

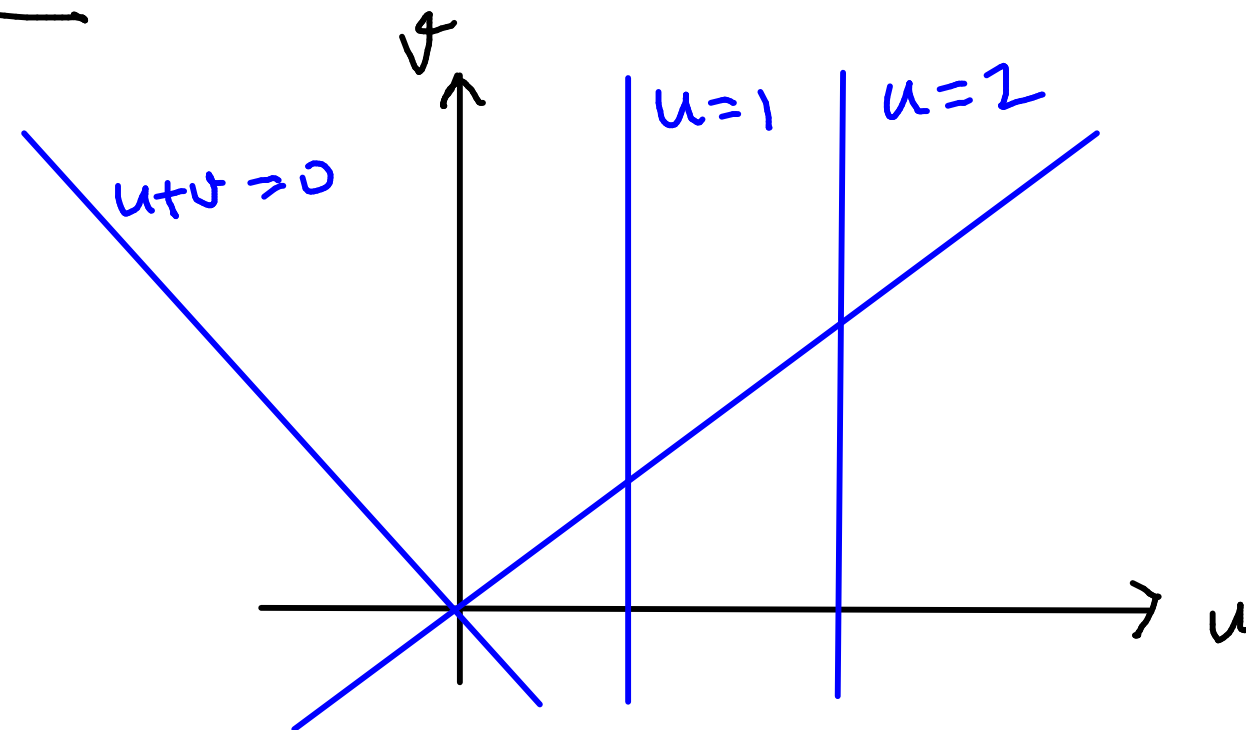
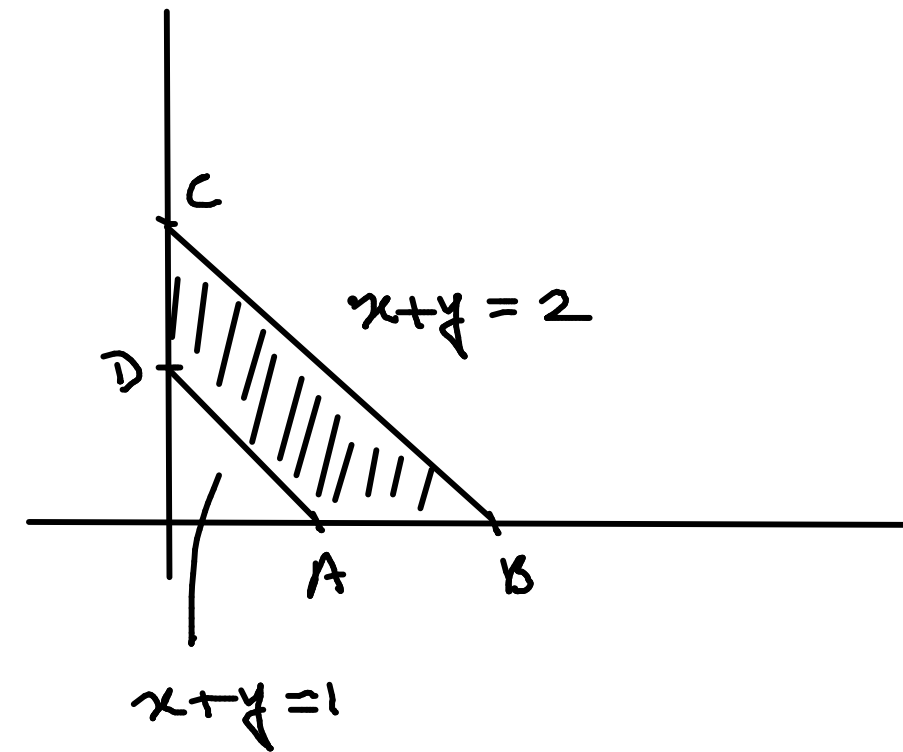
Evaluate the integral by making appropriate change of variables.

$\iint_R \cos\left(\frac{y-x}{y+x}\right) dA$, where R is the trapezoidal region with vertices $(1, 0)$, $(2, 0)$, $(0, 2)$, and $(0, 1)$

$$\begin{aligned} u &= x+y \\ v &= y-x \end{aligned} \quad \left| \quad \begin{aligned} v &= (u+v)/2 \\ x &= (u-v)/2 \end{aligned} \right.$$

AD $\begin{aligned} x+v &= 1 \\ u &= 1 \end{aligned}$

BC $\begin{aligned} x+v &= 2 \\ u &= 2 \end{aligned}$

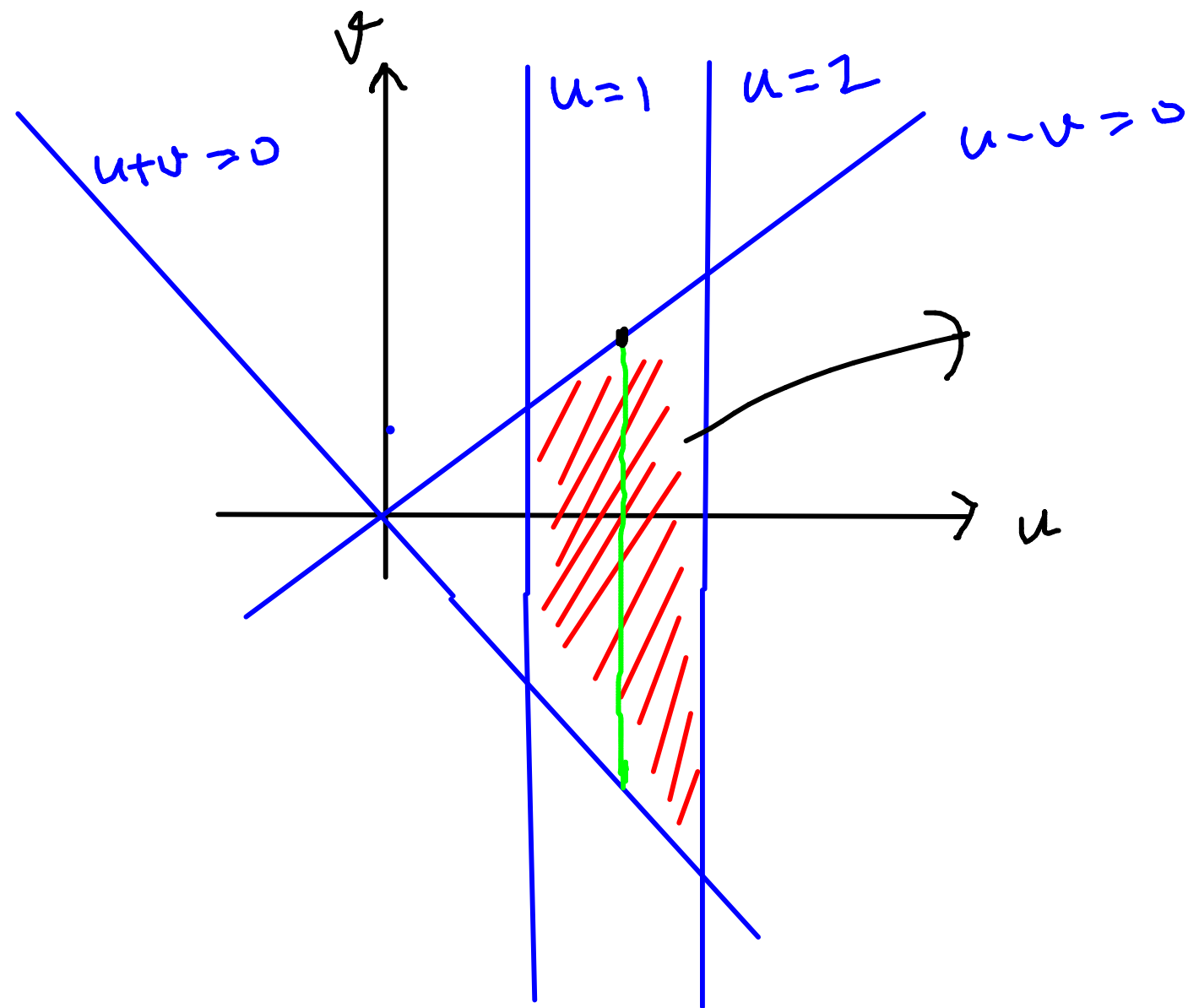


$$\text{AB:} \quad \begin{array}{l} y = 0 \\ (u+v)/2 = 0 \end{array} \quad | \quad u+v = 0$$

$$\text{DC:} \quad \begin{array}{l} x = 0 \\ u-v = 0 \end{array}$$

$$\begin{array}{l} u = x+y \\ v = y-x \end{array} \quad | \quad \begin{array}{l} y = (u+v)/2 \\ x = (u-v)/2 \end{array}$$

$$\begin{aligned} J &= \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 1/2 & -1/2 \\ 1/2 & 1/2 \end{vmatrix} \\ &= \frac{1}{2} \end{aligned}$$



$$= \int_1^2 \int_{-u}^u \cos\left(\frac{v}{u}\right) \left(\frac{1}{2}\right) dv du$$

$$\int_1^2 \int_{-u}^u \left(\frac{1}{2}\right) \cos\left(\frac{v}{u}\right) dv du$$



Go

$$= \sin(1) \frac{3}{2}$$