Recall: we were discussing exact ODE:

$$M + N\frac{dy}{dx} = 0$$

if exact: we find $u(x, y(x))$

$$\frac{du}{dx} = M + N\frac{dy}{dx}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$\Downarrow$

exact

if not find I s.t.

$$IM + IN\frac{dy}{dx} = 0 \quad \text{is exact}$$

1.5 Linear ODEs.

$$y' + p(x)y = r(x)$$

$$I(x) = e^{\int p(x)\,dx}$$

$$I(x)y' + I(x)p(x)y \stackrel{?}{=} \frac{d}{dx}\left(I(x)\,y\right)$$

RHS: $\frac{d}{dx}\left(I(x)\,y\right) = \frac{dI}{dx}y + I\frac{dy}{dx}$

$$I(x) = e^{\int p(x)dx}$$

$$\frac{dI}{dx} = e^{\int p(x)dx} \cdot \frac{d}{dx}\left(\int p(x)dx\right) = e^{\int p(x)dx}\, p(x) \cdot = I\, p(x)$$

$$y' + p(x)\,y = r(x)$$

$$y' e^{\int p(x)\,dx} + p(x)\,y\,e^{\int p(x)\,dx} = r(x)\,e^{\int p(x)\,dx}$$

$$\frac{d}{dx}\left(y e^{\int p(x)\,dx} \right) = r(x)\,e^{\int p(x)\,dx}$$

$$y e^{\int p(x)\,dx} = \int r(x)\,e^{\int p(x)\,dx} + c$$

& solve for y

Q: But how did anybody come up with the formula of $IF = e^{\int p(x)dx}$

Recall the routine

$$M + N\frac{dy}{dx} = 0$$

$$\left[y' + p(x)y = r(x) \right]$$

$$\downarrow$$

$$y' + p(x)y = 0$$

$$\underbrace{p(x)y}_{M} + \underbrace{y'}_{N} = 0$$

→ Try to find IF → $e^{\int p(x)dx}$

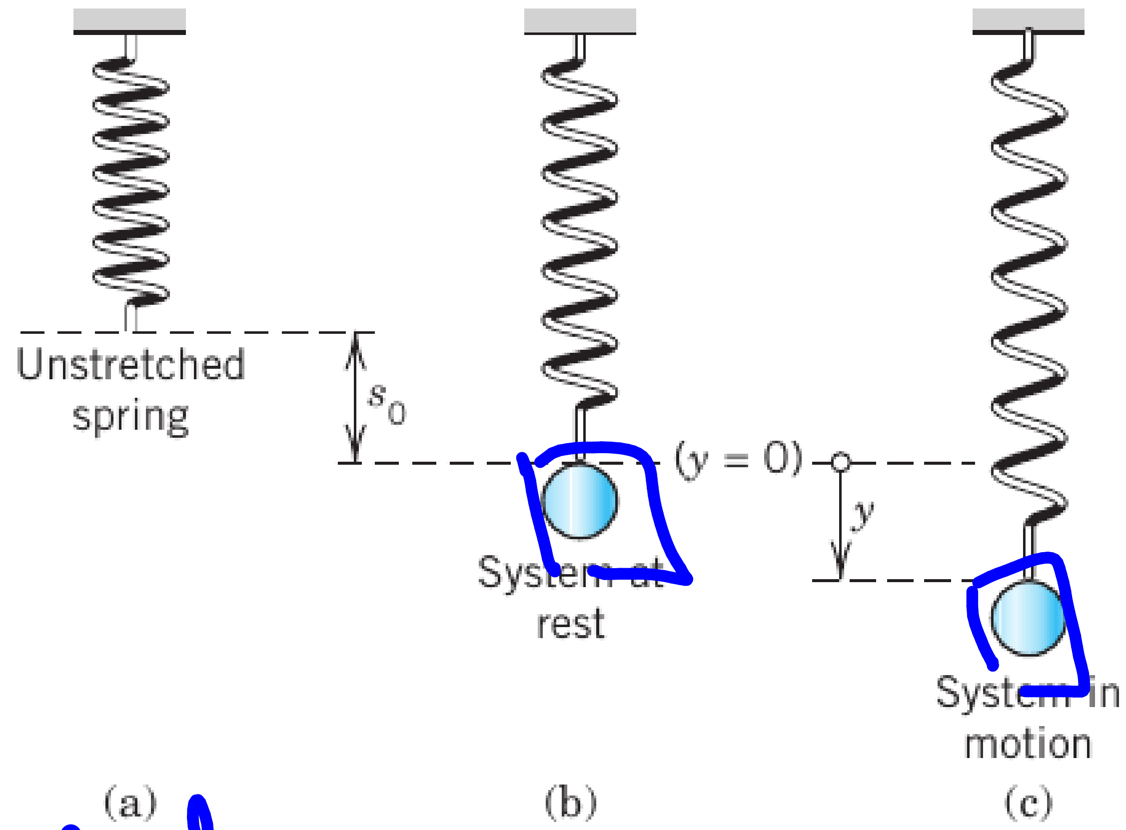
CHAPTER 2

Second-Order Linear ODEs

=

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = r(x)$$

2.4 Modeling of Free Oscillations of a Mass–Spring System



→ mechanical :

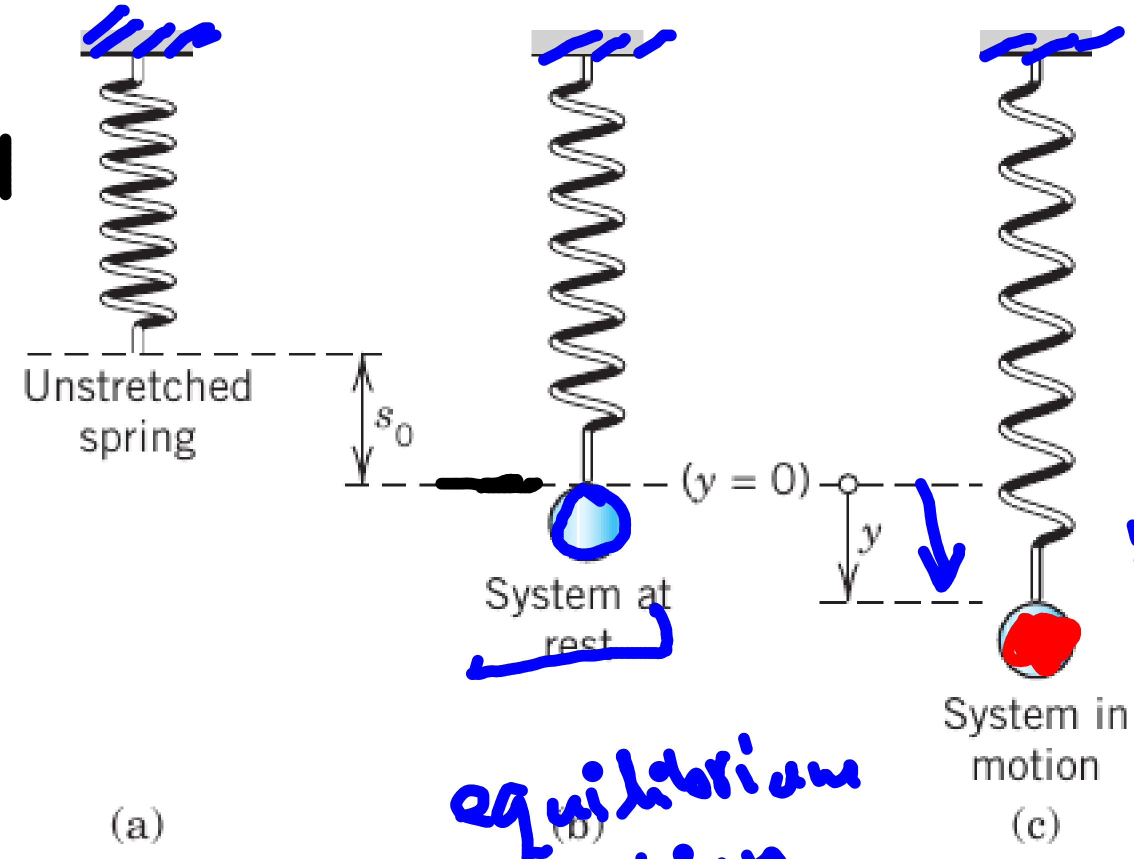
mechanical vibration

& strength of material.

$$F = m \frac{d^2y}{dx^2}$$

2.4 Modeling of Free Oscillations of a Mass–Spring System

+ve dir



y(t)

equilibrium position of mass

m : attached

y(t) : position of the body at time t

Aim: find Eqn for y(t) :

$$m\frac{d^2y}{dt^2} = \text{net force on the body at time } t.$$

$$m\frac{d^2y}{dt^2} = \text{gravity} + \text{spring force} + \text{medium resistance}$$

$$m\frac{d^2y}{dt^2} = mg - k(y+s_0) - \underbrace{c\frac{dy}{dt}}_{\text{experimental}}$$

$\rightarrow$ this eq^n completes the eq^n for y

$$m\frac{d^2y}{dt^2} + c\frac{dy}{dt} + ky = mg - ks_0 \quad , \quad \left.\begin{array}{l} y(0) = y_0 \\ y'(0) = v_0 \end{array}\right] \text{2 extra conditions required}$$

$$\frac{d^2y}{dt^2} = 5$$

2.9 Modeling: Electric Circuits

$I(t) = ??$

$E_0 \sin\omega(t) =$ sum of voltage drop accross each component

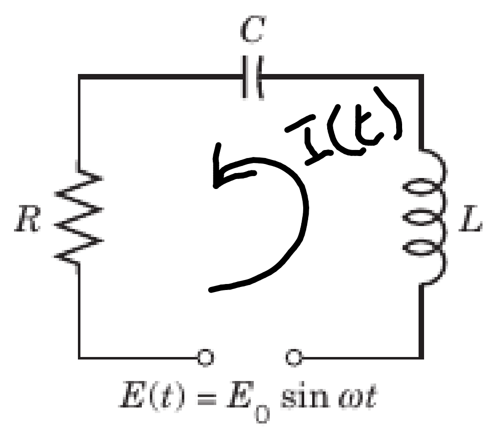


Fig. 61. *RLC*-circuit

$$E_0 \sin\omega(t) = L\frac{dI}{dt} + \frac{1}{C}q + RI$$

Name	Symbol	Notation		Unit	Voltage Drop
Ohm's Resistor	resistor	R	Ohm's Resistance	ohms (Ω)	RI
Inductor	inductor	L	Inductance	henrys (H)	$L\frac{dI}{dt}$
Capacitor	capacitor	C	Capacitance	farads (F)	Q/C

Fig. 62. Elements in an *RLC*-circuit

$I = \frac{dq}{dt}$

$q = \int I(t)dt$

$$E_0 \sin\omega(t) = L\frac{dI}{dt} + \frac{1}{C}\int I(\tau)d\tau + RI$$

$$\boxed{\omega E_0 \cos(\omega t) = L\frac{d^2I}{dt^2} + \frac{1}{C}I + R\frac{dI}{dt}}$$

2.2 Homogeneous Linear ODEs with Constant Coefficients

next time