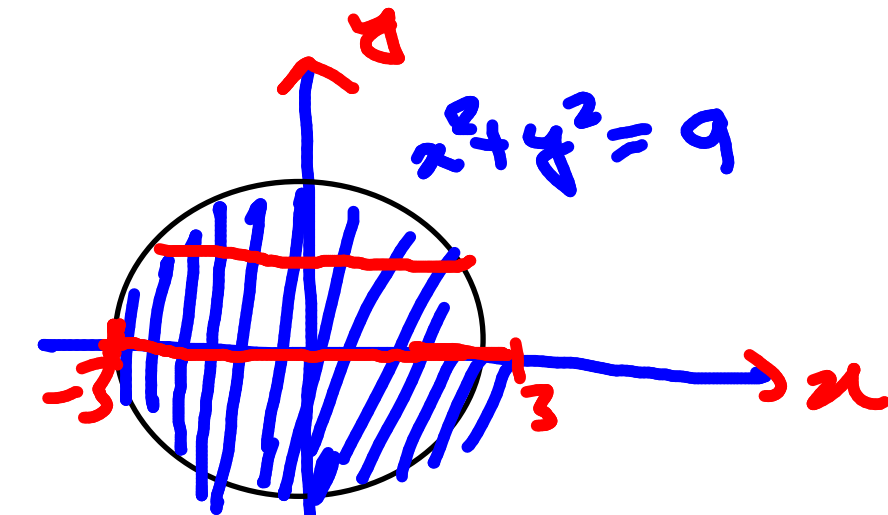**EXAMPLE 2** Find the domain and range of $g(x, y) = \sqrt{9 - x^2 - y^2}$.
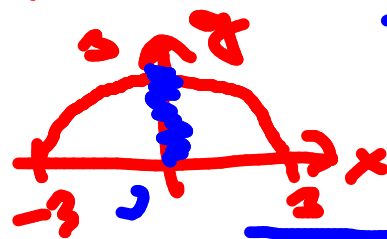
$x^2 + y^2 = 9$

domain: collection of all (x, y) s.t.

$9 - x^2 - y^2 \geq 0$

i.e. $\boxed{x^2 + y^2 \leq 9}$

disc

g: disc $\longrightarrow$ real numbers

$g = \sqrt{9-x^2}$

range $= \left\{ g(x,y) \mid (x, y) \in \text{domain} \right\}$
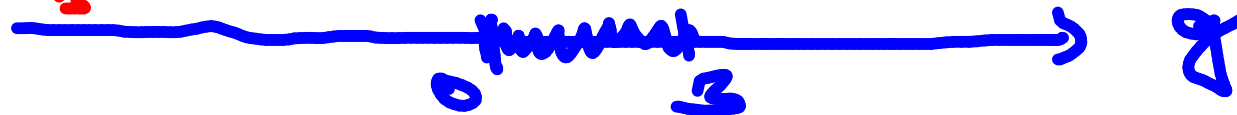
$\text{Range}(g) = [0, 3]$
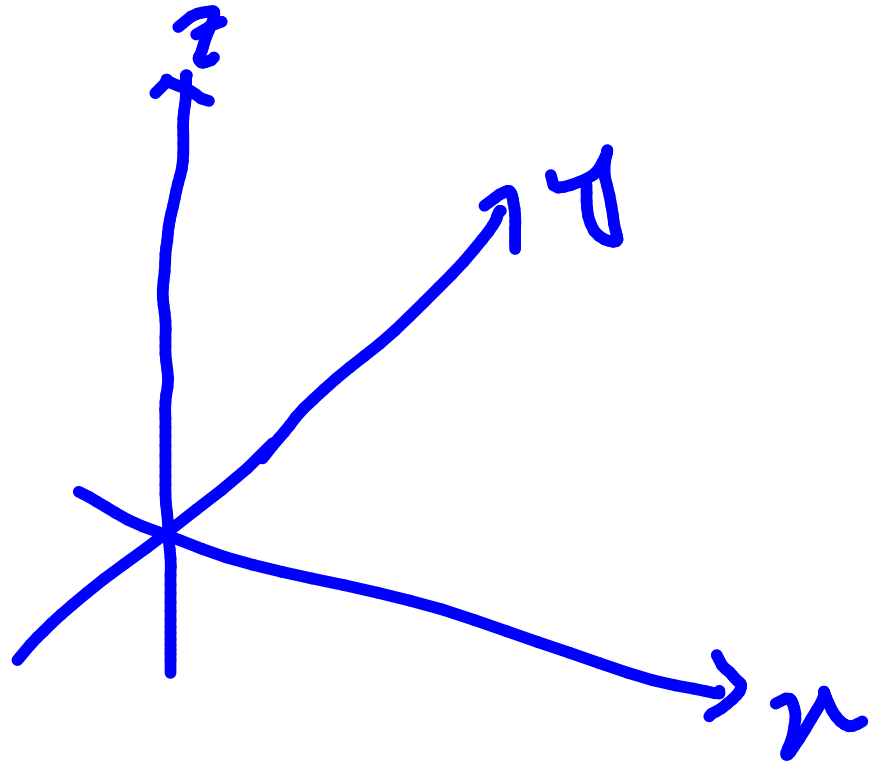
whats a multivariable function ?!

$f(x, y) = xy$

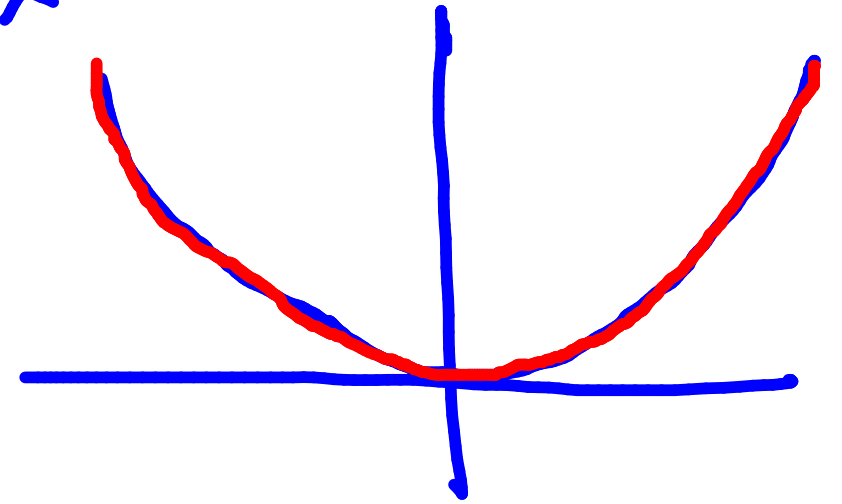$f(x, y, z) = xyz^2$

$V(l, b, h) = lbh$

$V(r, h) = \pi r^2 h$

$T(x, y, z) = $ temperature at point x, y, z

GRAPHS

$$f(x) = x^2$$
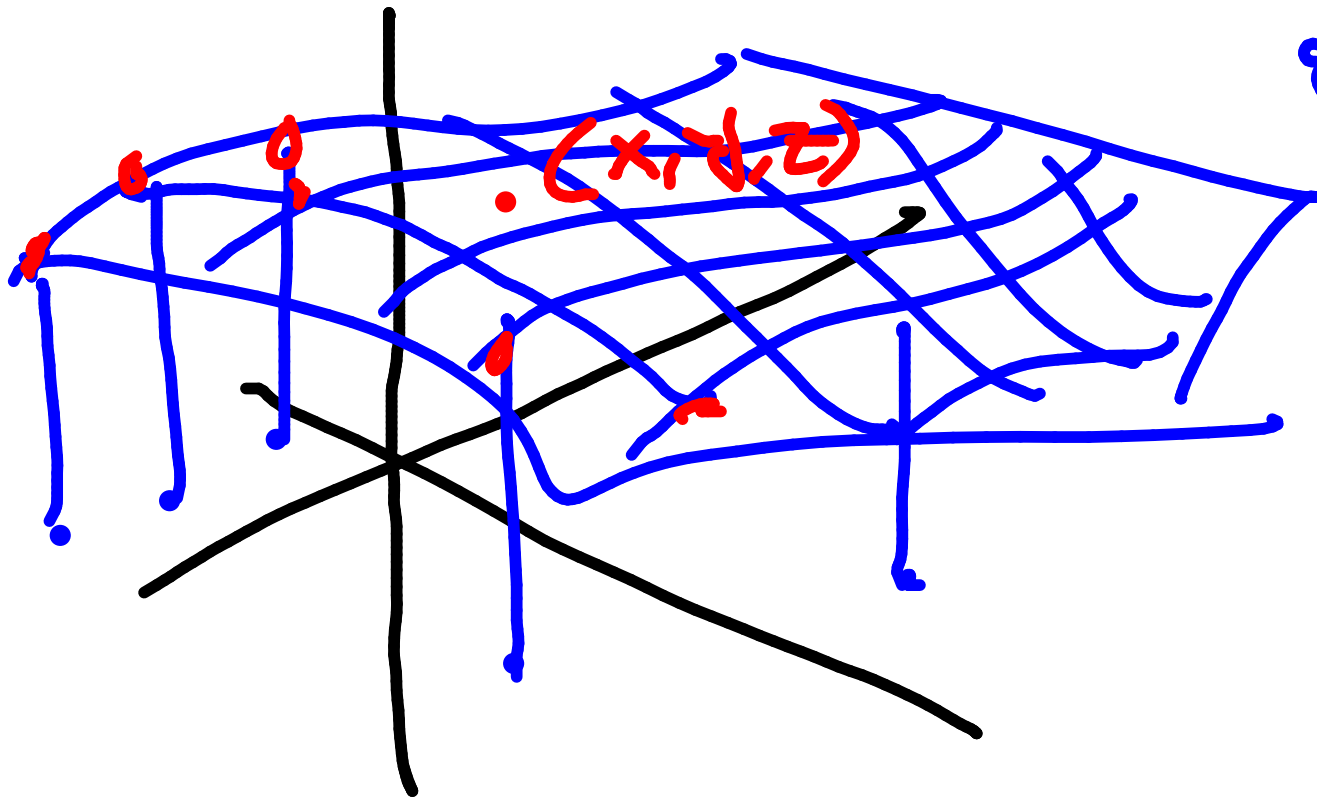
$$= \left[(x, y) \mid y = x^2 \right]$$

DEFINITION If f is a function of two variables with domain D, then the **graph** of f is the set of all points (x, y, z) in $\mathbb{R}^3$ such that $z = f(x, y)$ and (x, y) is in D.

3d space

$$f(x,y) = x^2 + y^2 \qquad \text{graph ??}$$
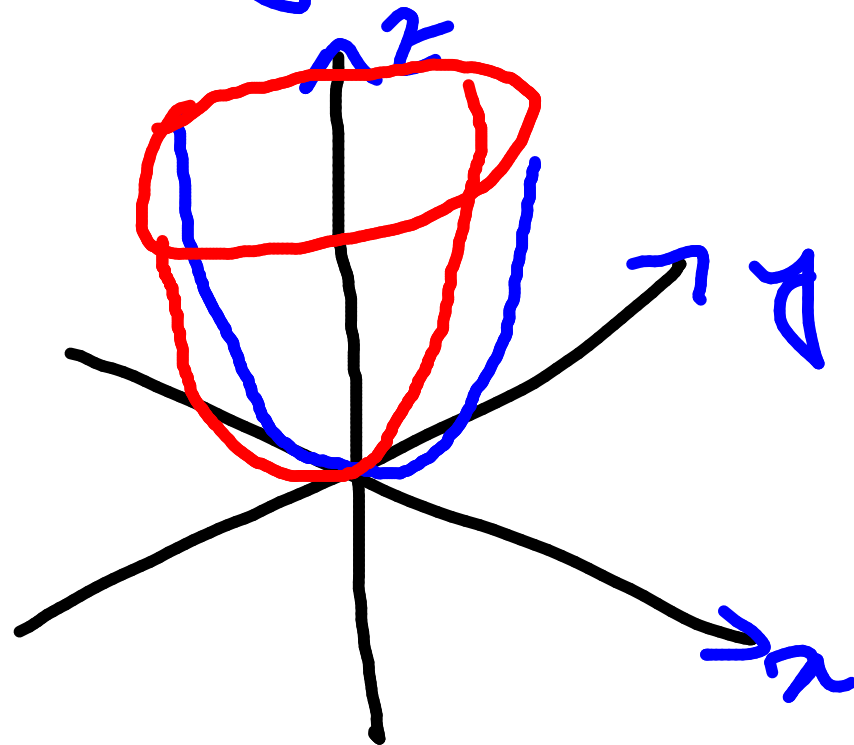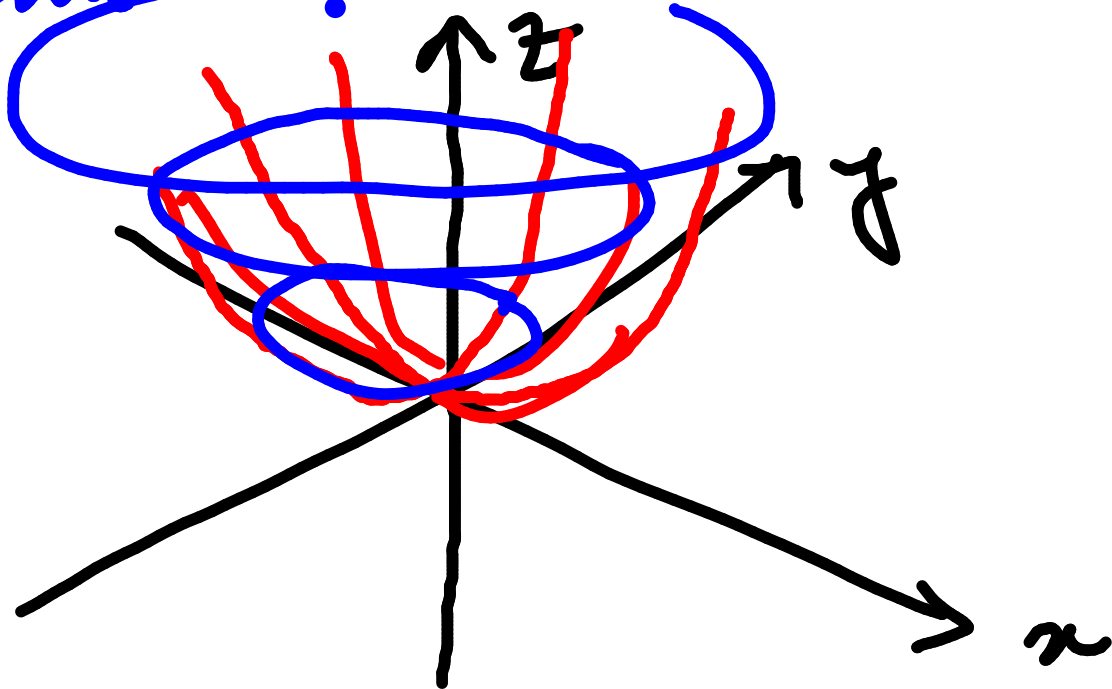
graph will lie in 3d space
$$(x, y, f(x,y))$$

graph
$$= \{ (x,y,z) \mid z = x^2 + y^2 \}$$

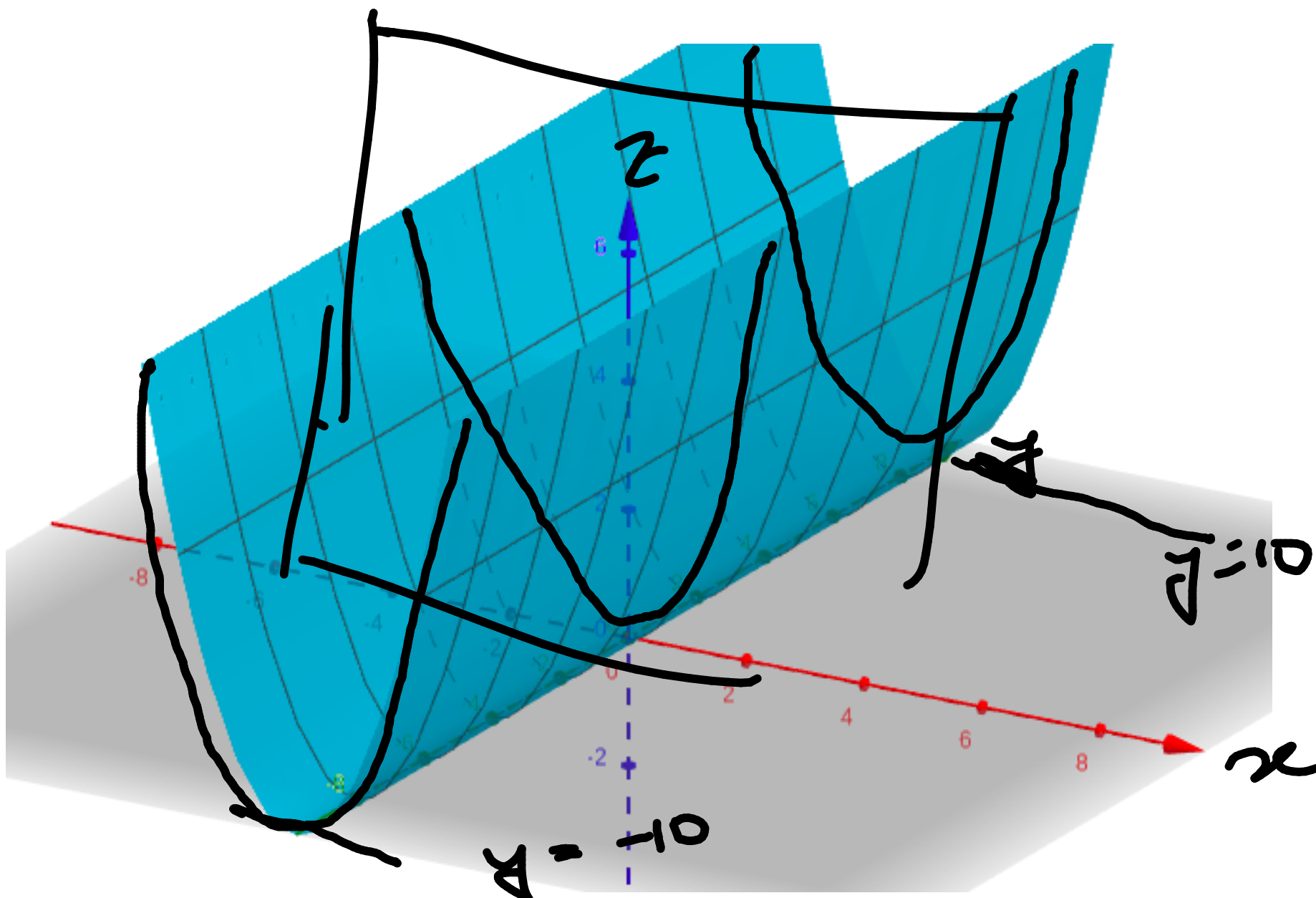# Q: Draw the graph of
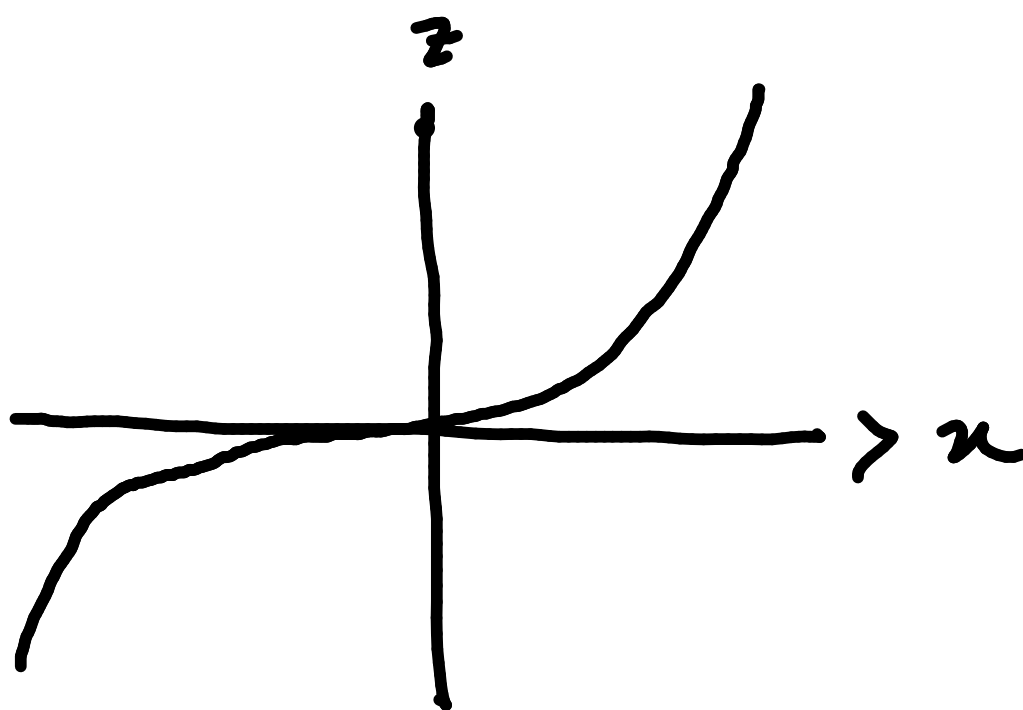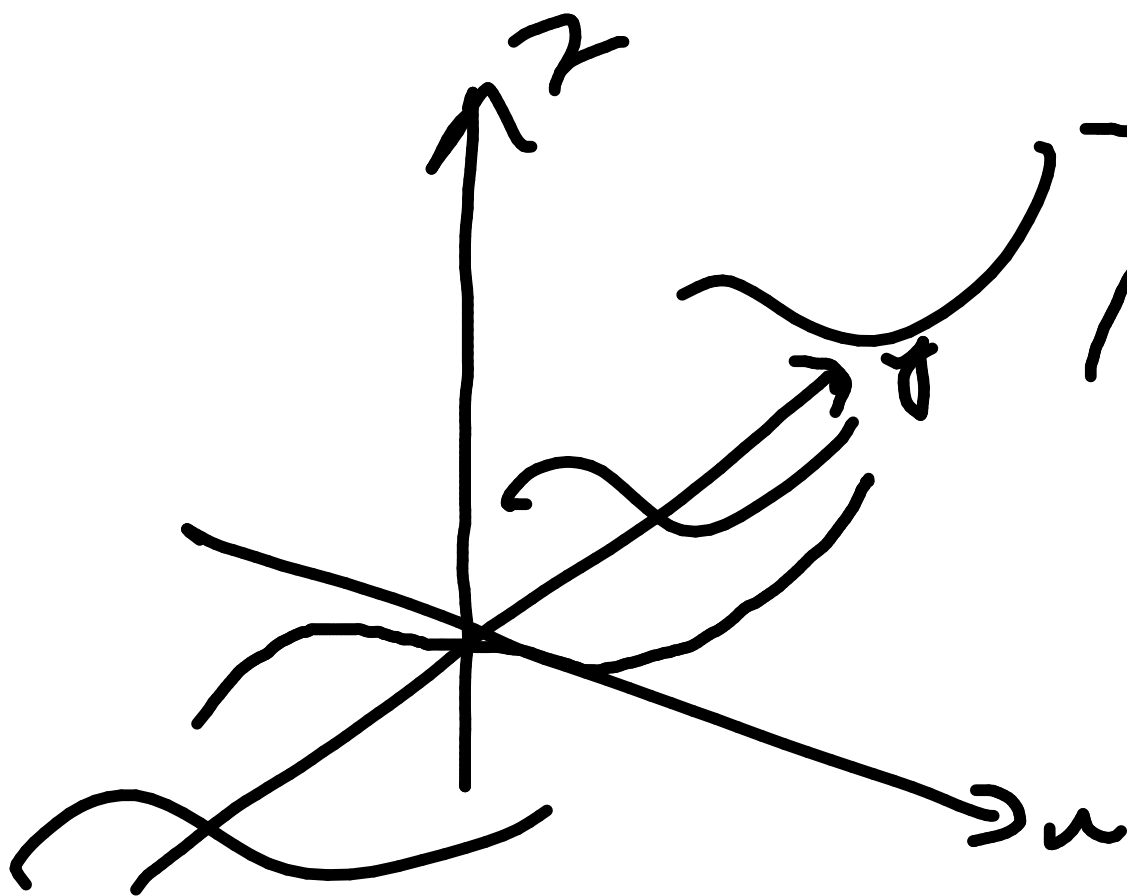
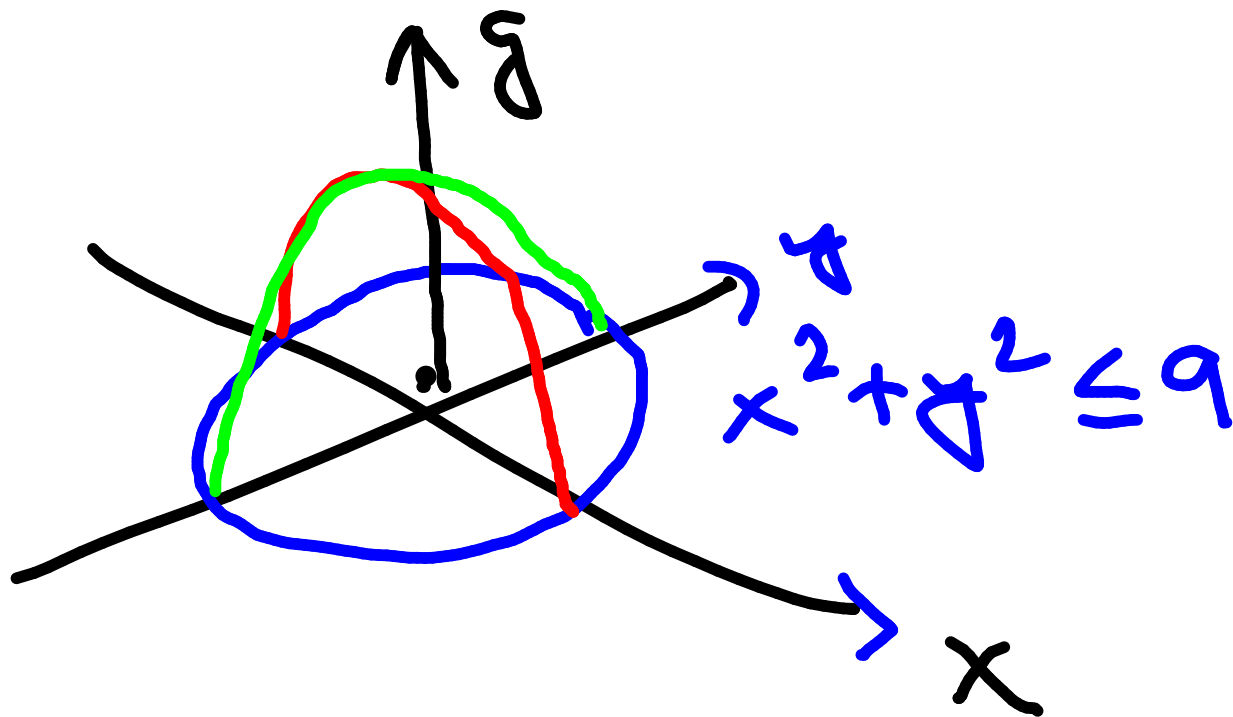$$f(x,y) = x^2 + y^2$$

domain? the entire xy plane

$f(x, y) = x^2$

Q. $f(x, y) = x^3$

EXAMPLE 4 Sketch the graph of $g(x, y) = \sqrt{9 - x^2 - y^2}$.

domain : $\qquad$ $x^2 + y^2 \le 9$

$x^2 + y^2 \le 9$

above x-axis

$g = \sqrt{9 - x^2}$

$x^2 + y^2 = 9$

(a) $f(x, y) = (x^2 + 3y^2)e^{-x^2-y^2}$
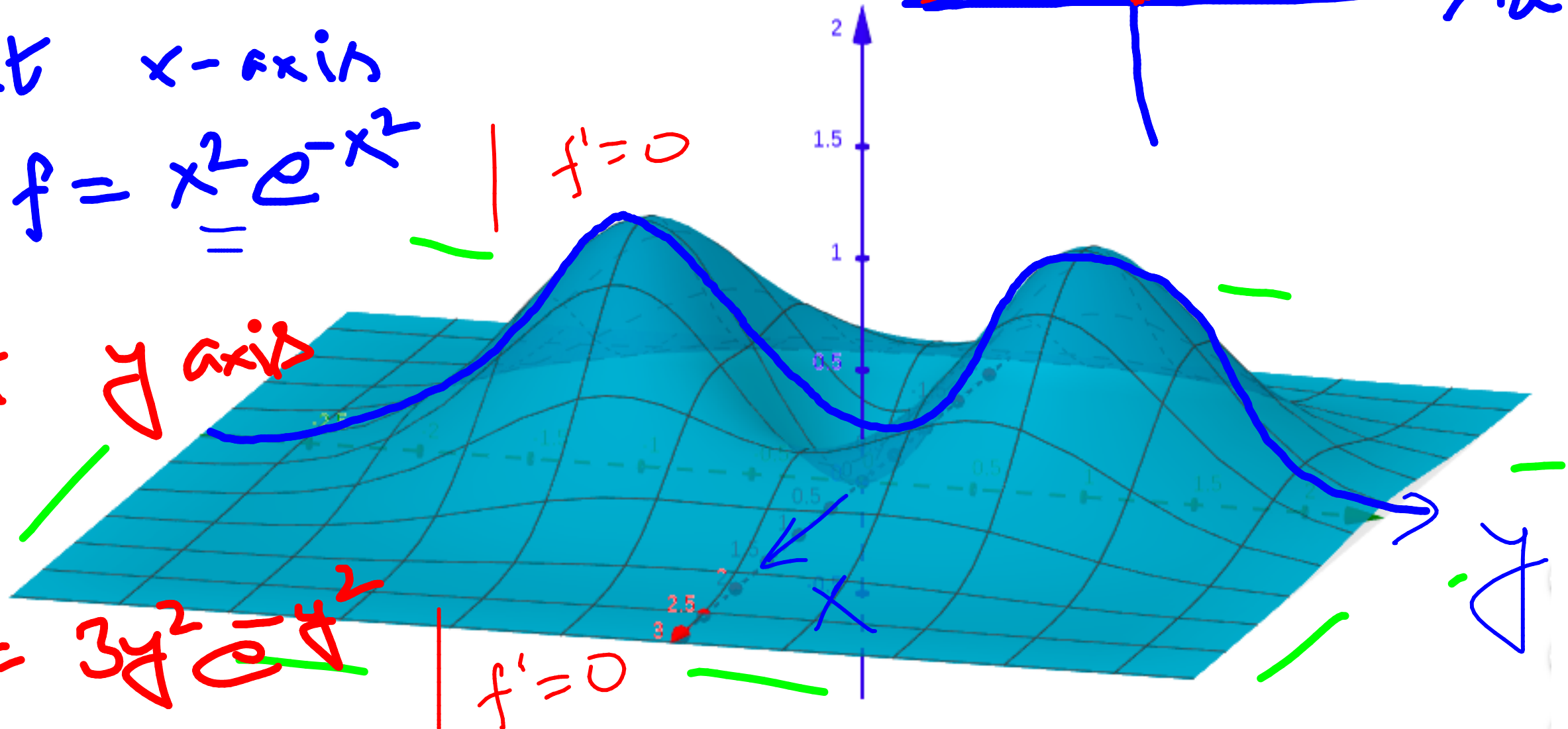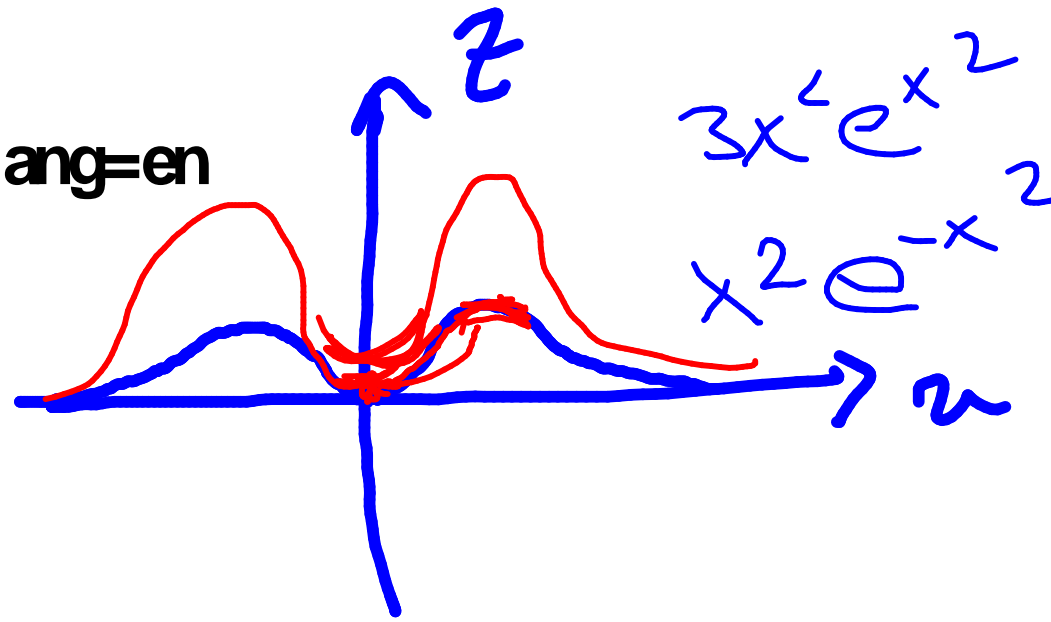
$3x^2 e^{x^2}$

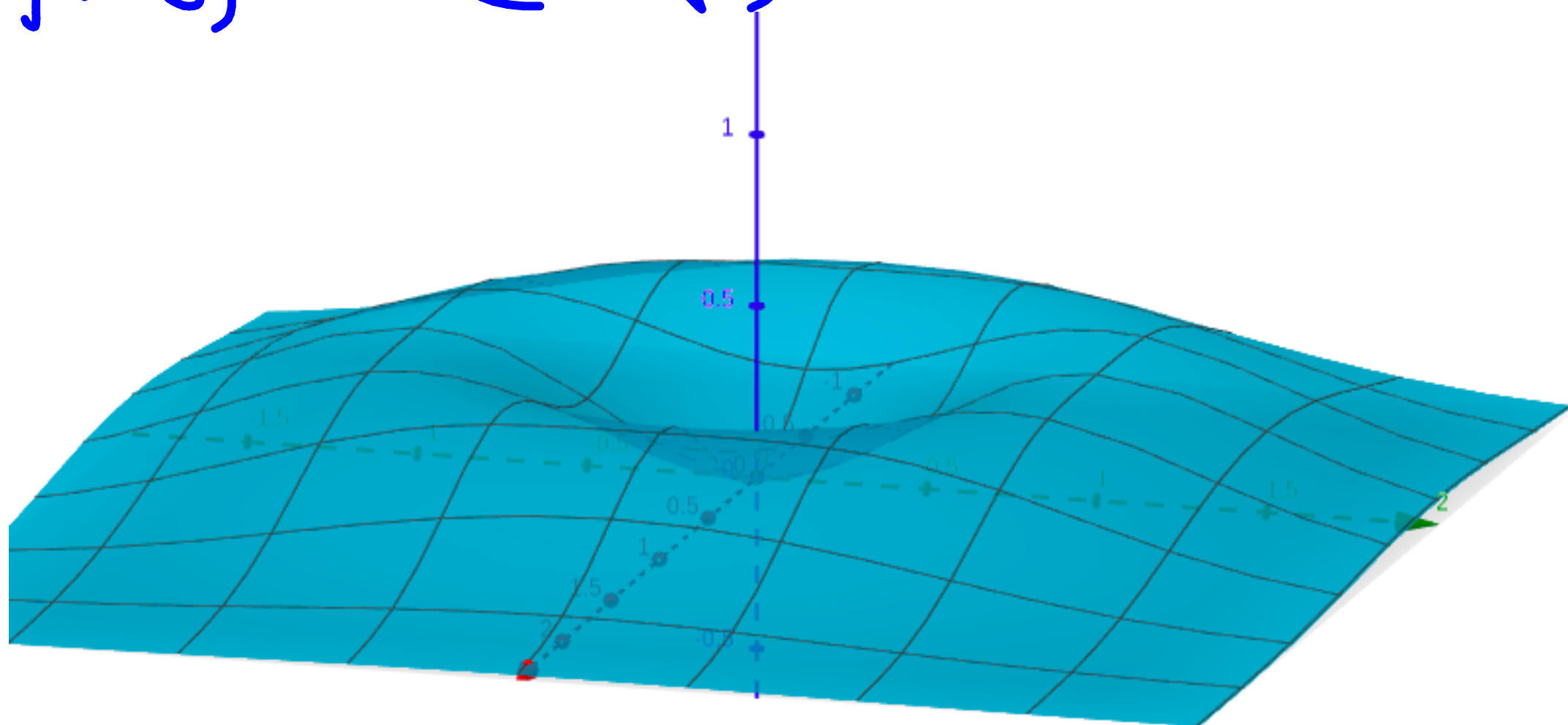$x^2 e^{-x^2}$

at x-axis

$f = x^2 e^{-x^2}$ | $f' = 0$

at y axis

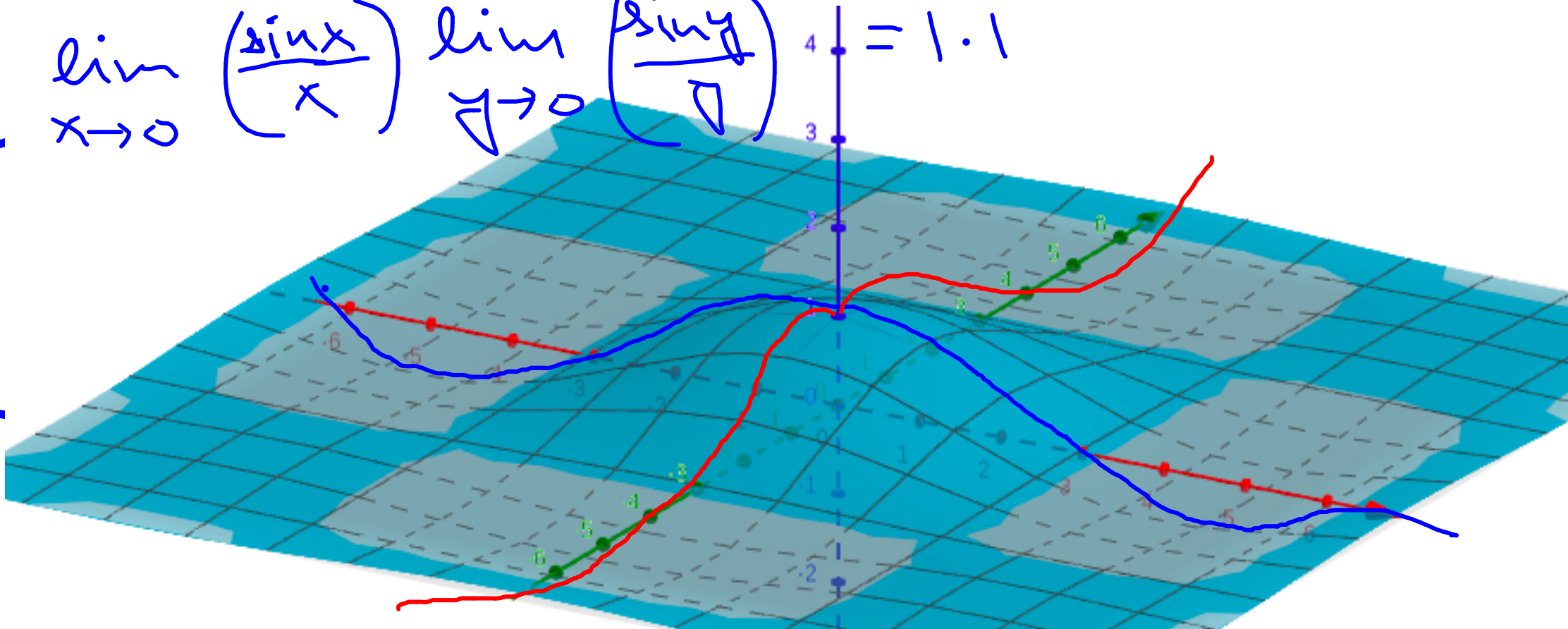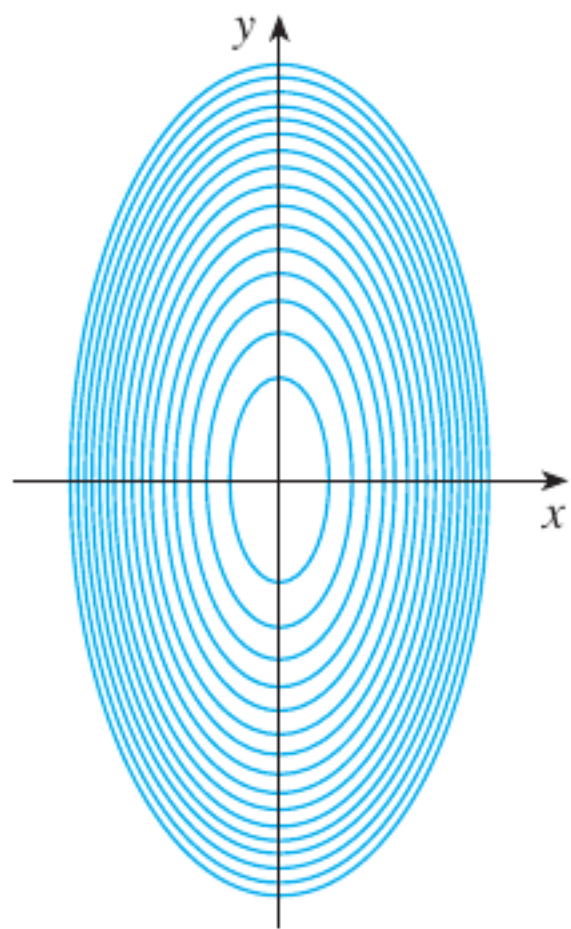$f = 3y^2 e^{-y^2}$ | $f' = 0$

$$f(x,y) = (x^2 + y^2)e^{-x^2-y^2}$$

waves with decreasing amplitude.
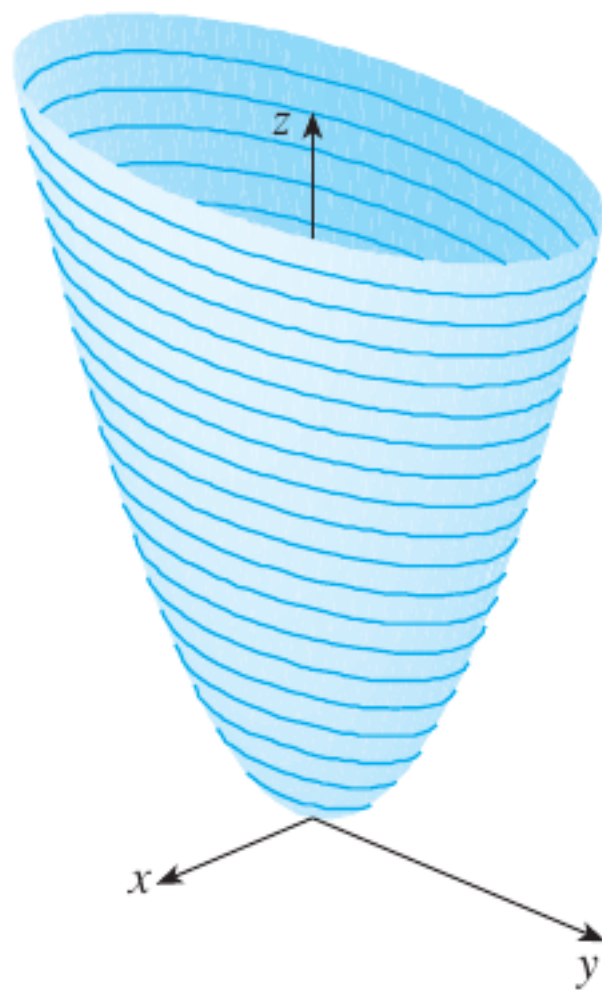
(d) $f(x, y) = \dfrac{\sin x \, \sin y}{xy}$

$\lim\limits_{x \to 0} \left(\dfrac{\sin x}{x} \right) \lim\limits_{y \to 0} \left(\dfrac{\sin y}{y} \right) = 1 \cdot 1$
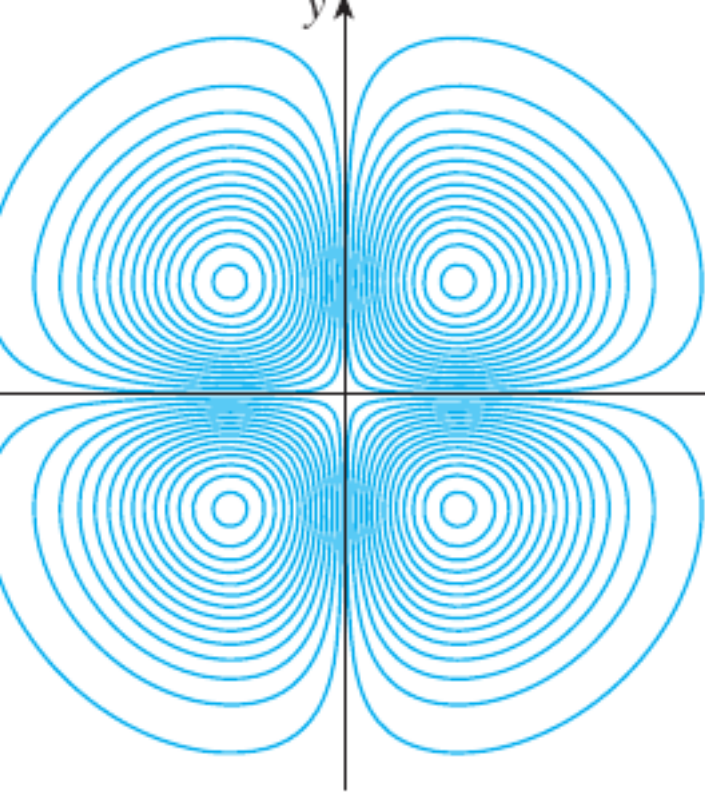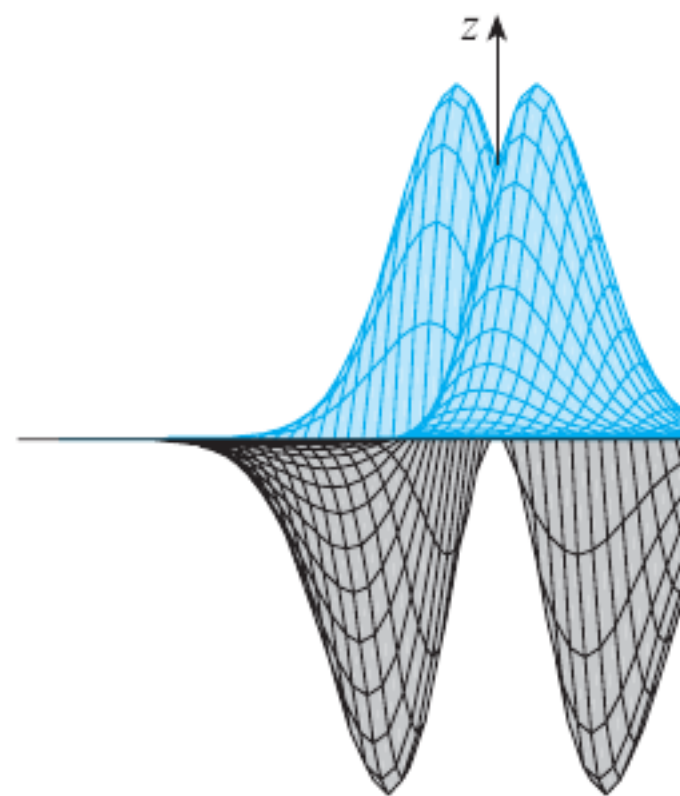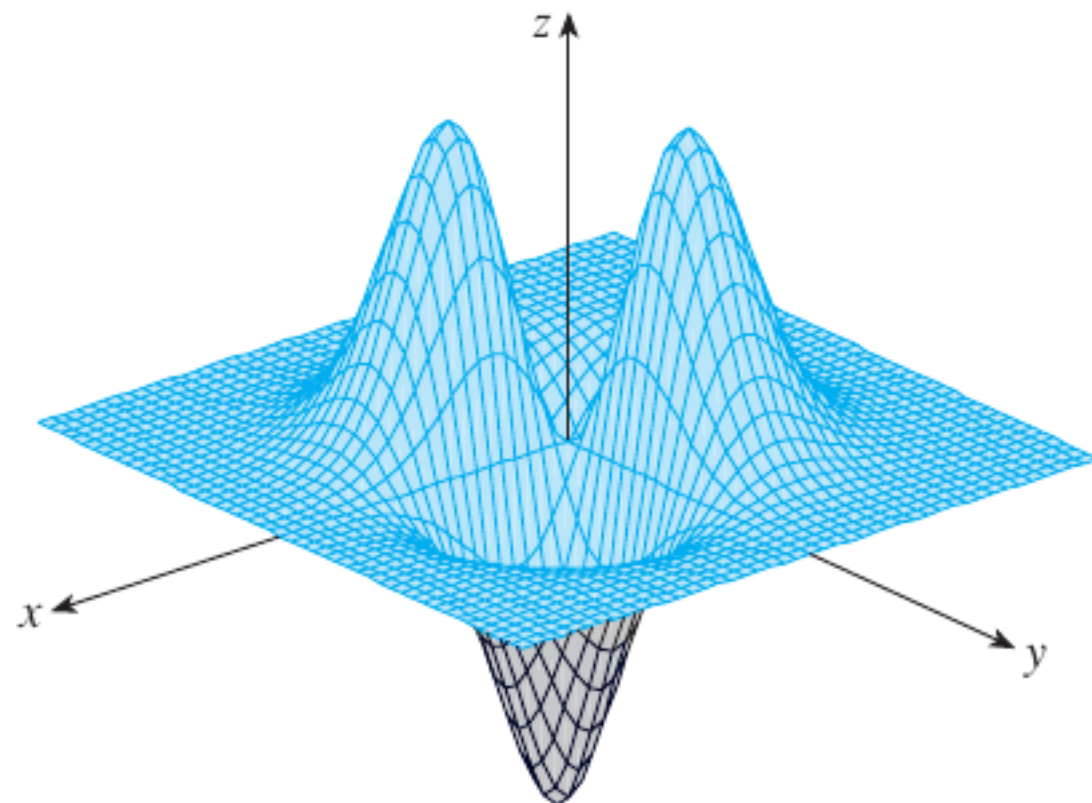
contour curves, or *level curves.*

(a) Contour map

(b) Horizontal traces are raised level curves

(a) Level curves of $f(x, y) = -xye^-$

(b) Two views of $f(x, y) = -xye^{-x^2-y^2}$

EXAMPLE 10 Find the domain of f if $f(x, y, z) = \ln(z - y) + xy \sin z$.

EXAMPLE 11 Find the level surfaces of the function $f(x, y, z) = x^2 + y^2 + z^2$.

Match the function