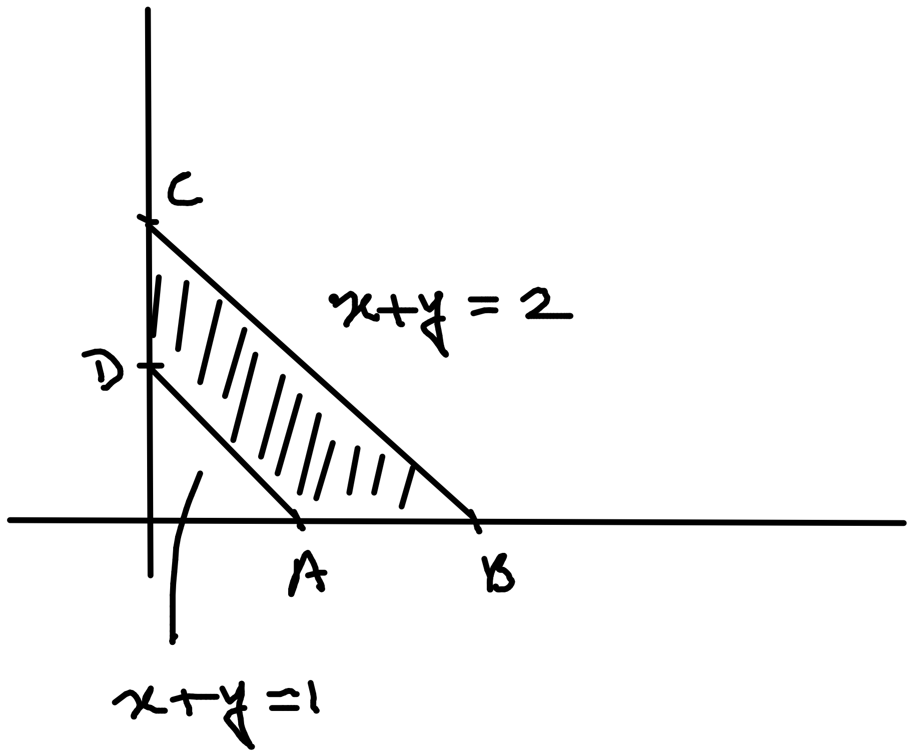
Evaluate the integral by making appropriate change of variables.

$$\iint_R \cos\left(\frac{y-x}{y+x}\right) dA, \text{ where } R \text{ is the trapezoidal region}$$
with vertices (1, 0), (2, 0), (0, 2), and (0, 1)



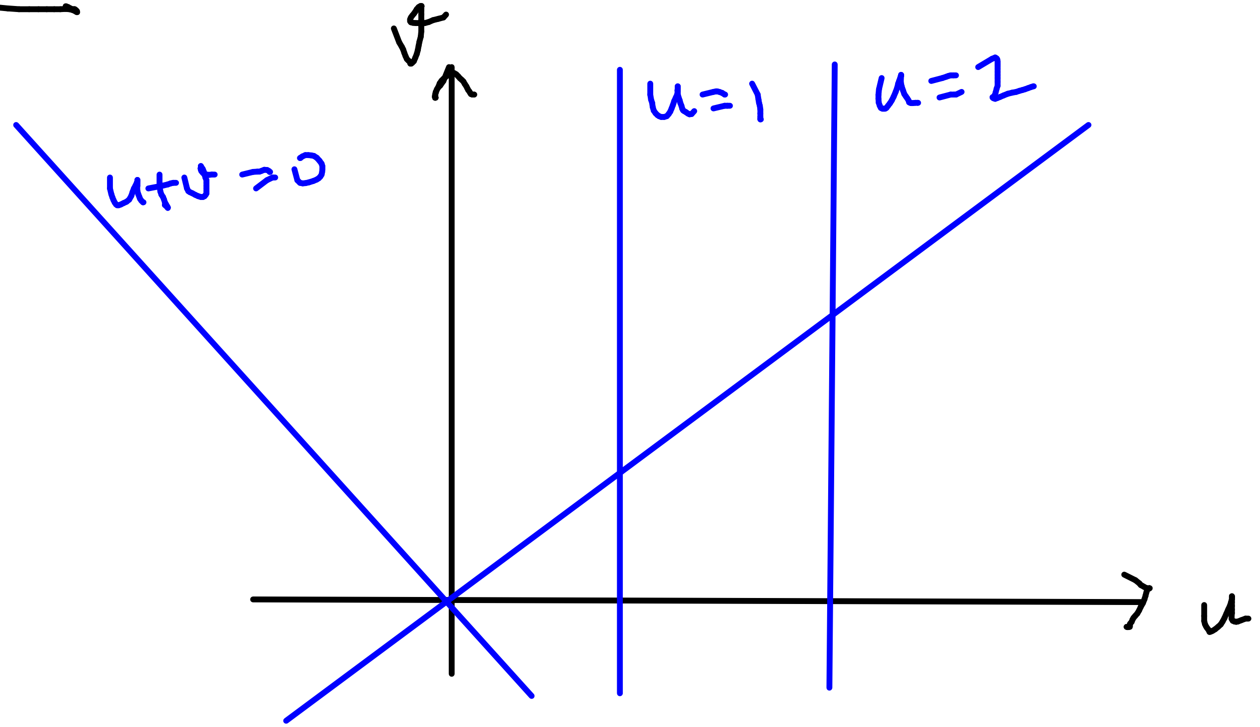
$u = x+y$	$y = (u+v)/2$
$v = y-x$	$x = (u-v)/2$

AD $\quad x+y=1$

$u=1$

BC $\quad x+y=2$

$u=2$



AB: $y = 0$

$(u+v)/2 = 0 \quad | \quad u+v = 0$

DC: $x = 0$

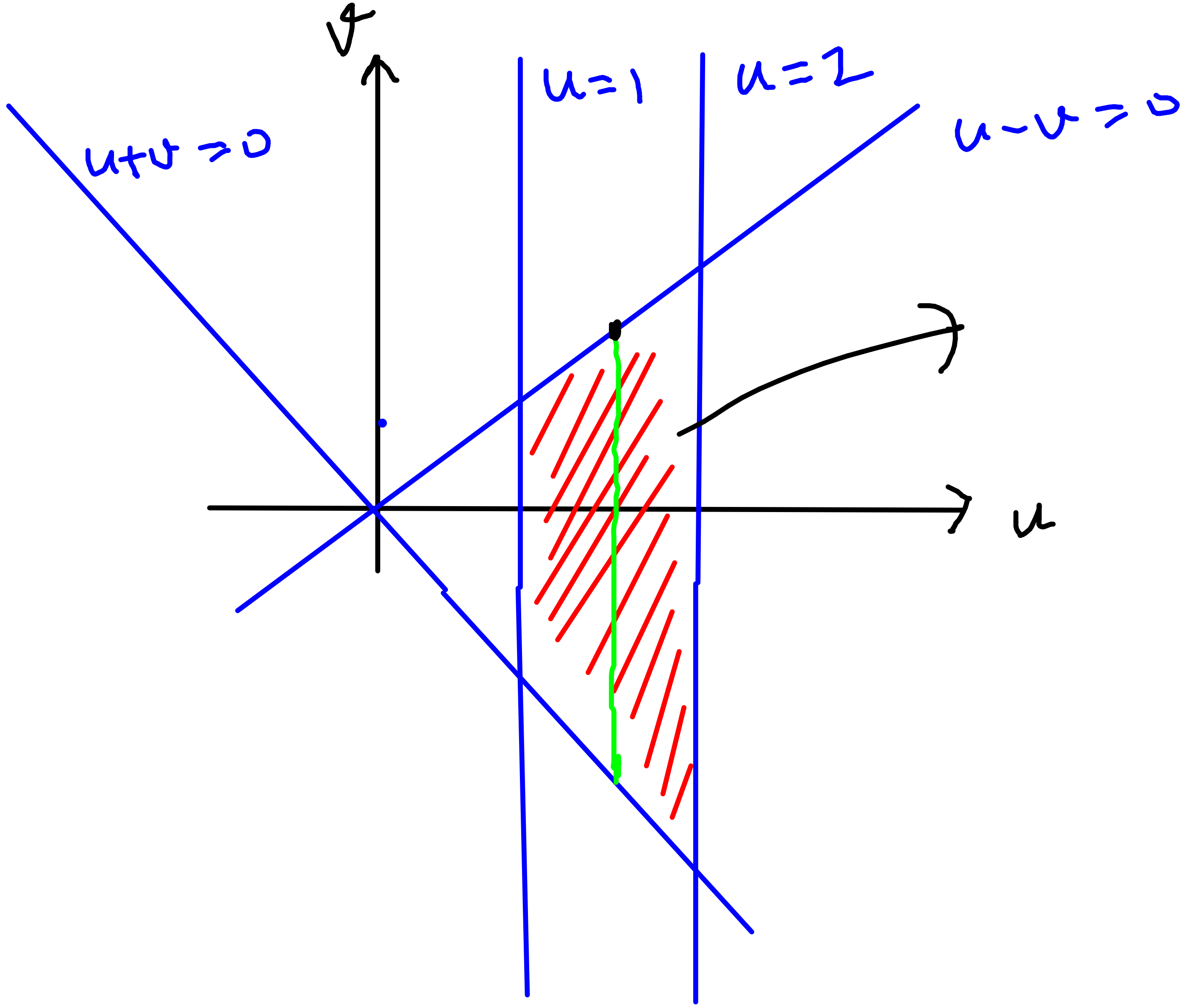
$u - v = 0$

$u = x+y \quad | \quad y = (u+v)/2$

$v = y-x \quad | \quad x = (u-v)/2$

$$J = \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 1/2 & -1/2 \\ 1/2 & 1/2 \end{vmatrix}$$

$$= \frac{1}{2}$$



$$\int_1^2 \int_{-u}^{u} \cos\left(\frac{v}{u}\right) \left(J \right) dv\,du$$

$$= \int_1^2 \int_{-u}^{u} \cos\left(\frac{v}{u}\right) \frac{1}{2} \, dv\,du$$

$$\int_1^2 \int_{-u}^{u} \left(\frac{1}{2}\right)\cos\left(\frac{v}{u}\right)\, dv\, du$$

Go

$$= \sin(1)\ \frac{3}{2}$$