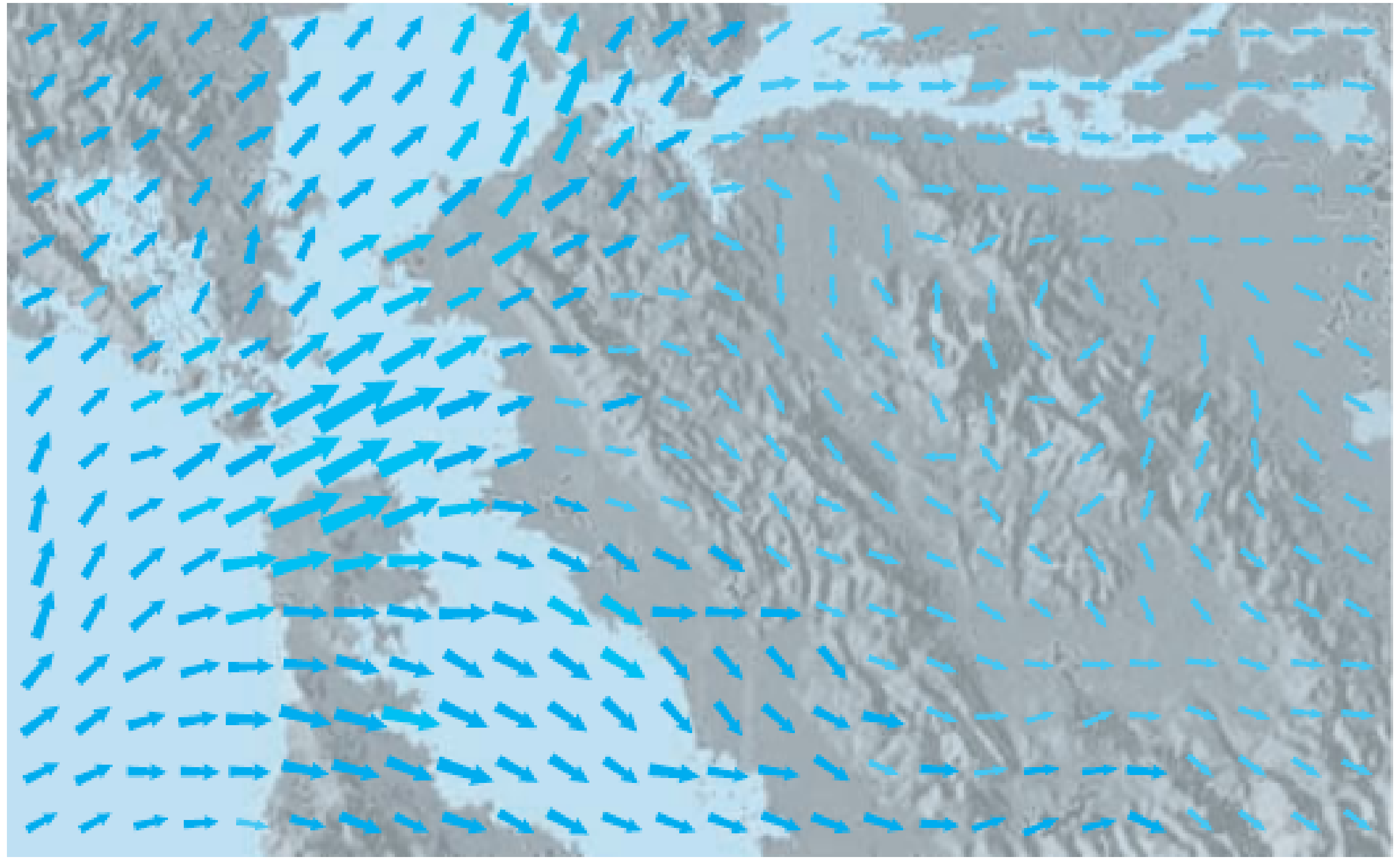
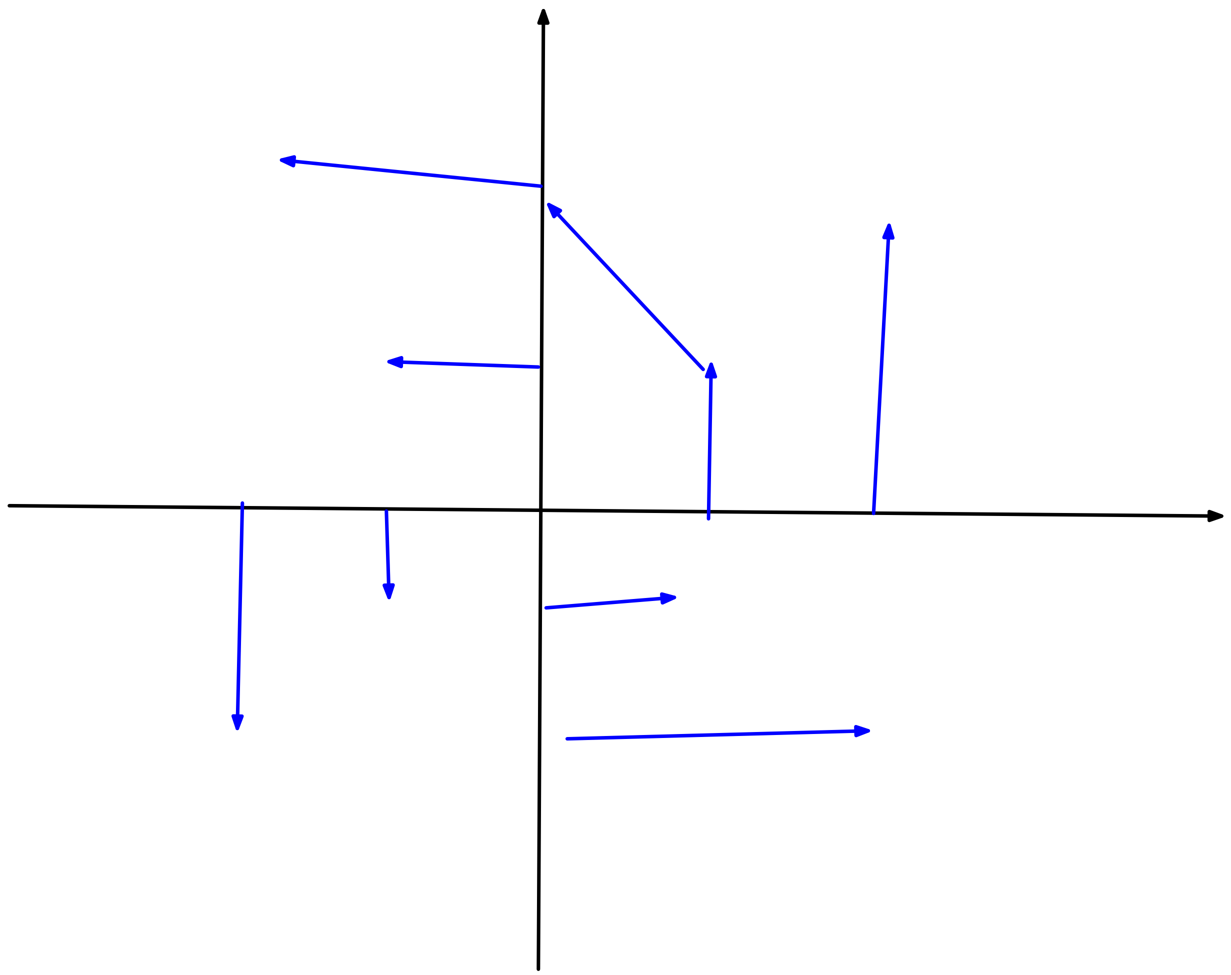
13 VECTOR CALCULUS

13.1 VECTOR FIELDS



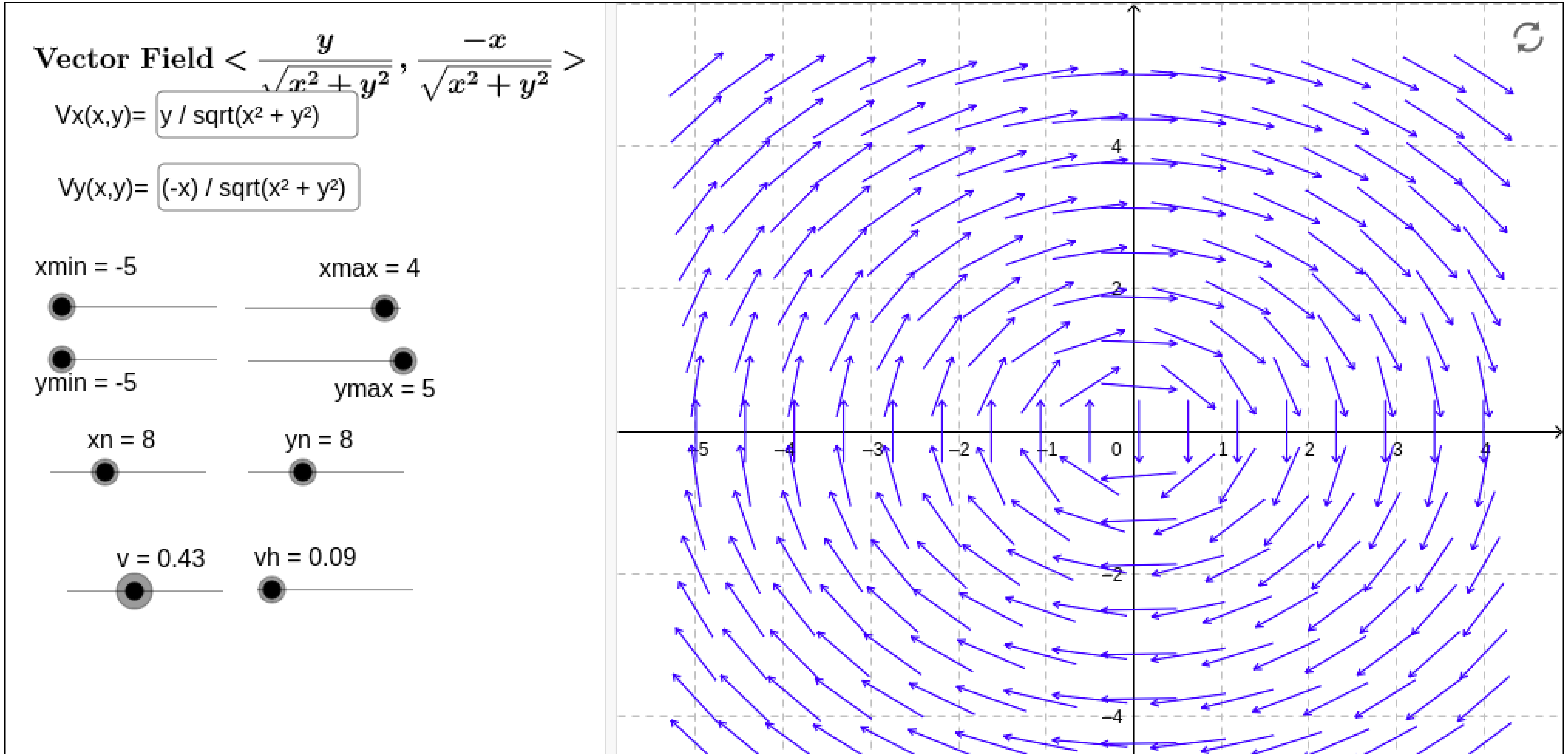
Sketch the vector field **F**

$$\mathbf{F}(x, y) = -y\,\mathbf{i} + x\,\mathbf{j}.$$



Sketch the vector field $\mathbf{F}$ $\quad \mathbf{F}(x, y) = \dfrac{y\,\mathbf{i} - x\,\mathbf{j}}{\sqrt{x^2 + y^2}}$

geogebra.org/m/QPE4PaDZ

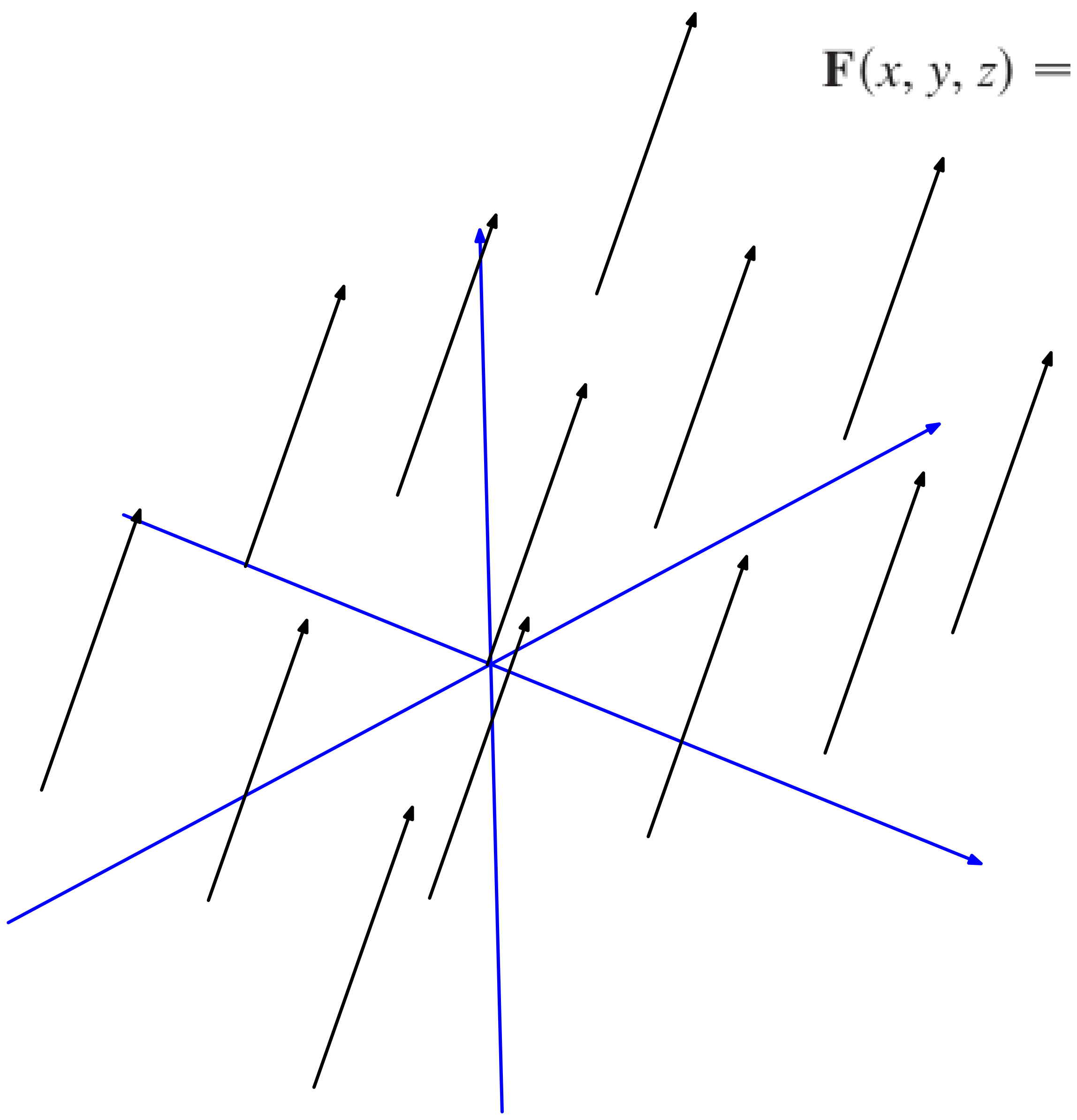


Sketch the vector field **F**

$$\mathbf{F}(x, y) = (x - y)\,\mathbf{i} + x\,\mathbf{j}$$

Sketch the vector field $\mathbf{F}$

$$\mathbf{F}(x, y, z) = \mathbf{i} + 2\,\mathbf{j} + 3\,\mathbf{k}$$



Sketch the vector field $\mathbf{F}$

$$\mathbf{F}(x, y, z) = \mathbf{i} + 2\,\mathbf{j} + z\,\mathbf{k}$$

Vector Fields

$$\mathbb{R}^2 \longrightarrow \mathbb{R}^2$$

$$\vec{F}(x,y) = P\hat{i} + Q\hat{j}$$

$$= F_1\hat{i} + F_2\hat{j}$$

$$\mathbb{R}^3 \longrightarrow \mathbb{R}^3$$

$$\vec{F}(x,y,z) = P\hat{i} + Q\hat{j} + R\hat{k}$$

$$= F_1\hat{i} + F_2\hat{j} + F_3\hat{k}$$

GRADIENT FIELDS

If f is a scalar function of two variables, recall from Section 11.6 that its gradient ∇f (or grad f) is defined by

$$\nabla f(x, y) = f_x(x, y)\,\mathbf{i} + f_y(x, y)\,\mathbf{j}$$

Therefore, ∇f is really a vector field on $\mathbb{R}^2$ and is called a **gradient vector field**. Likewise, if f is a scalar function of three variables, its gradient is a vector field on $\mathbb{R}^3$ given by

$$\nabla f(x, y, z) = f_x(x, y, z)\,\mathbf{i} + f_y(x, y, z)\,\mathbf{j} + f_z(x, y, z)\,\mathbf{k}$$

13.1

later

EXAMPLE 6 Find the gradient vector field of $f(x, y) = x^2y - y^3$. Plot the gradient vector field together with a contour map of f. How are they related?

A vector field $\mathbf{F}$ is called a **conservative vector field** if it is the gradient of some scalar function, that is, if there exists a function f such that $\mathbf{F} = \nabla f$. In this situation f is called a **potential function** for $\mathbf{F}$.

A particle moves in a velocity field $\mathbf{V}(x, y) = \langle x^2, x + y^2 \rangle$. If it is at position $(2, 1)$ at time $t = 3$, estimate its location at time $t = 3.01$.

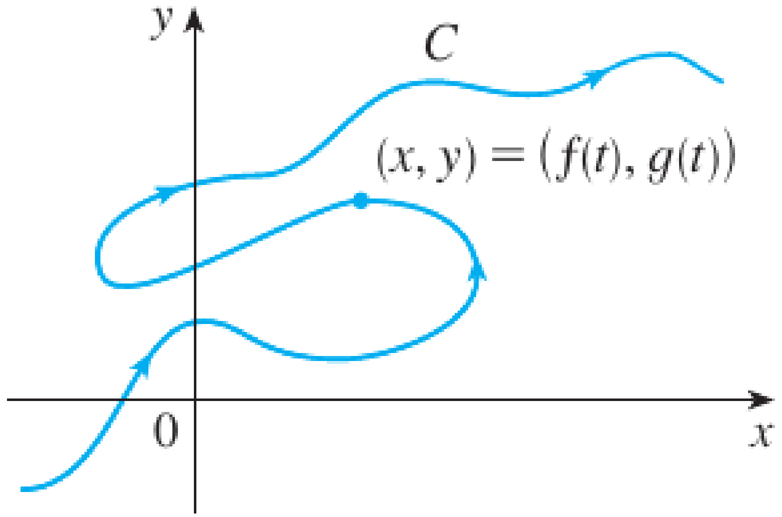
13.2 LINE INTEGRALS

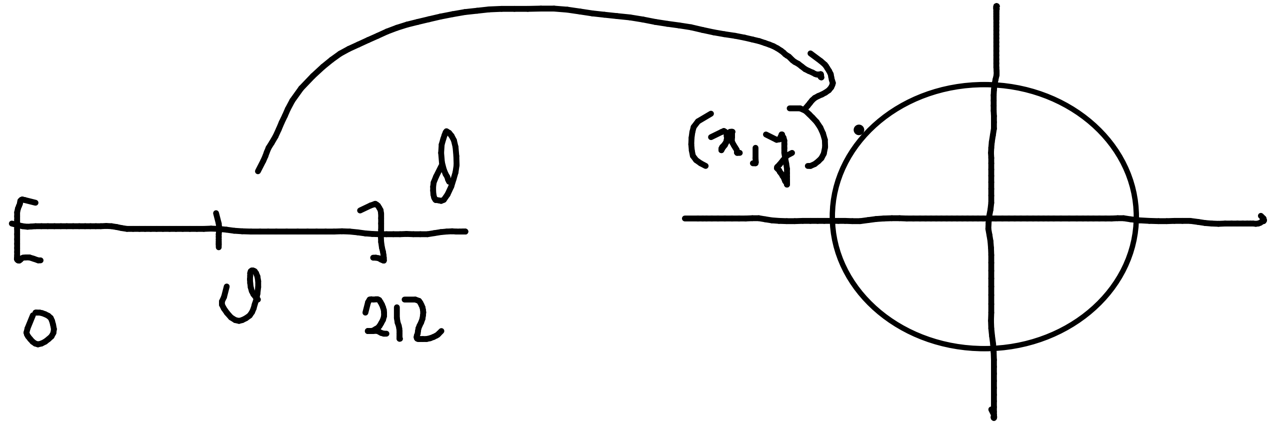
next time

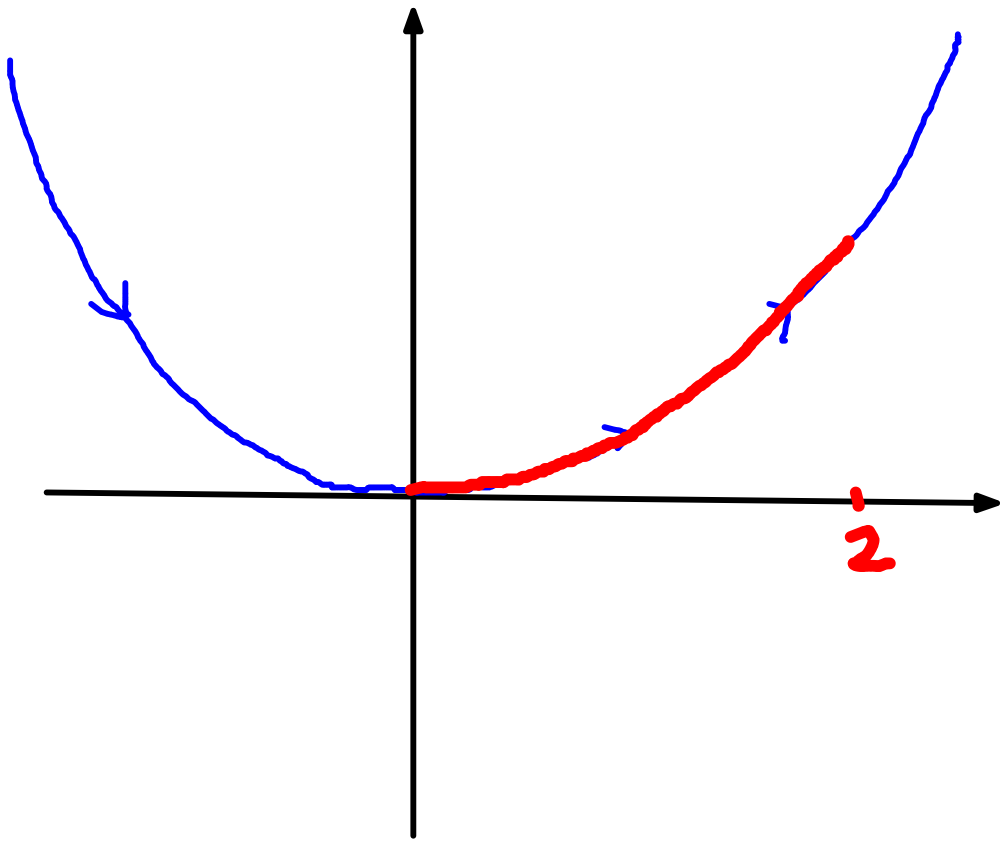
9.1 PARAMETRIC CURVES

:

$$\boxed{\begin{aligned} x &= \cos\theta \\ y &= \sin\theta \end{aligned} \qquad 0 \le \theta \le 2\pi}$$



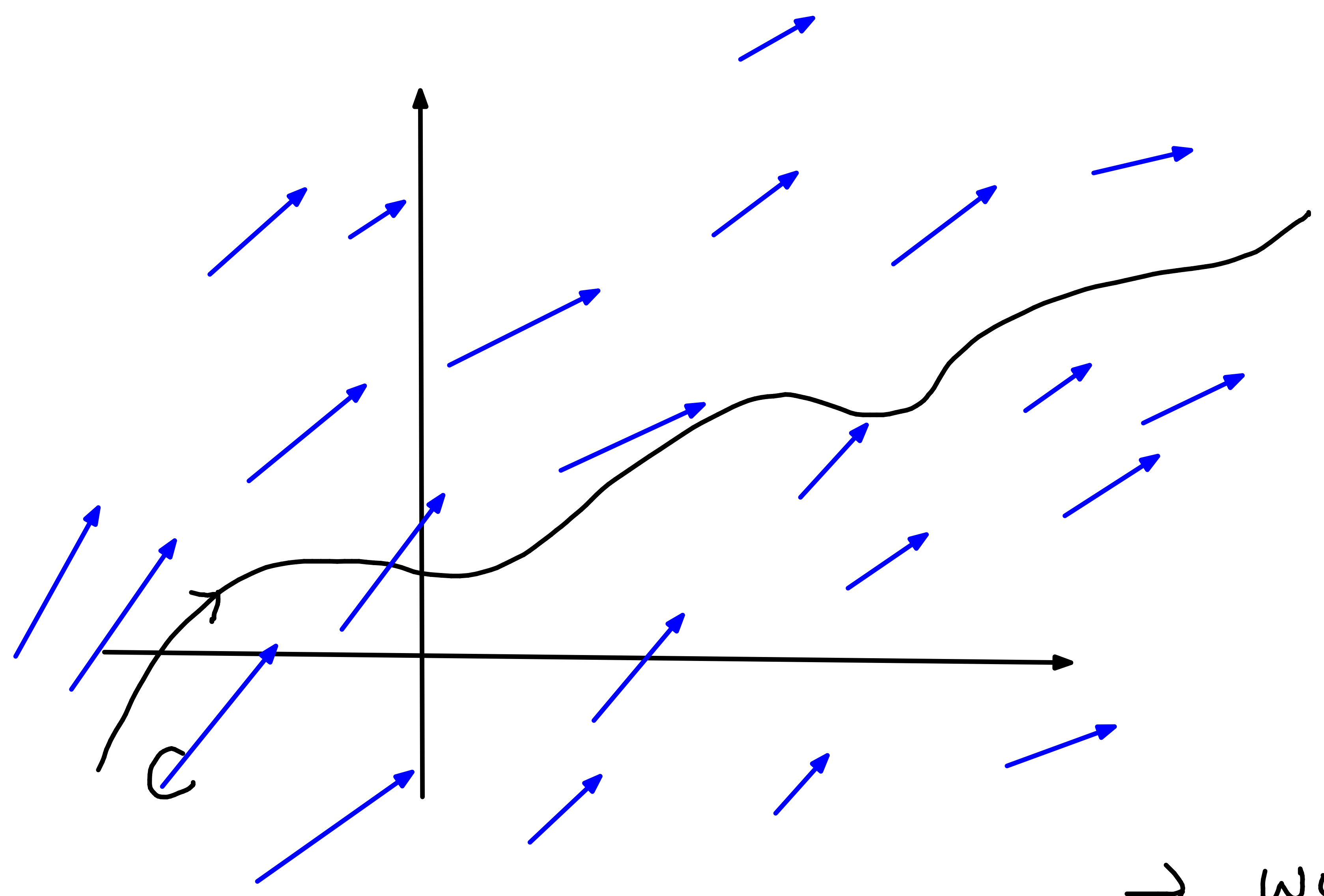




$y = x^2$

$$\left[\begin{array}{l} -\infty \le t \le \infty \\ x = t \\ y = t^2 \end{array}\right]$$

$$\boxed{\begin{array}{l} 0 \le t \le 2 \\ x = t \\ y = t^2 \end{array}} \longrightarrow \text{what curve ??}$$



- Force field $\vec{F}$
- a particle is moving along a curve C

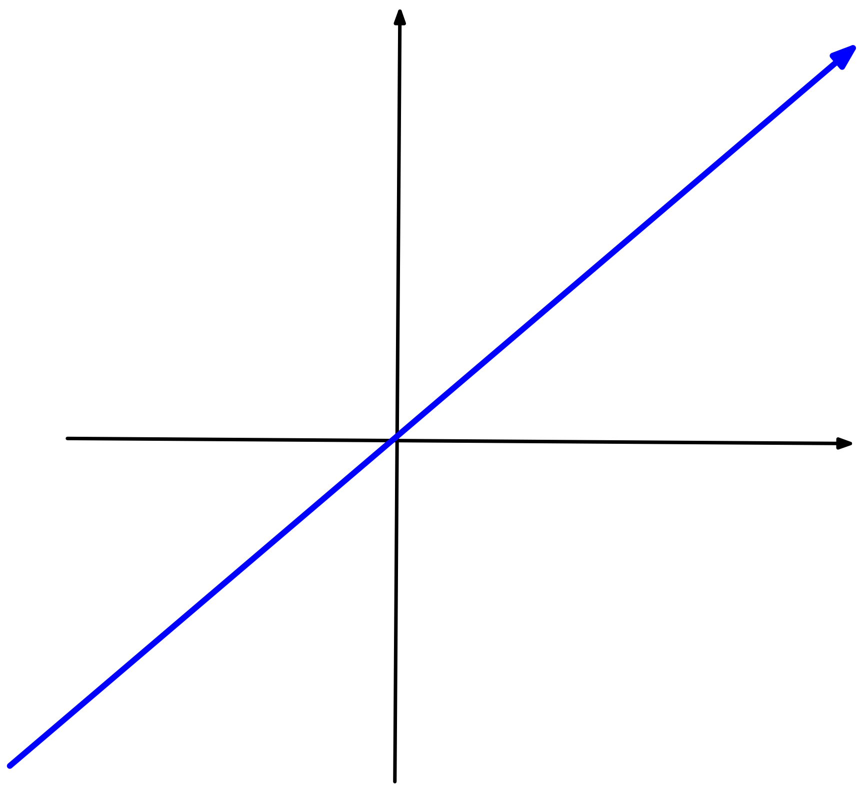
Q: find work done by $\vec{F}$ in moving the particle along the curve ??

→ we need a better precise description of whats a curve or path.

$$-6\pi \le t \le 6\pi$$

$$x = t$$

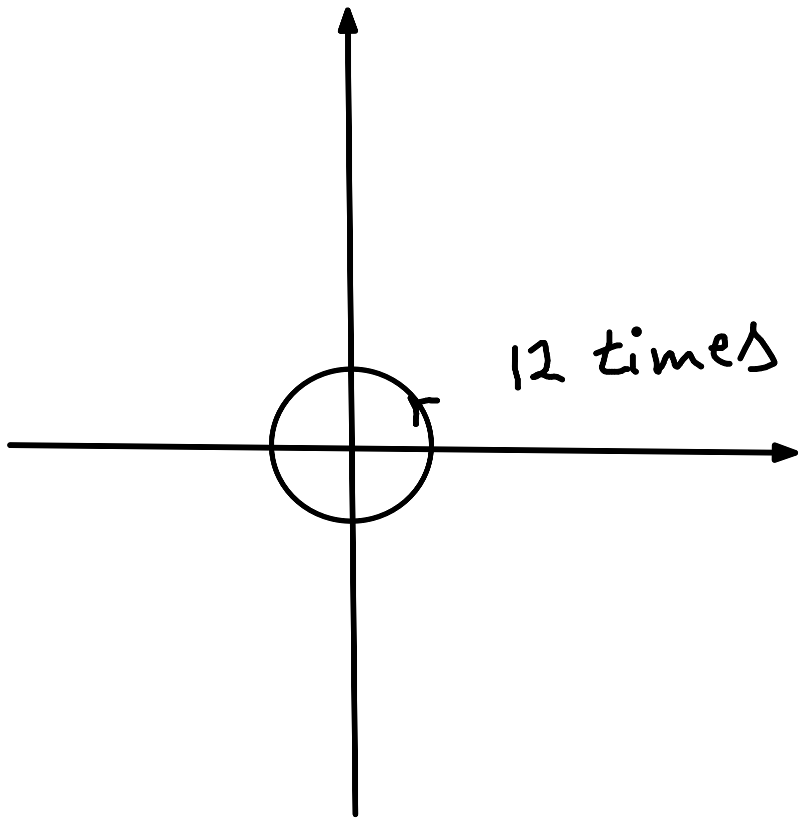
$$y = t$$



$$-6\pi \le t \le 6\pi$$

$$x = 0.1\cos(2t)$$

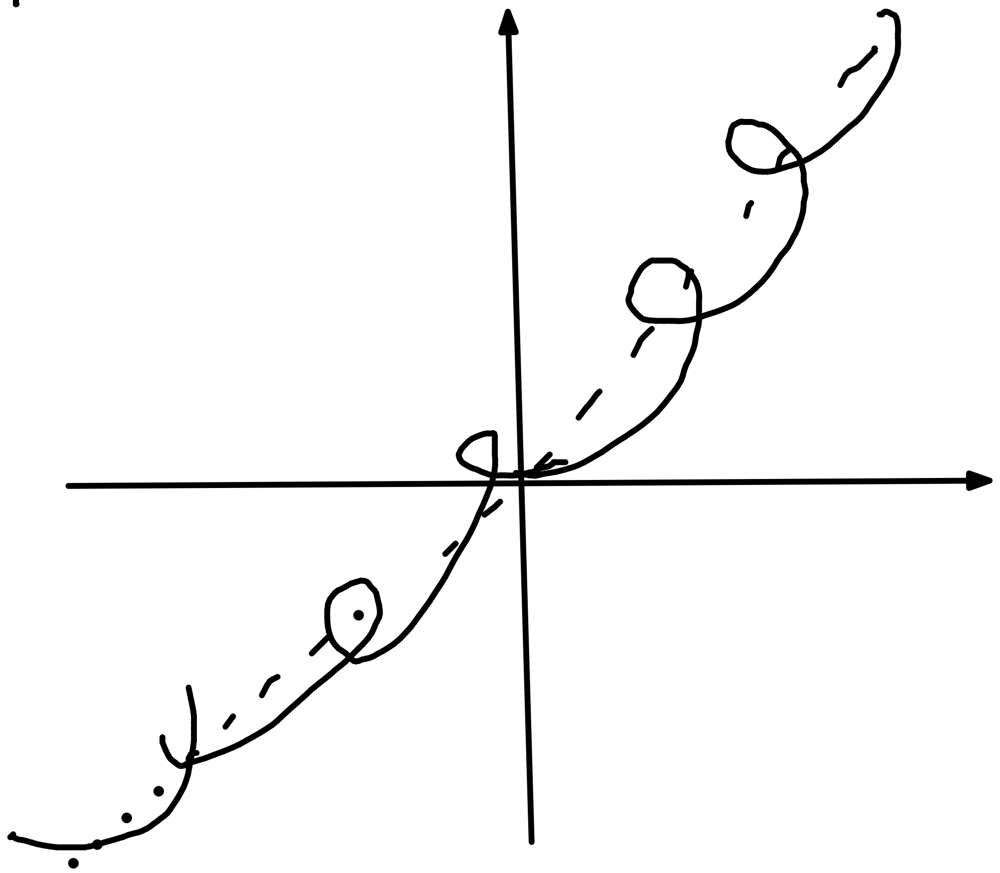
$$y = 0.1\sin(2t)$$



$$-6\pi \le t \le 6\pi$$

$$x = t + 0.1\cos(2t)$$

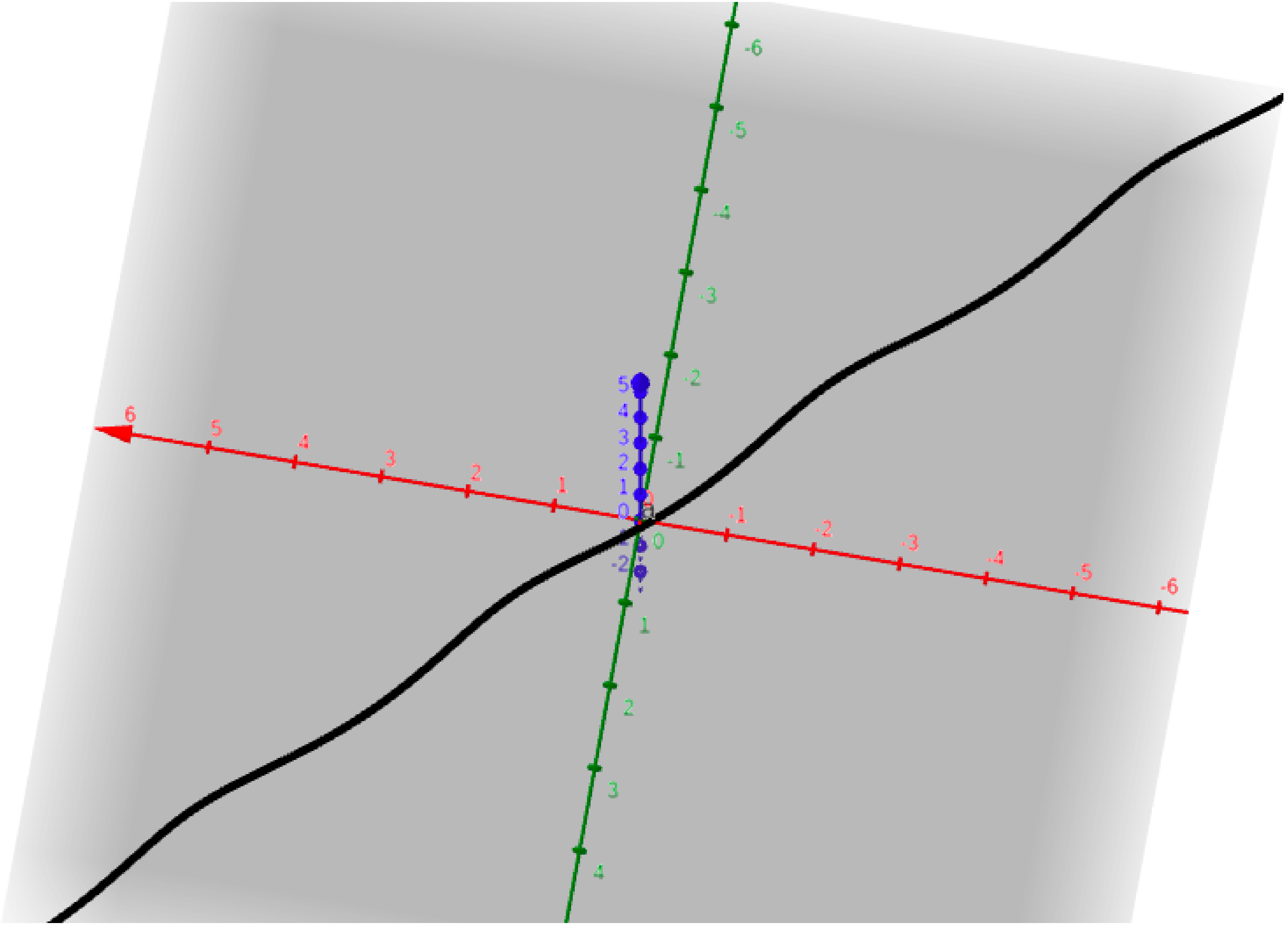
$$y = t + 0.1\sin(2t)$$



$a = \text{Curve}(t + 0.1\ \sin(2\ t), t + 0.1\ \cos(2\ t), t, -6\ \pi, 6\ \pi$

$$\rightarrow \left.\begin{array}{l} x = t + 0.1\ \sin(2\ t) \\ y = t + 0.1\ \cos(2\ t) \end{array}\right\} -18.85 \leq t \leq 18.85$$

Input...



EXAMPLE 1 Sketch and identify the curve defined by the parametric equations

$$x = t^2 - 2t \qquad y = t + 1$$

$$x = \cos t \qquad y = \sin t \qquad 0 \leqslant t \leqslant 2\pi$$

$$x = t + 2\sin 2t$$
$$y = t + 2\cos 5t$$

Evaluate the line integral, where C is the given curve.

$$\int_C y\, ds, \quad C: x = t^2,\ y = t,\ 0 \leqslant t \leqslant 2$$

Evaluate the line integral, where C is the given curve.

$$\int_C xy^3\, ds,$$
$$C\colon x = 4 \sin t,\ \ y = 4 \cos t,\ \ z = 3t,\ \ 0 \leqslant t \leqslant \pi/2$$