13.6 PARAMETRIC SURFACES AND THEIR AREAS

EXAMPLE 1 Identify and sketch the surface with vector equation

$$\mathbf{r}(u, v) = 2\cos u\,\mathbf{i} + v\,\mathbf{j} + 2\sin u\,\mathbf{k}$$

A bit of practice with matlab

→ LIVE SCRIPTS

EXAMPLE 2 Use a computer algebra system to graph the surface

$$\mathbf{r}(u, v) = \langle (2 + \sin v) \cos u, (2 + \sin v) \sin u, u + \cos v \rangle$$

Which grid curves have u constant? Which have v constant?

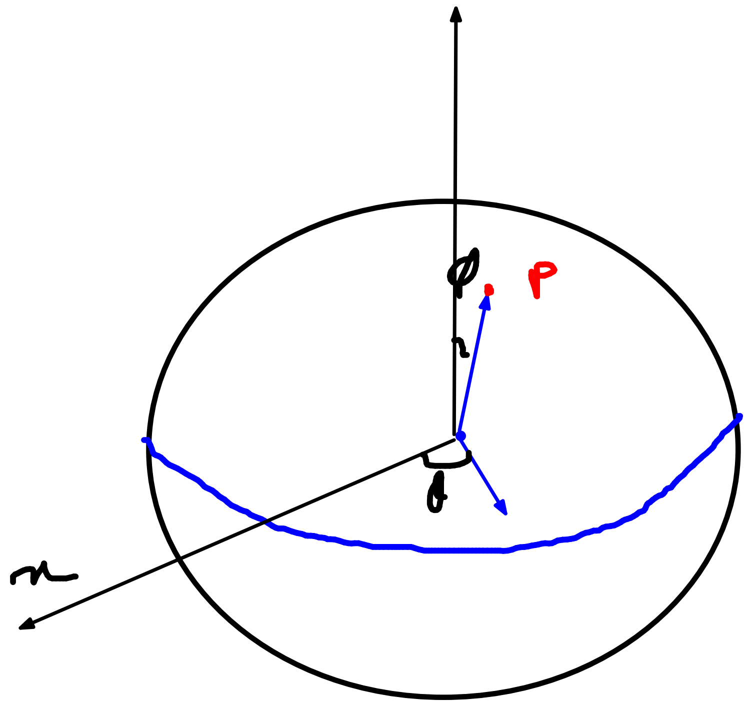
→ plot this in matlab

EXAMPLE 4 Find a parametric representation of the sphere $x^2 + y^2 + z^2 = a^2$.

$$x = a\cos\theta\sin\phi$$

$$y = a\sin\theta\sin\phi$$

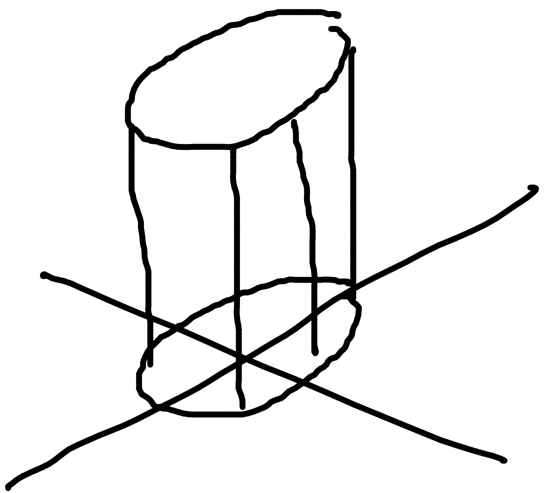
$$z = a\cos\phi$$



recall spherical coordinates

EXAMPLE 5 Find a parametric representation for the cylinder

$$x^2 + y^2 = 4 \qquad 0 \leqslant z \leqslant 1$$



$x = 2\cos\theta$

$y = 2\sin\theta$

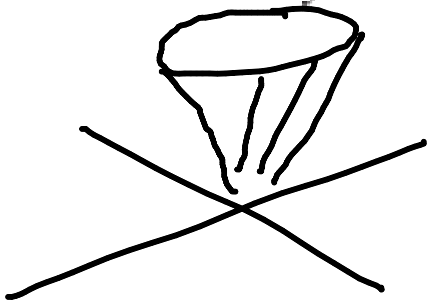
$z = z$

parameters

$0 \le \theta \le 2\pi$

$0 \le z \le 1$

EXAMPLE 6 Find a parametric representation for the surface $z = 2\sqrt{x^2 + y^2}$, that is, the top half of the cone $z^2 = 4x^2 + 4y^2$.

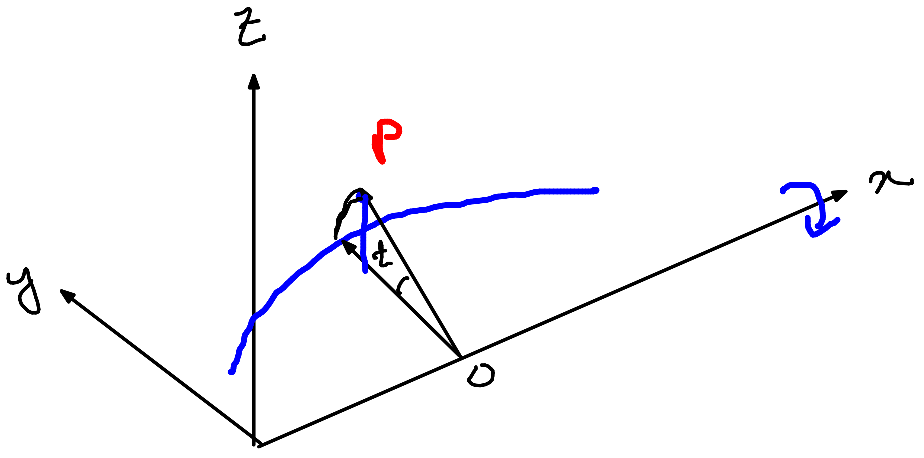


Drawing ??

parametric eqn

$$x = x \qquad x = r\cos\theta$$
$$y = y \qquad y = r\sin\theta$$
$$z = 2\sqrt{x^2+y^2} \qquad z = 2r$$

$y = f(x)$



$OP = f(x)$

OP's y coordinate

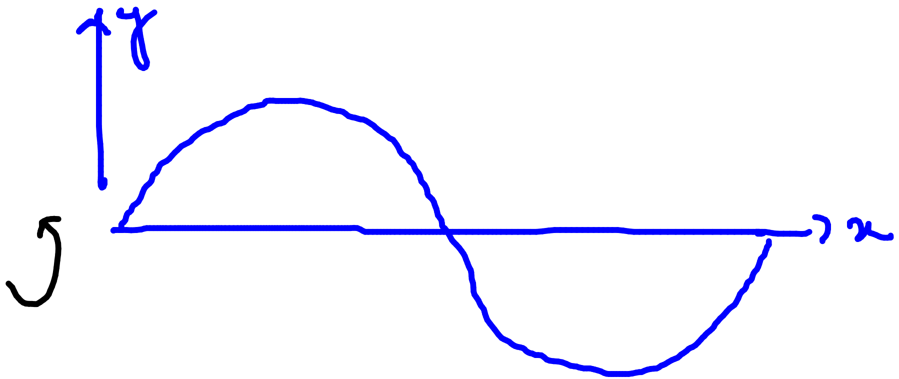
P: a point on the surface formed by revolving the graph of $y = f(x)$ about x axis

$\rightarrow$ x coordinate of P is x

y " " " $f(x)\cos(t)$

z " " " $f(x)\sin(t)$

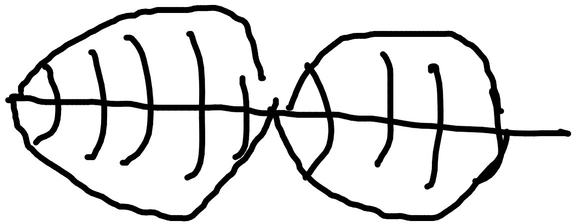
EXAMPLE 7 Find parametric equations for the surface generated by rotating the curve $y = \sin x$, $0 \leqslant x \leqslant 2\pi$, about the x-axis. Use these equations to graph the surface of revolution.

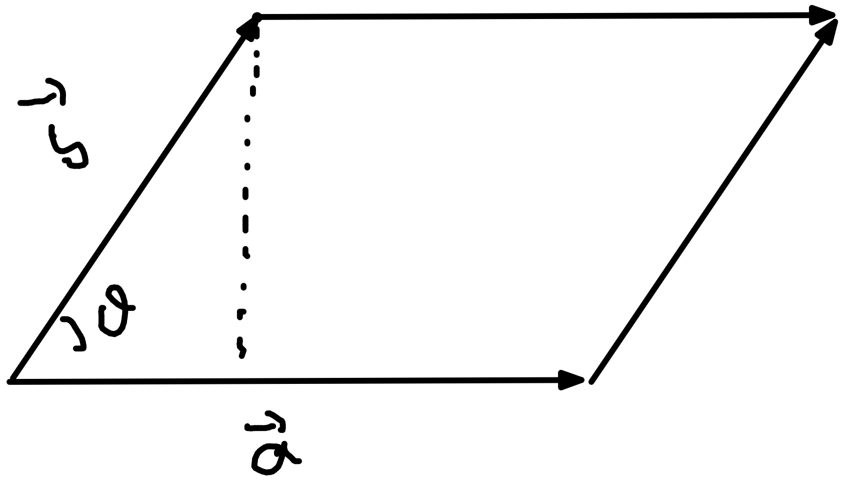


$$x = x$$

$$y = \sin(x)\cos(t)$$

$$z = \sin(x)\sin(t)$$



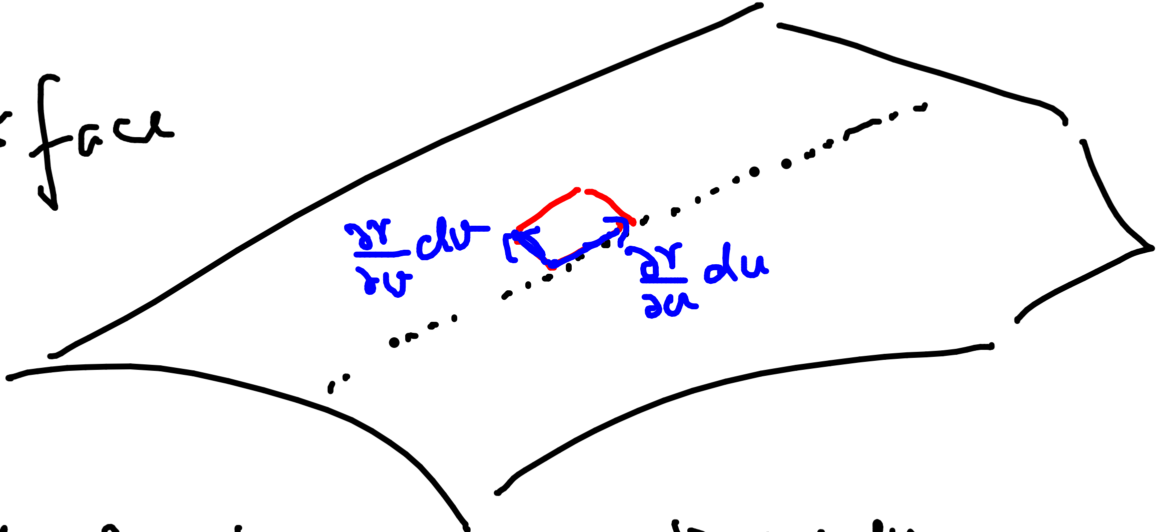


$$\text{area} = |\vec{a} \times \vec{b}|$$

$$= |a|\ |b| \sin\theta$$

Area of parametric surfaces?

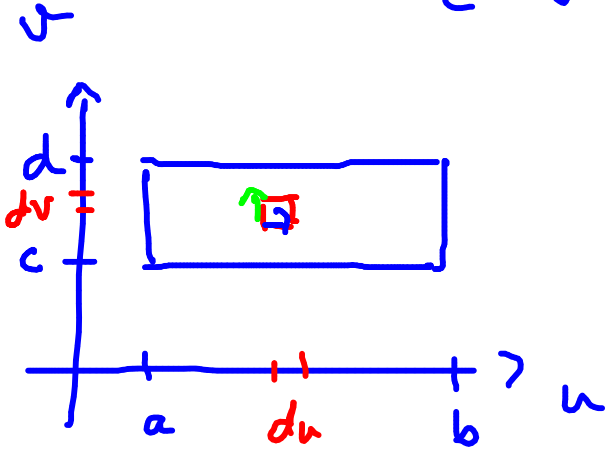
Suppose we have a surface



$$r(u,v) = x\hat{i} + y\hat{j} + z\hat{k}$$

$$a \le u \le b$$

$$c \le v \le d$$



→ the effect of change in u to u + du

$$\frac{\partial r}{\partial u} du$$

→ the effect of change in v to v + dv

$$\frac{\partial r}{\partial v} dv$$

Area on the surface swiped by changing

u to $u + du$ & v to $v + dv$

$$= \left| \frac{\partial r}{\partial u} du \times \frac{\partial r}{\partial v} dv \right|$$

$$\boxed{dA = \left| \frac{\partial r}{\partial u} \times \frac{\partial r}{\partial v} \right| du \, dv}$$

$$A = \iint dA = \int_c^d \int_a^b \left| \frac{\partial r}{\partial u} \times \frac{\partial r}{\partial v} \right| du \, dv$$