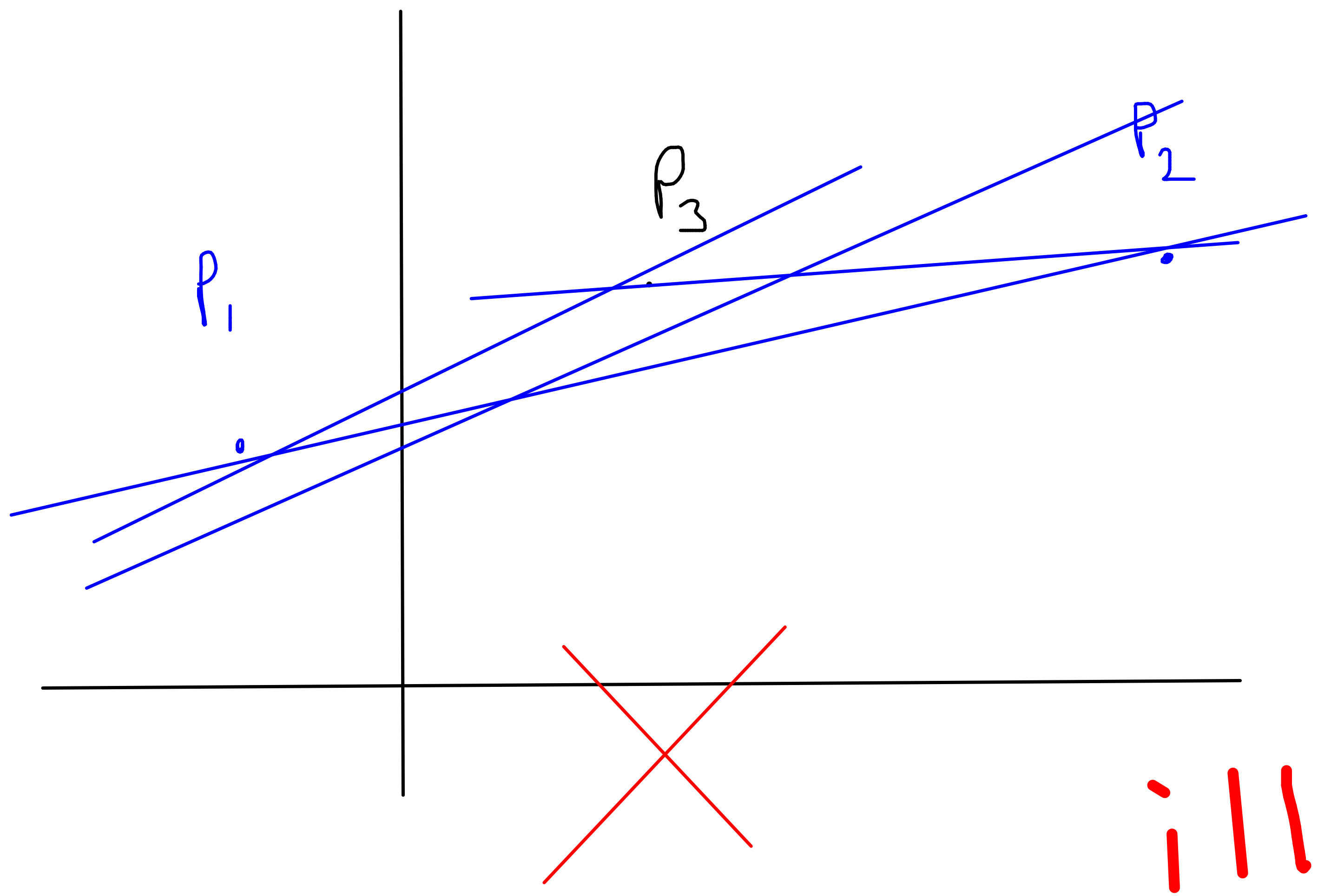
Today's agenda

- → use of max/min problem for finding a best fit line
- → Lagrange multipliers

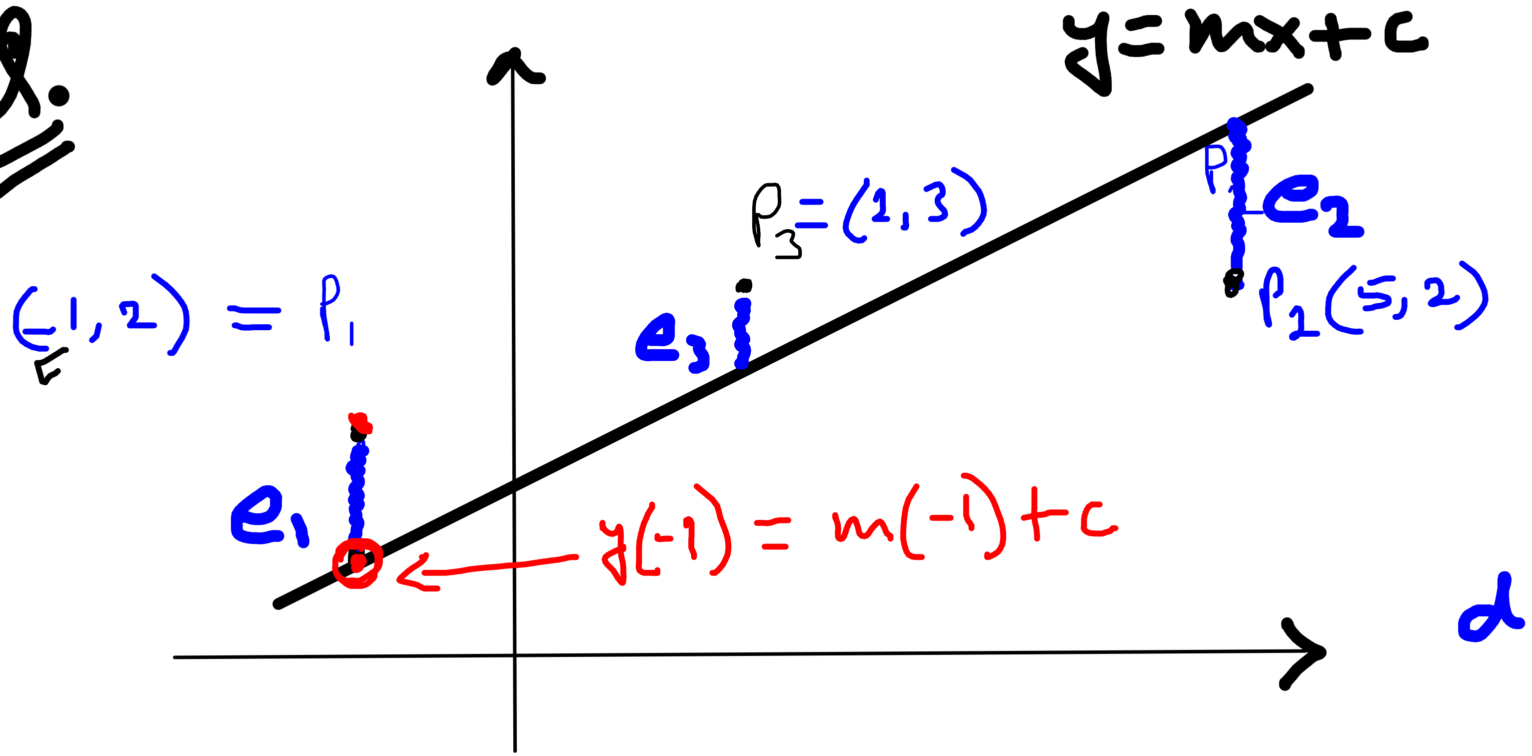
Q.



find the line passing through P_1, P_2, P_3

ill posed question

Q.



find a line $y = mx + c$

which passes nearby P_1, P_2, P_3 as close as possible.

$d(m, c) = \boxed{\text{distance between the line } y = mx + c \text{ \& the points } P_1, P_2, P_3}$

kind of

we will minimize the sum of errors at all points

$$(\text{error})^2 = e_1^2 + e_2^2 + e_3^2$$

$$\text{error}(m, c) = \left[2 - [m(-1) + c]\right]^2 + \left[2 - (m(5) + c)\right]^2 + \left[3 - (m(2) + c)\right]^2$$

$$= A m^2 + B c^2 + D mc + F \quad (\text{after simplification})$$

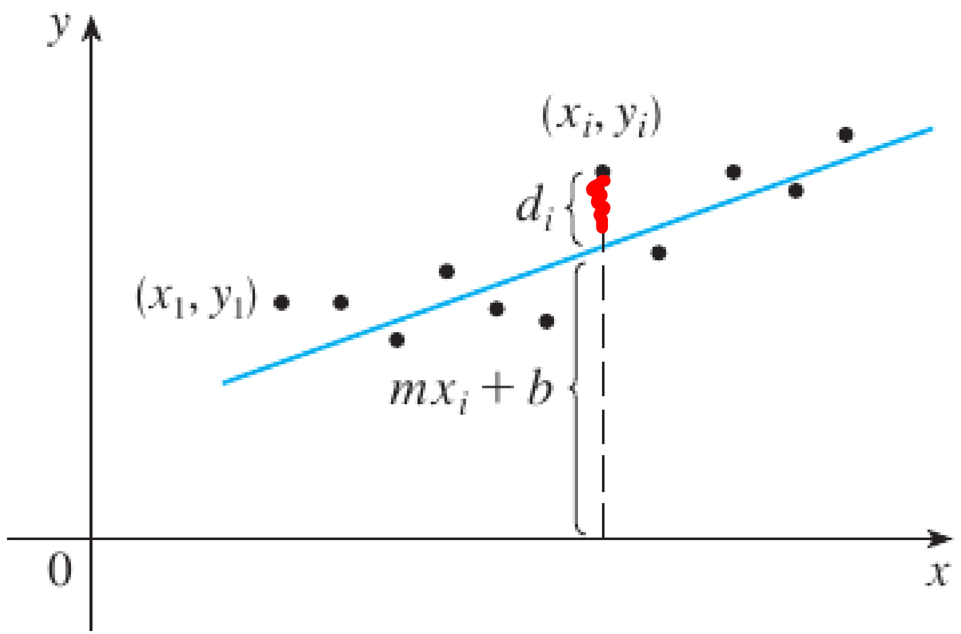
Q. find the best fit line $y = mx + c$

strategy : minimize the error(m,c) as a function of m, c

$$\frac{\partial}{\partial m}(\text{error}) = 0 \qquad \frac{\partial}{\partial c}(\text{error}) = 0$$

& solve for m & c

47. Suppose that a scientist has reason to believe that two quantities x and y are related linearly, that is, $y = mx + b$, at least approximately, for some values of m and b. The scientist performs an experiment and collects data in the form of points $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$, and then plots these points. The points don't lie exactly on a straight line, so the scientist wants to find constants m and b so that the line $y = mx + b$ "fits" the points as well as possible. (See the figure.)



error(m,b)

$$E(m,b) = \sum_{i=1}^{n} \left[y_i - (mx_i + b)\right]^2$$

Aim: minimize $E(m,b)$

$$E(m,b) = \sum_{i=1}^{n}\left[y_i - (mx_i+b)\right]^2$$

Aim: minimize $E(m,b)$ & find a formula for m, b.

$\rightarrow \dfrac{\partial E}{\partial m} = 0$ & $\dfrac{\partial E}{\partial b} = 0$

$$\rightarrow E = \sum_{i=1}^{n}\left[y_i^2 + m^2 x_i^2 + b^2 - 2y_i m x_i - 2y_i b + 2mbx_i\right]$$

$$= \sum y_i^2 + m^2 \sum x_i^2 + nb^2 - 2m\sum x_i y_i - 2b\sum y_i + 2mb\sum x_i$$

(this a formula of m, b)

$$\frac{\partial E}{\partial m} = 0 \qquad \sum x_i^2\, m + \sum x_i\, b = \sum x_i y_i$$

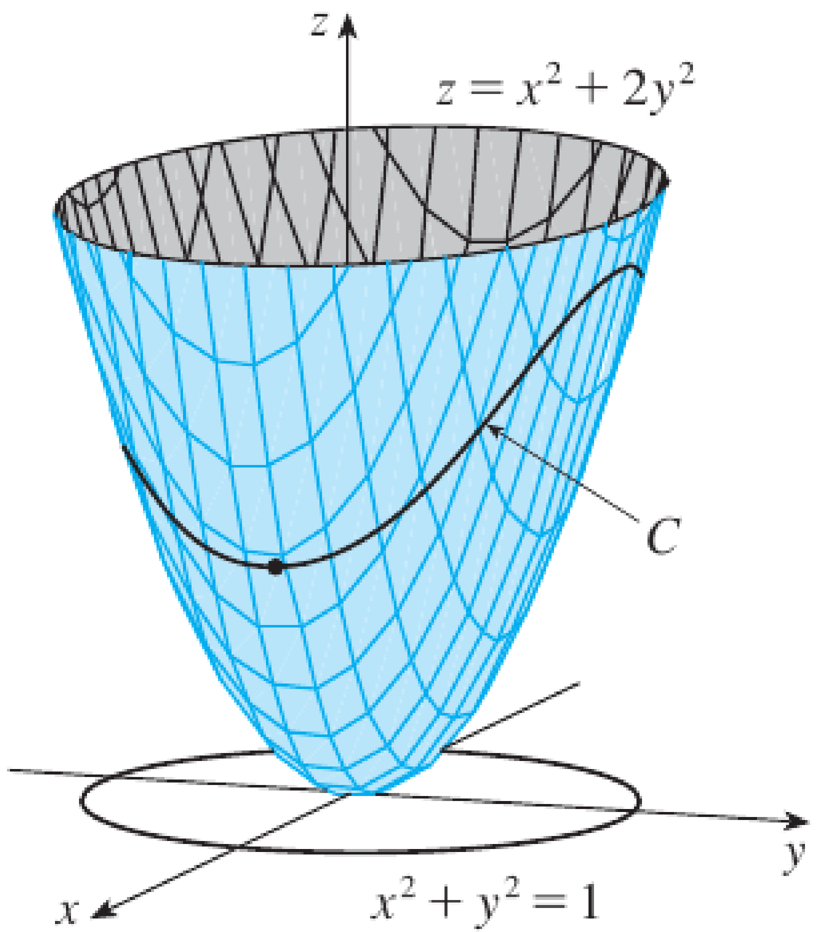
$$\frac{\partial E}{\partial b} = 0 \qquad \sum x_i\, m + n\, b = \sum y_i$$

$\rightarrow$ solve for m & b & that gives us the best fit line.

H.W. find m & b for points (x_i, y_i) given on google spreadsheet.

11.8 LAGRANGE MULTIPLIERS

EXAMPLE 2 Find the extreme values of the function $f(x, y) = x^2 + 2y^2$ on the circle $x^2 + y^2 = 1$.



$$f(x, y) = x^2y; \quad x^2 + 2y^2 = 6$$