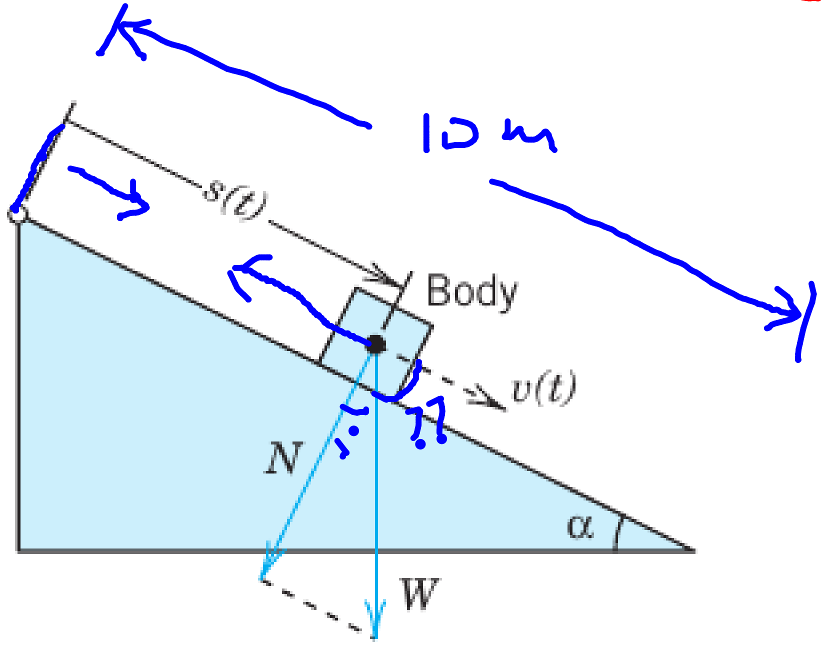
$$\frac{2v}{v^2+1}\,dv = -\frac{dx}{x} \;\rightarrow\; \ln(v^2+1) = -\ln x + \ln c$$

$$v^2+1 = \frac{c}{x}$$

$$\boxed{\frac{y^2}{x^2} + 1 = \frac{c}{x}}$$

y is given implicitly by this eq^n

32. Friction. If a body slides on a surface, it experiences friction F (a force against the direction of motion). Experiments show that $|F| = \mu|N|$ (*Coulomb's*[6] *law of kinetic friction without lubrication*), where N is the normal force (force that holds the two surfaces together; see Fig. 15) and the constant of proportionality μ is called the *coefficient of kinetic friction*. In Fig. 15 assume that the body weighs 45 nt (about 10 lb; see front cover for conversion). $\mu = 0.20$ (corresponding to steel on steel), $\alpha = 30°$, the slide is 10 m long, the initial velocity is zero and air resistance is negligible. Find the velocity of the body at the end of the slide.



$W = 45\ N$

$\mu = 0.2$

$\alpha = 30°$

$s(t)$: distance from the peak of the slide

→ we use $F = ma$ to get an eqn for $s(t)$

$$m\frac{d^2s}{dt^2} = W\cos(60°) - \mu W\cos(30°)$$

$$\frac{\cancel{W}}{g}\frac{d^2s}{dt^2} = \cancel{W}\cos(60°) - \mu\cancel{W}\cos(30°)$$

$$\frac{d^2s}{dt^2} = \underbrace{g\left[\frac{1}{2} - 0.2\frac{\sqrt{3}}{2}\right]}_{} = A \approx 3.202$$

$$\frac{d^2s}{dt^2} = A, \qquad s(0) = 0$$

$$s'(0) = 0$$

Q: what $\frac{ds}{dt}$ when $s = 10$??] Can you complete this ??

$$\frac{d^2s}{dt^2} = A$$

$$\frac{ds}{dt} = At + C$$

, $C = ??$ [$\frac{ds}{dt}(0) = 0$, $C = 0$

$$v(t) = \boxed{\frac{ds}{dt} = At}$$

&

$$s(t) = \frac{At^2}{2} + D$$

$D = ??$ [$s(0) = 0$, $D = 0$]

$$\boxed{s(t) = At^2/2}$$

$$\frac{At^2}{2} = 10$$

$$t = \sqrt{\frac{20}{A}}$$

$$v\left(\sqrt{20/A}\right) = A\sqrt{\frac{20}{A}} = \sqrt{20A}$$

1.4 Exact ODEs. Integrating Factors

$$M(x, y) + N(x, y)y' = 0,$$

next time

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}.$$

$$\cos(x + y)\,dx + (3y^2 + 2y + \cos(x + y))\,dy = 0.$$

$$(\cos y \sinh x + 1)\,dx - \sin y \cosh x\,dy = 0, \qquad y(1) = 2.$$

Reduction to Exact Form. Integrating Factors

$$-y\,dx + x\,dy = 0.$$

$$(e^{x+y} + ye^{y})\,dx + (xe^{y} - 1)\,dy = 0$$