MAT103

Mathematical Methods – I

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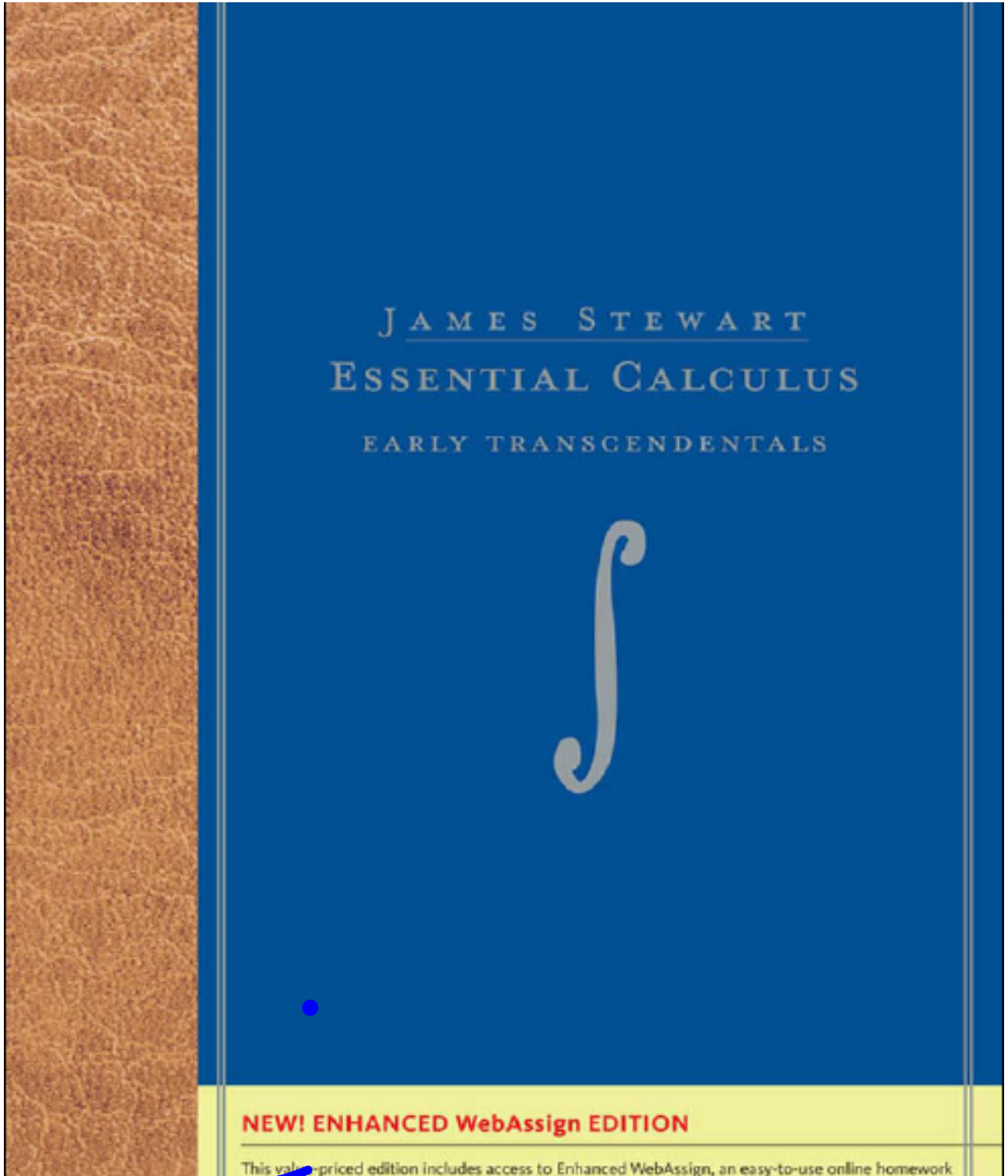
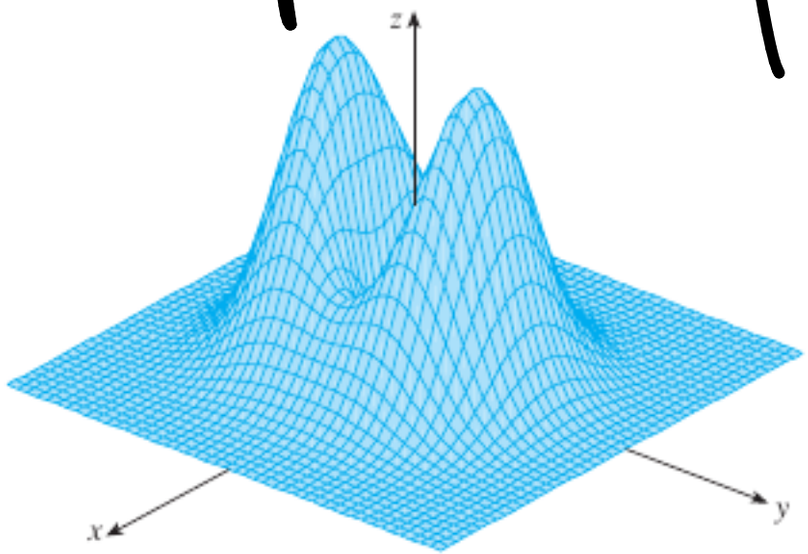
& Teaching Assistants (TAs)



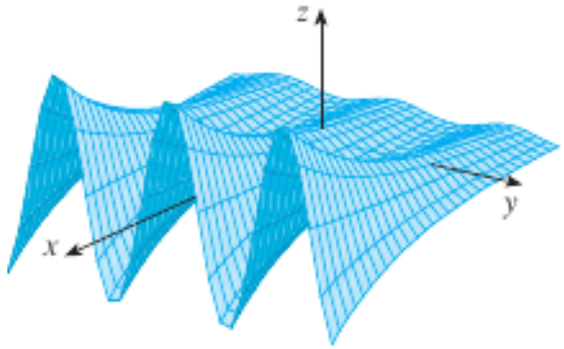
- 1 FUNCTIONS AND LIMITS
- 2 DERIVATIVES
- 3 INVERSE FUNCTIONS: Exponential, Logarithmic, a…
- 4 APPLICATIONS OF DIFFERENTIATION
- 5 INTEGRALS
- 6 TECHNIQUES OF INTEGRATION
- 7 APPLICATIONS OF INTEGRATION
- 8 SERIES
- 9 PARAMETRIC EQUATIONS AND POLAR COORDIN…
- 10 VECTORS AND THE GEOMETRY OF SPACE
- 11 PARTIAL DERIVATIVES
- 12 MULTIPLE INTEGRALS
- 13 VECTOR CALCULUS

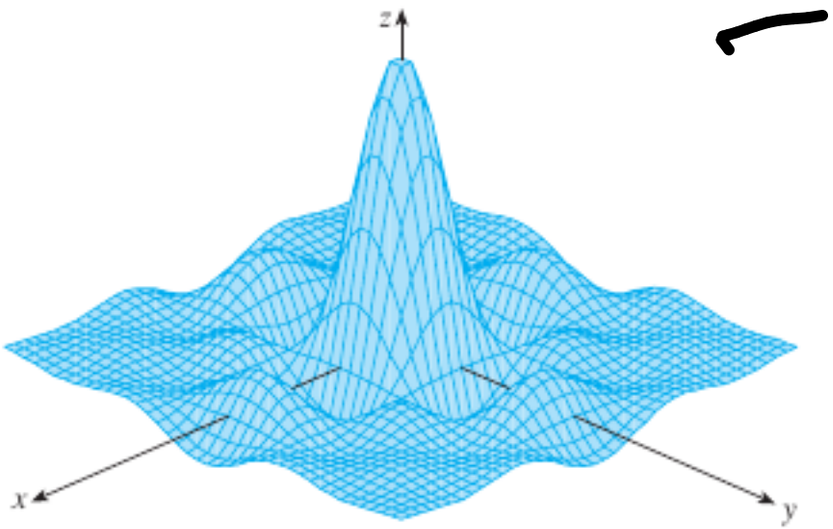
Syllabus for this Sem

Graphs of multivariable fns.

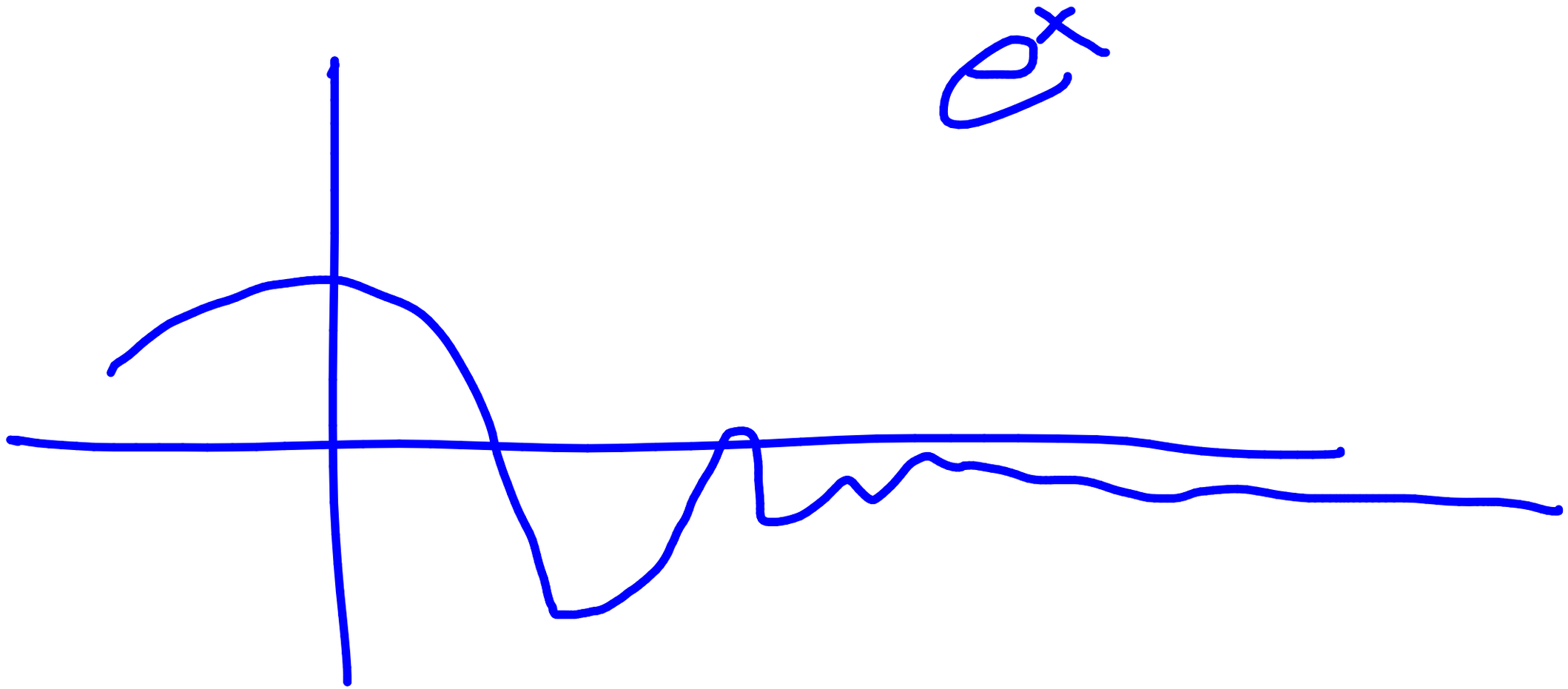


(a) $f(x, y) = (x^2 + 3y^2)e^{-x^2-y^2}$

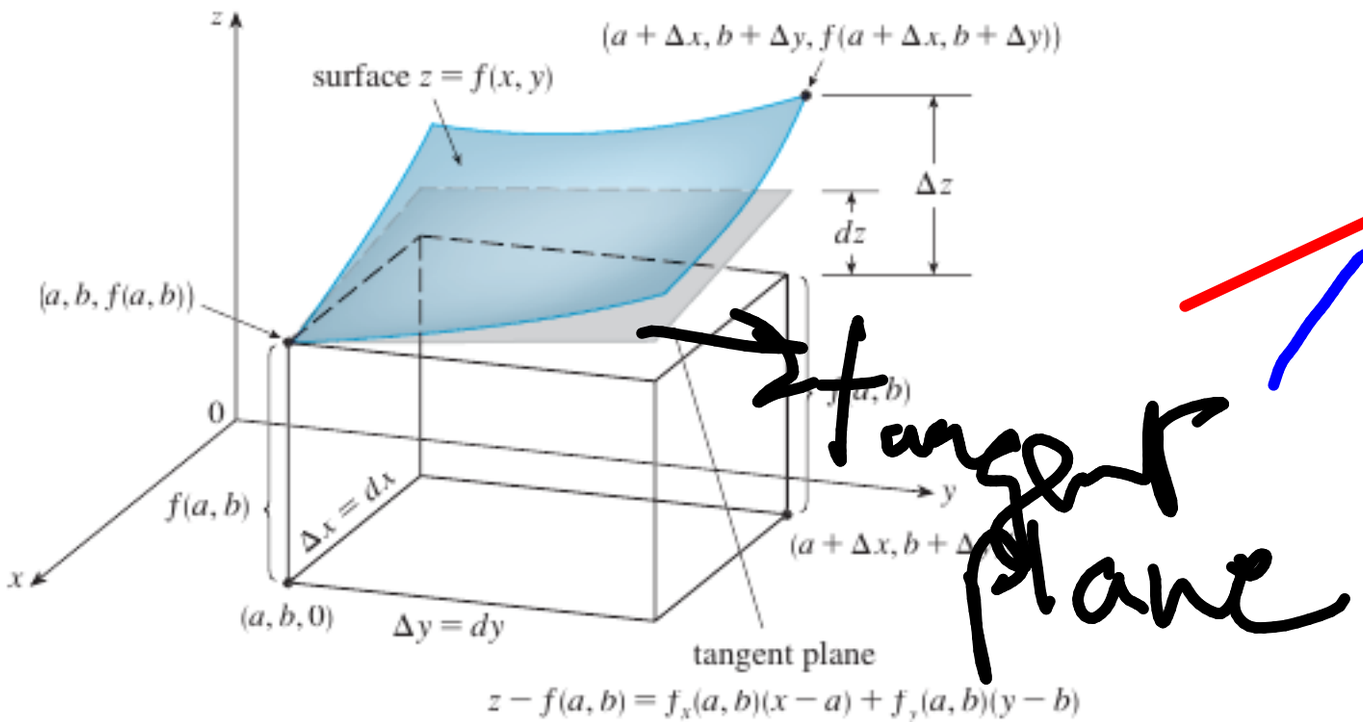


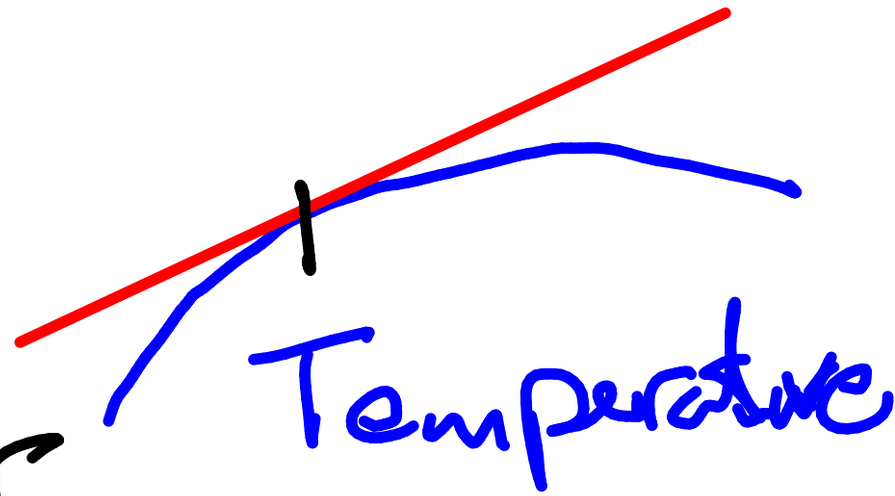


(d) $f(x, y) = \dfrac{\sin x \, \sin y}{xy}$


e^x

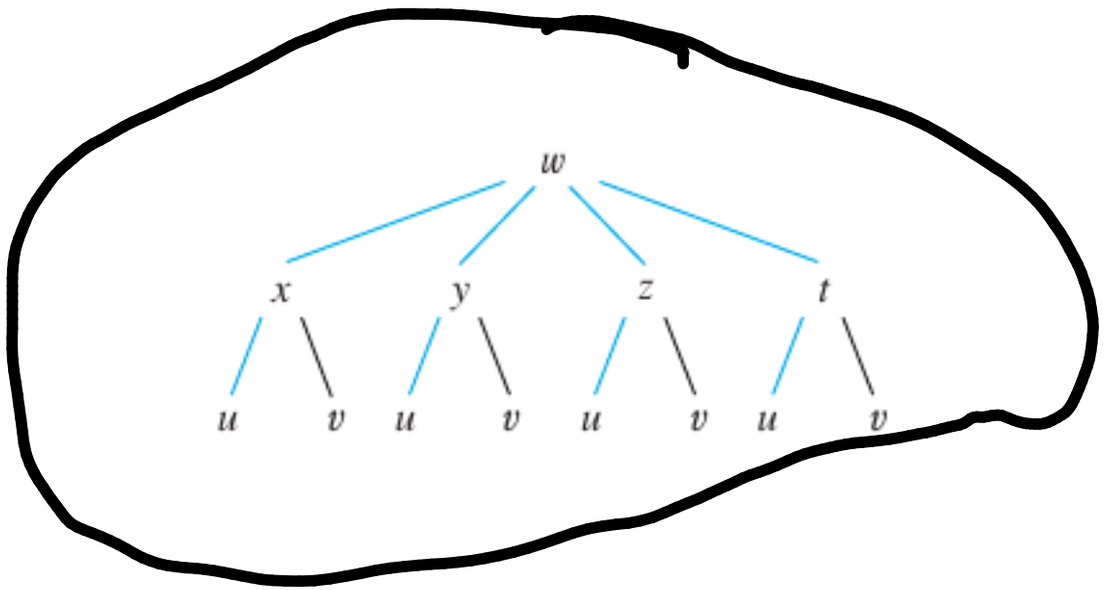
derivatives of $f(x, y)$

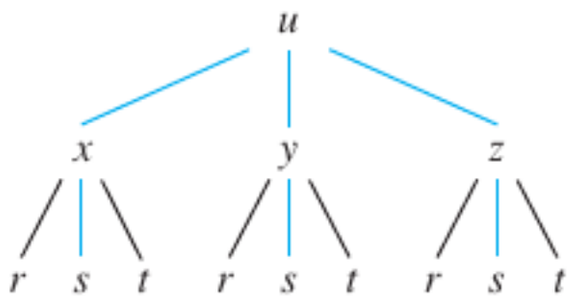


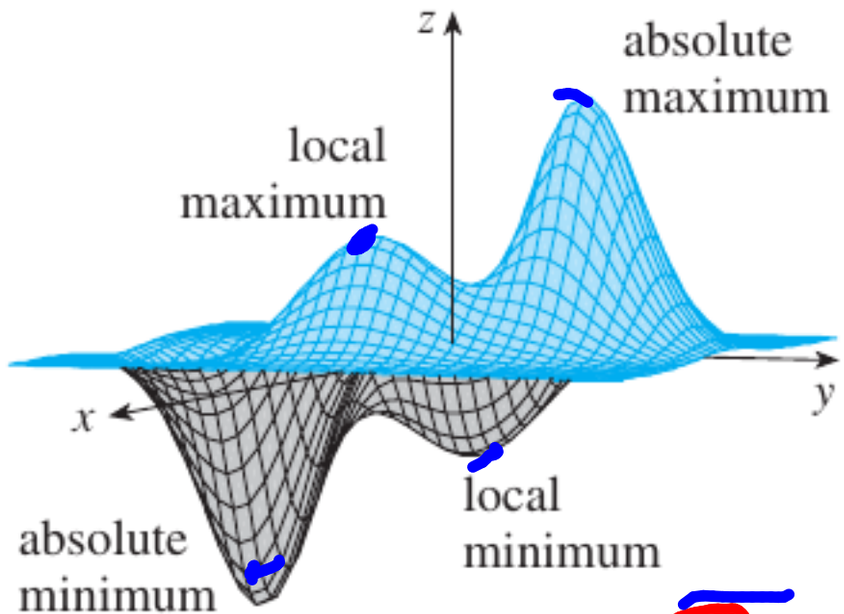


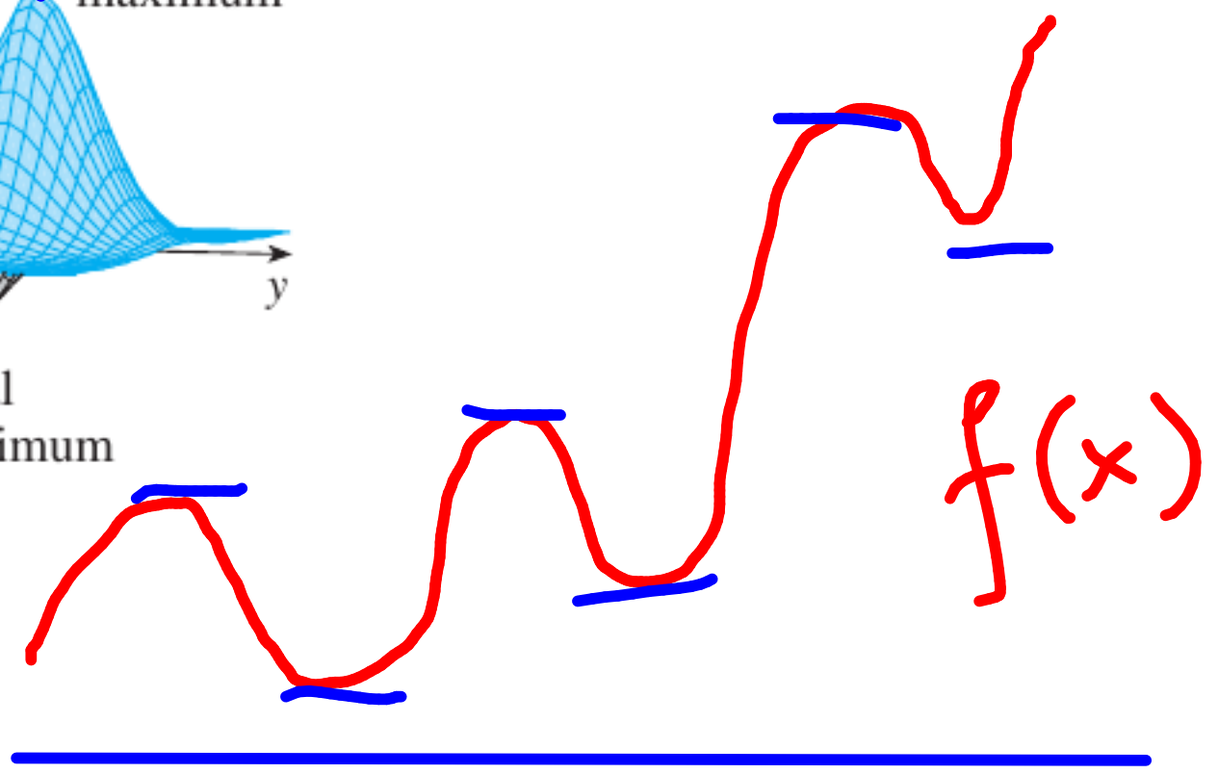
tangent plane

THE CHAIN RULE

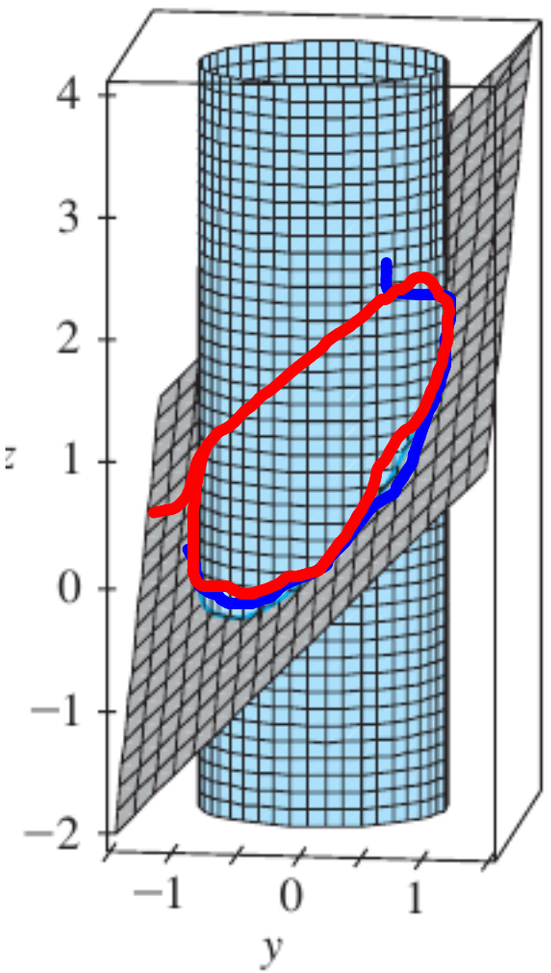




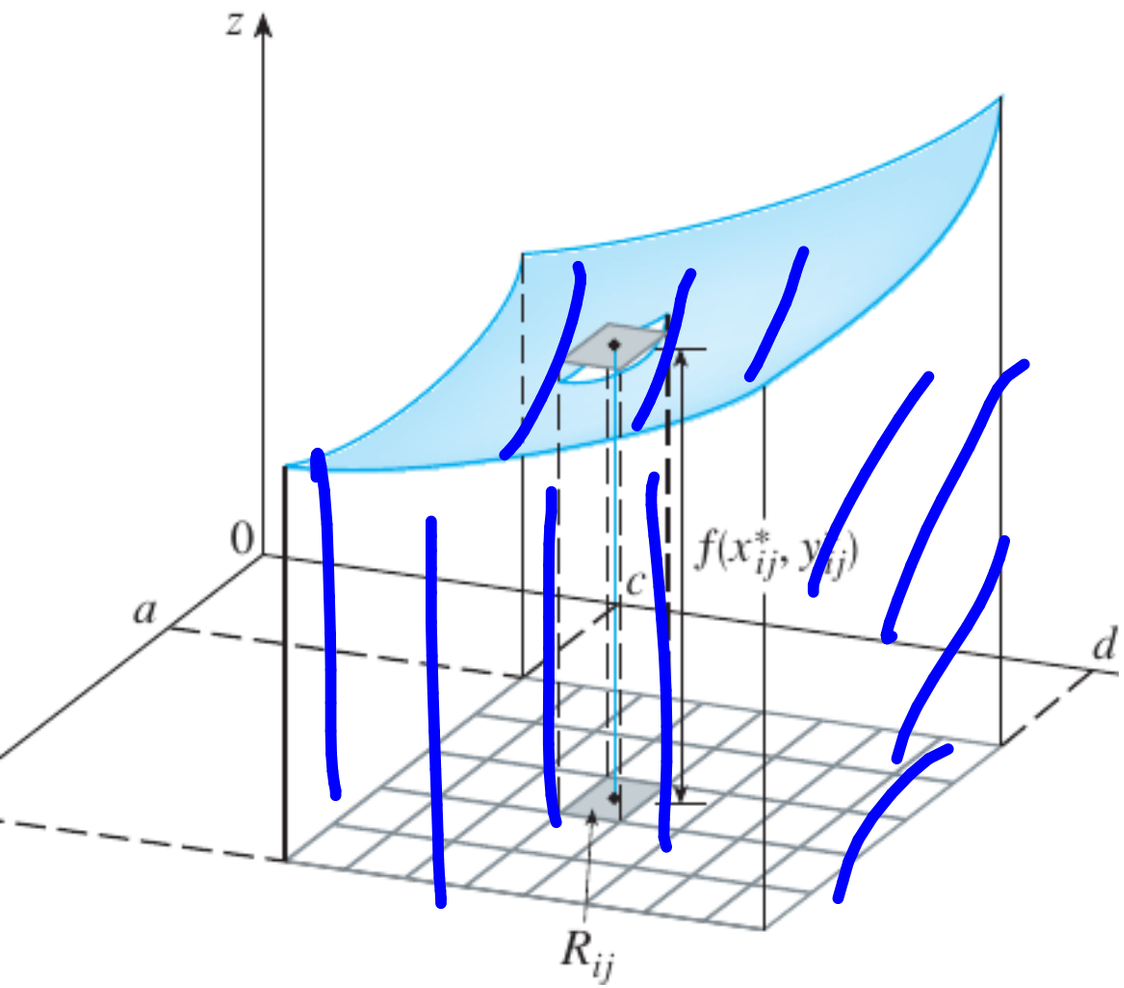

z
absolute
maximum
local
maximum
x
y
local
minimum
absolute
minimum

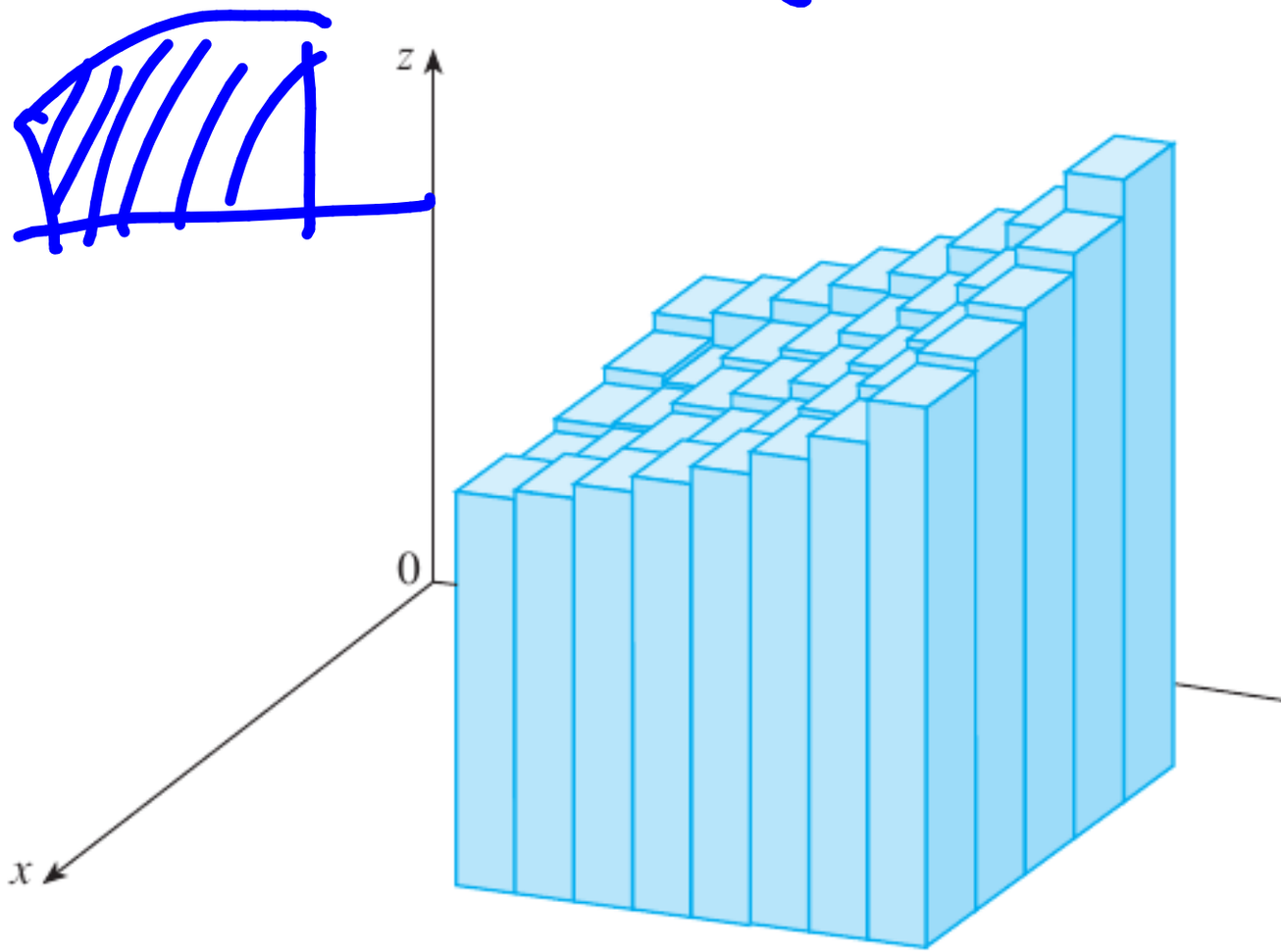

f(x)

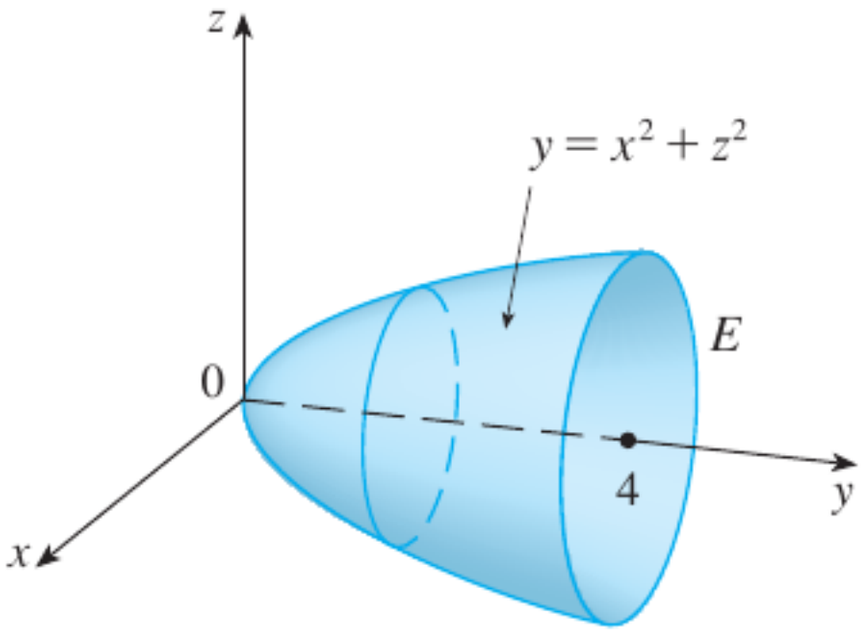
Lagrange multipliers

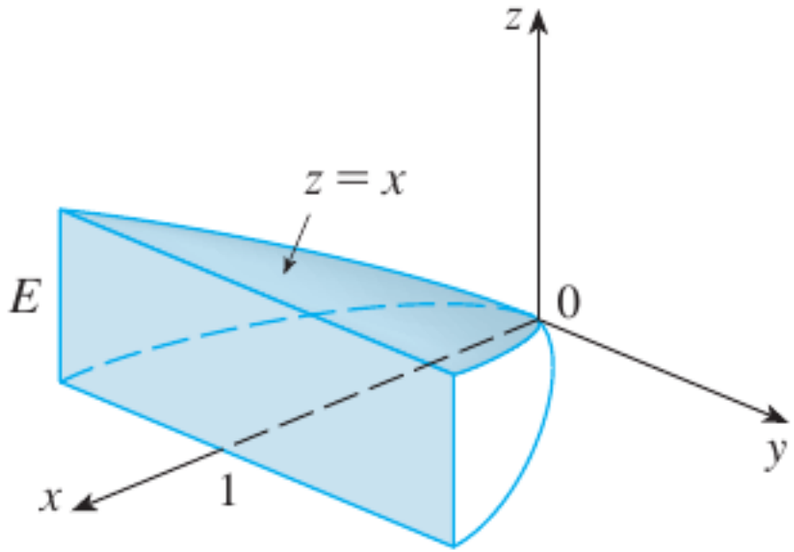


Chapter 12: Integration

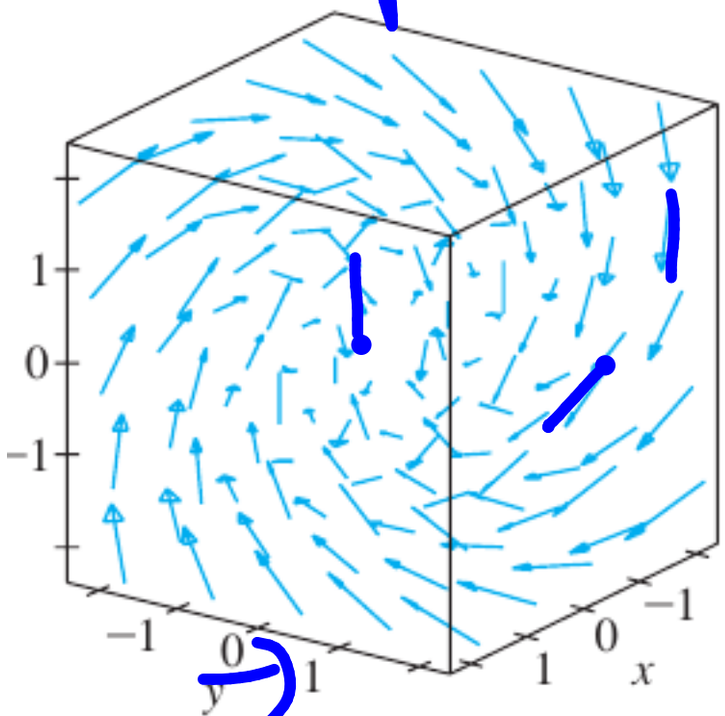




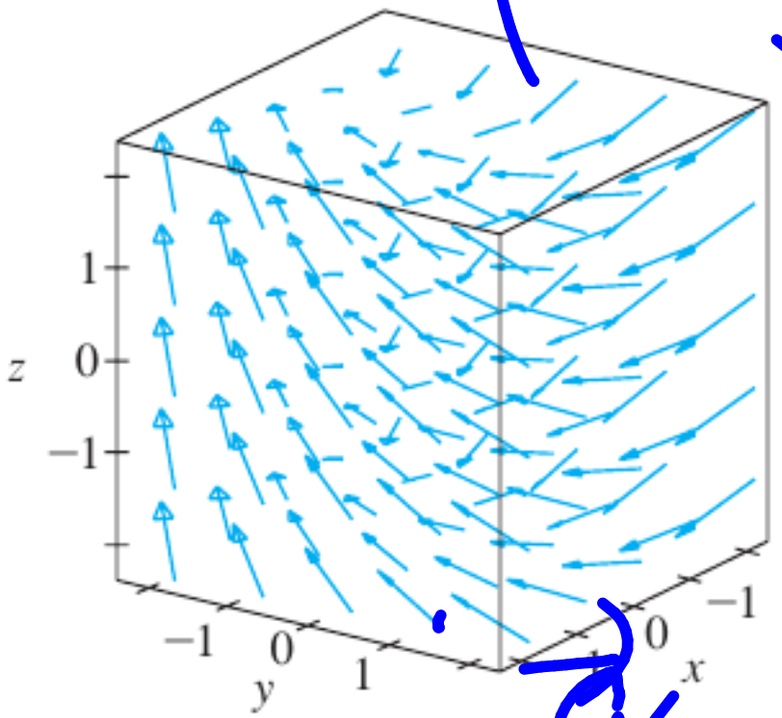

z
y = x² + z²
E
0
4
y
x


z
z = x
E
0
y
x
1

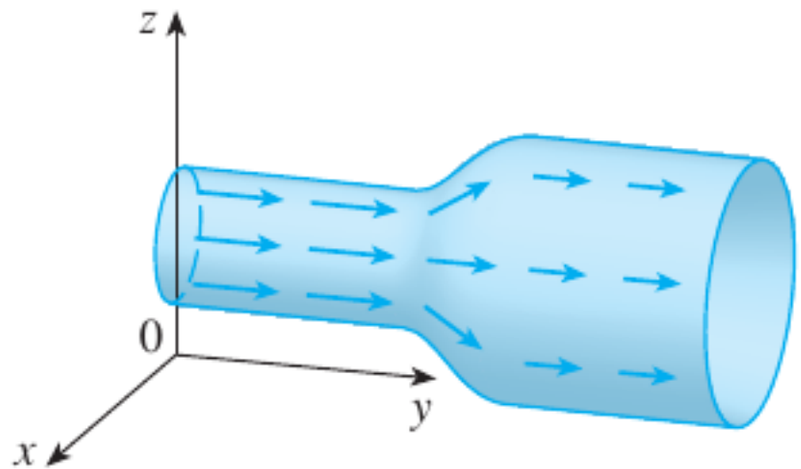
Chapter 13: Surfaces & Vector fields



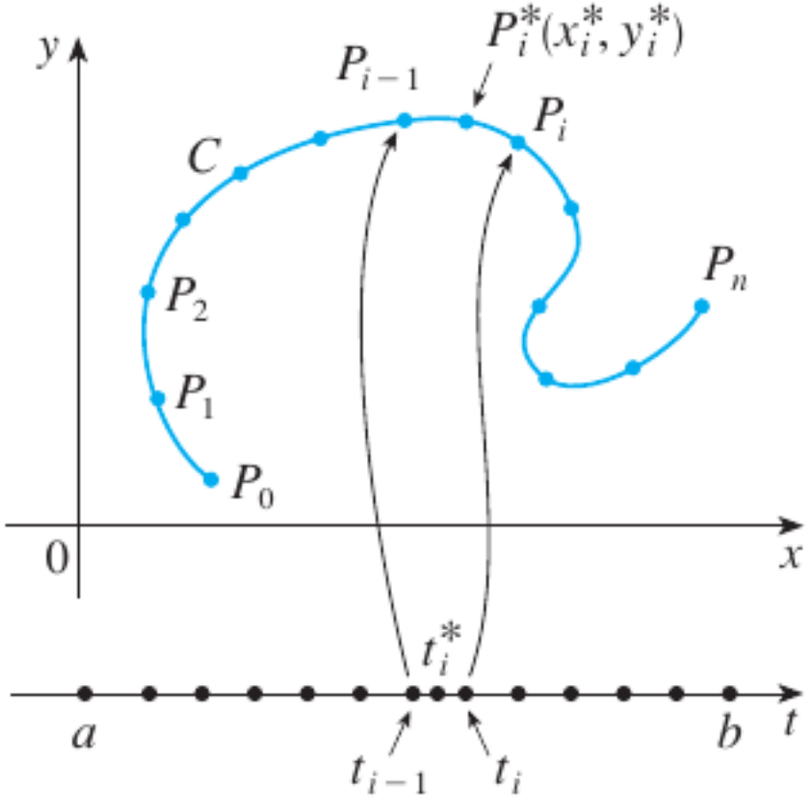
f



g

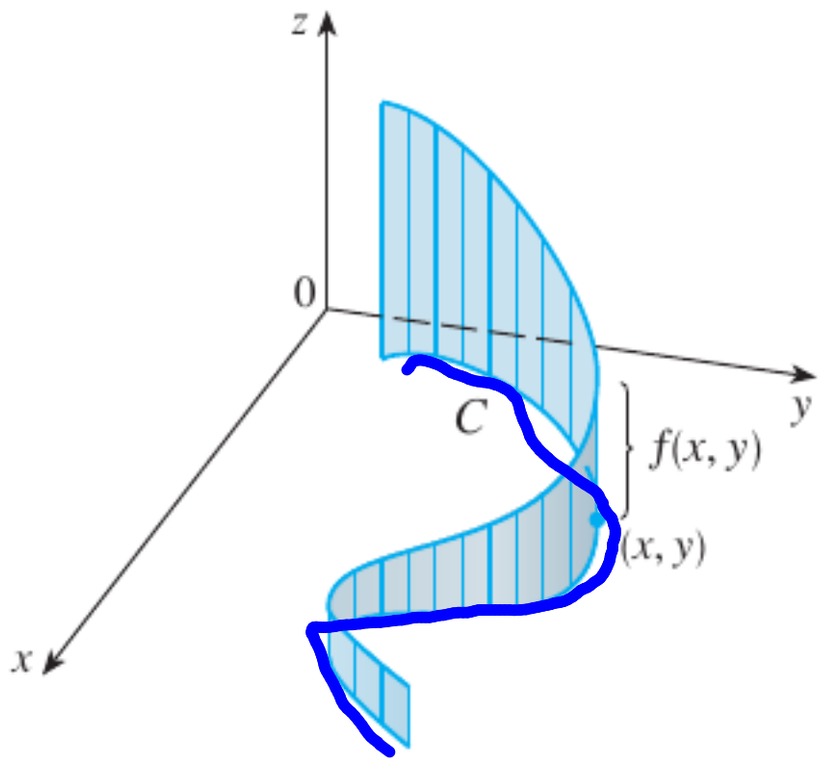


h

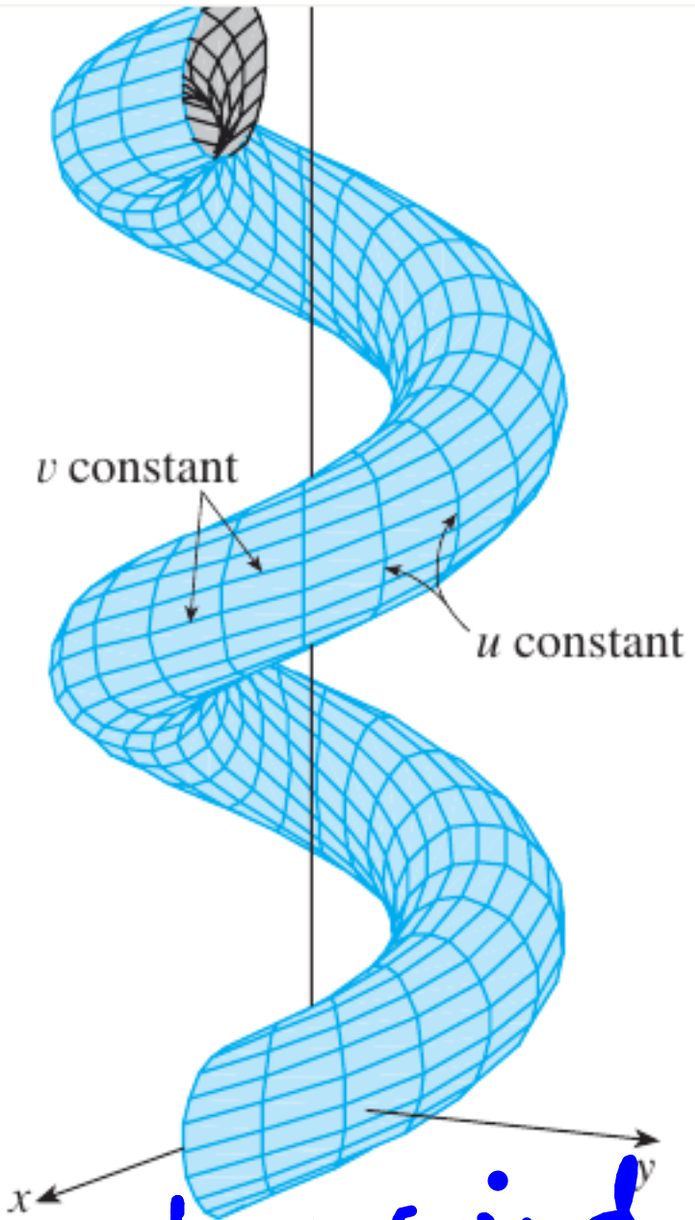


length of C

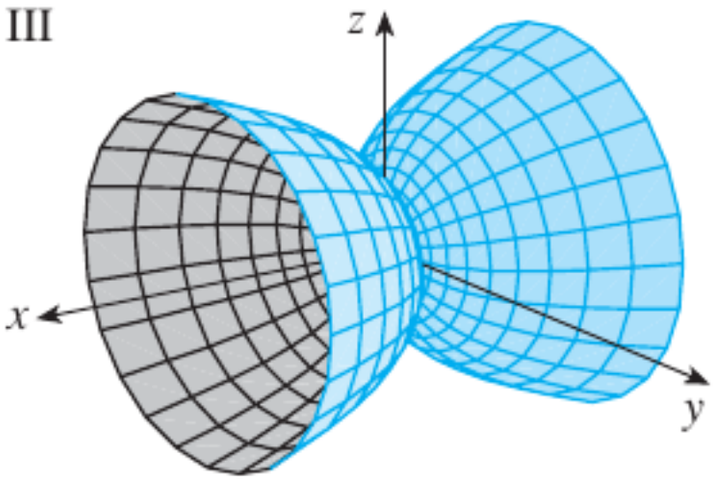
$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

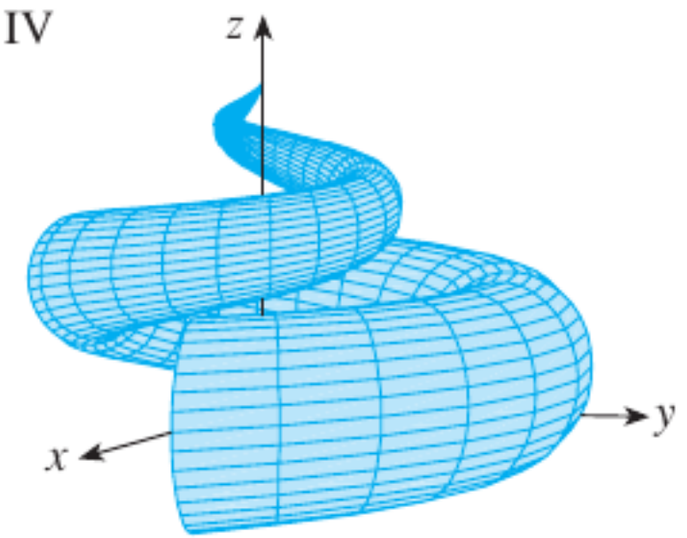

z
0
y
x
C
f(x, y)
(x, y)

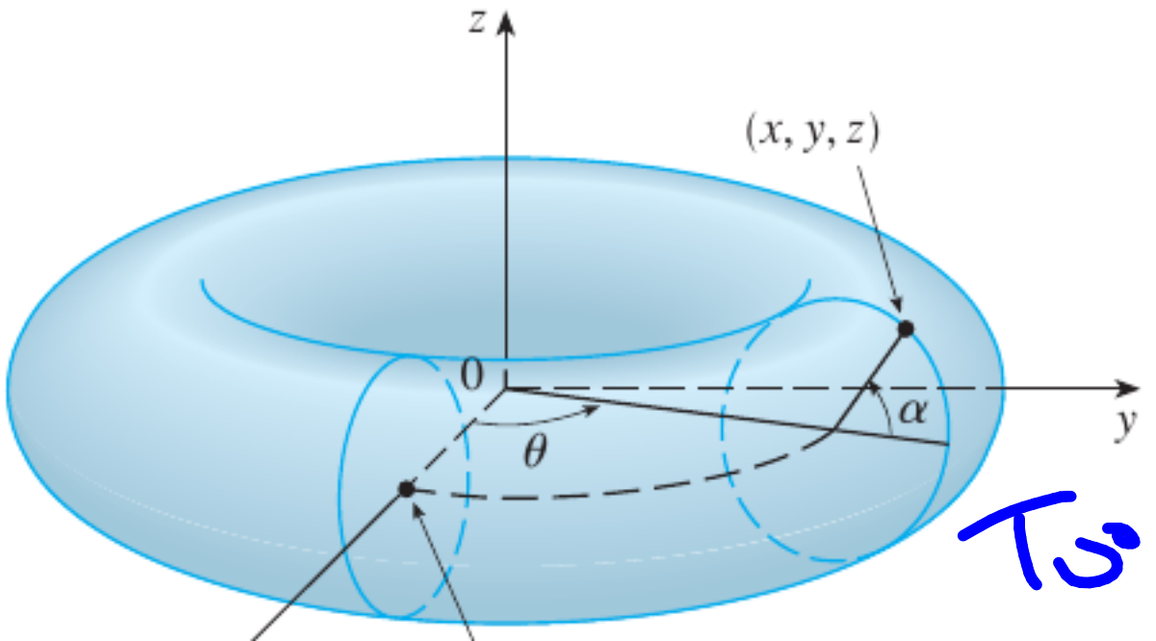
PARAMETRIC SURFACES



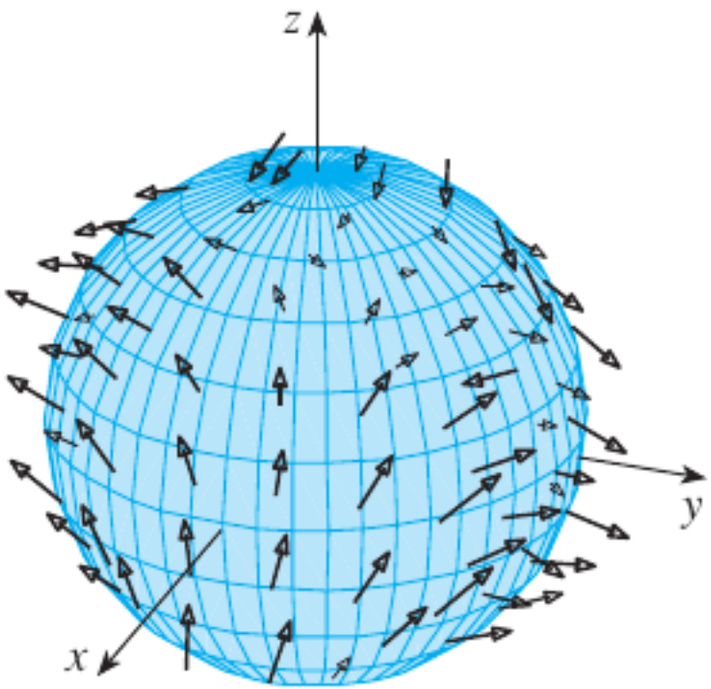
Helicoid







Torus


z
y
x

11 PARTIAL DERIVATIVES

$$f(x,y) = x^2 y$$

$$\frac{\partial f}{\partial x} = 2xy$$

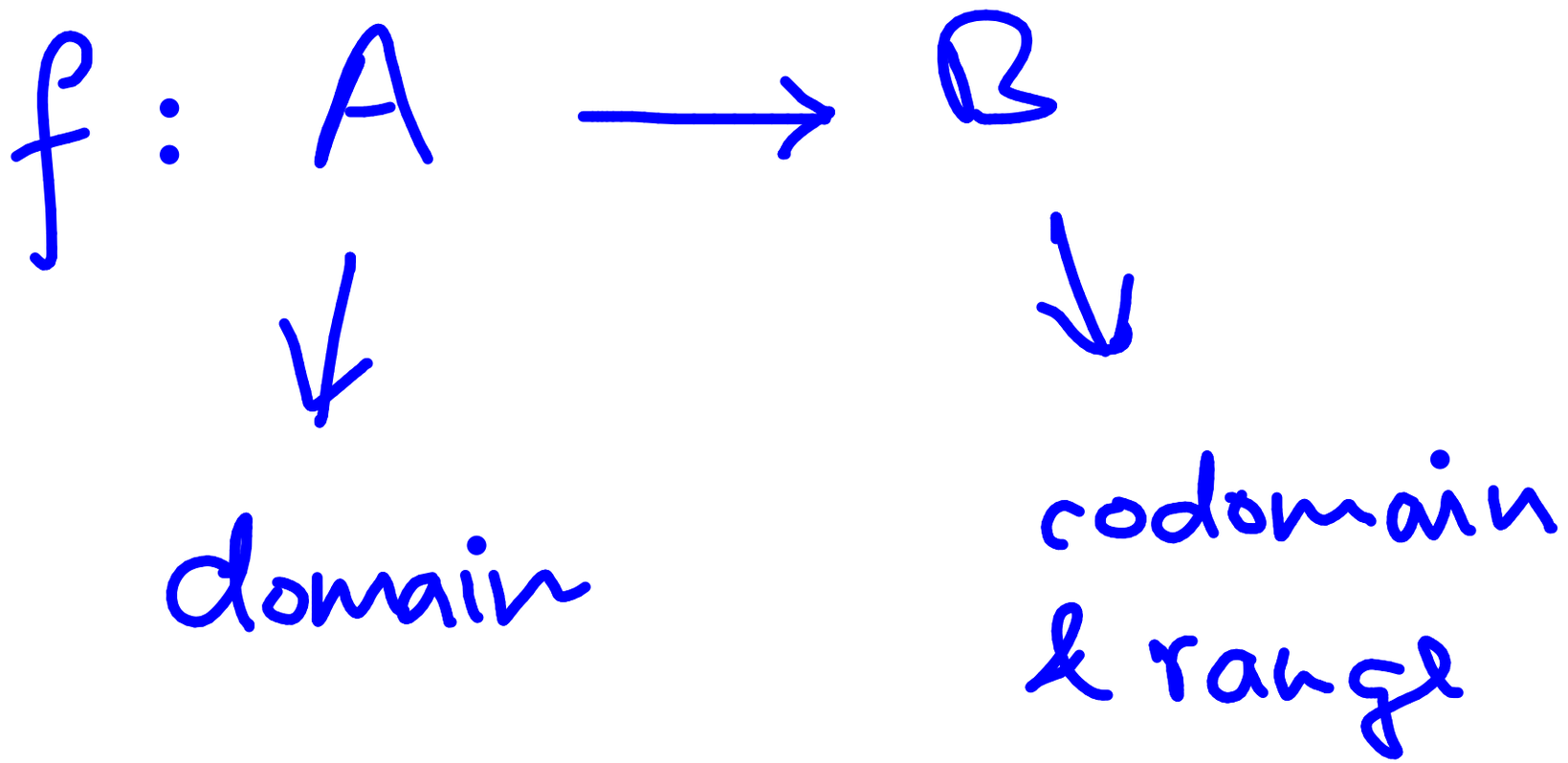
this chapter is about partial derivatives & their applications

11.1 FUNCTIONS OF SEVERAL VARIABLES

→ introduction of several variable function

→ graphs, contour plots, level surfaces

functions

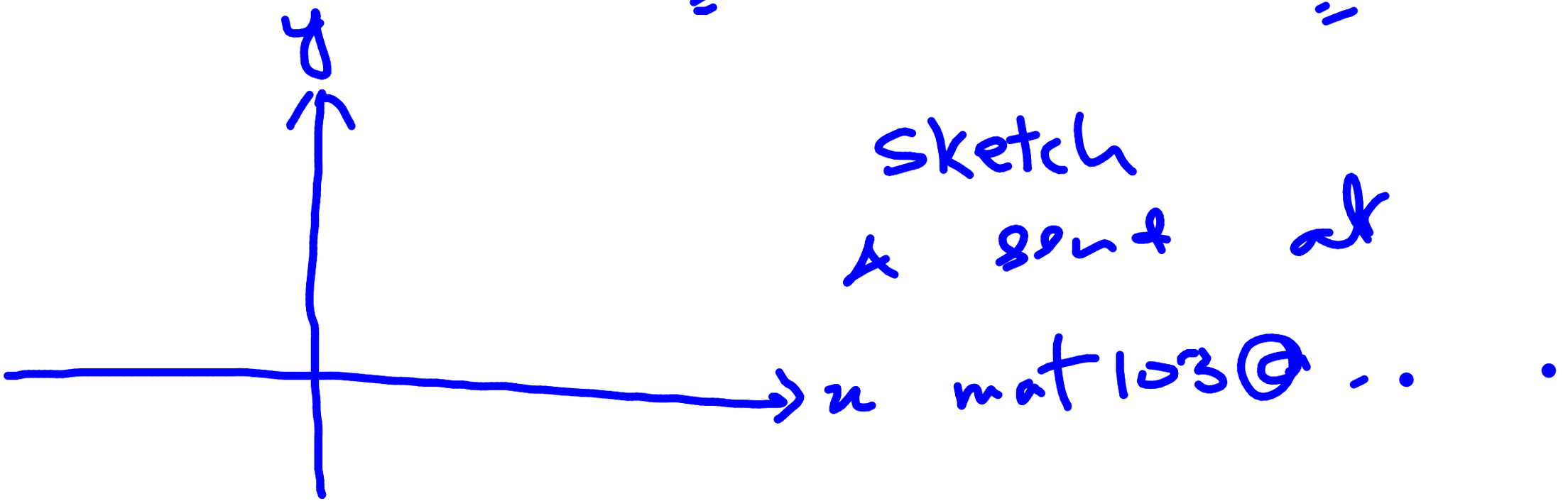


EXAMPLE 1 Find the domains of the following functions and evaluate $f(3, 2)$.

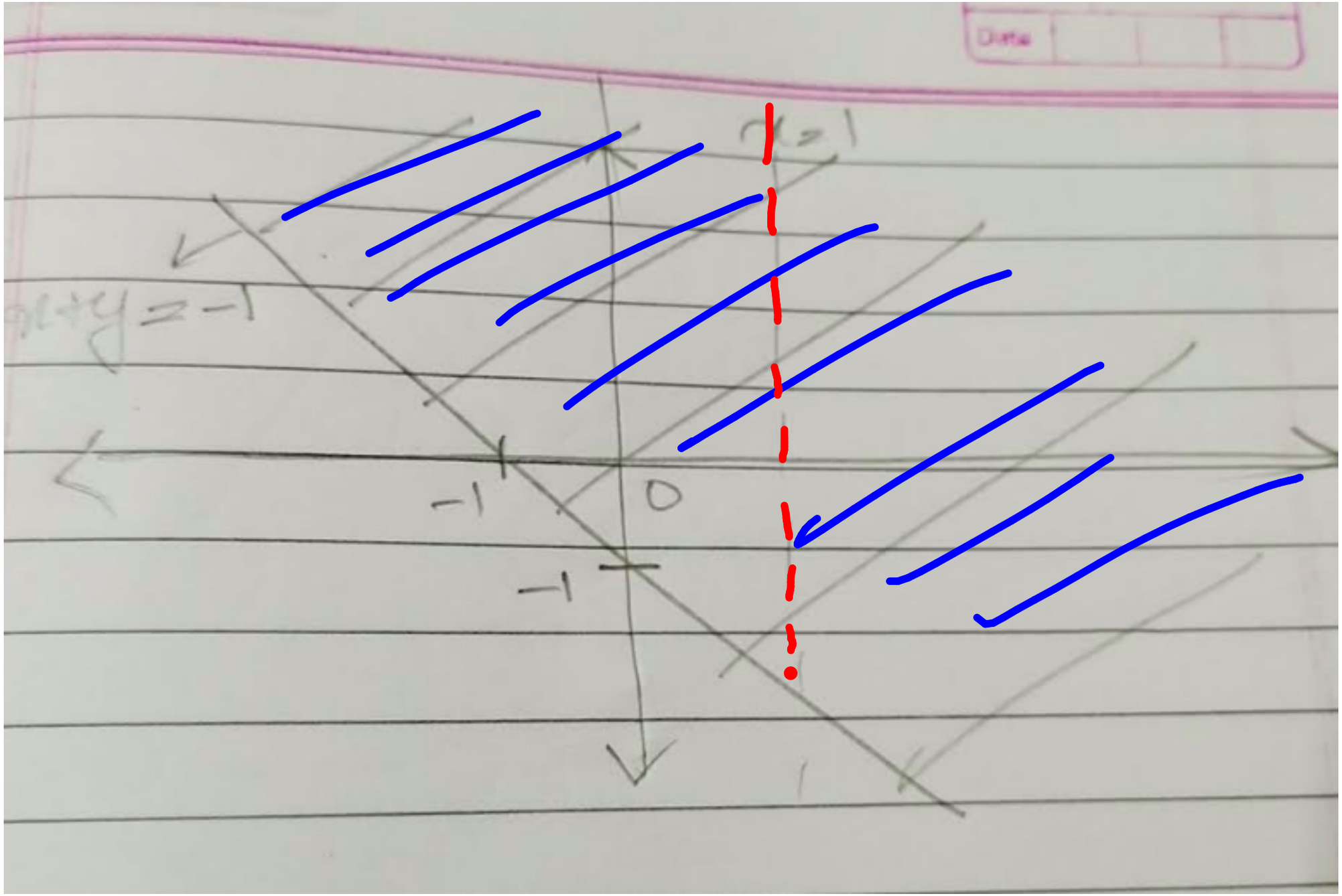
(a) $f(x, y) = \dfrac{\sqrt{x + y + 1}}{x - 1}$

domain: part of the xy plane where

$(x+y+1) \geq 0$ AND $x \neq 1$



sketch a graph of ...

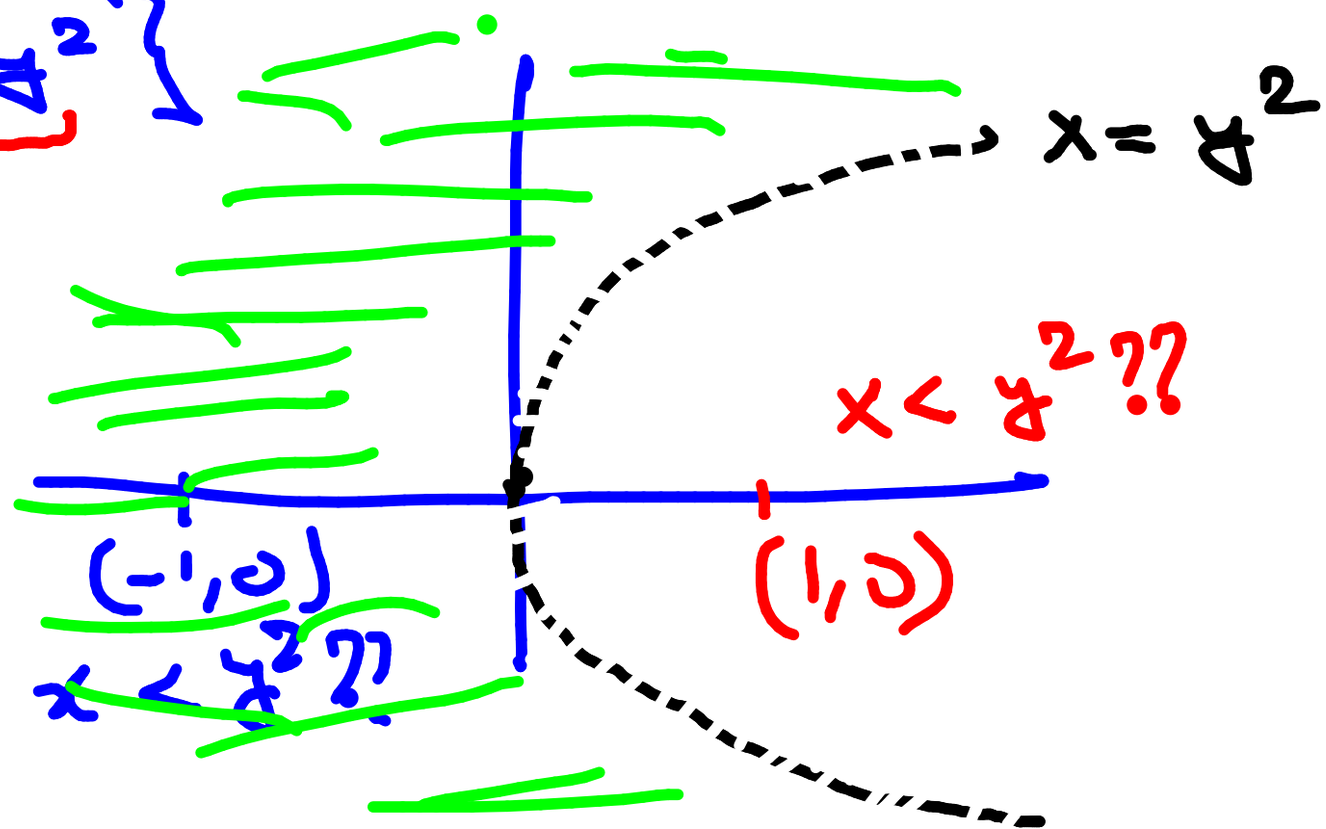

x+y=-1
x=1
-1
0
-1

EXAMPLE 1 Find the domains of the following functions and evaluate $f(3, 2)$.

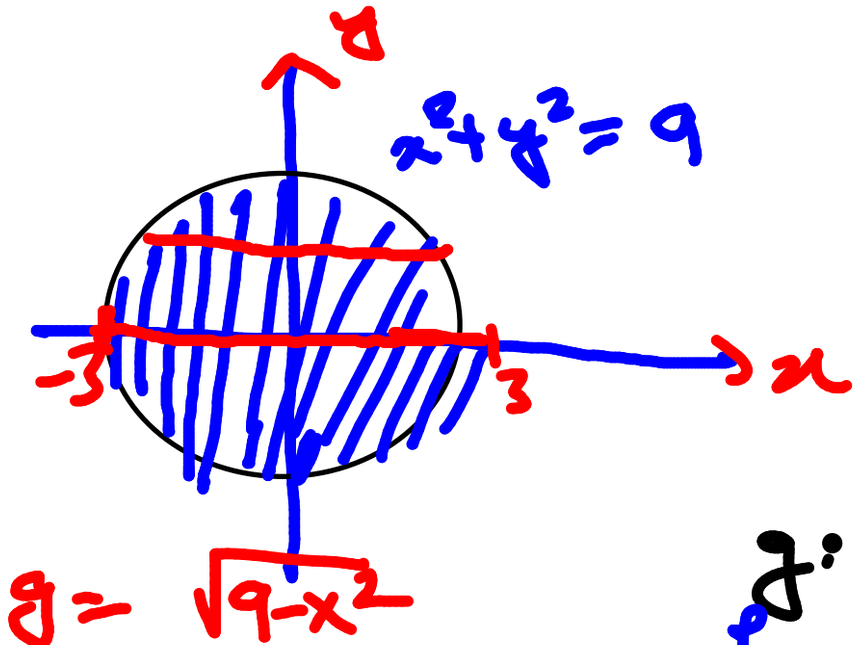
(b) $f(x, y) = x \ln(y^2 - x)$

domain will be all the points (x, y) s.t. $y^2 - x > 0$

$= \{(x, y) \mid x < y^2\}$



EXAMPLE 2 Find the domain and range of $g(x, y) = \sqrt{9 - x^2 - y^2}$.



$x^2 + y^2 = 9$

$g = \sqrt{9 - x^2}$

domain is collection of all (x, y) s.t.

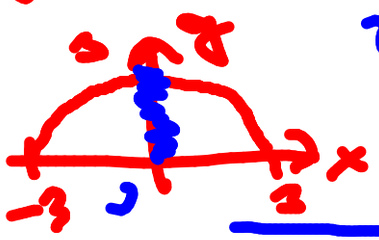
$9 - x^2 - y^2 \geq 0$

i.e. $\boxed{x^2 + y^2 \leq 9}$

disc

g: disc $\rightarrow$ real numbers

range $= \{ g(x, y) \mid (x, y) \in \text{domain} \}$

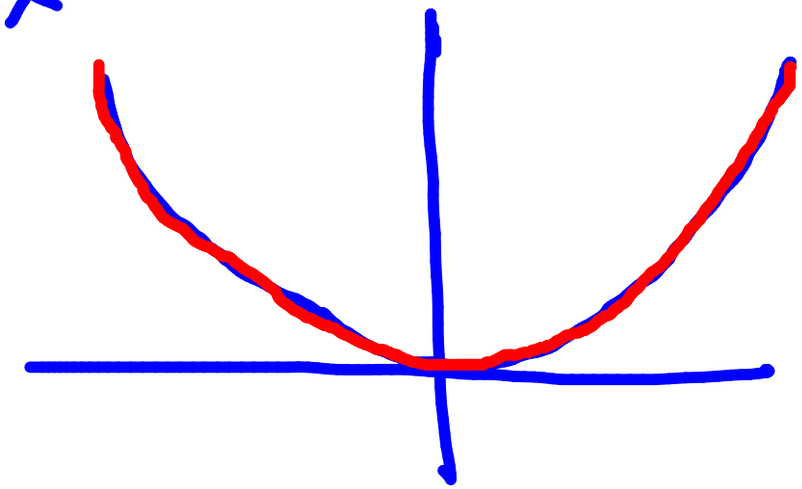


Range(g) $= [0, 3]$

GRAPHS

$f(x) = x^2$

$= \{ (x, y) \mid y = x^2 \}$



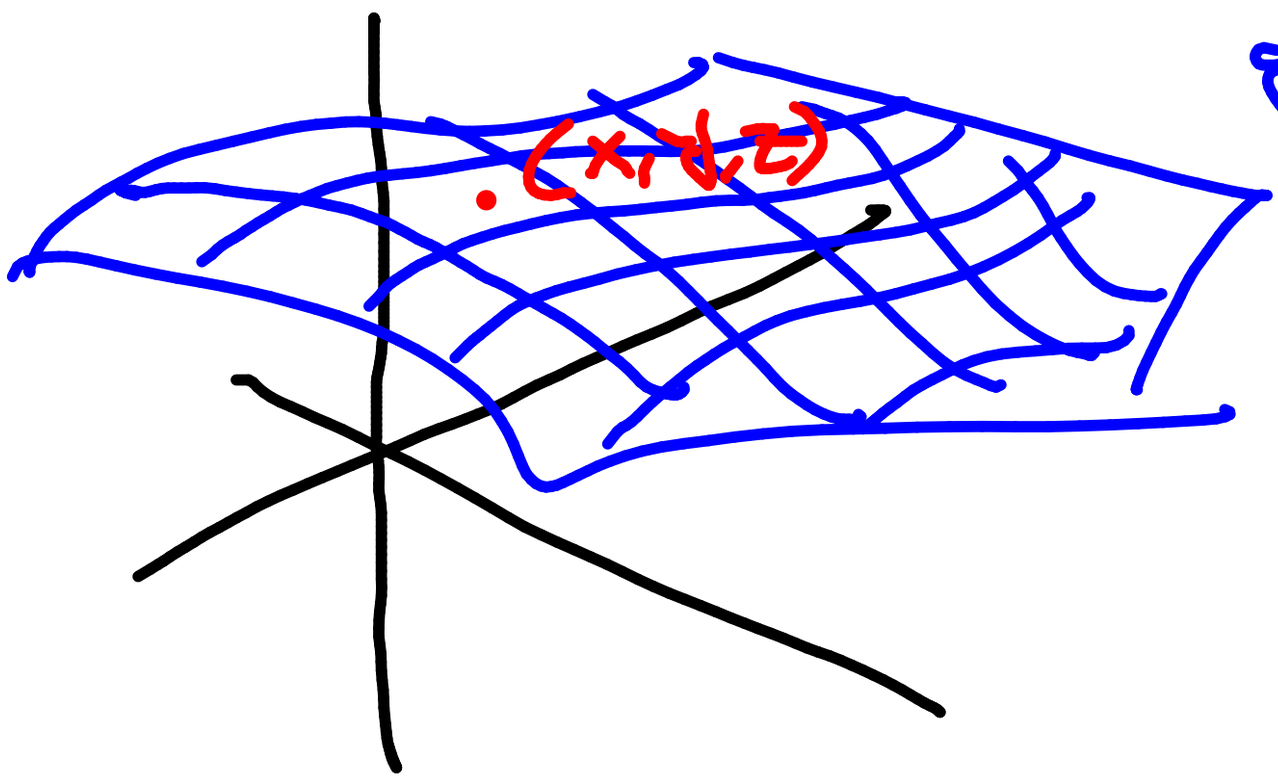
DEFINITION If f is a function of two variables with domain D, then the **graph** of f is the set of all points (x, y, z) in $\mathbb{R}^3$ such that $z = f(x, y)$ and (x, y) is in D.

$\mathbb{R}^3$ → 3d space

$f(x,y) = x^2 + y^2$ graph ??

graph will lie in 3d space

$(x, y, f(x,y))$



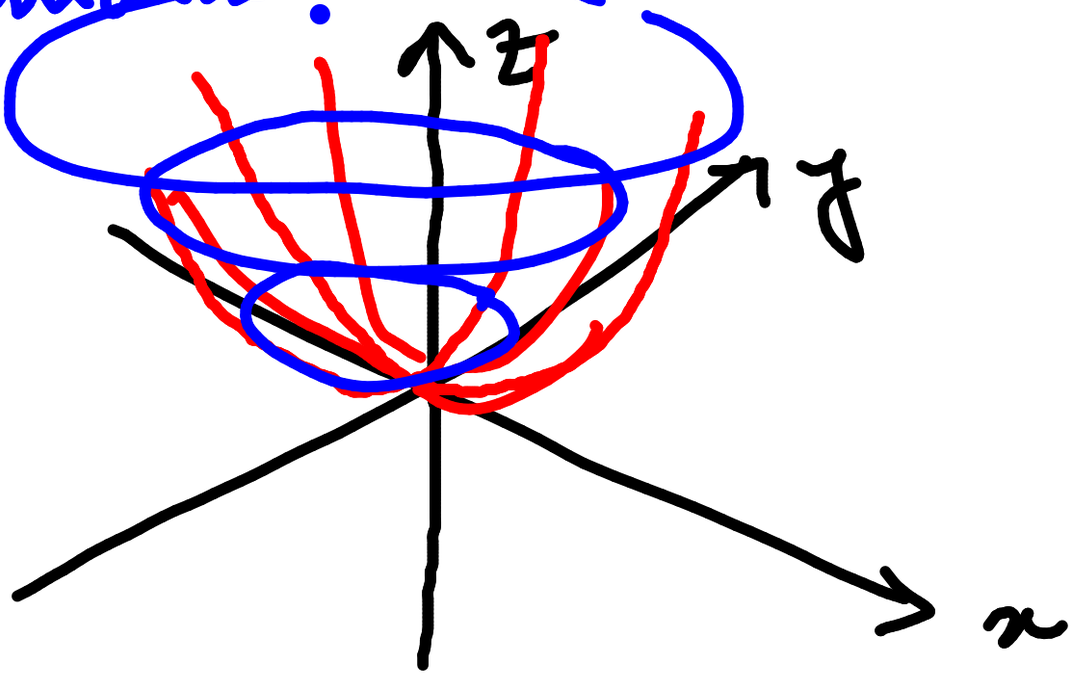
graph

$= \{(x,y,z) \mid z = x^2 + y^2\}$

Q. Draw the graph of

$$f(x,y) = x^2 + y^2$$

domain? the entire xy plane



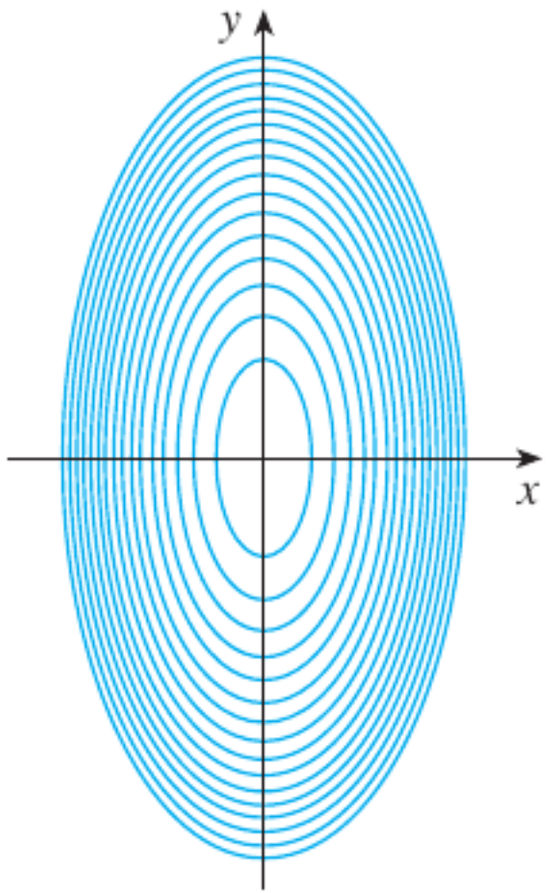
EXAMPLE 4 Sketch the graph of $g(x, y) = \sqrt{9 - x^2 - y^2}$.

https://www.geogebra.org/3d?lang=en

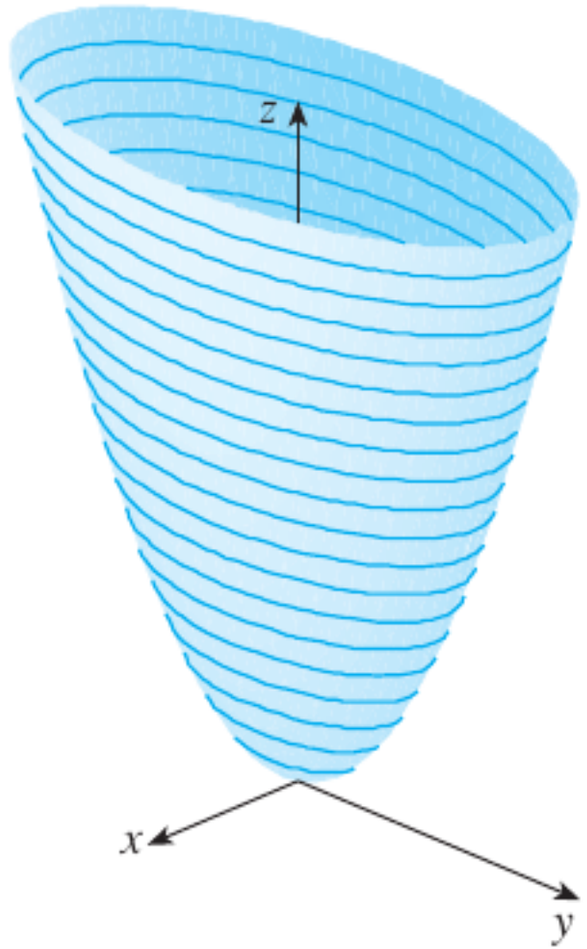
(a) $f(x, y) = (x^2 + 3y^2)e^{-x^2-y^2}$

(d) $f(x, y) = \dfrac{\sin x \ \sin y}{xy}$

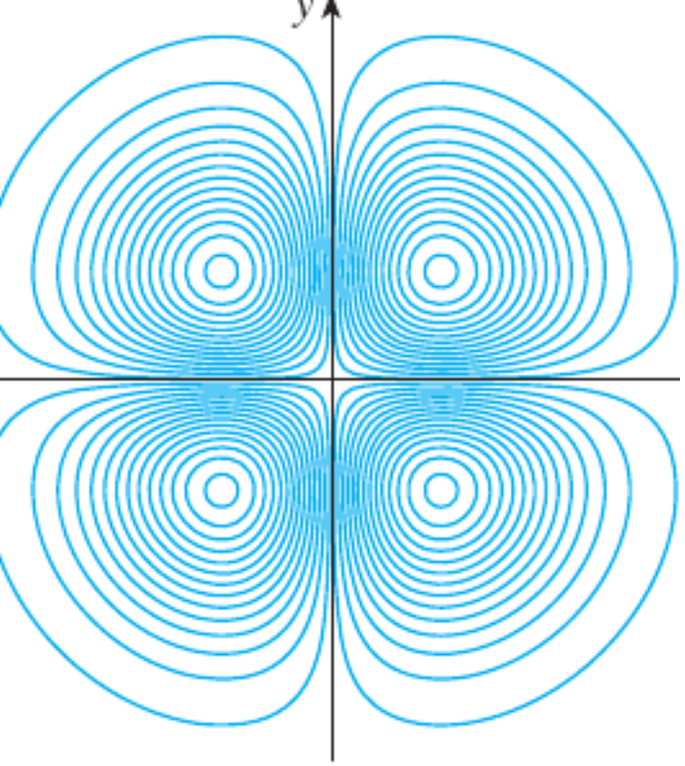
contour curves, or *level curves.*



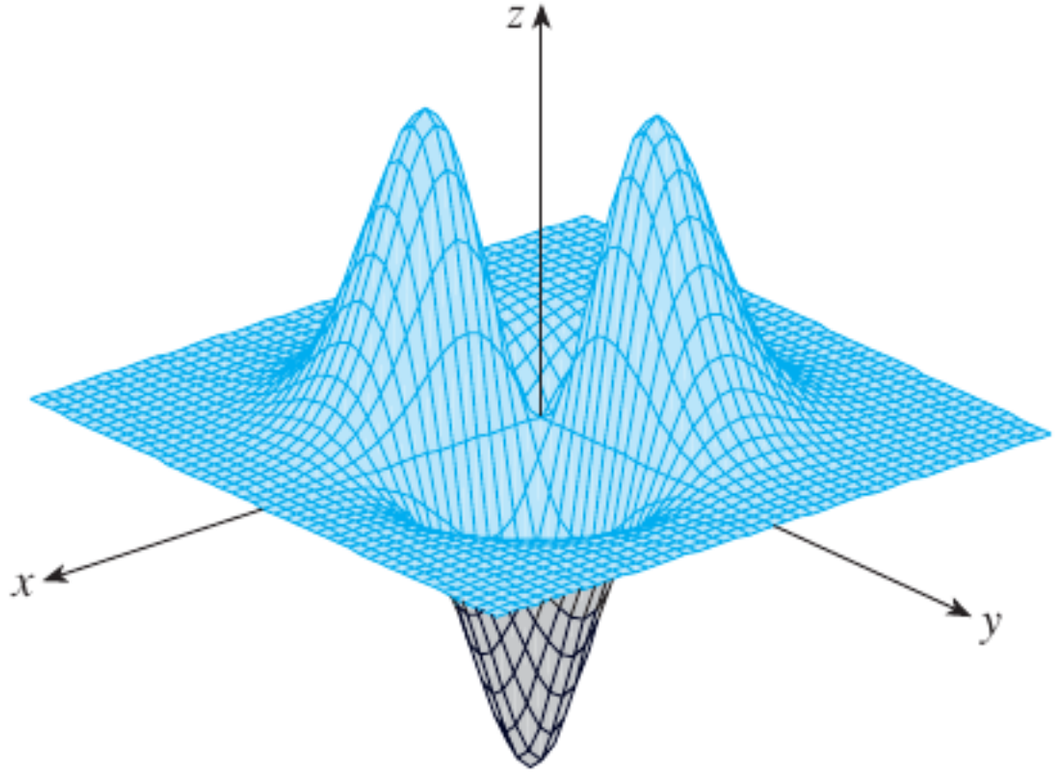
(a) Contour map

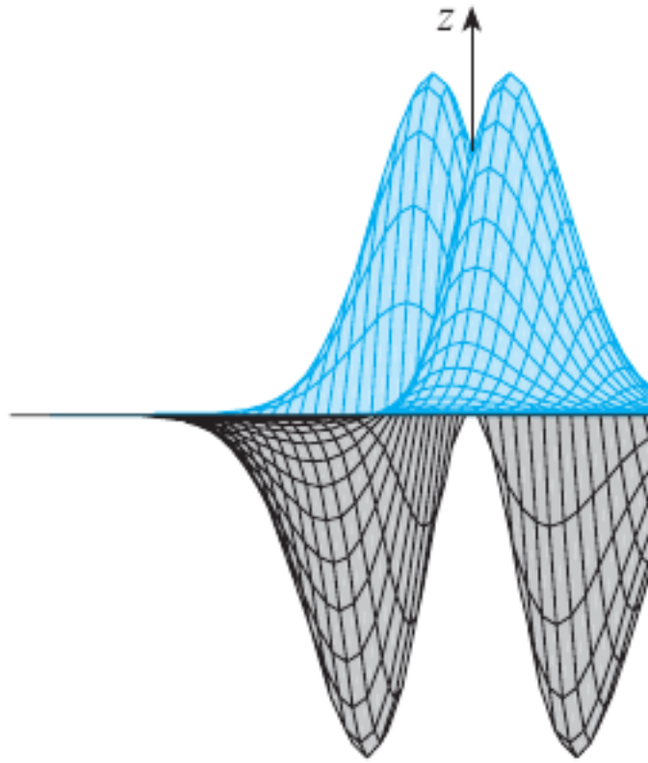


(b) Horizontal traces are raised level curves



a) Level curves of $f(x, y) = -xye^{-}$





(b) Two views of $f(x, y) = -xye^{-x^2-y^2}$

EXAMPLE 10 Find the domain of f if $f(x, y, z) = \ln(z - y) + xy \sin z$.

EXAMPLE 11 Find the level surfaces of the function $f(x, y, z) = x^2 + y^2 + z^2$.

Match the function