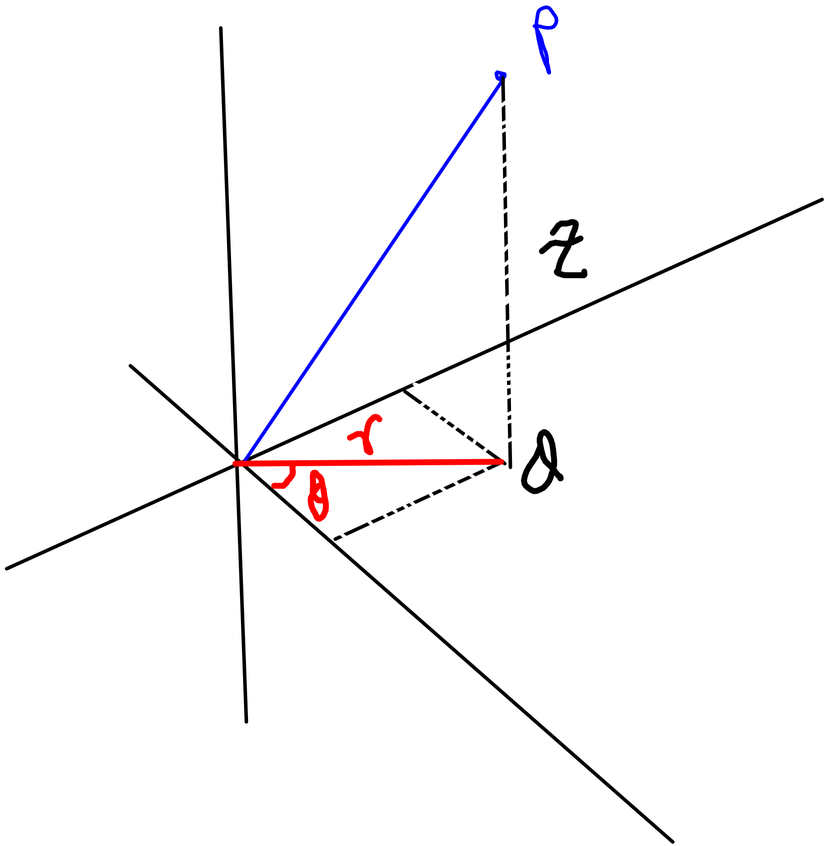
Section 12.6

## 12.6 TRIPLE INTEGRALS IN CYLINDRICAL COORDINATES

Section 12.7

## 12.7 TRIPLE INTEGRALS IN SPHERICAL COORDINATES

# CYLINDRICAL COORDINATES

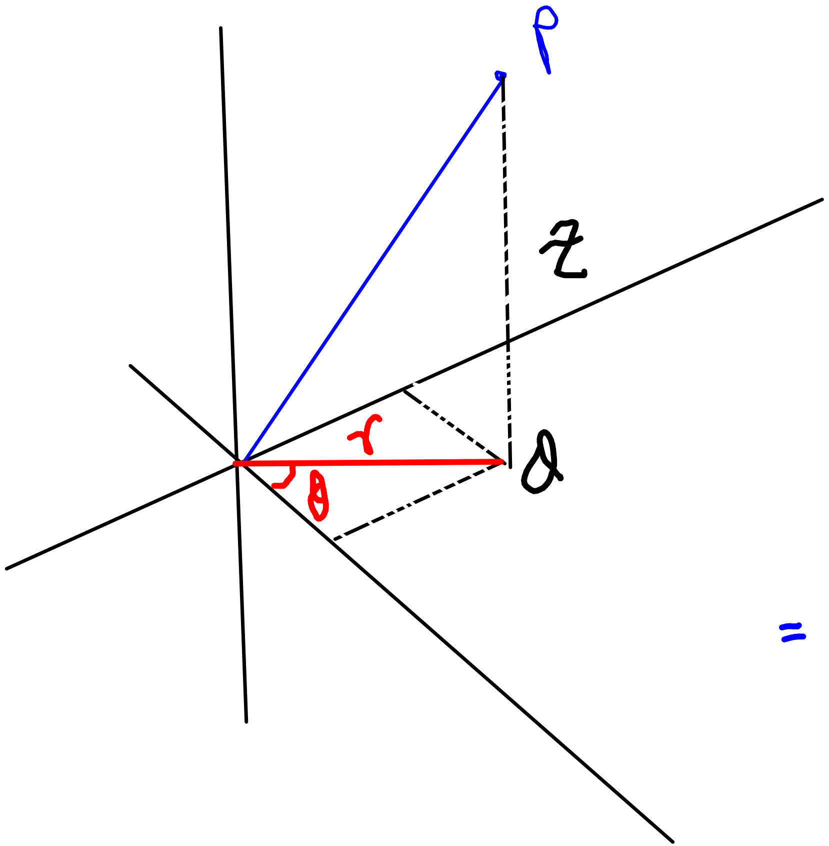



$(r, \theta, z)$

$(5, \frac{\pi}{3}, 10)$

$(r, \theta)$ = polar coordinates of projection of P on the xy plane

# CYLINDRICAL COORDINATES




$$dx\,dy\,dz = r\,dr\,d\theta\,dz$$

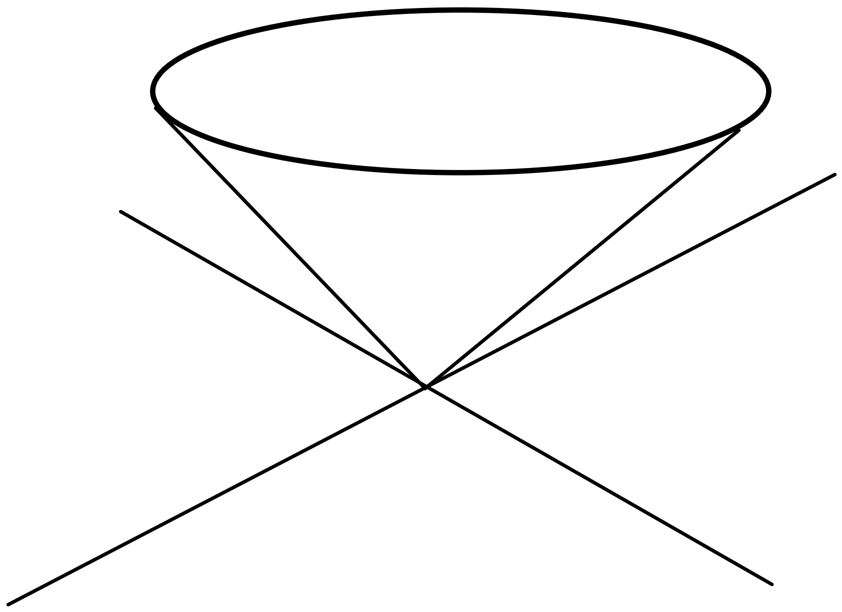
$$x = r\cos\theta \qquad y = r\sin\theta \qquad z = z$$

$$\text{Jacobian} = \frac{\partial(x,y,z)}{\partial(r,\theta,z)}$$

$$= \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} & \frac{\partial x}{\partial z} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} & \frac{\partial y}{\partial z} \\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial \theta} & \frac{\partial z}{\partial z} \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\sin\theta & 0 \\ \sin\theta & r\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = r$$

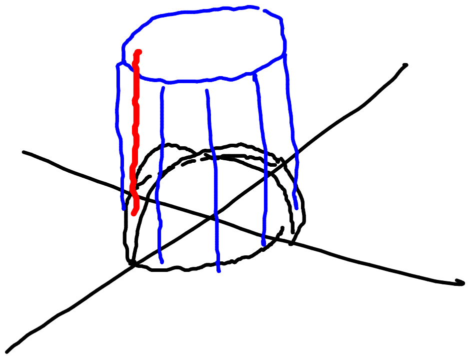
**EXAMPLE 2** Describe the surface whose equation in cylindrical coordinates is $z = r$.

$z = \sqrt{x^2+y^2}$



**V EXAMPLE 3** A solid $E$ lies within the cylinder $x^2 + y^2 = 1$, below the plane $z = 4$, and above the paraboloid $z = 1 - x^2 - y^2$. (See Figure 8.) The density at any point is proportional to its distance from the axis of the cylinder. Find the mass of $E$.

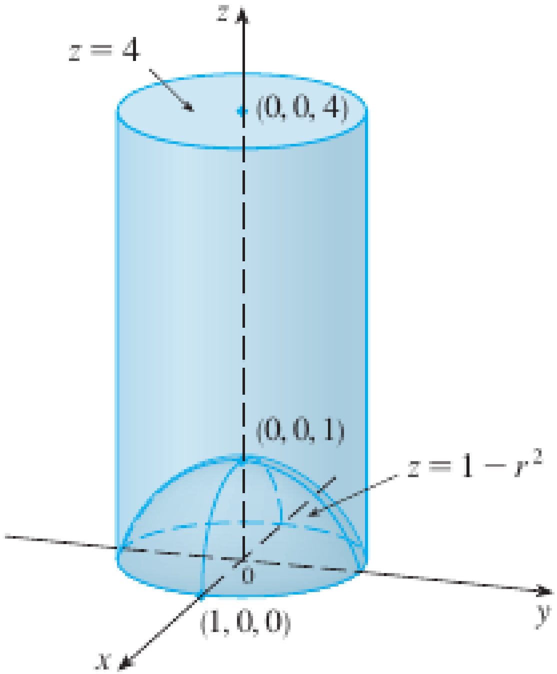
Sketch the domain



$z = 1 - r^2$

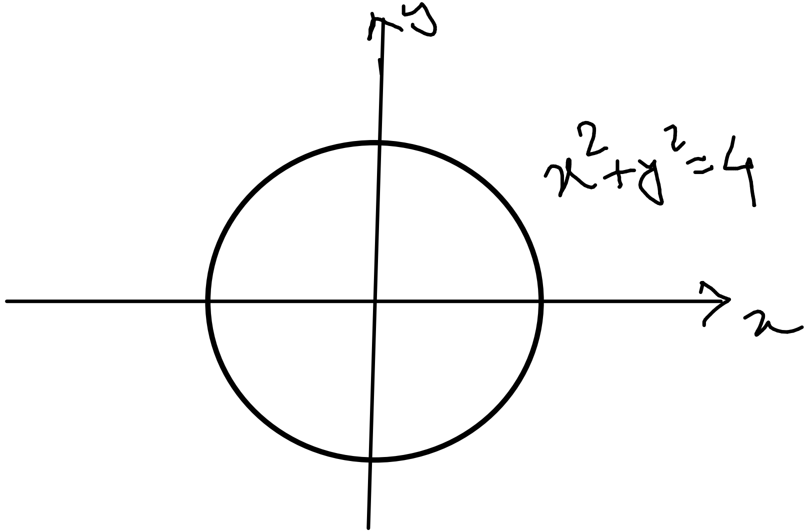
$$\int_0^1 \int_0^{2\pi} \int_{1-r^2}^{4} (\alpha r)\, r \, dz\, d\theta\, dr$$

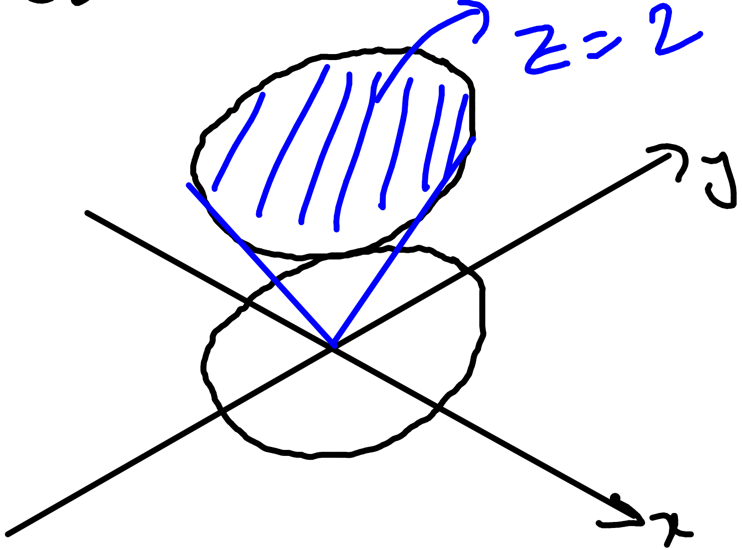
let density, $\rho = \alpha r$



**EXAMPLE 4** Evaluate $\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{2}(x^2+y^2)\,dz\,dy\,dx. = \int_0^{2\pi}\int_0^{2}\int_r^{2} r^2\, r \quad dz\,dr\,d\theta$

Identify the region of integration and swithch to cylindrical coordinates.



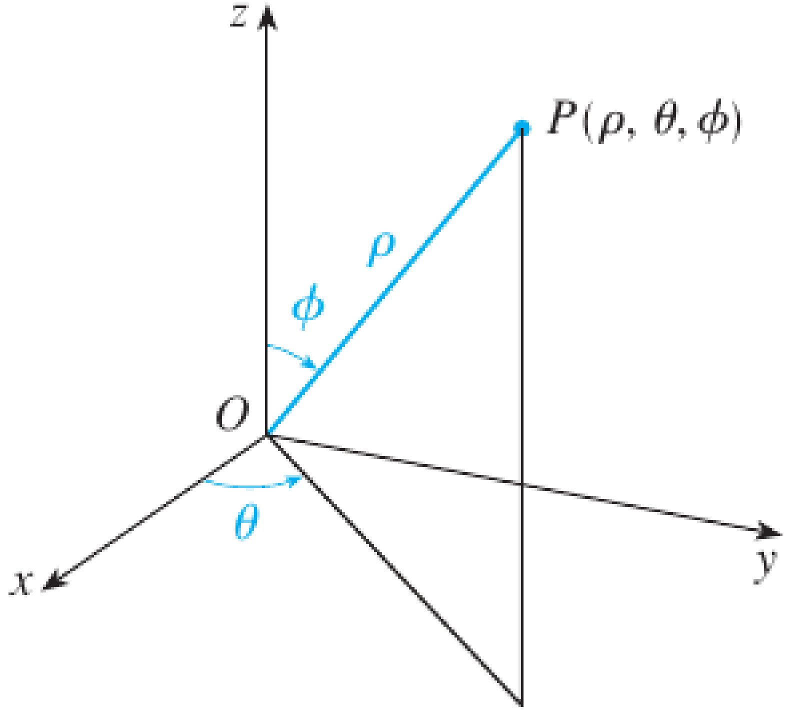




$z=\sqrt{x^2+y^2}$

$z=r$

## SPHERICAL COORDINATES

next time




$$x = \rho \sin \phi \cos \theta \qquad y = \rho \sin \phi \sin \theta \qquad z = \rho \cos \phi$$

**EXAMPLE 3** Evaluate $\iiint_B e^{(x^2+y^2+z^2)^{3/2}}\,dV$, where $B$ is the unit ball:

$$B = \left\{(x, y, z) \mid x^2 + y^2 + z^2 \leqslant 1\right\}$$

**EXAMPLE 4** Use spherical coordinates to find the volume of the solid that lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = z$. (See Figure 9.)