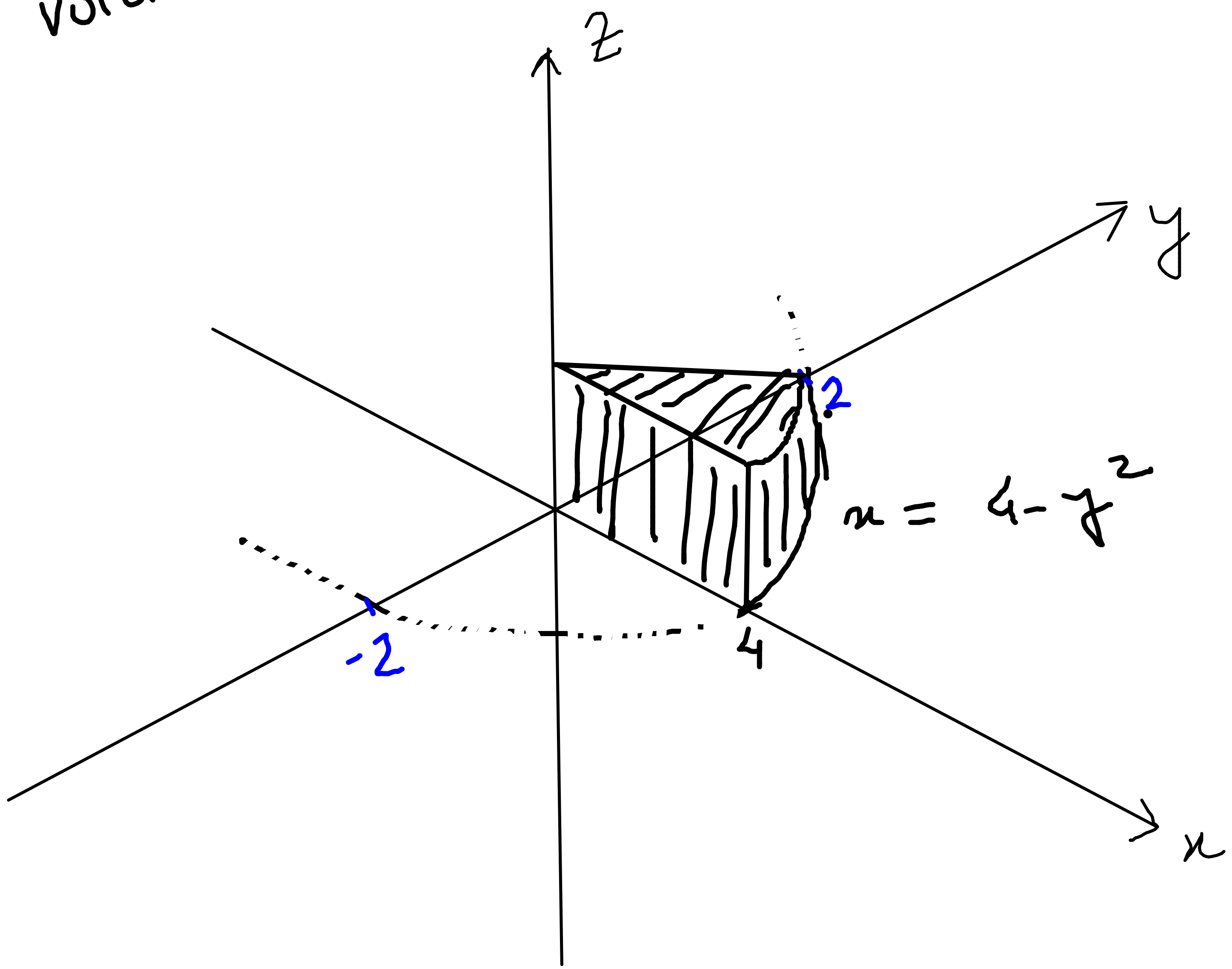
Sketch the solid whose volume is given by the iterated integral.

volume $= \int_0^2 \int_0^{2-y} \int_0^{4-y^2} 1 \, dx \, dz \, dy$



$z = 2 - y$

$z + y = 2$

$= ??$

symbolab.com/solver/triple-integrals-calculator/%5Cint_%7B0%7D%5E%7B2%7D%5Cint_%7B0%7D%5E%7B2-y%7D%5C

Indefinite Integrals
Definite Integrals
Specific-Method
Improper Integrals
Antiderivatives
Double Integrals
Triple Integrals
Multiple Integrals

Integral Applications

Riemann Sum (new)

Series

ODE

Multivariable Calculus (new)

Laplace Transform

Taylor/Maclaurin Series

Fourier Series

$\int_0^2\int_0^{2-y}\int_0^{4-y^2} dxdzdy$

Go

Examples »

Solution

Keep Practicing >

Show Steps

$\int_0^2\int_0^{2-y}\int_0^{4-y^2} 1dxdzdy = \frac{20}{3}$ (Decimal: $6.66666\ldots$)

Steps

$\int_0^2\int_0^{2-y}\int_0^{4-y^2} 1dxdzdy$

$\int_0^{4-y^2} 1dx = -y^2+4$ *Show Steps*

$= \int_0^2\int_0^{2-y}\left(-y^2+4\right)dzdy$

$\int_0^{2-y}\left(-y^2+4\right)dz = (-y+2)\left(-y^2+4\right)$ *Show Steps*

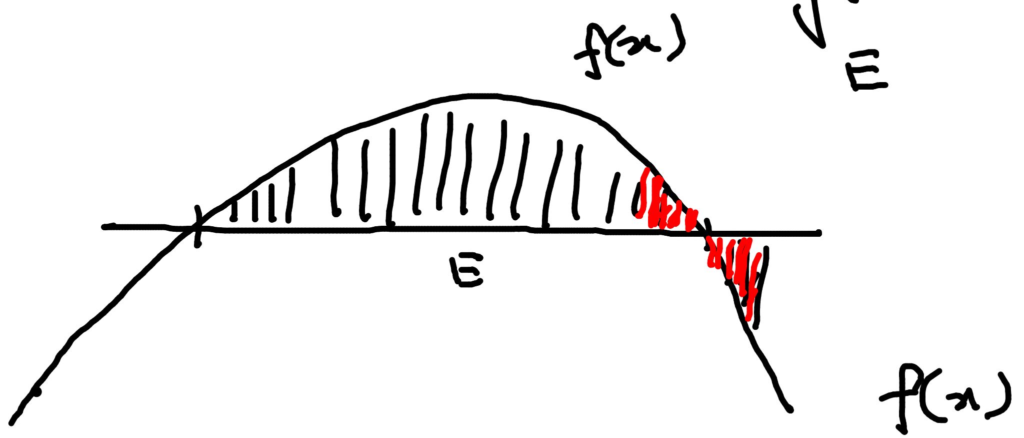
$= \int_0^2(-y+2)\left(-y^2+4\right)dy$

$= \frac{20}{3}$

47. Find the region E for which the triple integral

$$\iiint_E (1 - x^2 - 2y^2 - 3z^2)\, dV$$

is a maximum.



$\int_E f\,dx$

find E s.t.

$\int_E f\,du$ is as large as possible.

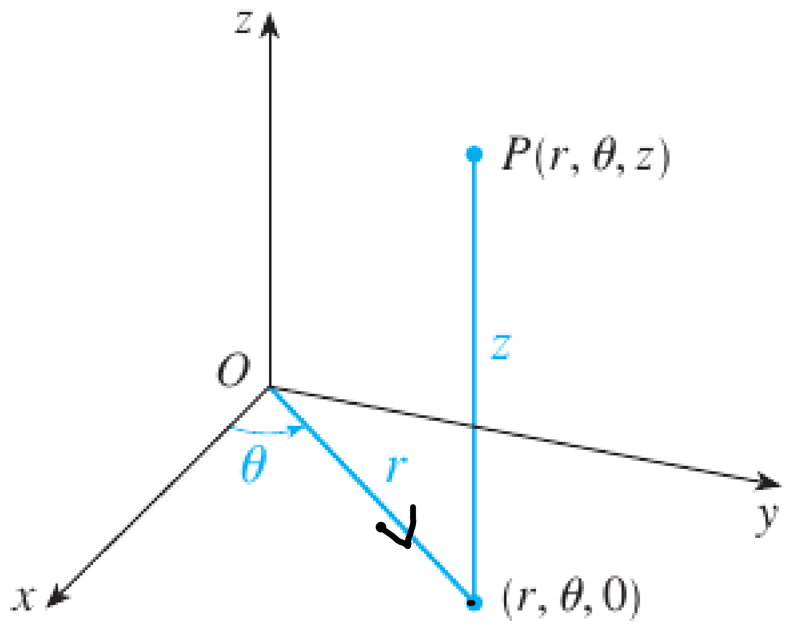
E:

$$1 - x^2 - 2y^2 - 3z^2 \geq 0$$

$$x^2 + 2y^2 + 3z^2 \leq 1$$

$$x^2 + \frac{y^2}{(1/\sqrt{2})^2} + \frac{z^2}{(1/\sqrt{3})^2} \leq 1$$

12.6 TRIPLE INTEGRALS IN CYLINDRICAL COORDINATES



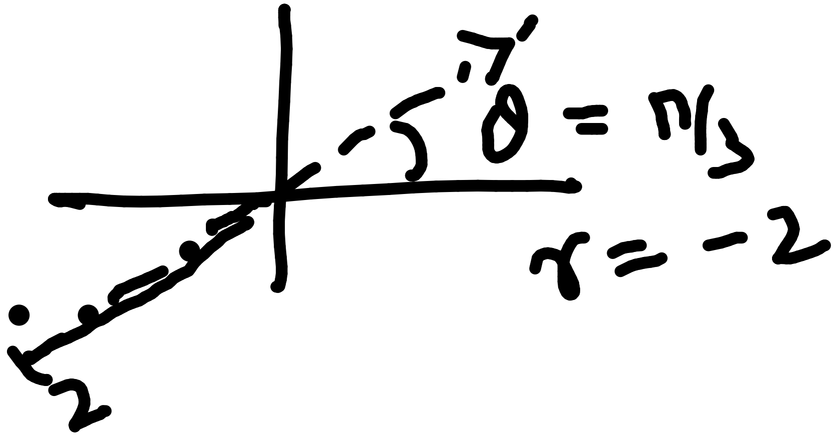
$$x = r \cos \theta \qquad y = r \sin \theta \qquad z = z$$

EXAMPLE 2 Describe the surface whose equation in cylindrical coordinates is $z = r$.

what surface is this ??

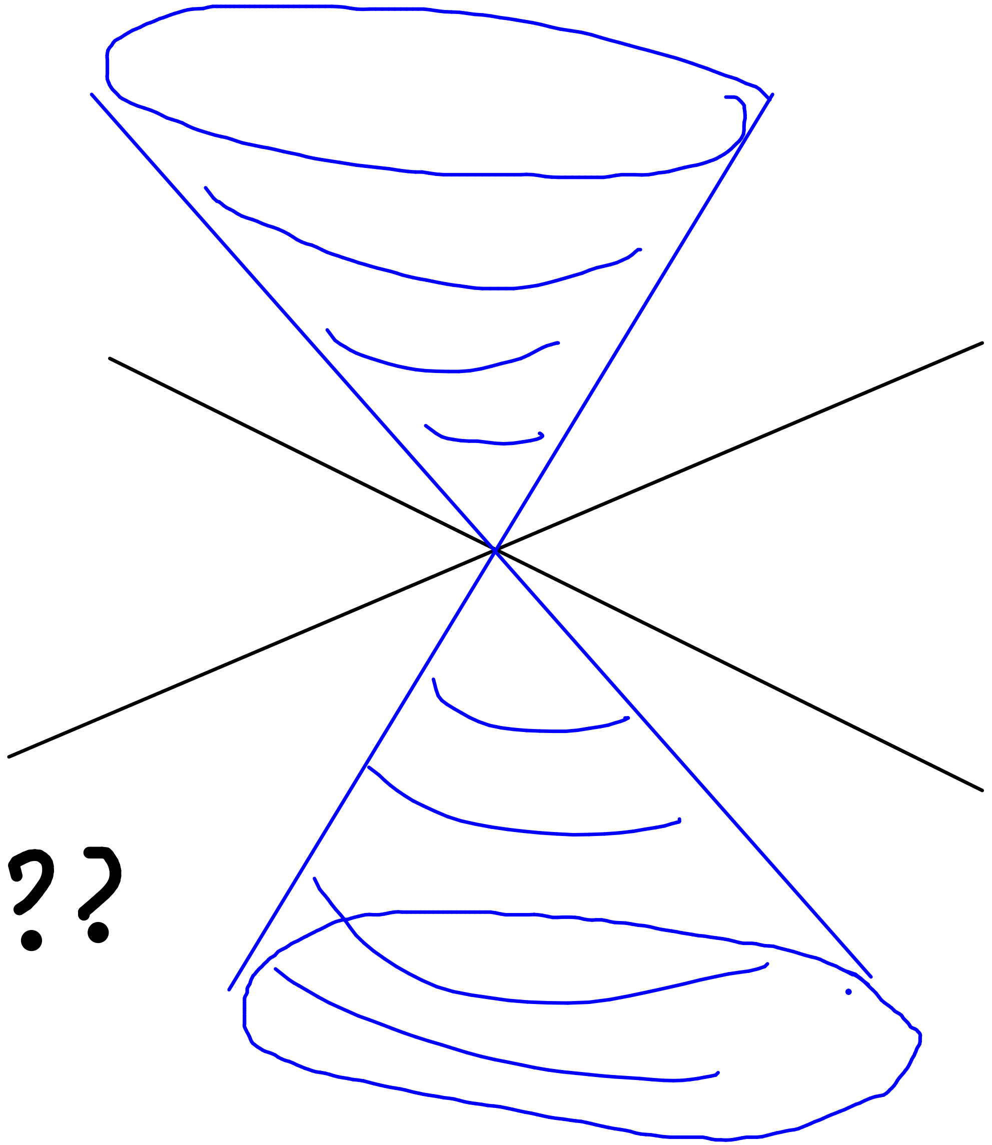
$r = \sqrt{x^2+y^2}$

$z = \pm\sqrt{x^2+y^2}$



$y = 0$

$$z = \sqrt{x^2}$$
$$= |x|$$



why bottom cone as well??

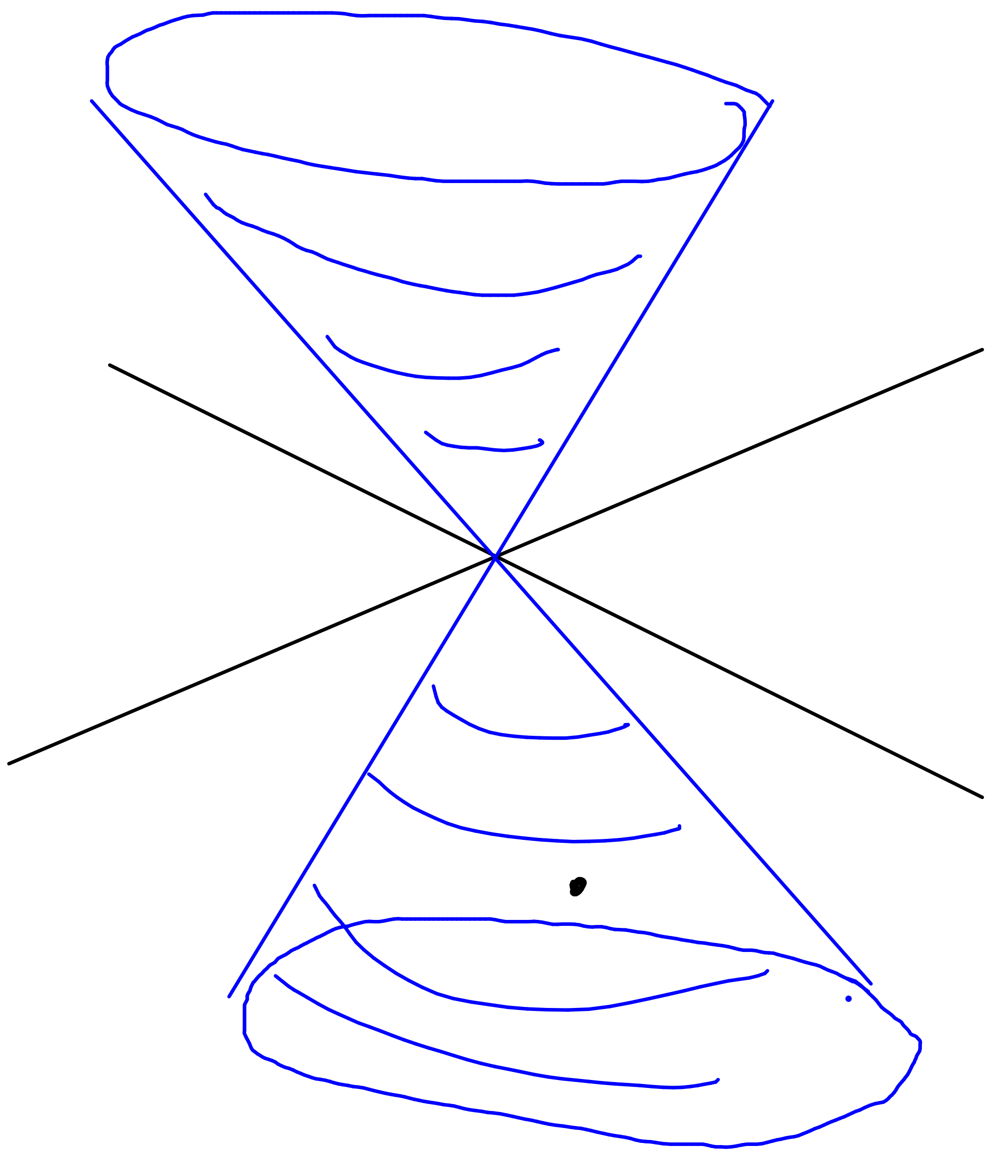
$z = r$

EXAMPLE 2 Describe the surface whose equation in cylindrical coordinates is $z = r$.

$r = \sqrt{x^2+y^2}$

surface ??

collection of all points (r, θ, z) which satisfy the equation $z = r$



Recall :

$$\iint \text{—}\; dx\,dy \longleftrightarrow \iint \text{—}\; du\,dv$$

$$dx\,dy = ?? \; du\,dv$$

Jacobian

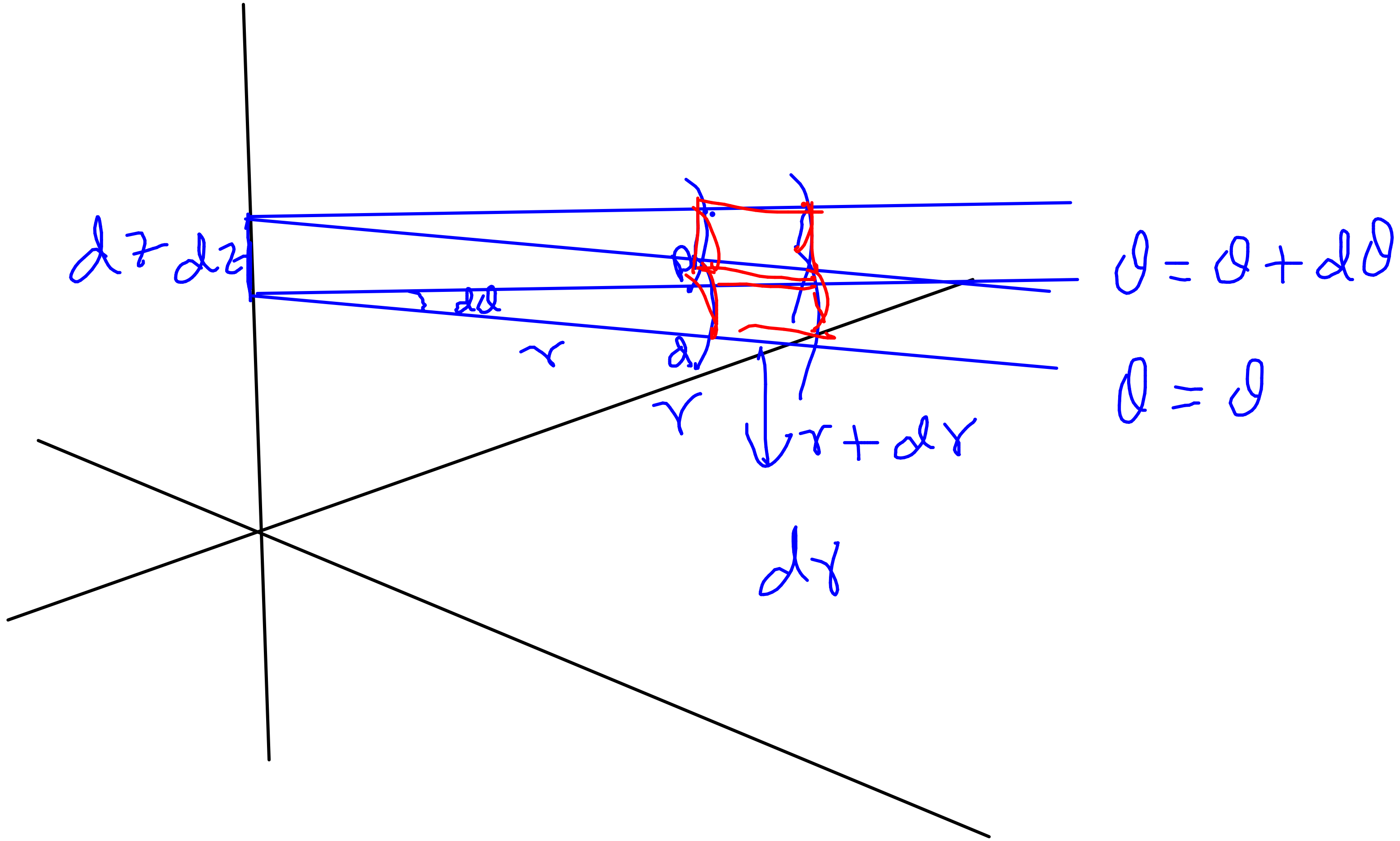
$$dx\,dy\,dz \longrightarrow ???\; dr\,d\theta\,dz$$

Jacobian of switching from (x, y, z) to cylindrical coordinates

$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$z = z$$

$$J = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} & \frac{\partial x}{\partial z} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} & \frac{\partial y}{\partial z} \\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial \theta} & \frac{\partial z}{\partial z} \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\sin\theta & 0 \\ \sin\theta & r\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = r$$

$$\therefore \quad dx\,dy\,dz = r\,dr\,d\theta\,dz$$



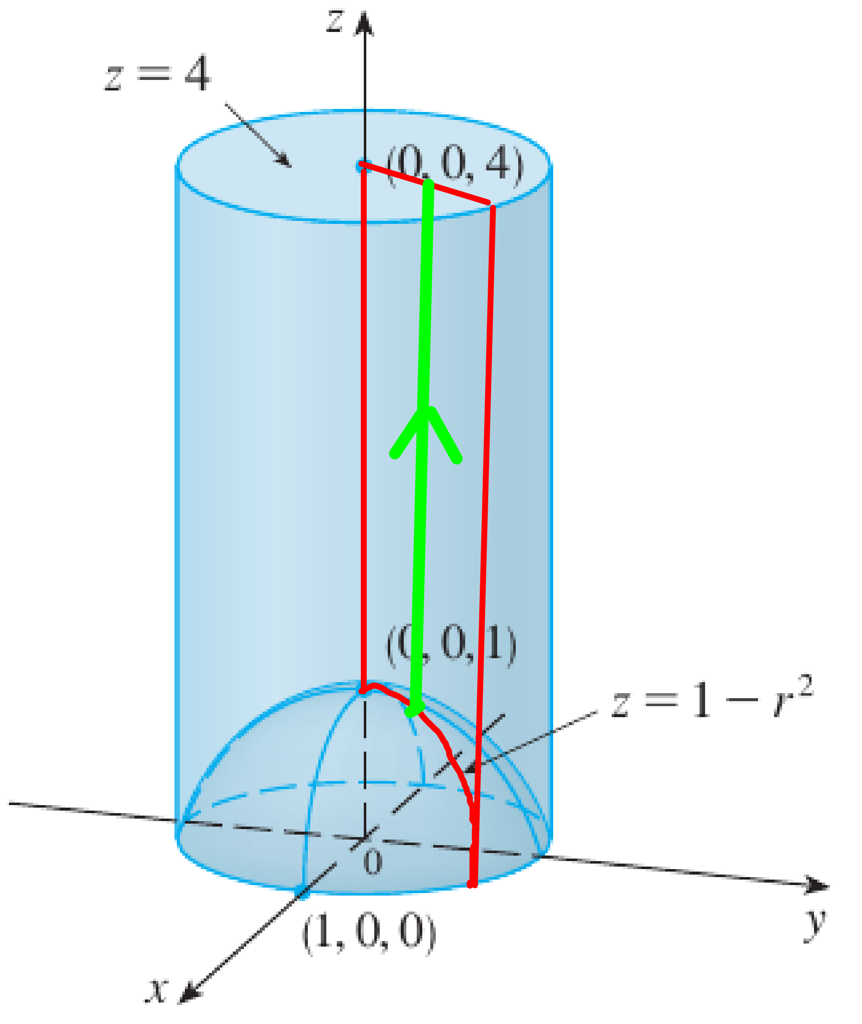
$\widehat{pq} = r\,d\theta$

$r\,dr\,d\theta\,dz = (r\,d\theta)(dr)(dz)$

volume swiped
for small change
$d\theta$, dr, dz

$r^2 = 1 \qquad r = 1$

EXAMPLE 3 A solid E lies within the cylinder $x^2 + y^2 = 1$, below the plane $z = 4$, and above the paraboloid $z = 1 - x^2 - y^2$. (See Figure 8.) The density at any point is proportional to its distance from the axis of the cylinder. Find the mass of E.



K: some constant not given

$$\rho(r,\theta,z) = K r$$

$$z = 1 - x^2 - y^2 = 1 - r^2$$

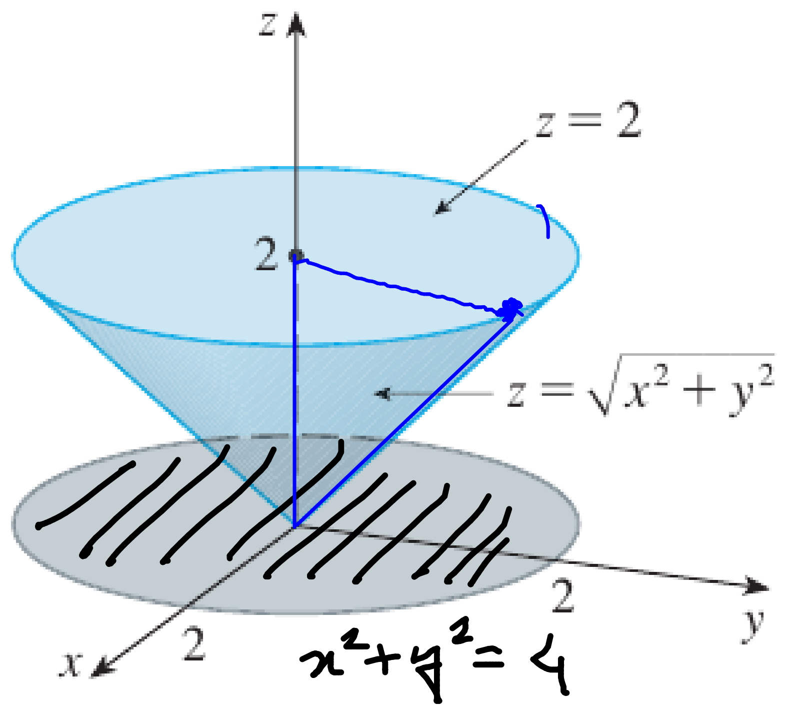
$$\text{mass} = \iiint_E \rho \, dV$$

$$= \int_0^{2\pi} \int_0^1 \int_{1-r^2}^{4} (Kr) \underbrace{r}_{} \, dz\, dr\, d\theta = K \frac{24}{10} \pi$$

EXAMPLE 4 Evaluate $\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{2} (x^2 + y^2)\, dz\, dy\, dx.$

Rewrite this integration in cylindrical coordinates

$$\int_0^{2\pi} \int_0^{2} \int_r^{2} r^2 \; r \; dz\, dr\, d\theta$$



$r^2 = 4$

$r = 2$

5–6 ■ Describe in words the surface whose equation is given.

5. $r = 3$

6. $\theta = \pi/3$

9–10 ▪ Write the equations in cylindrical coordinates.

9. (a) $z = x^2 + y^2$ (b) $x^2 + y^2 = 2y$

Evaluate $\iiint_E (x^3 + xy^2)\, dV$, where E is the solid in the first octant that lies beneath the paraboloid $z = 1 - x^2 - y^2$.