Partial Derivatives:

$$f(x,y) = x^2 \sin(y)$$

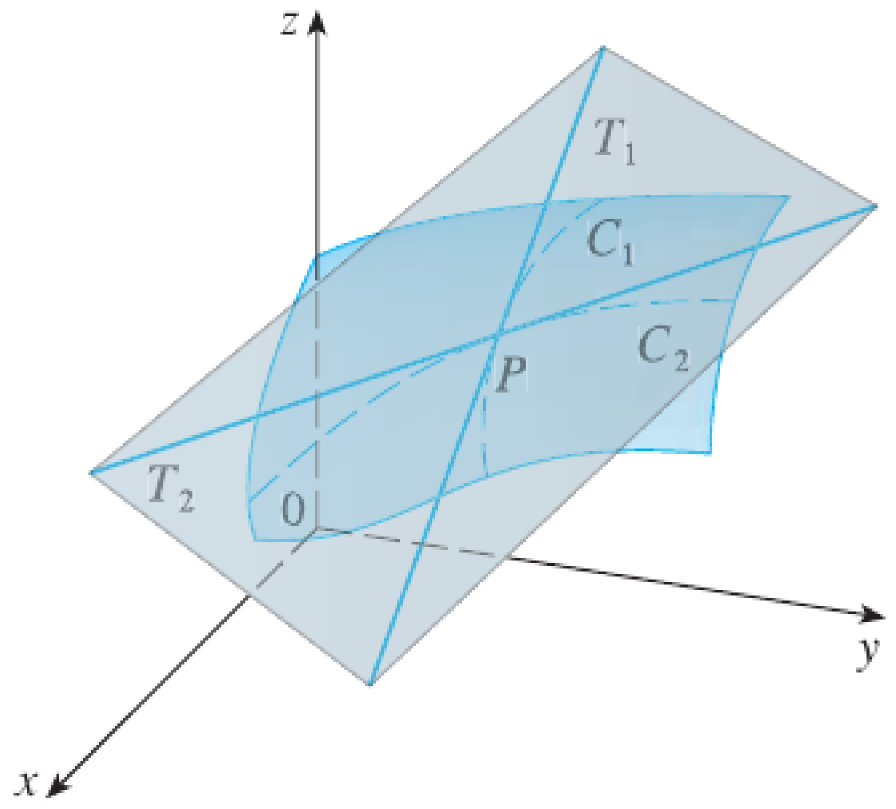
$$\frac{\partial f}{\partial x} = 2x \sin(y)$$

Clairaut's Theorem

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$$

if both LHS & RHS exist and are continuous

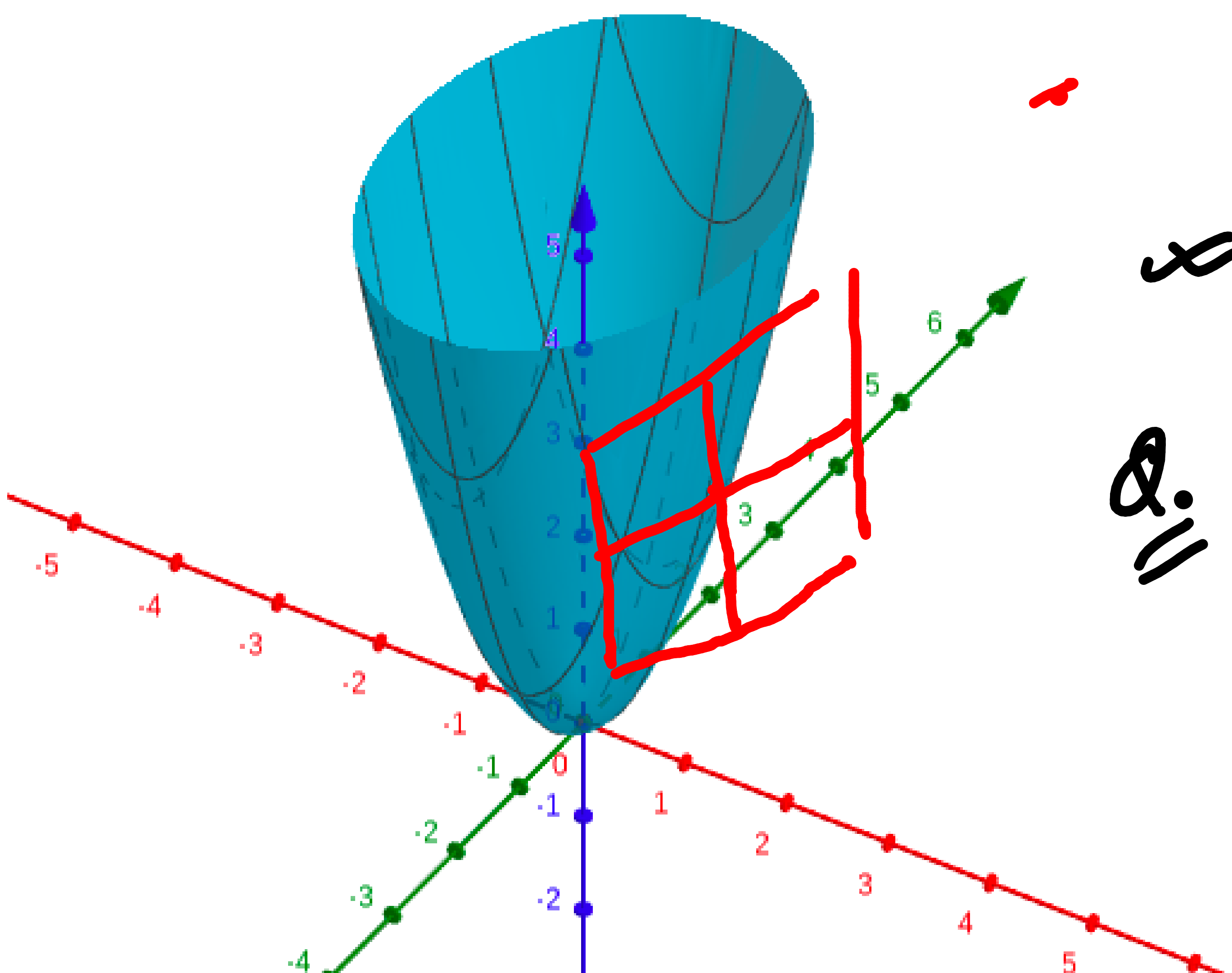
11.4 TANGENT PLANES AND LINEAR APPROXIMATIONS



V EXAMPLE 1 Find the tangent plane to the elliptic paraboloid $z = 2x^2 + y^2$ at the point $(1, 1, 3)$.

Q: is (1,1,3) a point on the graph??

Q: give general eqn of planes passing through (1,1,3)??

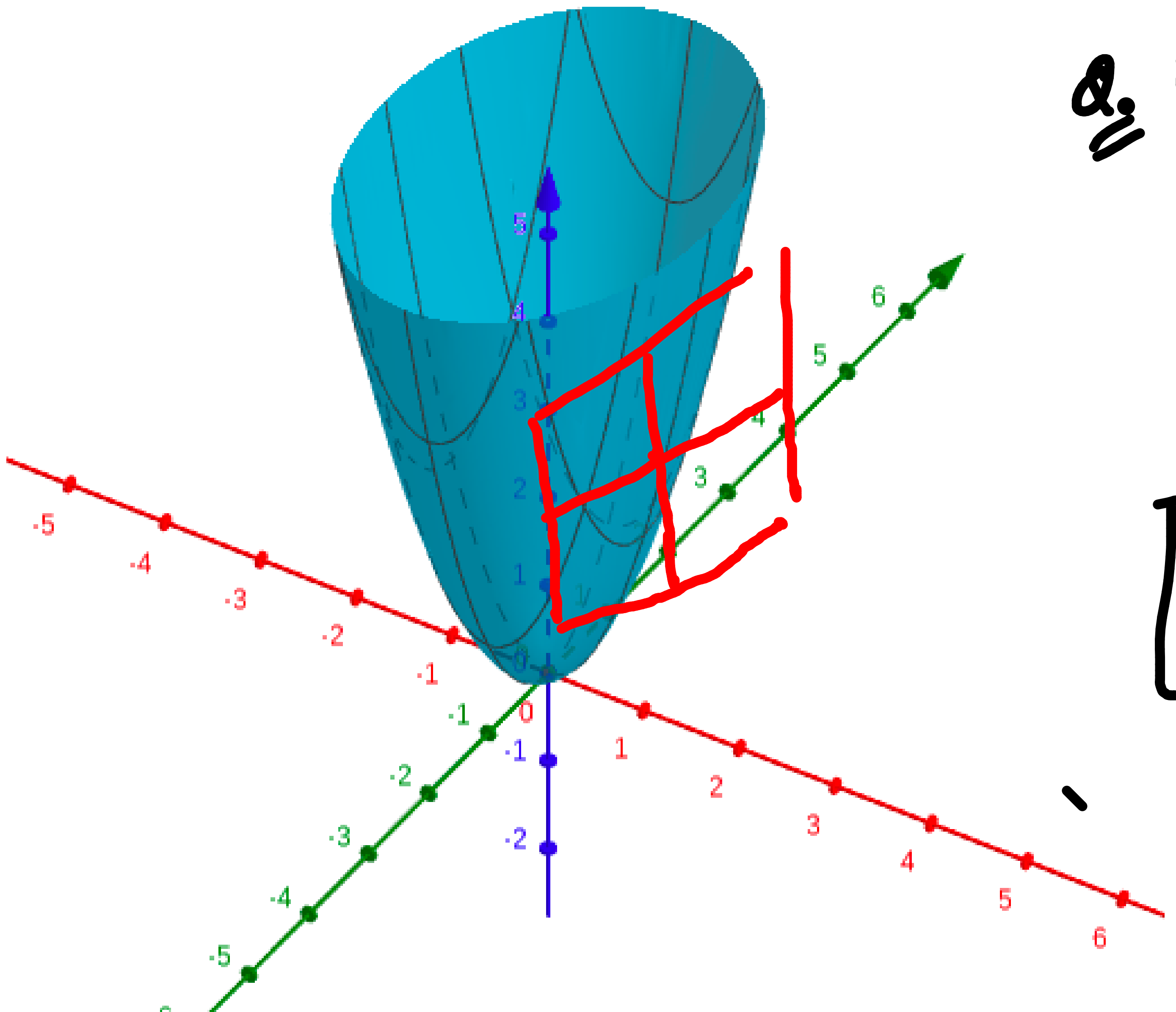


$\propto$ $A(x-1) + B(y-1) + C(z-3) = 0$

Q: if $C = 0$??

the tangent plane becomes vertical

EXAMPLE 1 Find the tangent plane to the elliptic paraboloid $z = 2x^2 + y^2$ at the point $(1, 1, 3)$.



$$\propto \quad A(x-1) + B(y-1) + C(z-3) = 0$$

Q: if $C = 0$??

the tangent plane becomes vertical

$$\boxed{z - 3 = -\frac{A}{C}(x-1) + \left(-\frac{B}{C}\right)(y-1)}$$

$$\boxed{z - 3 = A(x-1) + B(y-1)}$$

$$A = \left.\frac{\partial z}{\partial x}\right|_{\substack{x=1 \\ y=1}} \qquad B = \left.\frac{\partial z}{\partial y}\right|_{\substack{x=1 \\ y=1}}$$

$z = 2x^2 + y^2$

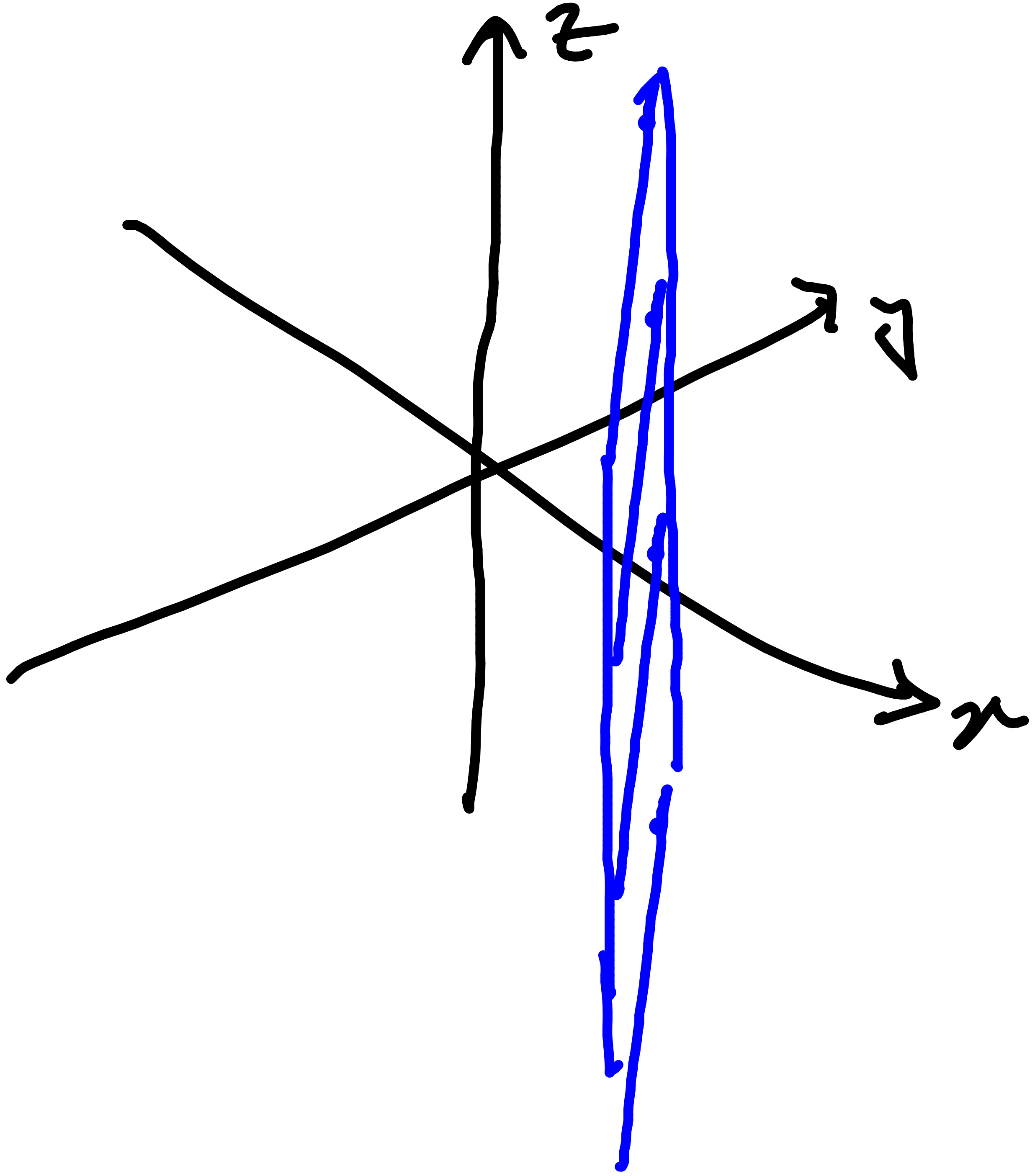
$\frac{\partial z}{\partial x} = 4x$

$\frac{\partial z}{\partial y} = 2y$

$\frac{\partial z}{\partial x}(1,1) = 4 = A$

$\frac{\partial z}{\partial y}(1,1) = 2 = B$

$z - 3 = 4(x-1) + 2(y-1)$



$$x + y + 0z = 1$$

Rule: To find the tangent plane of $z = f(x,y)$ at point (a,b,c), the eqn:

$$z - c = A(x-a) + B(y-b)$$

where $A = \frac{\partial z}{\partial x}(a,b)$, $B = \frac{\partial z}{\partial y}(a,b)$

Q: find Tangent plane

$f(x,y) = x\sin(y)$ at point $(1, \frac{\pi}{2}, 1)$

$$z-1 = A(x-1) + B\left(y-\frac{\pi}{2}\right)$$

$$A = ? = \frac{\partial f}{\partial x} = \sin(y)\Big|_{\substack{x=1\\ y=\pi/2}} = 1$$

$$B = ? = \frac{\partial f}{\partial y} = x\cos(y)\Big|_{\substack{x=1\\ y=\pi/2}} = 0$$

Tangent plane:

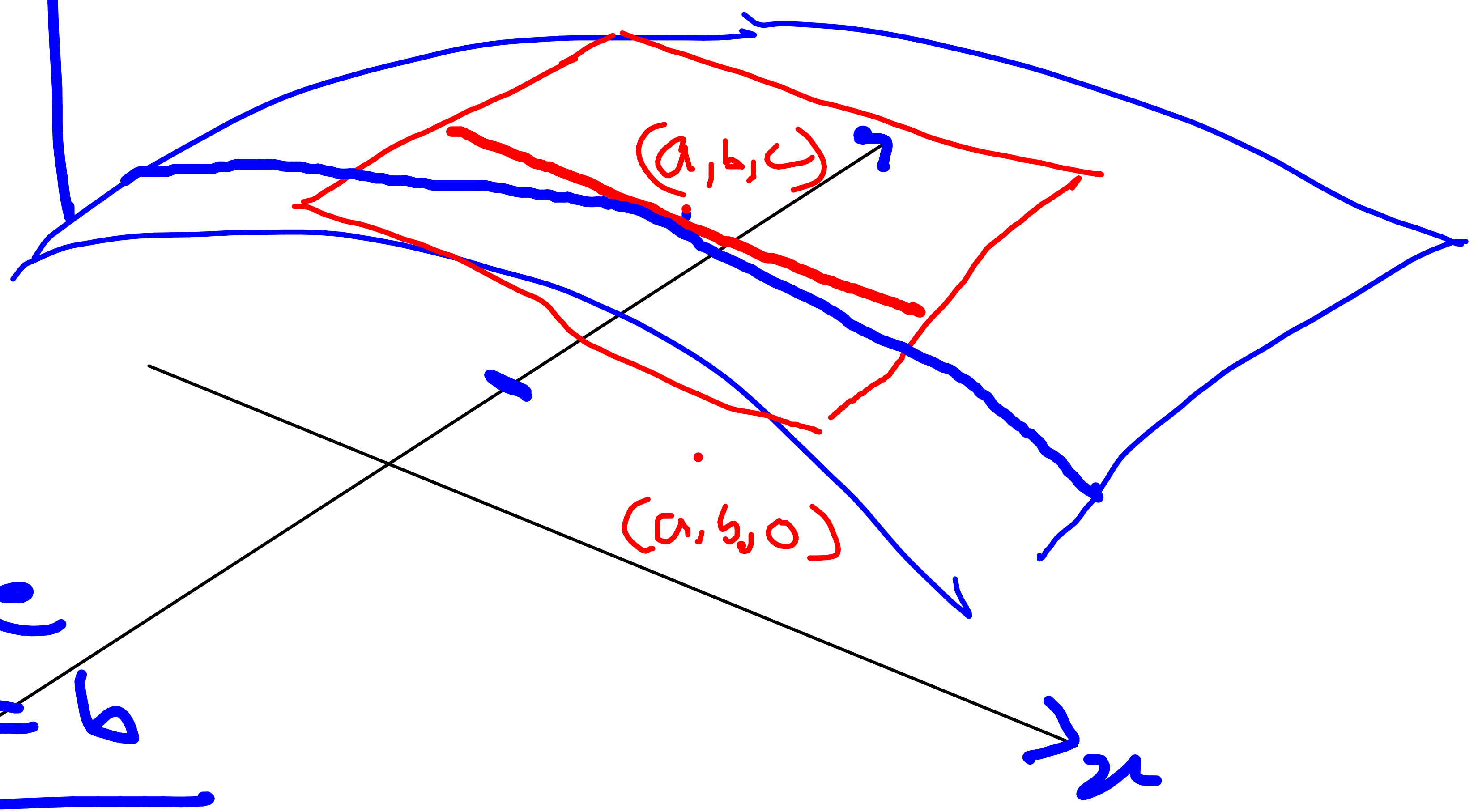
$$z-1 = x-1$$

or $$z = x$$

Rule: To find the tangent plane of $z=f(x,y)$ at point (a,b,c), the eqn:

$$\boxed{z-c = A(x-a) + B(y-b) \quad \text{where } A = \frac{\partial z}{\partial x}(a,b), \ B = \frac{\partial z}{\partial y}(a,b)}$$

why $A = \frac{\partial z}{\partial x}$, $B = \frac{\partial z}{\partial y}$



Q. Explain what geometry you will get if you intersect the tangent plane with a vertical plane $y=b$

$\boxed{y=1}$

$$z - c = A(x-a) + \underbrace{B(y-b)} \rightarrow 0$$

if intersect this with $y=b$

$$z - c = A(x-a)$$

8 THEOREM If the partial derivatives f_x and f_y exist near (a, b) and are continuous at (a, b), then f is differentiable at (a, b).

$\rightarrow$ f_x f_y

EXAMPLE 2 Show that $f(x, y) = xe^{xy}$ is differentiable at $(1, 0)$ and find its linearization there. Then use it to approximate $f(1.1, -0.1)$.

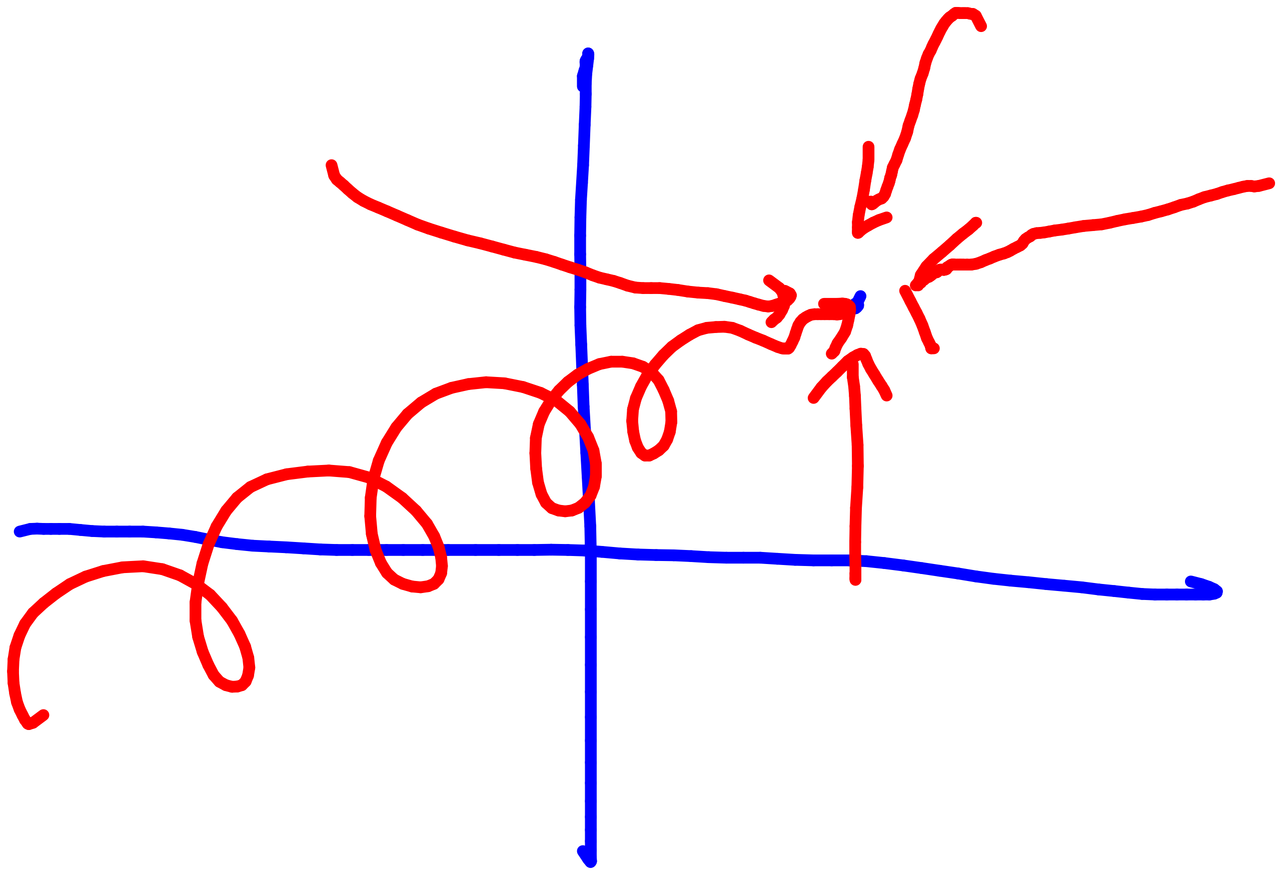
a.

$$f_x = xye^{xy} + e^{xy}$$

$$f_y = x^2e^{xy}$$

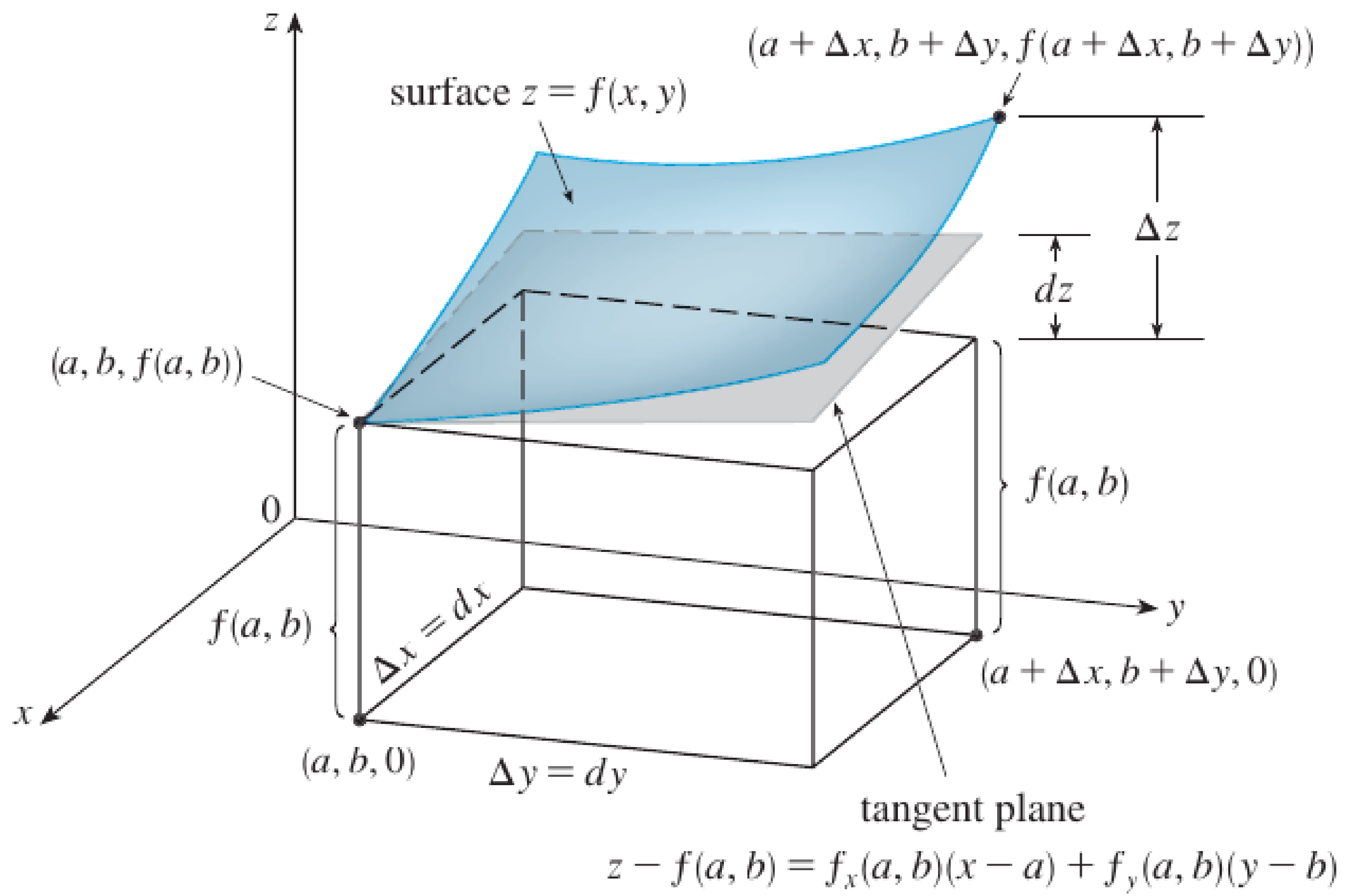
both exist & are continuous

we can say that $f(x,y)$ is differentiale



DIFFERENTIALS

$$dz = f_x(x, y)\, dx + f_y(x, y)\, dy = \frac{\partial z}{\partial x}\, dx + \frac{\partial z}{\partial y}\, dy$$

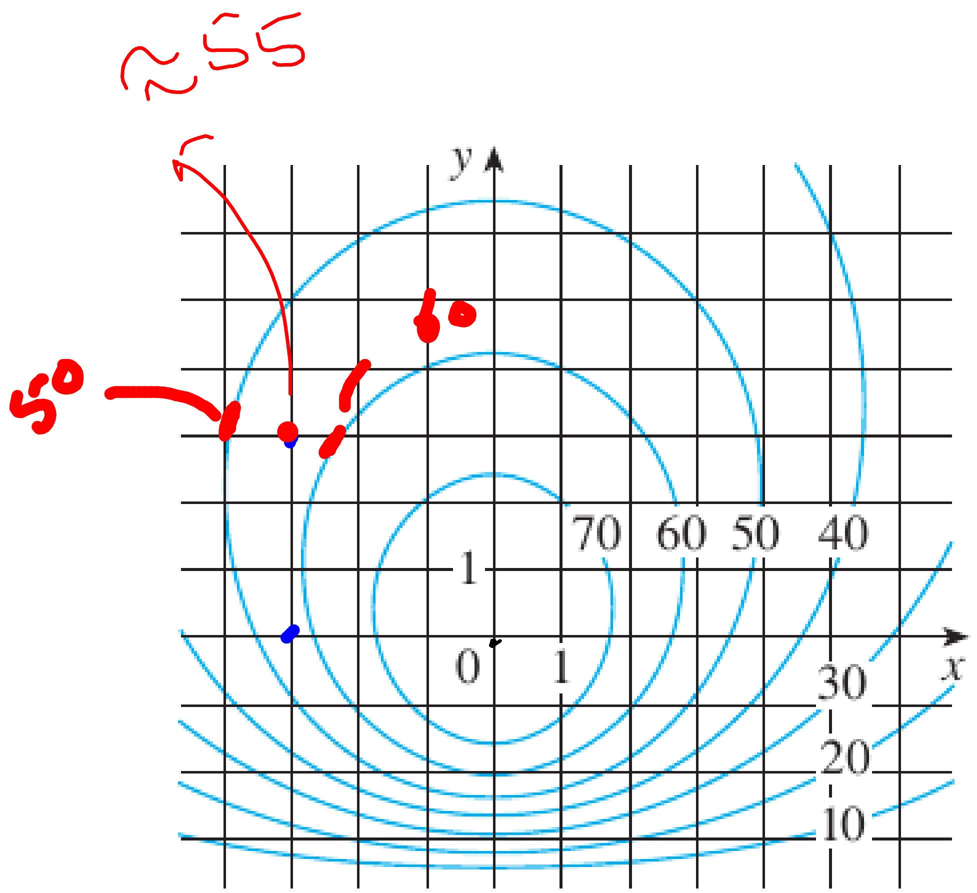


EXAMPLE 3

(a) If $z = f(x, y) = x^2 + 3xy - y^2$, find the differential dz.

(b) If x changes from 2 to 2.05 and y changes from 3 to 2.96, compare the values of Δz and dz.

EXAMPLE 4 The base radius and height of a right circular cone are measured as 10 cm and 25 cm, respectively, with a possible error in measurement of as much as 0.1 cm in each. Use differentials to estimate the maximum error in the calculated volume of the cone.



≈55
50
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20
10