Exam possibility: (kind of assignment)

→ 5-6 questions posted
(slightly non trivial) ex

→ need to make a video of yourself
explaining the solutions
(upload youtube, etc)

→ post the link on BB

→ videos will be peer evaluated

Trial Run:

→ one simple qustion will be given

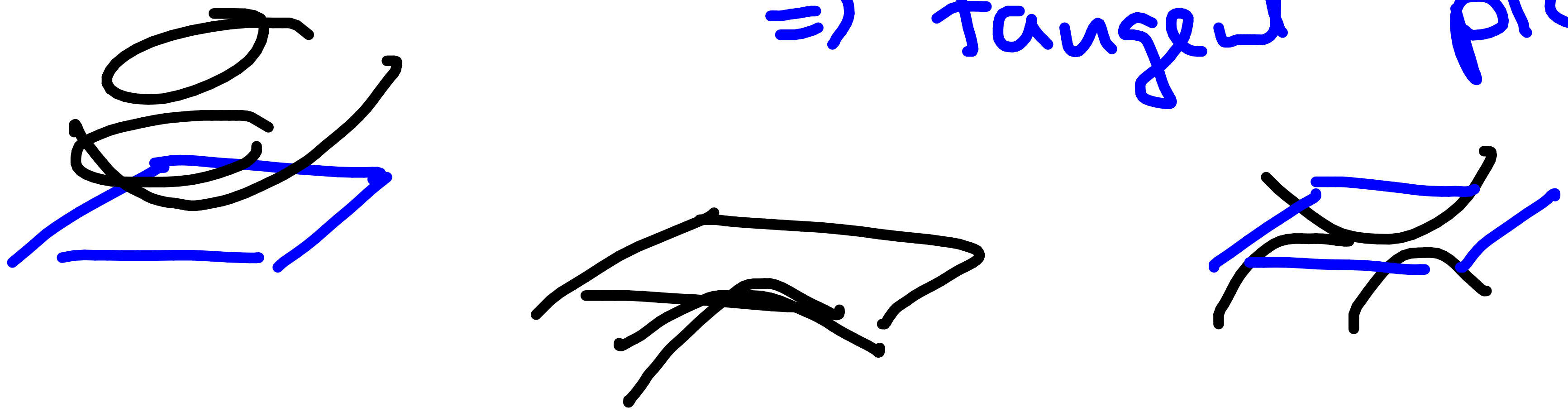
→ I will give demo of making a video of just the phone

Recall 11.7 $f(x,y)$

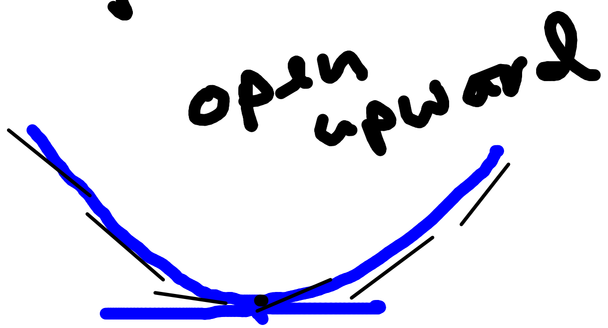
→ max/min of multivariable f's

→ critical point $\frac{\partial f}{\partial x} = 0 \quad \frac{\partial f}{\partial y} = 0$

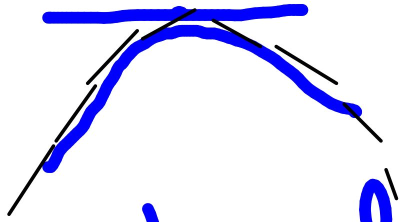
$\Rightarrow$ tangent plane is horizontal

→ 

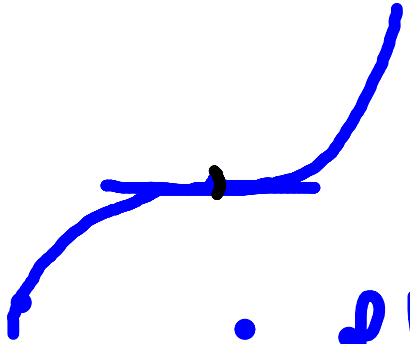
Recall one variable function criteria for classification of critical points



local min



local max



inflection pt.

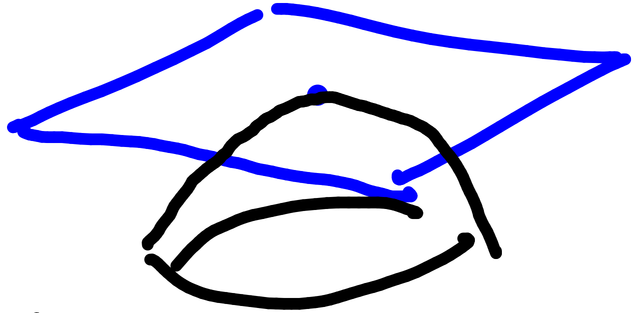
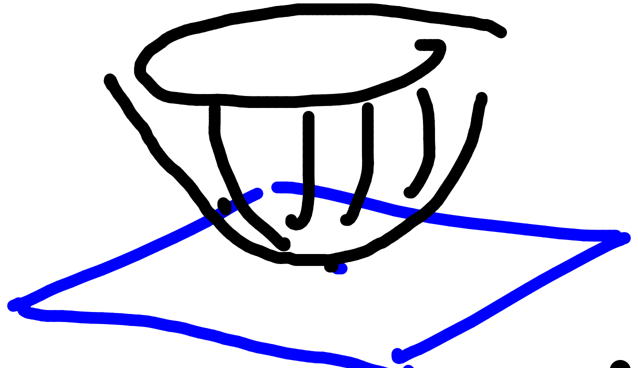
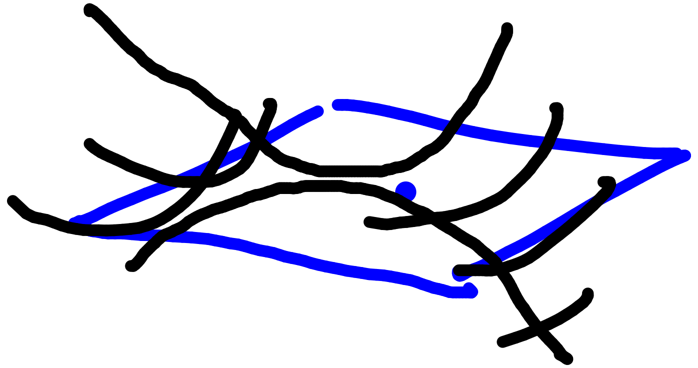
$f'' > 0$ $\iff$ f' is increasing $\iff$ f'' is open upward

$f'' < 0$ $\iff$ f' is decreasing $\iff$ f is open downward

$f'' = 0$

$f(x,y)$ critical point $\frac{\partial f}{\partial x} = 0$ $\frac{\partial f}{\partial y} = 0$

$$D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = f_{xx} f_{yy} - (f_{xy})^2$$

$D > 0$		$D < 0$
$\frac{\partial^2 f}{\partial x^2} = f_{xx} < 0$	$f_{xx} > 0$	
		
local max	local min	neither

Points to think.

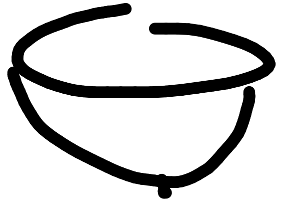
What can happen if $D = 0$.

find some $f(x,y)$ whose $D = 0$,

plot the graph around such critical points

Q: classify critical points of

$f = x^2 + y^2$

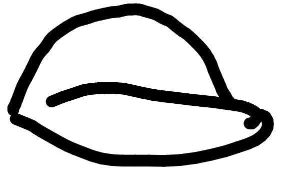


critical point = $(0,0)$

$$D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{yy} \end{vmatrix} = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = 4$$

$D > 0$ & $f_{xx} > 0$
$\Rightarrow (0,0)$ is a point of local min

$f = -x^2 - y^2$

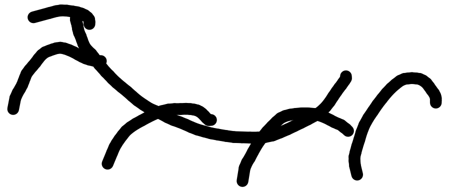


critical point = $(0,0)$

$$D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{yy} \end{vmatrix} = \begin{vmatrix} -2 & 0 \\ 0 & -2 \end{vmatrix} = 4$$

$D > 0$ & $f_{xx} < 0$.
$\Rightarrow (0,0)$ is a point of local max

$f = x^2 - y^2$



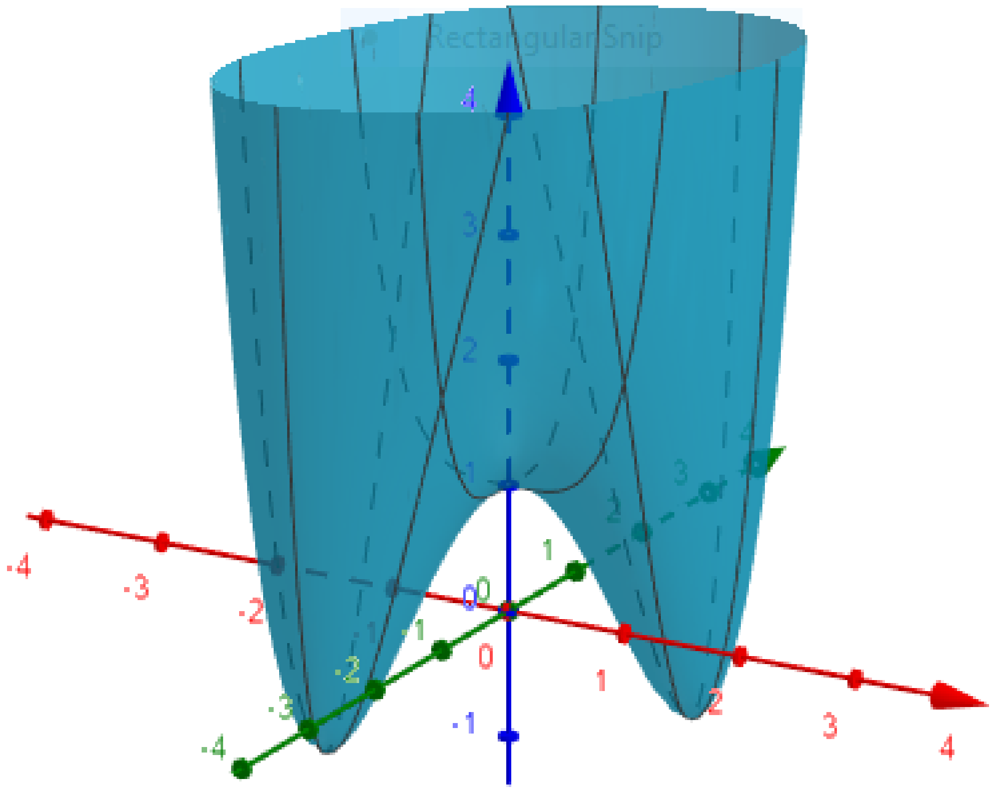
critical point = $(0,0)$

$$D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{yy} \end{vmatrix} = \begin{vmatrix} 2 & 0 \\ 0 & -2 \end{vmatrix} = -4$$

$(0,0)$ is a saddle point.

V EXAMPLE 3 Find the local maximum and minimum values and saddle points of $f(x, y) = x^4 + y^4 - 4xy + 1$.

& check your ans on Geogebra



$f_x = 0$ $\qquad$ $f_y = 0$

$4x^3 - 4y = 0$ $\qquad$ $4y^3 - 4x = 0$

$x^3 - y = 0$

$y^3 - x = 0$

$y = x^3$

$(x^3)^3 - x = 0$

$x^9 - x = 0$

$$D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{yy} \end{vmatrix} = \begin{vmatrix} 12x^2 & -4 \\ -4 & 12y^2 \end{vmatrix}$$

x	$y = x^3$	D	f_{xx}	
0	0	$-16 < 0$	whatever	saddle pt
-1	-1	$128 > 0$	$12 > 0$	local min
1	1	$128 > 0$	$12 > 0$	local min

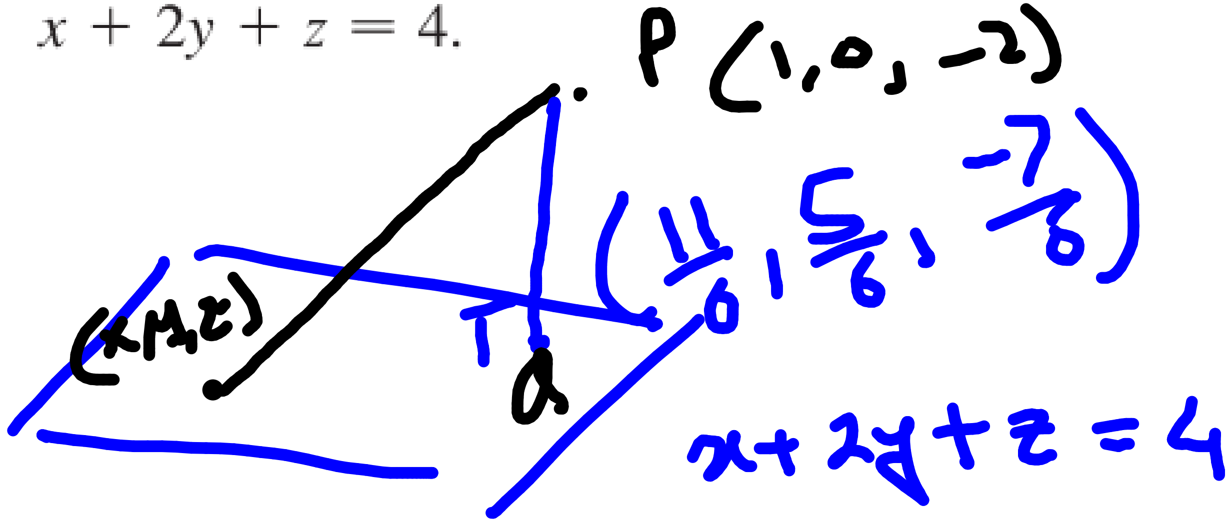
$$x(x^8 - 1) = 0$$

$x = 0$ & $x^8 - 1 = 0$

$x^8 = 1$

$x = \pm 1$

EXAMPLE 4 Find the shortest distance from the point $(1, 0, -2)$ to the plane $x + 2y + z = 4$.



minimize :

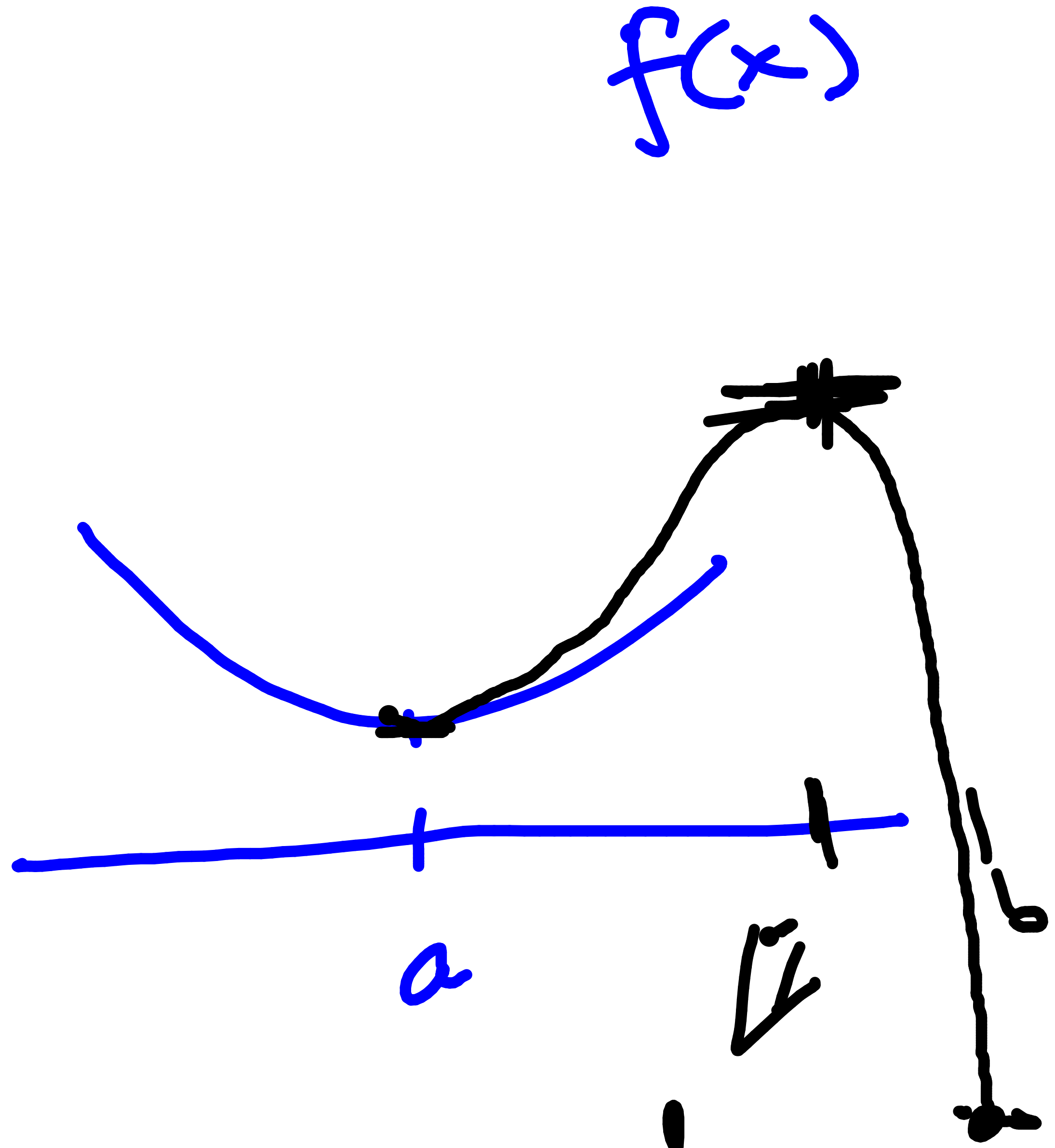
$$d^2 = (x-1)^2 + y^2 + (z+2)^2$$

$\Leftrightarrow$ minimize

$$f(x,y) = (x-1)^2 + y^2 + (\underbrace{4-x-2y}_{=z} + 2)^2$$

$\rightarrow$ find the critical point

$$\frac{\partial f}{\partial x} = 0 \quad , \quad \frac{\partial f}{\partial y} = 0$$



a : only possible critical point of f

f is local min at a

Then a is in fact a absolute min

need for new critical points on the way

$$2(x-1) + 2(6-x-2y)(-1) = 0 \quad \Big] \ \frac{\partial f}{\partial x} = 0$$

$$2y + 2(6-x-2y)(-2) = 0 \quad \Big] \ \frac{\partial f}{\partial y} = 0$$

$$\rightarrow \left.\begin{aligned} 4x + 4y &= 14 \\ 4x + 10y &= 24 \end{aligned}\right\} \Rightarrow x = \frac{11}{6} \qquad y = \frac{10}{6}$$

$$\& \quad z = 4 - x - 2y$$
$$= -7/6$$

now the candidate for the point on the plane which is nearest to P is $\left(11/6, \frac{10}{6}, -7/6\right)$

→ local min becomes absolute min because

→ hear the recording

$$2(x-1) + 2(6-x-2y)(-1) = 0$$

$$2y + 2(6-x-2y)(-2) = 0$$

$\rightarrow$ $4x + 4y$

$4x + 10y$

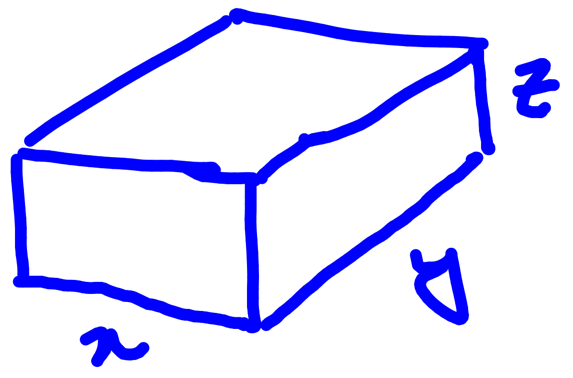
Classify the critical point

$$D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = \begin{vmatrix} 4 & 4 \\ 4 & 10 \end{vmatrix}$$

$D = 24 > 0$ & $f_{xx} = 4 > 0$

$\Rightarrow \left(x = \frac{11}{6}, y = \frac{10}{6}\right)$ is a local min point of $f(x,y)$

EXAMPLE 5 A rectangular box without a lid is to be made from 12 m^2 of cardboard. Find the maximum volume of such a box.



maximize xyz

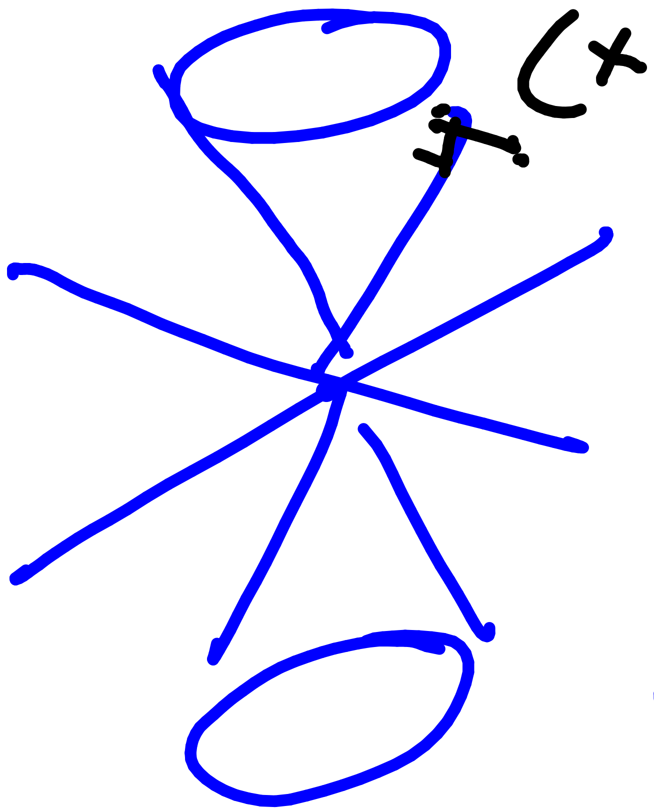
$$xy + 2xz + 2yz = 12$$

$\rightarrow$ maximize $f(y,z) = \left(\frac{12 - 2yz}{y + 2z}\right) yz$

$\rightarrow$ find critical points $\frac{\partial f}{\partial y} = 0$ $\frac{\partial f}{\partial z} = 0$

$\rightarrow$ argue that the critical points is a max point.

33. Find the points on the cone $z^2 = x^2 + y^2$ that are closest to the point $(4, 2, 0)$.



minimize $(x-4)^2 + (y-2)^2 + z^2$

eliminate z

$z^2 = x^2 + y^2$

minimize $f(x,y) = (x-4)^2 + (y-2)^2 + (x^2+y^2)$

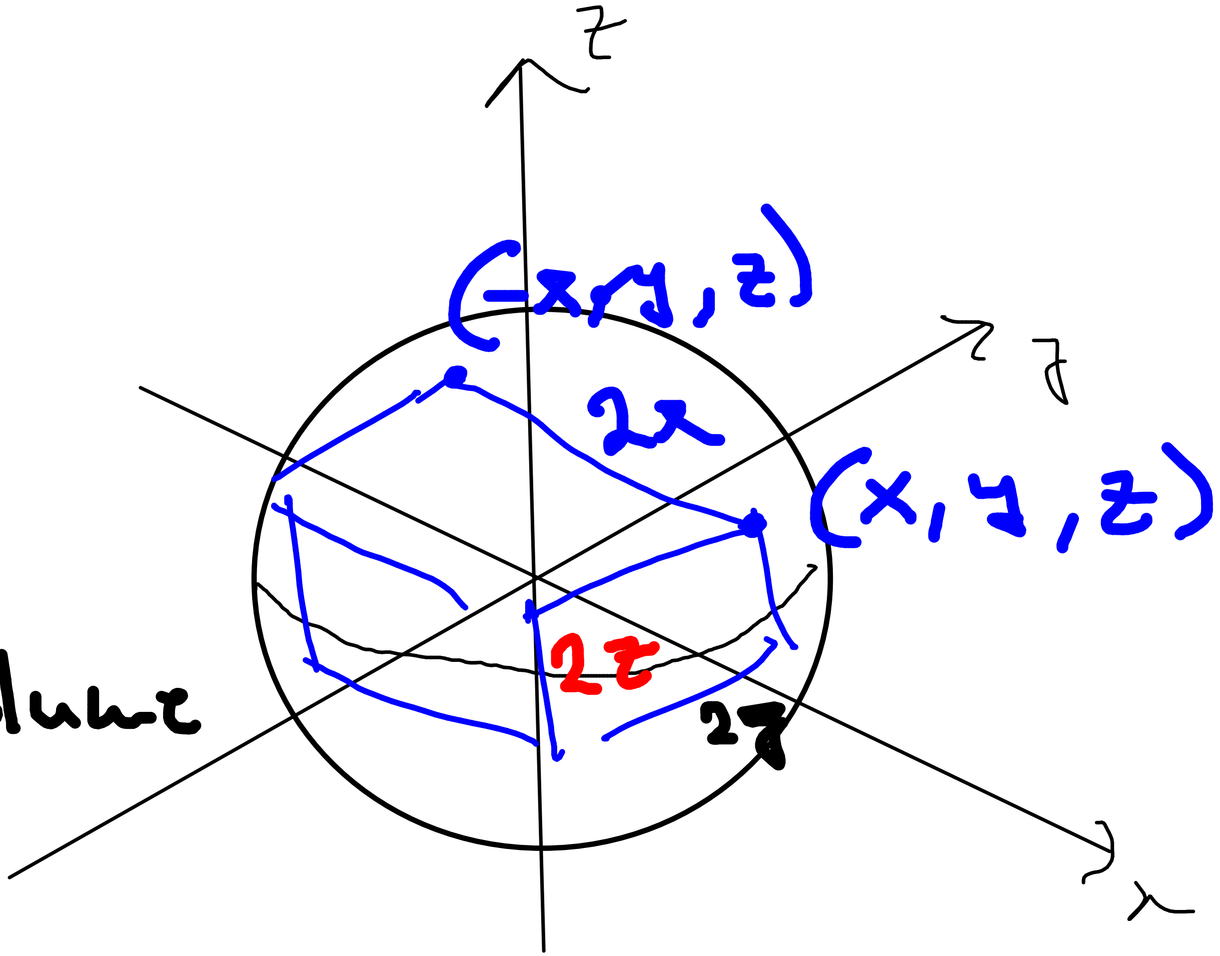
$\rightarrow$ find & classify the critical points

Find the volume of the largest rectangular box with edges parallel to the axes that can be inscribed in the ellipsoid

$$9x^2 + 36y^2 + 4z^2 = 36$$

ellipsoid ??

$$\frac{x^2}{4} + \frac{y^2}{1} + \frac{z^2}{9} = 1$$



a: consider such box

find a formula for the volume

$$V = 8xyz$$

maximize $V(x,z) = 8x\sqrt{\left(1-\frac{x^2}{4}-\frac{z^2}{9}\right)}\, z$

$\to$ find & classify critical points

Do yourself.