SIR model

(epidemic model)

S: susceptible
I: infected
R: Recovered

I(t): number of infected person at time t.

R(t): number of person recovered at time t

S(t): total population − (I(t) + R(t))

Aim: Estimate $I(t)$ $\forall\, t \geq 0$

$$\frac{dS(t)}{dt} = -\beta S(t) I(t)$$

$$\frac{dI(t)}{dt} = \beta S(t) I(t) - \gamma I(t)$$

$$\frac{dR(t)}{dt} = \gamma I(t)$$

$I(t)$: number of infected person at time t.

$R(t)$: number of person recovered at time t

$S(t)$: total population $- (I(t) + R(t))$

[no dying allowed in SIR]

$$\frac{dS(t)}{dt} = -\beta S(t)I(t)$$
$$\frac{dI(t)}{dt} = \beta S(t)I(t) - \gamma I(t)$$
$$\frac{dR(t)}{dt} = \gamma I(t)$$

<u>Aim</u>: Estimate $I(t)$ $\forall t > 0$

$$\frac{dS}{dt} = -\beta SI$$

$$\frac{dI}{dt} = \underbrace{\beta SI}_{\text{interaction}} - \overbrace{\gamma I(t)}^{\text{recovery rate}}$$

bigger β mean more interaction

$$\frac{dR}{dt} = \gamma I(t)$$

Q: mathematical question

Suppose we are given

$S(0) = S_0$, $I(0) = 100$, $R(0) = 0$

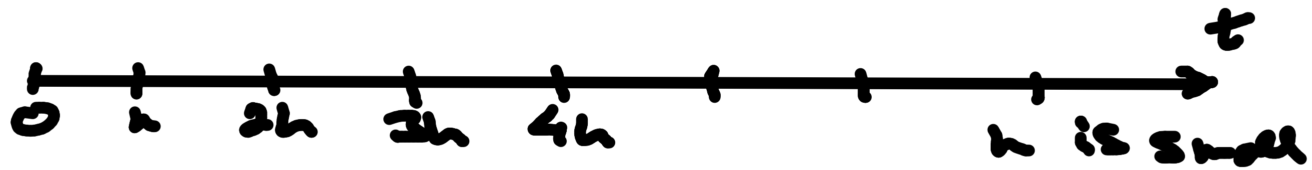
& $\beta = 0.01$, $\gamma = 0.2$

$$\boxed{\begin{aligned} \frac{dS(t)}{dt} &= -\beta S(t)I(t) \\ \frac{dI(t)}{dt} &= \beta S(t)I(t) - \gamma I(t) \\ \frac{dR(t)}{dt} &= \gamma I(t) \end{aligned}}$$

Q: find $S(t)$, $I(t)$, $R(t)$ for any future time t

Explaination of the code for solving SIR diff eq^n

$$\frac{dS}{dt} = -\beta S I \qquad \frac{dI}{dt} = \beta S I - \gamma I \qquad \frac{dR}{dt} = \gamma I$$



$$\rightarrow \frac{I(h) - I(0)}{h} \approx \left.\frac{dI}{dt}\right|_{t=0} = \beta S(0) I(0) - \gamma I(0)$$

$$I(h) = I(0) + h \left.\frac{dI}{dt}\right|_{t=0}$$

$$S(h) = S(0) + h \left.\frac{dS}{dt}\right|_{t=0}$$

$$R(h) = R(0) + h \left.\frac{dR}{dt}\right|_{t=0}$$

→ once we know $I(h), R(h), S(h)$

similarly we can find

$I(2h), R(2h), S(2h)$

> This is solving diff. equation using Numerical methods

— (approximate solution)