ERWIN KREYSZIG
ADVANCED ENGINEERING MATHEMATICS

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Ordinary Differential Equations (ODEs)

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- Chap. 6 Laplace Transforms

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GUIDES AND MANUALS
Maple Computer Guide
Mathematica Computer Guide

Student Solutions Manual and Study Guide

Instructor's Manual

Same topics next sem

PART A Ordinary Differential Equations (ODEs)

Whats ODE ?

$\underbrace{\text{Ordinary } \underbrace{\text{Differential } \underbrace{\text{Equation}}}}$.

Idea: we know something about $\frac{df}{dx}$ or $\frac{d^2f}{dx^2}$

& we wish to find out about f.

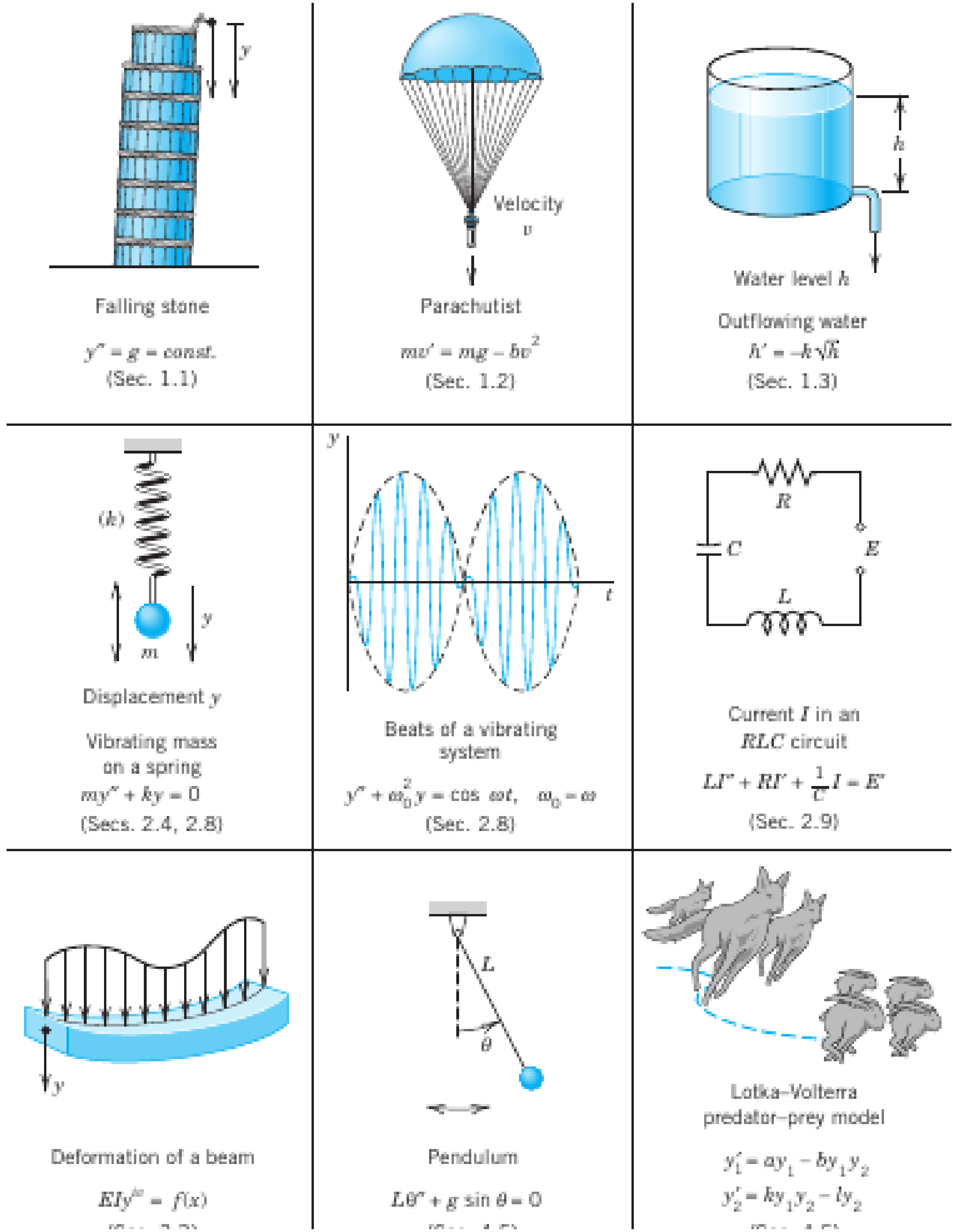
$\frac{df}{dx} = 2$, find $f(x)$

$\frac{df}{dx} + f = 5$

$\frac{d^2f}{dx^2} + \frac{df}{dx} + f = e^x$

find $f(x)$

Why care about knowing to solve ODEs:



H.W. → Just read section 1.1 & 1.2.

→ Don't solve exercise problems.

mathematical modelling: converting real world scenarios into mathematical equations.

↳ in particular into ODE eqns.

1.3 Separable ODEs

1st order ODEs
& separable

$$\left[\frac{dy}{dx} + y^2 = \sin(x)\right] \quad \text{, solve for } y$$

$$\left[\cancel{\frac{d^2y}{dx^2} + y^2 = \sin(x)}\right] \quad \text{2nd order ODE}$$

$$\left.\frac{dy}{dx} + y = \frac{d^2}{dx^2}(e^x)\right] \quad \text{1st order ODE}$$

Q.

$$y' = (x+1)e^{-x}y^2$$

Use separation of variables
↳ try to move all x in one side
& all y in other side
& integrate

$$\frac{dy}{dx} = (1+x)e^{-x}y^2$$

$$\frac{1}{y^2}dy = (1+x)e^{-x}\,dx$$

$$\int \frac{1}{y^2}\,dy = \int (1+x)e^{-x}\,dx$$

$$-\frac{1}{y} = -(x+2)e^{-x} + c \quad ,$$

c is an arbitrary constant

&

$$\boxed{y = \frac{1}{(x+2)e^{-x} + c}}$$

d.

Solve $y' = -2xy$, $y(0) = 1.8$.

initial condition

Solve using separation of variables

use the extra condition to eliminate the arbitrary constant C

$$\frac{dy}{dx} = -2xy$$

$$\frac{1}{y}\,dy = -2x\,dx$$

$$\ln y = -x^2 + C$$

$y(0) = 1.8$

$$\ln 1.8 = -0^2 + C$$

$$C = \ln 1.8$$

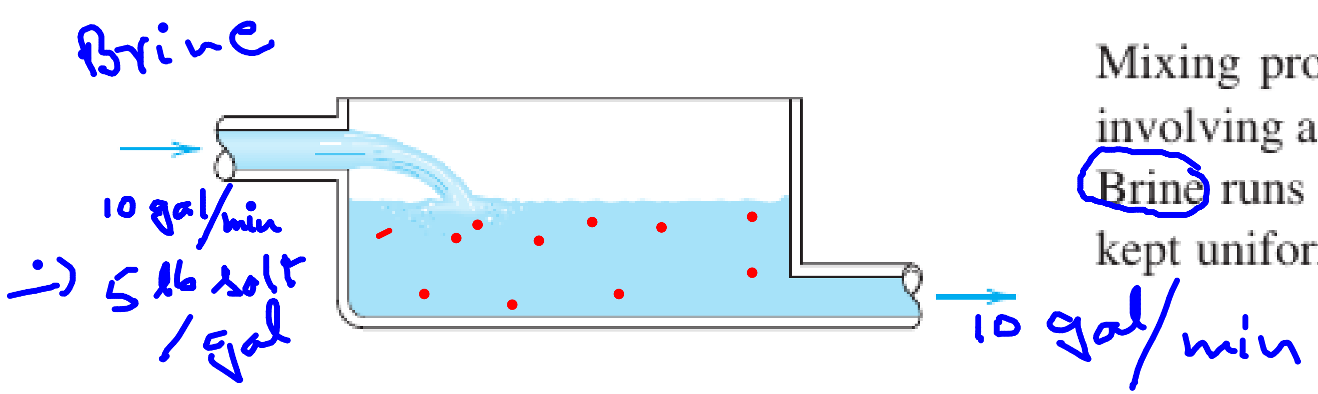
$$\ln y = -x^2 + \ln 1.8$$

$$\ln\left(\frac{y}{1.8}\right) = -x^2$$

$$\frac{y}{1.8} = e^{-x^2}$$

$$\boxed{y = 1.8e^{-x^2}}$$

EXAMPLE 5 Mixing Problem



Mixing problems occur quite frequently in chemical industry. We explain here how to solve the basic model involving a single tank. The tank in Fig. 11 contains 1000 gal of water in which initially 100 lb of salt is dissolved. Brine runs in at a rate of 10 gal/min, and each gallon contains 5 lb of dissoved salt. The mixture in the tank is kept uniform by stirring. Brine runs out at 10 gal/min. Find the amount of salt in the tank at any time t.

Brine $\Leftrightarrow$ salt + water

$y(t)$: amount of salt in the tank at time t

initial salt density ??

0.1 lb/gal

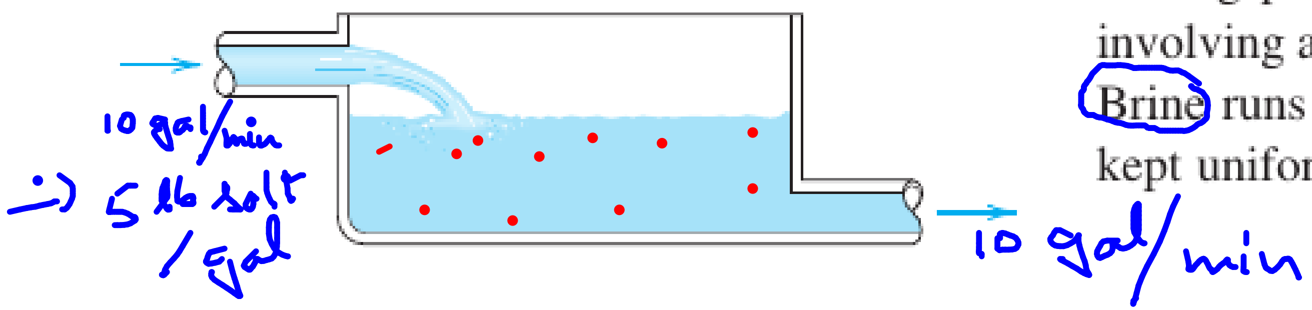
$y(t) = ??$

$y(0) = 100$

$\lim_{t\to\infty} y(t) =$ guess $= 5000$ lb

EXAMPLE 5 Mixing Problem

Brine



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$y(t) = ??$

$y(0) = 100$

$\frac{dy}{dt}$ = rate of change of salt mass

= inflow rate − outflow rate

$= 50 - \frac{10}{1000} y$

Solve y from

$\frac{dy}{dt} = 50 - \frac{y}{100}$, $y(0) = 100$

Solve $\frac{dy}{dt} = 50 - \frac{y}{100}$ using separation of variables.

$$\frac{dy}{dt} = \frac{5000 - y}{100}$$

$$\frac{1}{5000 - y}\, dy = \frac{1}{100}\, dt$$

$$\int \frac{1}{5000 - y}\, dy = \int \frac{1}{100}\, dt$$

$$-\ln(5000 - y) = \frac{t}{100} + C$$

$t = 0$

$y = 100$

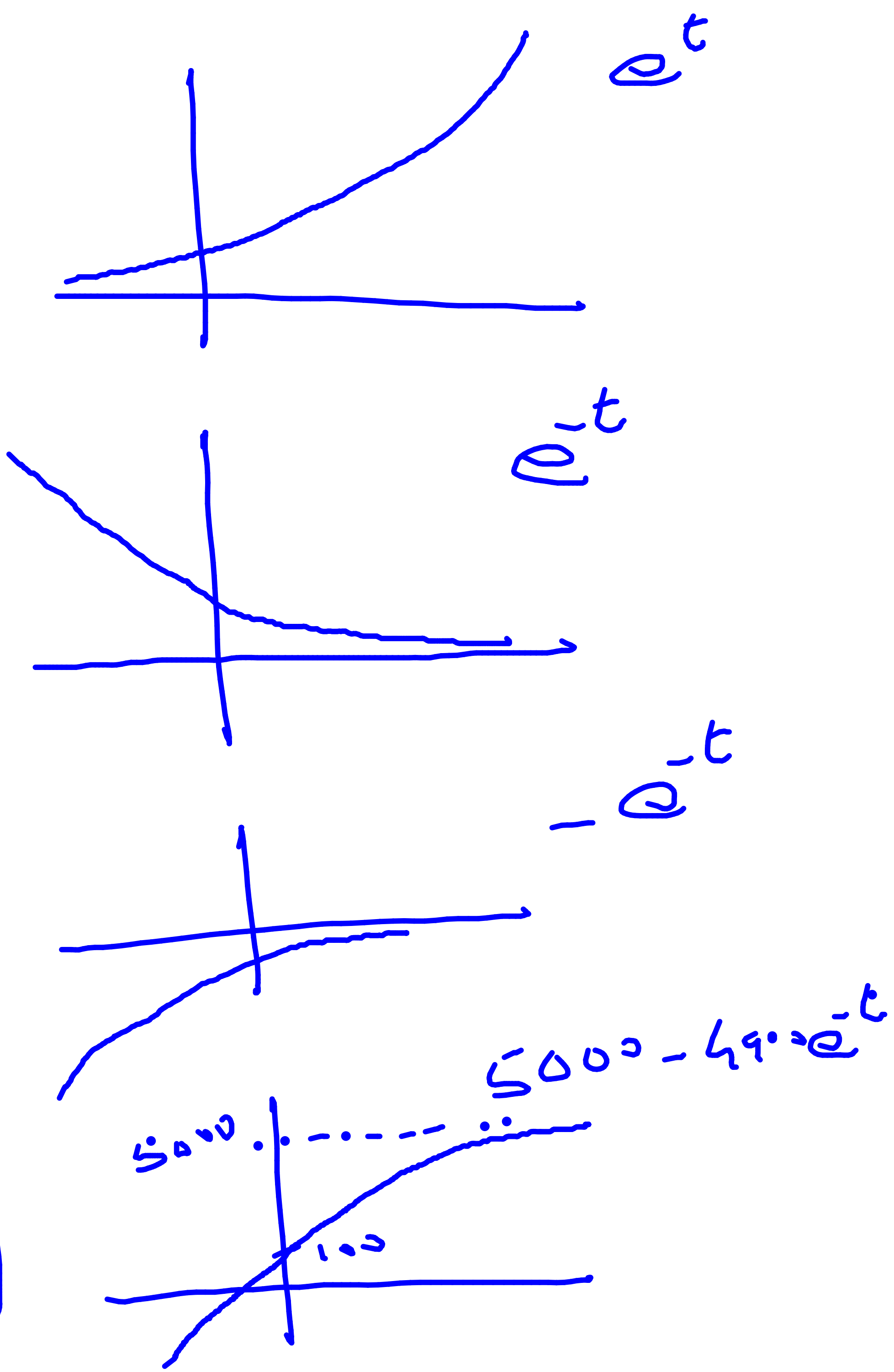
$C = -\ln 4900$

$$\ln\left(\frac{5000 - y}{4900}\right) = -\frac{t}{100}$$

$y = ??$

$$5000 - y = 4900\, e^{-t/100}$$

$$\boxed{y = 5000 - 4900\, e^{-t/100}}$$



Extended Method: Reduction to Separable Form

$$y' = f\left(\frac{y}{x}\right)$$

$$2xyy' = y^2 - x^2.$$

32. Friction. If a body slides on a surface, it experiences friction F (a force against the direction of motion). Experiments show that $|F| = \mu|N|$ (*Coulomb's*[6] *law of kinetic friction without lubrication*), where N is the normal force (force that holds the two surfaces together; see Fig. 15) and the constant of proportionality μ is called the *coefficient of kinetic friction.* In Fig. 15 assume that the body weighs 45 nt (about 10 lb; see front cover for conversion). $\mu = 0.20$ (corresponding to steel on steel), $a = 30°$, the slide is 10 m long, the initial velocity is zero, and air resistance is negligible. Find the velocity of the body at the end of the slide.