ERWIN KREYSZIG

ADVANCED ENGINEERING MATHEMATICS

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GUIDES AND MANUALS
Maple Computer Guide
Mathematica Computer Guide

Student Solutions Manual and Study Guide

Instructor's Manual

Same topics next sem

PART A Ordinary Differential Equations (ODEs)

Whats ODE ?

Ordinary Differential Equation.

Idea: we know something about $\frac{df}{dx}$ or $\frac{d^2f}{dx^2}$

& we wish to find out about f.

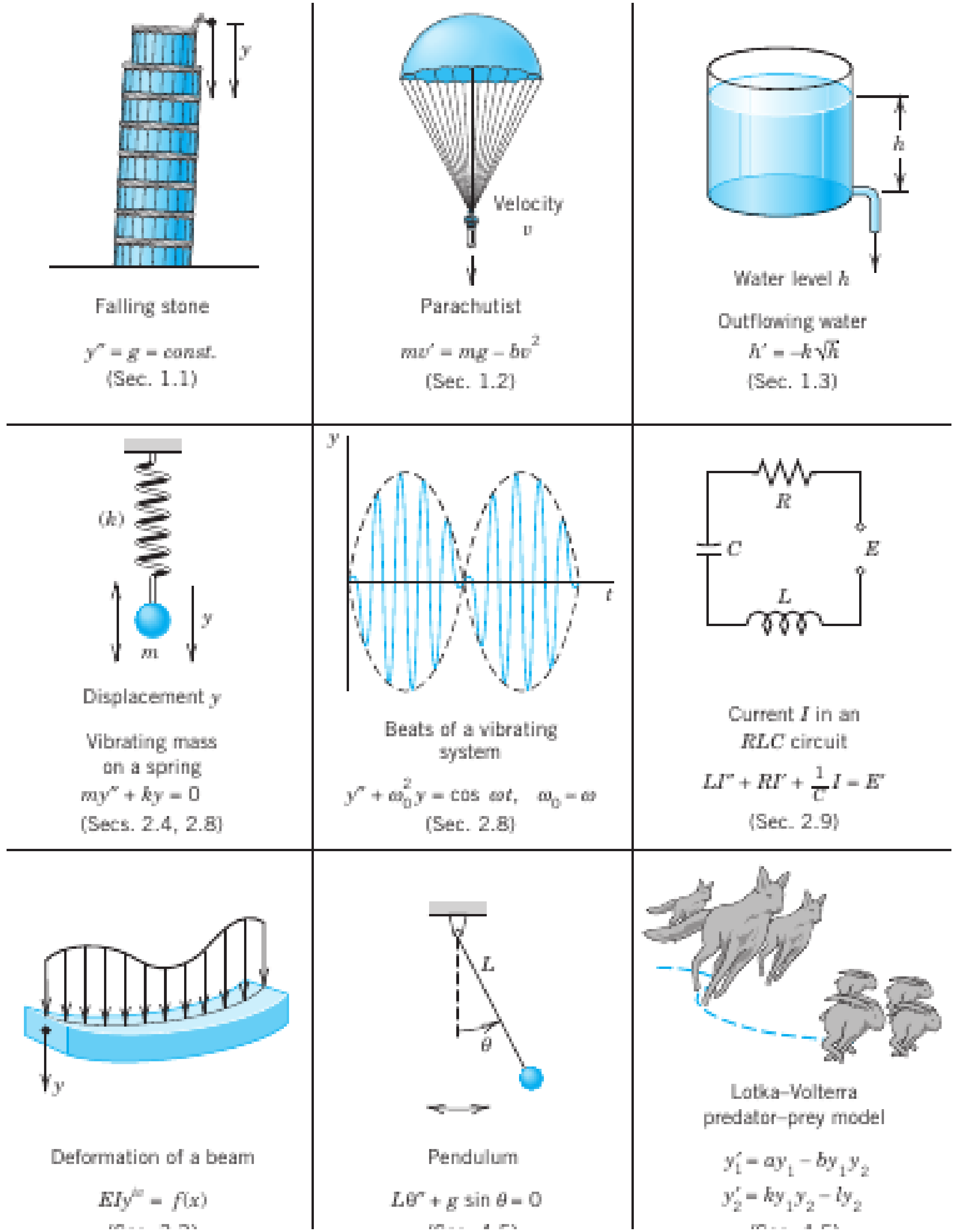
$\frac{df}{dx} = 2$, find $f(x)$

$\frac{df}{dx} + f = 5$

$\frac{d^2f}{dx^2} + \frac{df}{dx} + f = e^x$

find $f(x)$

Why care about knowing to solve ODEs:



H.W. → Just read section 1.1 & 1.2.

→ Don't solve exercise problems.

mathematical modelling: converting real world senarios into mathematical equations.

↳ in particular into ODE eqns.

Q. Whats ODE ??

solving for a function $f(x)$ from an equation where $f'(x)$, $f''(x)$, etc comes into picture

1.3 Separable ODEs

1st order ODEs

& separable

$$\left[\frac{dy}{dx} + y^2 = \sin(x) \right.$$, solve for y

$$\left[\cancel{\frac{d^2y}{dx^2} + y^2 = \sin(x)} \right]$$ 2nd order ODE

$$\left. \frac{dy}{dx} + y = \frac{d^2}{dx^2}(e^x) \right]$$ 1st order ODE

8.

$$y' = (x+1)e^{-x}y^2$$

Use separation of variables

→ try to move all x in one side

& all y in other side

& integrate

$$\frac{dy}{dx} = (1+x)e^{-x}y^2$$

$$\frac{1}{y^2}dy = (1+x)e^{-x}\,dx$$

$$\int \frac{1}{y^2}\,dy = \int (1+x)e^{-x}\,dx$$

$$-\frac{1}{y} = -(x+2)e^{-x} + c \,,$$

c is an arbitrary constant

&

$$\boxed{y = \frac{1}{(x+2)e^{-x} + c}}$$

d.

Solve $y' = -2xy$, $y(0) = 1.8$.

Solve using separation of variables

initial condition

use the extra condition to eliminate the arbitrary constant C

$$\frac{dy}{dx} = -2xy$$

$$\frac{1}{y}\,dy = -2x\,dx$$

$$\ln y = -x^2 + C$$

$y(0) = 1.8$

$$\ln 1.8 = -0^2 + C$$

$$C = \ln 1.8$$

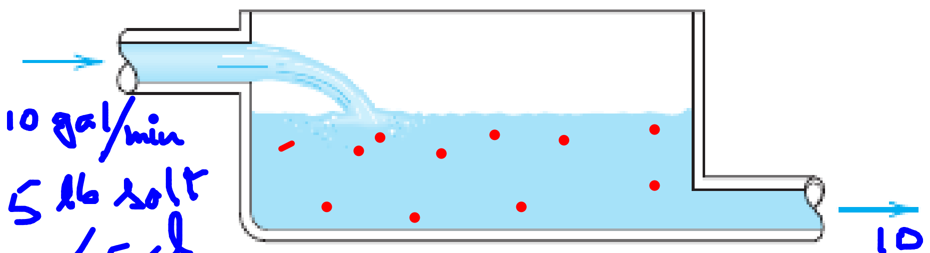
$$\ln y = -x^2 + \ln 1.8$$

$$\ln\left(\frac{y}{1.8}\right) = -x^2$$

$$\frac{y}{1.8} = e^{-x^2}$$

$$\boxed{y = 1.8e^{-x^2}}$$

EXAMPLE 5 Mixing Problem



Mixing problems occur quite frequently in chemical industry. We explain here how to solve the basic model involving a single tank. The tank in Fig. 11 contains 1000 gal of water in which initially 100 lb of salt is dissolved. Brine runs in at a rate of 10 gal/min, and each gallon contains 5 lb of dissoved salt. The mixture in the tank is kept uniform by stirring. Brine runs out at 10 gal/min. Find the amount of salt in the tank at any time t.

Brine ⇔ salt + water

$y(t)$: amount of salt in the tank at time t

initial salt density ??

0.1 lb/gal

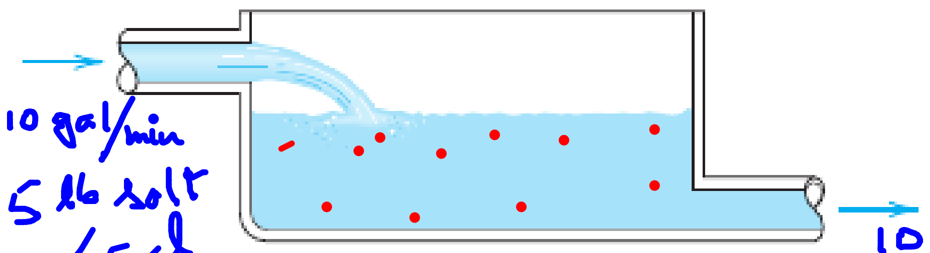
$y(t) = ??$

$y(0) = 100$

$\lim_{t \to \infty} y(t) =$ guess $= 5000$ lb

EXAMPLE 5 Mixing Problem

Brine



10 gal/min

$\Rightarrow$ 5 lb salt / gal

10 gal/min

Mixing problems occur quite frequently in chemical industry. We explain here how to solve the basic model involving a single tank. The tank in Fig. 11 contains 1000 gal of water in which initially 100 lb of salt is dissolved. Brine runs in at a rate of 10 gal/min, and each gallon contains 5 lb of dissoved salt. The mixture in the tank is kept uniform by stirring. Brine runs out at 10 gal/min. Find the amount of salt in the tank at any time t.

Brine $\Leftrightarrow$ salt + water

$y(t)$: amount of salt in the tank at time t

$y(t) = ??$

$y(0) = 100$

initial salt density ??

0.1 lb/gal

$$\frac{dy}{dt} = \text{rate of change of salt mass}$$

$$= \text{inflow rate} - \text{outflow rate}$$

$$= 50 - \frac{10}{1000} y$$

Solve y from

$$\frac{dy}{dt} = 50 - \frac{y}{100}, \quad y(0) = 100$$

Solve $\frac{dy}{dt} = 50 - \frac{y}{100}$ using separation of variables.

$$\frac{dy}{dt} = \frac{5000 - y}{100}$$

$$\frac{1}{5000-y}\,dy = \frac{1}{100}\,dt$$

$$\int \frac{1}{5000-y}\,dy = \int \frac{1}{100}\,dt$$

$$-\ln(5000-y) = \frac{t}{100} + C$$

$t = 0$

$y = 100$

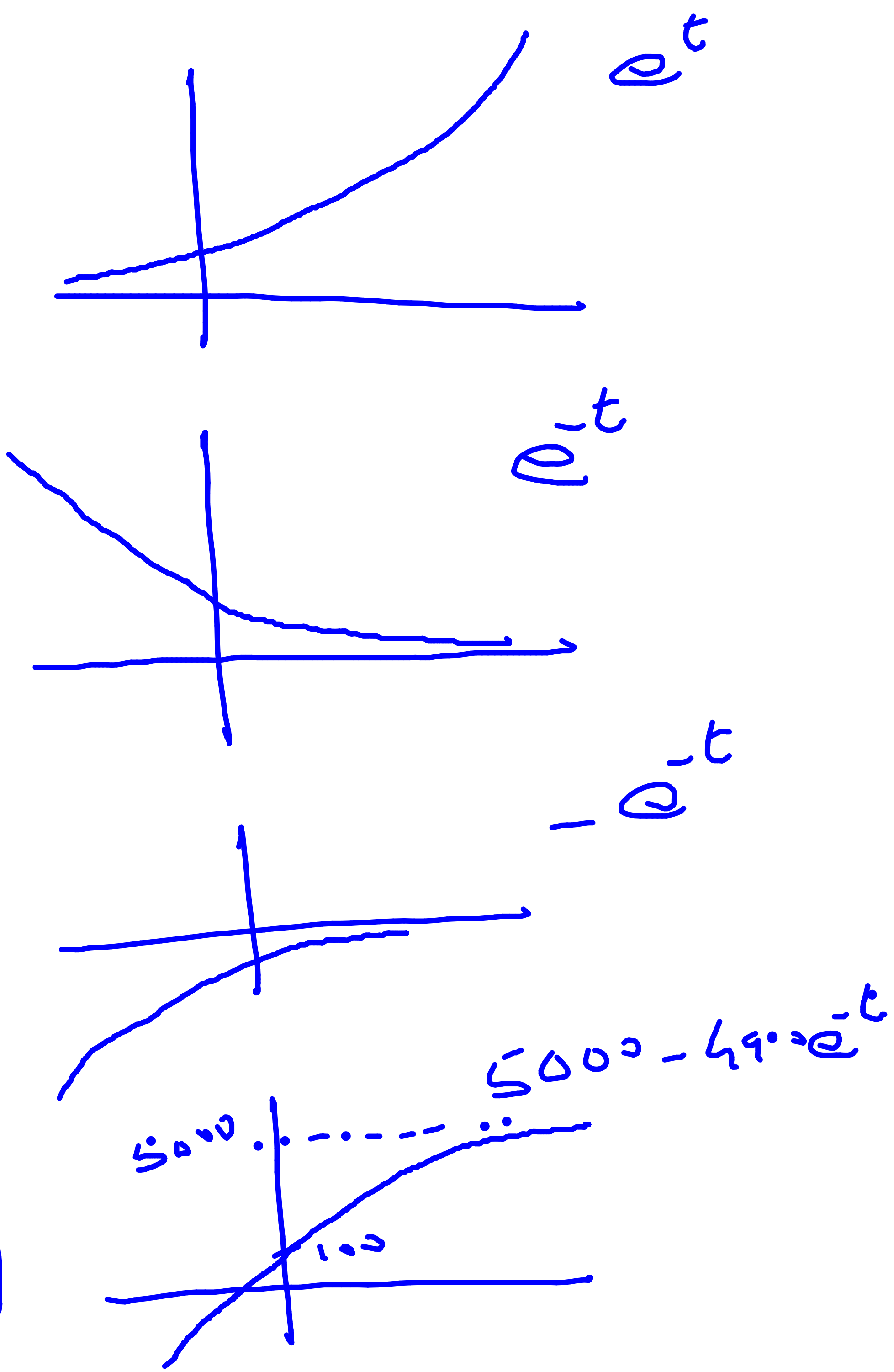
$C = -\ln 4900$

$$\ln\left(\frac{5000-y}{4900}\right) = -\frac{t}{100}$$

$y = ??$

$$5000 - y = 4900\,e^{-t/100}$$

$$\boxed{y = 5000 - 4900\,e^{-t/100}}$$



Recall : what did we start ?? ODE

In differential equations, we solve for a function :

↳ we have some information about derivatives

e.g. $\frac{dx}{dt} = 2$, find $x(t)$.

$2\frac{d^2x}{dt^2} = 7$, find $x(t)$

→ We learnt last time : the first thing we should try when solving an ODE

Variable separable

$$\boxed{x\frac{dy}{dx} = 5x^2y}$$

is this an ODE ??

$$\frac{1}{y}\,dy = \frac{5x^2}{x}\,dx$$

$$\int \frac{1}{y}\,dy = \int 5x\,dx$$

$$\sin\left(\frac{dy}{dx}\right) = x$$

$$\frac{dy}{dx} = \sin^{-1}(x)$$

Extended Method: Reduction to Separable Form

$$\boxed{y' = f\left(\frac{y}{x}\right)} \quad [\qquad]$$

$$\boxed{y/x = v}$$

$$2xyy' = y^2 - x^2.$$

$$2xy\frac{dy}{dx} = y^2 - x^2$$

try to move x & y in two sides

$\downarrow$??

$$\left(\begin{matrix}\text{only}\\ y \text{ terms}\end{matrix}\right) dy = \left(\begin{matrix}\text{only}\\ x \text{ terms}\end{matrix}\right) dx$$

$$\boxed{v = \frac{y}{x}}$$

& eliminate y

$$2\frac{dy}{dx} = \frac{y^2 - x^2}{xy}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\left(\frac{y}{x}\right) - \left(\frac{x}{y}\right)\right]$$

$$\frac{dy}{dx} = \frac{1}{2}\left[v - \frac{1}{v}\right]$$

$$y = vx$$

$$\frac{dy}{dx} = \frac{d}{dx}(vx)$$

$$= v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{1}{2}\left[v - \frac{1}{v}\right]$$

$\hookrightarrow$ is this separable ??

$$x\frac{dv}{dx} = -\frac{1}{2}\left[\frac{v^2+1}{v}\right]$$

$$\frac{2v}{v^2+1}\,dv = -\frac{dx}{x}$$

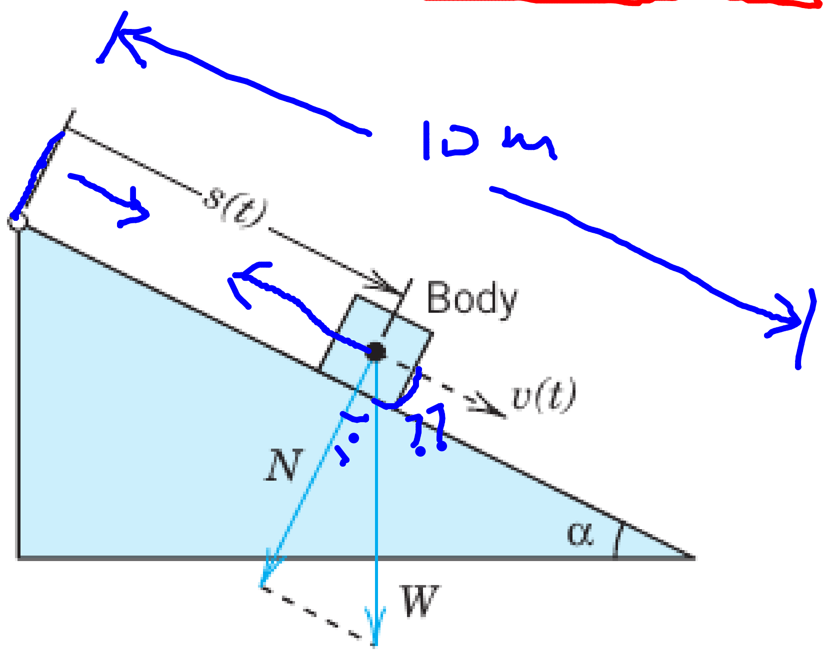
$$\rightarrow \ln(v^2+1) = -\ln x + \ln c$$

$$v^2+1 = \frac{c}{x}$$

$$\boxed{\frac{y^2}{x^2} + 1 = \frac{c}{x}}$$

y is given implicitly by this eq^n

32. Friction. If a body slides on a surface, it experiences friction F (a force against the direction of motion). Experiments show that $|F| = \mu|N|$ (*Coulomb's*[6] *law of kinetic friction without lubrication*), where N is the normal force (force that holds the two surfaces together; see Fig. 15) and the constant of proportionality μ is called the *coefficient of kinetic friction*. In Fig. 15 assume that the body weighs 45 nt (about 10 lb; see front cover for conversion). $\mu = 0.20$ (corresponding to steel on steel), $a = 30°$, the slide is 10 m long, the initial velocity is zero and air resistance is negligible. Find the velocity of the body at the end of the slide.



$W = 45\ N$

$\mu = 0.2$

$\alpha = 30°$

$s(t)$: distance from the peak of the slide

→ we use $F = ma$ to get an eqn for $s(t)$

$$m\frac{d^2s}{dt^2} = W\cos(60°) - \mu W\cos(30°)$$

$$\frac{\cancel{W}}{g}\frac{d^2s}{dt^2} = \cancel{W}\cos(60°) - \mu \cancel{W}\cos(30°)$$

$$\frac{d^2s}{dt^2} = \underbrace{g\left[\frac{1}{2} - 0.2\frac{\sqrt{3}}{2}\right]}_{}$$

$$= A \approx 3.202$$

$$\frac{d^2s}{dt^2} = A, \qquad s(0) = 0, \quad s'(0) = 0$$

Q: what $\frac{ds}{dt}$ when $s = 10$??] can you complete this ??

$$\frac{d^2s}{dt^2} = A$$

$$\frac{ds}{dt} = At + C \quad , \quad C = ?? \quad \left[\frac{ds}{dt}(0) = 0, \; C = 0 \right]$$

$$v(t) = \boxed{\frac{ds}{dt} = At}$$

&

$$s(t) = \frac{At^2}{2} + D \qquad D = ?? \quad \left[s(0) = 0, \; D = 0 \right]$$

$$\boxed{s(t) = At^2/2}$$

$$\frac{At^2}{2} = 10$$

$$t = \sqrt{\frac{20}{A}}$$

$$v\left(\sqrt{20/A}\right) = A\sqrt{\frac{20}{A}} = \sqrt{20A}$$

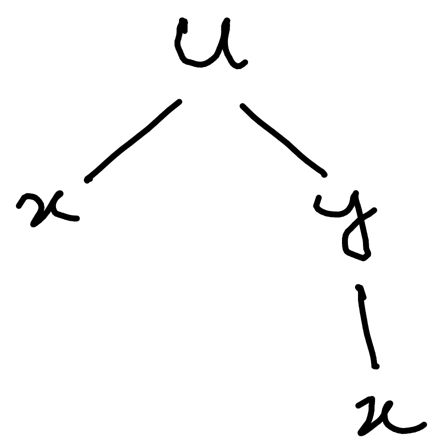
1.4 Exact ODEs. Integrating Factors

$$M(x,y) + N(x,y)\frac{dy}{dx} = 0 \quad \text{—— } (1)$$

lets hope that there exist an eqution

$$u(x,y) = c \quad \text{— } (2)$$

from which we get eq (1) by differentiation



$$\frac{du}{dx} = \underbrace{\frac{\partial u}{\partial x}}_{M} + \underbrace{\frac{\partial u}{\partial y}}_{N}\frac{dy}{dx}$$

find a formula $u(x,y)$ s.t. $\frac{du}{dx} = M + N\frac{dy}{dx}$

this helps because we will solve for y from

$$u(x,y) = C$$

Objective: Solve for y from eqn of the form

$$M + N\frac{dy}{dx} = 0$$

Plan: find a formula $u(x,y)$ s.t.

$$\frac{du}{dx} = M + N\frac{dy}{dx}$$

When can we do this??

$\rightarrow$ ODE: $\quad \frac{du}{dx} = 0$

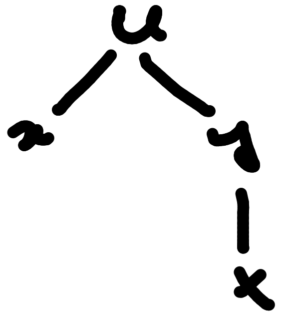
→ $u(x,y) = C$

→ solve for y from $u(x,y) = C$

$$M + N\frac{dy}{dx} = 0$$

1 **Aim:** find $u(x,y)$ s.t.

$$\frac{du}{dx} = M + N\frac{dy}{dx}$$



$$\frac{du}{dx} = \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\frac{dy}{dx}$$

$$M = \frac{\partial u}{\partial x} \qquad N = \frac{\partial u}{\partial y}$$

if $M + N\frac{dy}{dx} = \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\frac{dy}{dx}$ for some $u(x,y)$

or: if there exist $u(x,y)$ s.t.

$$\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x} = M\right) \quad \& \quad \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y} = N\right)$$

$$\frac{\partial^2 u}{\partial x \partial y} \stackrel{?}{=} \frac{\partial^2 u}{\partial y \partial x}$$

→ Check for existence of such $u(x,y)$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$M + N\frac{dy}{dx} = 0$$

$$\Leftrightarrow \quad M\,dx + N\,dy = 0$$

d. $\underbrace{\cos(x+y)}_{M}\,dx + \underbrace{(3y^2+2y+\cos(x+y))}_{N}\,dy = 0$

→ is it exact ??

→ $M\,dx + N\,dy = 0$ is exact $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$$\frac{\partial M}{\partial y} = -\sin(x+y) = \frac{\partial N}{\partial x}$$ exact

→ there exist $u(x,y)$ s.t. $\frac{\partial u}{\partial x} = M$ & $\frac{\partial u}{\partial y} = N$

→ now find this $u(x,y)$

$$\frac{\partial u}{\partial x} = \cos(x+y)$$

$$u = \underbrace{\sin(x+y) + g(y)}$$

[where $g(y)$ is to be found]

$$\rightarrow \quad \frac{\partial u}{\partial y} = M$$

$$\cancel{\cos(x+y)} + \frac{dg}{dy} = 3y^2 + 2y + \cancel{\cos(x+y)}$$

$$\frac{dg}{dy} = 3y^2 + 2y$$

$$g(y) = y^3 + y^2 + c$$

→ finally: $u(x,y) = \sin(x+y) + y^3 + y^2 + c$

→ solve for y from:

$$u(x,y) = \text{constant}$$

→ $$\boxed{\sin(x+y) + y^3 + y^2 = C}$$

↳ y is given implicitly by this eq^n.

Q:

(cos y sinh x + 1) dx − sin y cosh x dy = 0, $y(1) = 2$.

$$\underbrace{(\cos y \sinh x + 1)}_{M}\, dx - \sin y \cosh x \, dy = 0$$

check for exactness.

$\cosh x = \frac{e^x + e^{-x}}{2}$

$\sinh x = \frac{e^x - e^{-x}}{2}$

$\frac{d}{dx}(\cosh x) = \sinh x$

$\frac{d}{dx}(\sinh x) = \cosh x$

$\frac{\partial M}{\partial y} = -\sin y \sinh x = \frac{\partial N}{\partial x}$ exact

→ find u s.t. $\frac{\partial u}{\partial x} = M$, $\frac{\partial u}{\partial y} = N$

$$\frac{\partial u}{\partial x} = \cos y \ \sinh x \ + 1$$

$$u = \cos y \ \cosh x \ + x \ + \ g(y)$$

$$\frac{\partial u}{\partial y} = -\cancel{\sin y \ \cosh x} \ + \frac{dg}{dy} \ = \ \underbrace{-\cancel{\sin y \ \cosh x}}_{N}$$

$$\frac{dg}{dy} = 0 \quad \Rightarrow \quad g = c$$

$$\Rightarrow u(x,y) = \cos y \ \cosh x \ + x \ + c$$

$-$) y satisfies $u = c$

$$\boxed{\cos y \ \cosh x \ + x = C}$$

$$\boxed{\cos(2) \cosh(1) + 1 = C}$$

find C by $y(1) = 2$

Reduction to Exact Form. Integrating Factors

Next time

$$-y\,dx + x\,dy = 0.$$

$$(e^{x+y} + ye^{y})\,dx + (xe^{y} - 1)\,dy = 0$$