ABSOLUTE MAXIMUM AND MINIMUM VALUES

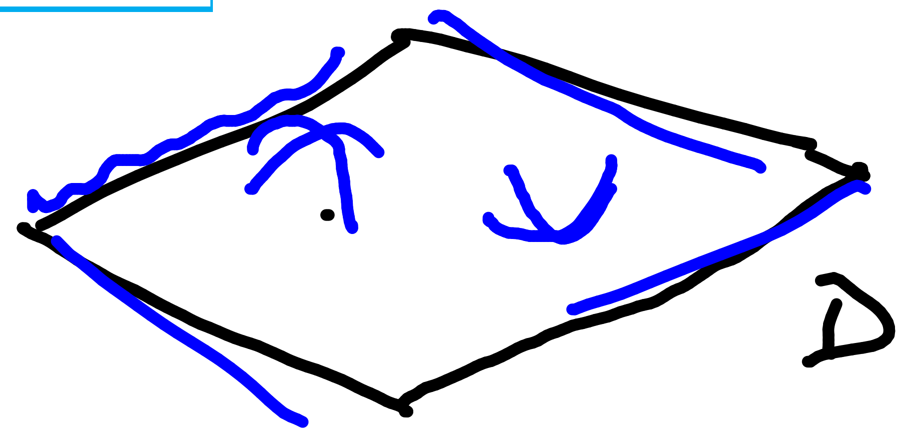
Section 11.7

continued

$f(x,y)$

5 To find the absolute maximum and minimum values of a continuous function f on a closed, bounded set D:

1. Find the values of f at the critical points of f in D.
2. Find the extreme values of f on the boundary of D.
3. The largest of the values from steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.



now we will maximize & minimize on a bounded domain:

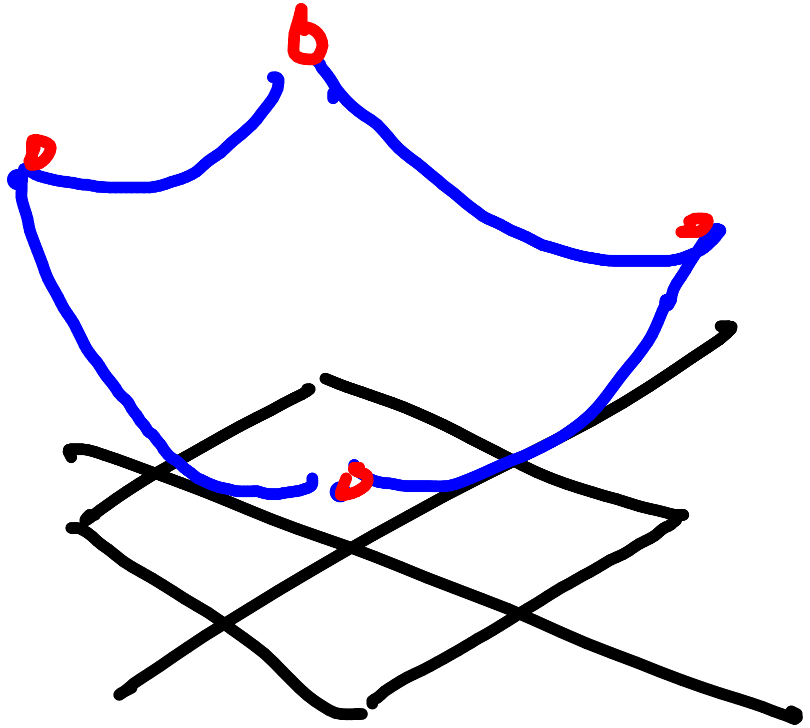
Q: $f(x,y) = x^2+y^2$

minimum $= 0$

& no maximumum

Q: $f(x,y) = x^2 + y^2$, & $-1 \le x \le 1$
$-1 \le y \le 1$

whats max & min of $f(x,y)$



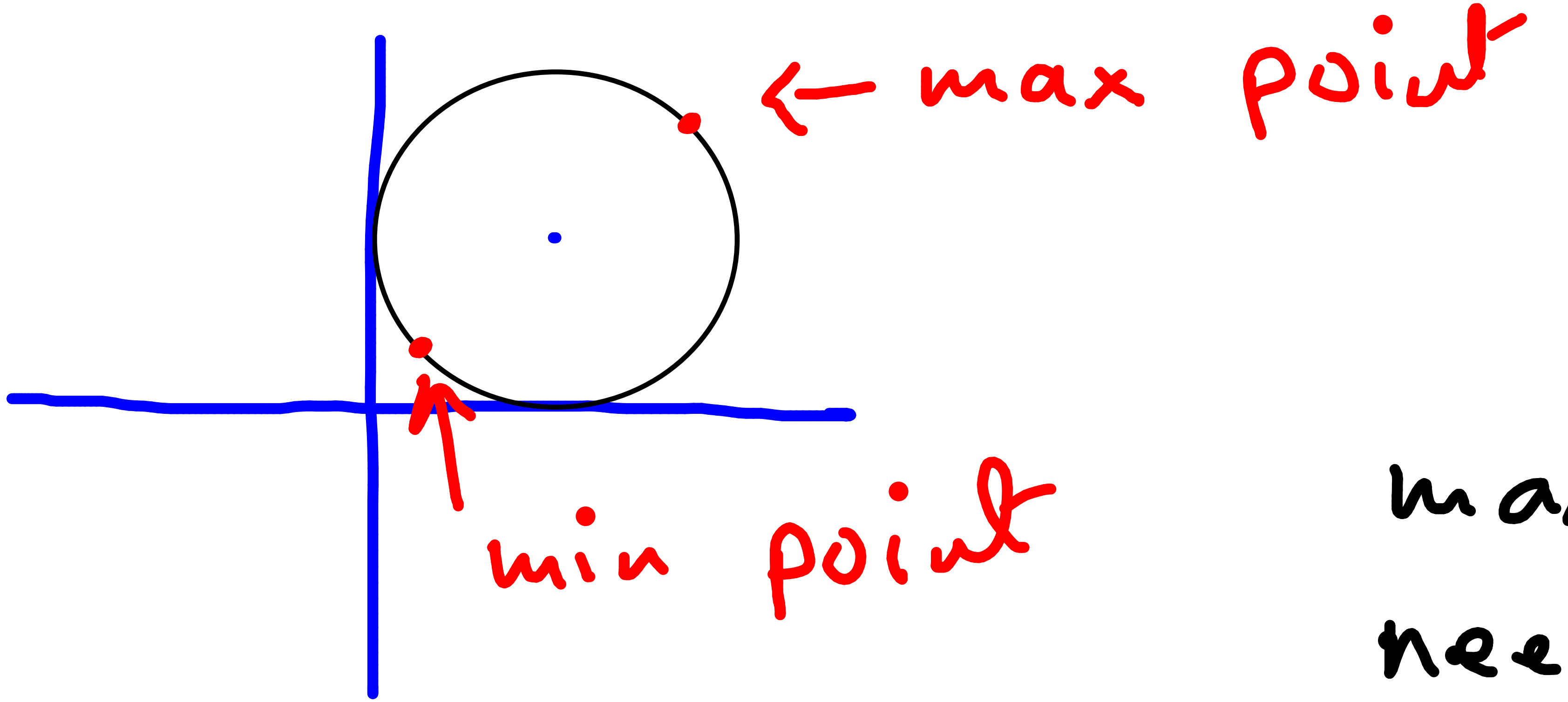
max : 2

& min $= 0$

Q: $f(x,y) = x^2 + y^2$

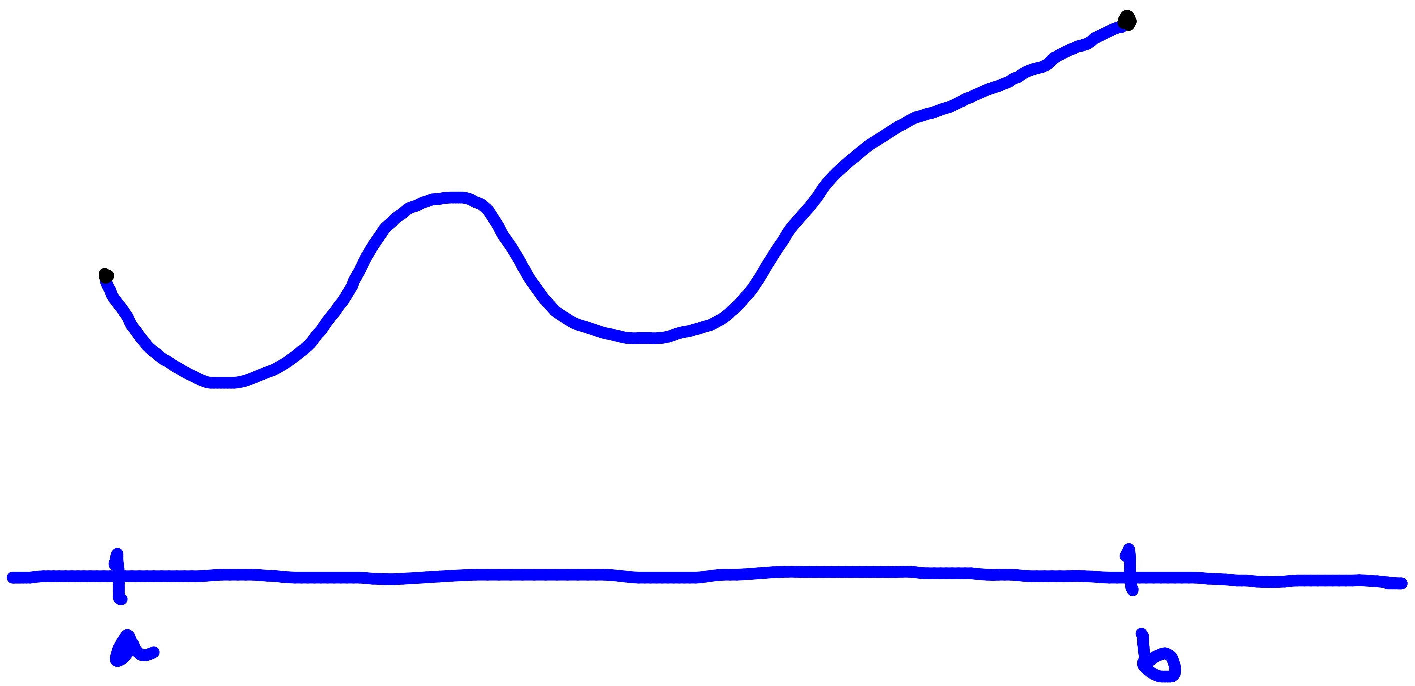
maximize $f(x,y)$ s.t.

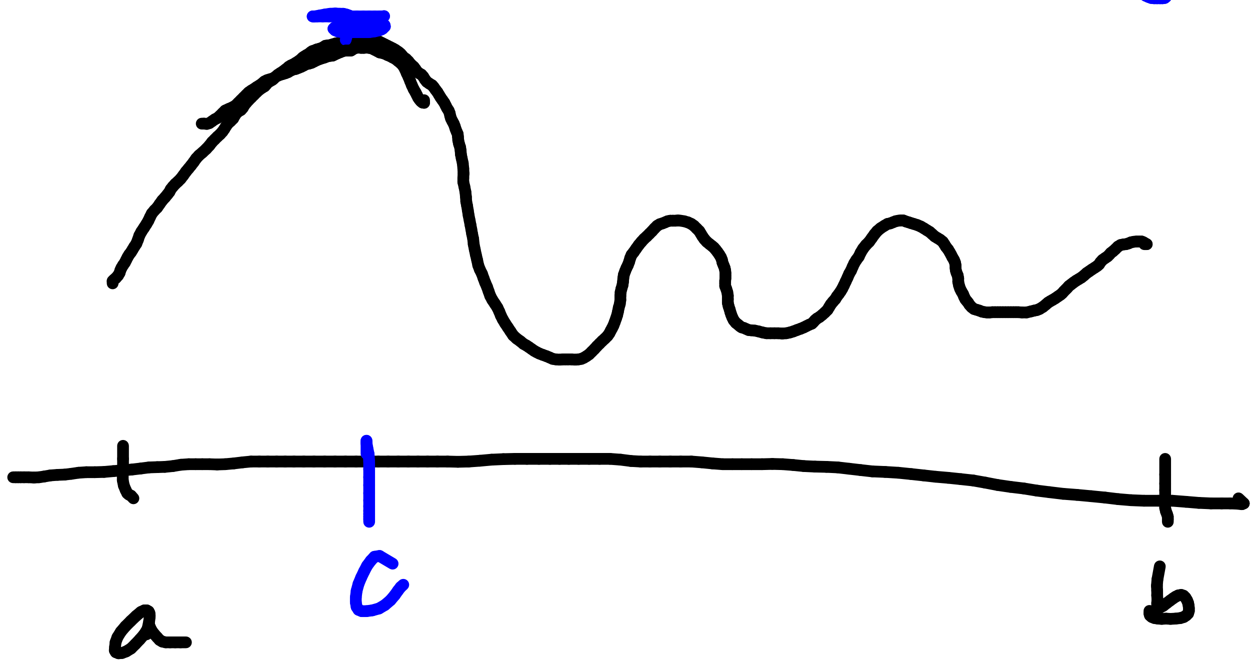
$$\underbrace{(x-1)^2 + (y-1)^2}_{\text{what shape is this}} \le 1$$



max points & min points need not be critical point if domain is bounded.

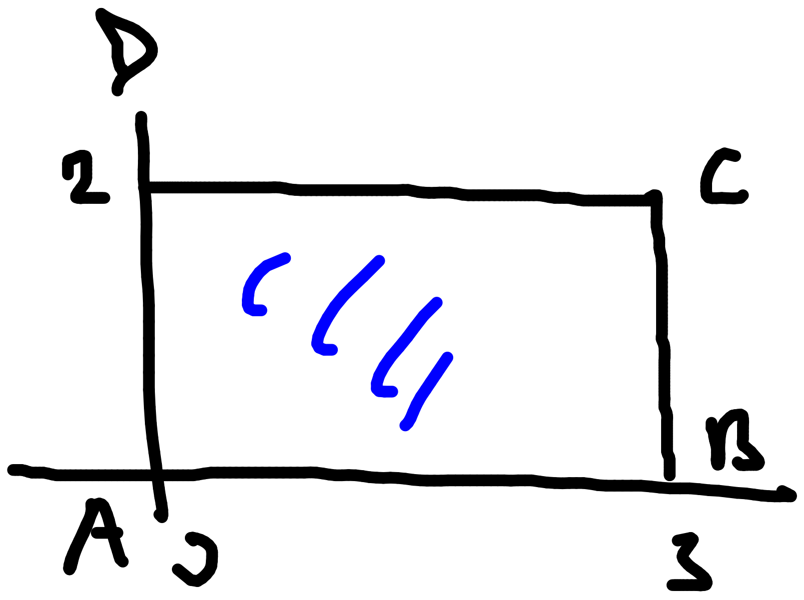
One variable calculus $f(x)$ defined on $[a,b]$





Q: is it possible that a max point in the interior is not a critical point

EXAMPLE 6 Find the absolute maximum and minimum values of the function $f(x, y) = x^2 - 2xy + 2y$ on the rectangle $D = \{(x, y) \mid 0 \leqslant x \leqslant 3, 0 \leqslant y \leqslant 2\}$.



the max can occur in the interior of the rectangle ABCD

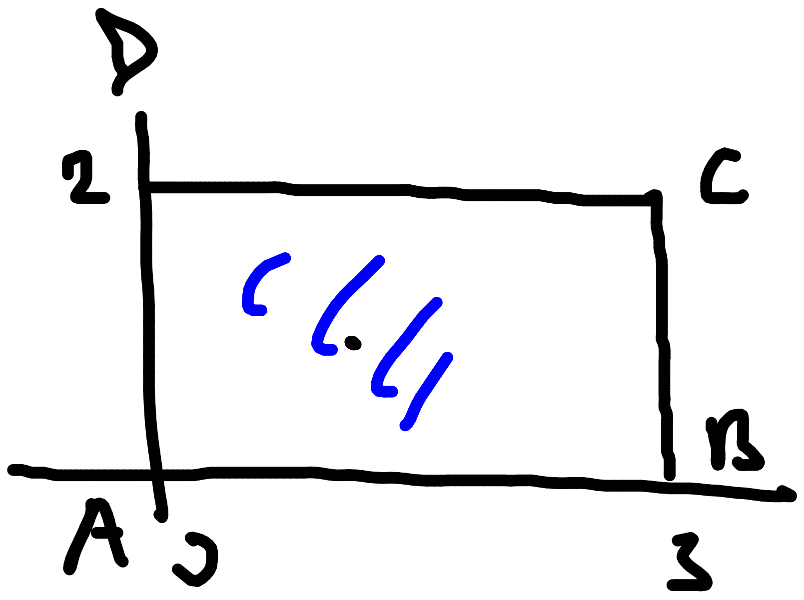
$$\frac{\partial f}{\partial x} = 0 \quad \& \quad \frac{\partial f}{\partial y} = 0$$

or the max can occur at any point on the line AB, BC, CD, or DA (not only only in corners)

EXAMPLE 6 Find the absolute maximum and minimum values of the function $f(x, y) = x^2 - 2xy + 2y$ on the rectangle $D = \{(x, y) \mid 0 \leqslant x \leqslant 3, 0 \leqslant y \leqslant 2\}$.

the max can occur in the interior of the rectangle ABCD

$\frac{\partial f}{\partial x} = 0$ & $\frac{\partial f}{\partial y} = 0$



critical points:

$$2x - 2y = 0$$
$$-2x + 2 = 0$$

$x = 1$, $y = 1$

if abs max occurs in the interior it will only occur at (1,1)

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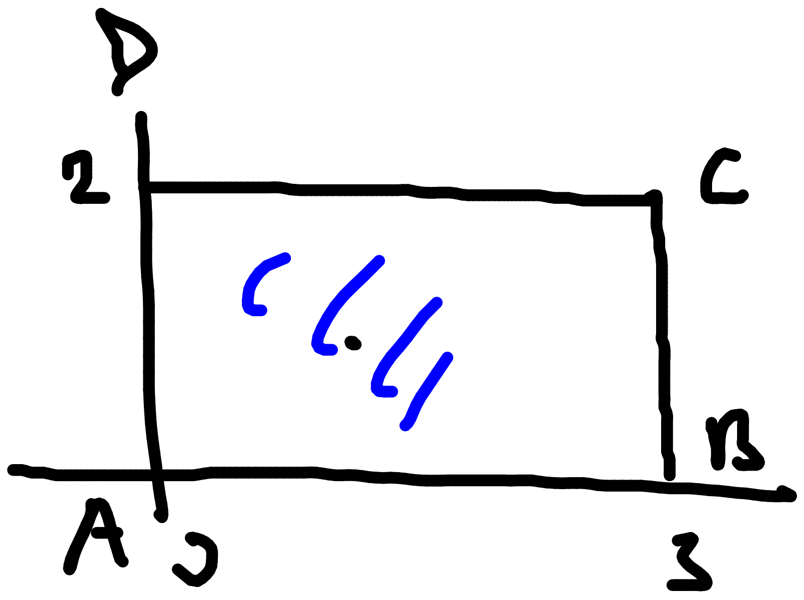


Q: what's max/min of $f(x,y)$ on the line AB

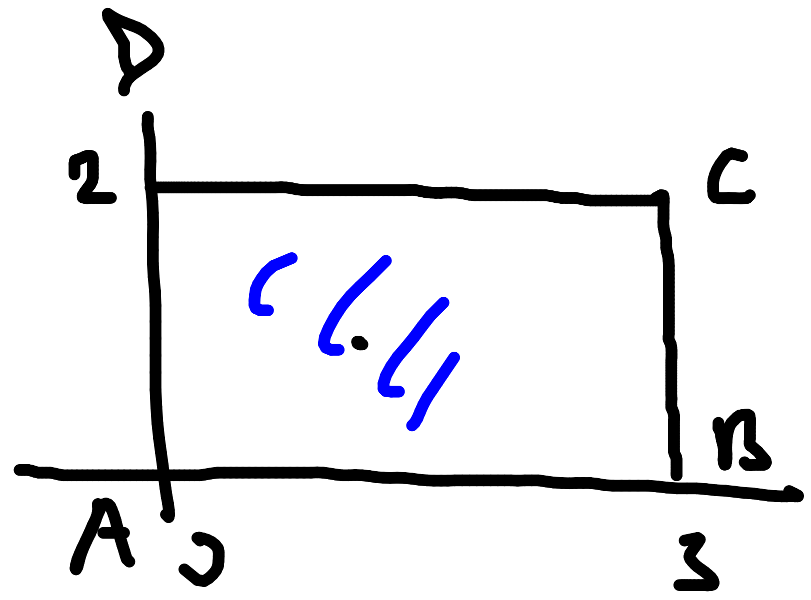
$0 \le x \le 3, \quad y = 0$

$f\big|_{AB} = x^2 \longrightarrow$ min at $x = 0$

max at $x = 3$

Repeat this for all line BC, CD, DA

EXAMPLE 6 Find the absolute maximum and minimum values of the function $f(x, y) = x^2 - 2xy + 2y$ on the rectangle $D = \{(x, y) \mid 0 \leqslant x \leqslant 3, 0 \leqslant y \leqslant 2\}$.



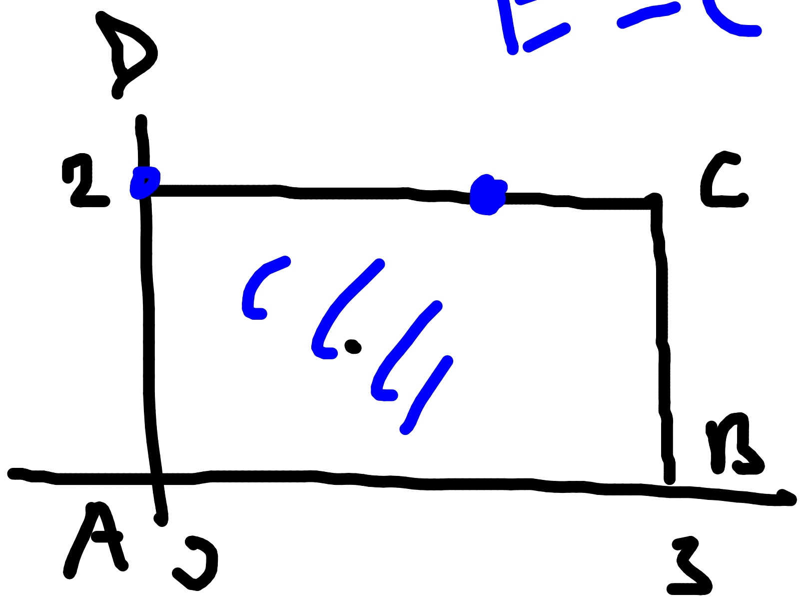
Q: what's max/min of $f(x,y)$ on line AD

$x = 0 \qquad 0 \le y \le 2$

$f\big|_{AD} = 2y$ → max → $y = 2$
→ min → $y = 0$

EXAMPLE 6 Find the absolute maximum and minimum values of the function $f(x, y) = x^2 - 2xy + 2y$ on the rectangle $D = \{(x, y) \mid 0 \leqslant x \leqslant 3, 0 \leqslant y \leqslant 2\}$.

$E = (2,2)$



Q: what's max/min of $f(x,y)$ on line DC

$0 \le x \le 3$, $y = 2$

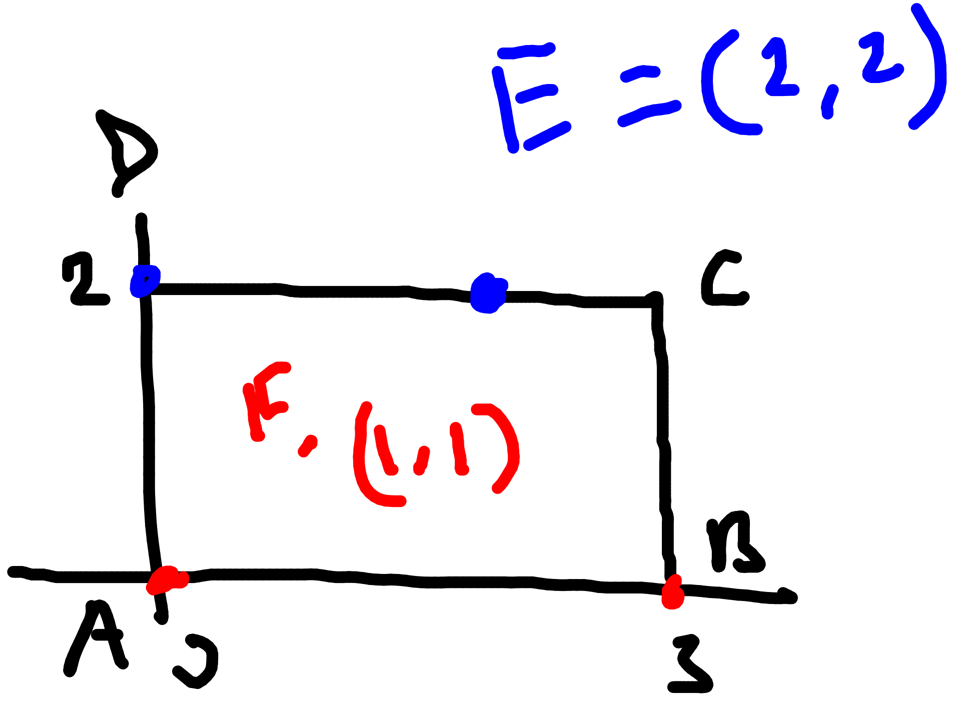
$$f\Big|_{DC} = x^2 - 4x + 4$$

Q: what is the max/min of $x^2 - 4x + 4$ when $0 \le x \le 3$

$\to$ min $\to$ $x = 2$, $y = 2$

max $\to$ $x = 0$, $y = 2$

EXAMPLE 6 Find the absolute maximum and minimum values of the function $f(x, y) = x^2 - 2xy + 2y$ on the rectangle $D = \{(x, y) \mid 0 \leq x \leq 3, 0 \leq y \leq 2\}$.



Q: what's max/min of $f(x,y)$ on line BC

$\to$ $x = 3$ $\quad 0 \leq y \leq 2$

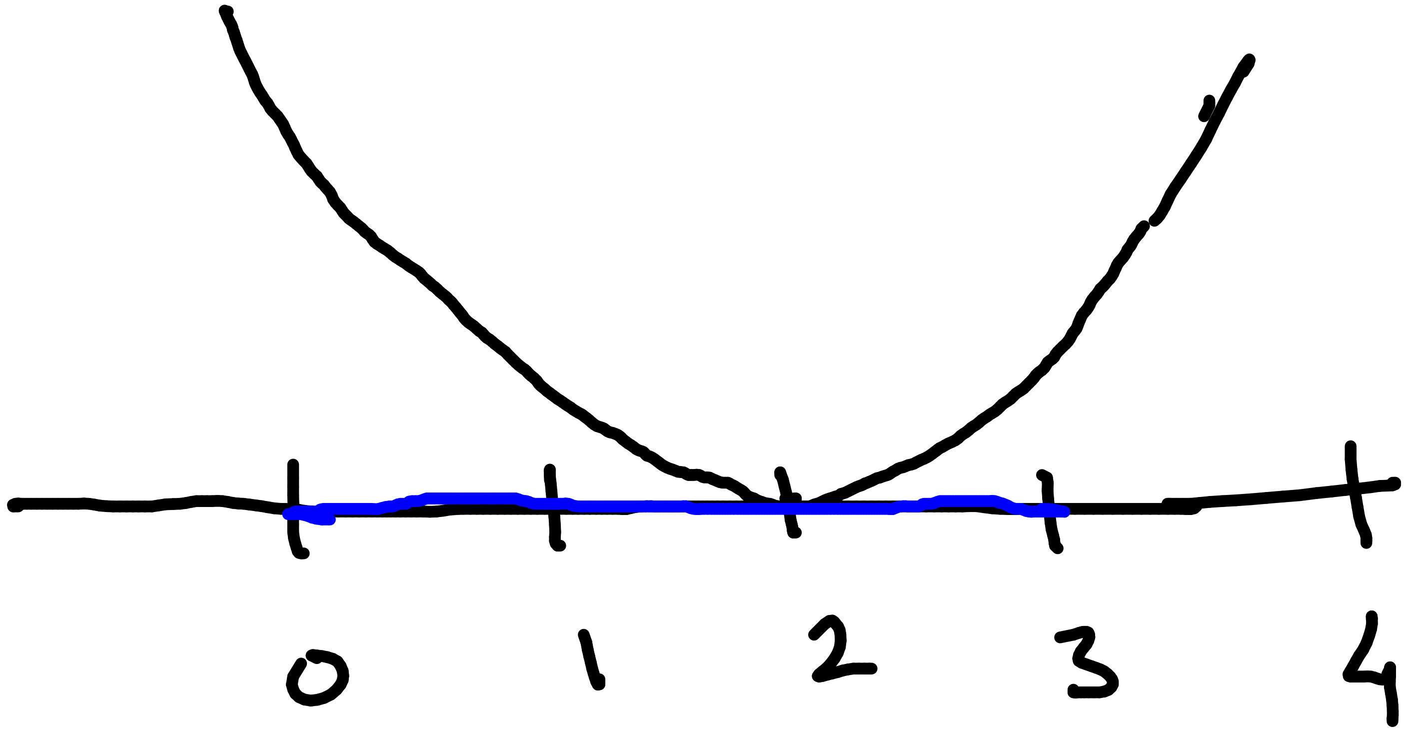
$$f\big|_{BC} = 9 - 6y + 2y = 9 - 4y$$

max $\to$ $y = 0, x = 3$

min at $y = 2, x = 3$

finally absolute max = max$\{f(A), f(B), f(C), f(D), f(E), f(F)\}$

Q: what is the max / min of x^2-4x+4 when
$0 \leq x \leq 3$

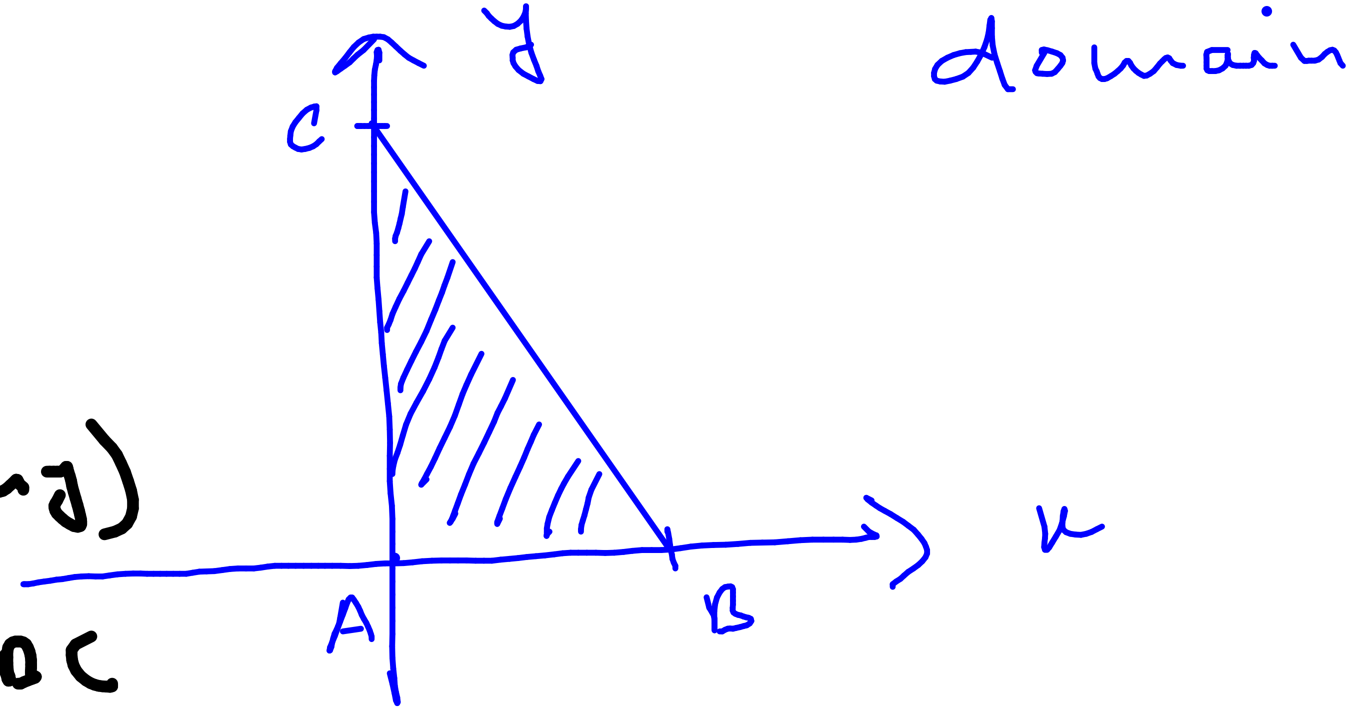


$x^2-4x+4 = (x-2)^2$

$\rightarrow$ min $x = 2$

& max $x = 0$

23. $f(x, y) = 1 + 4x - 5y$, D is the closed triangular region with vertices $(0, 0)$, $(2, 0)$, and $(0, 3)$

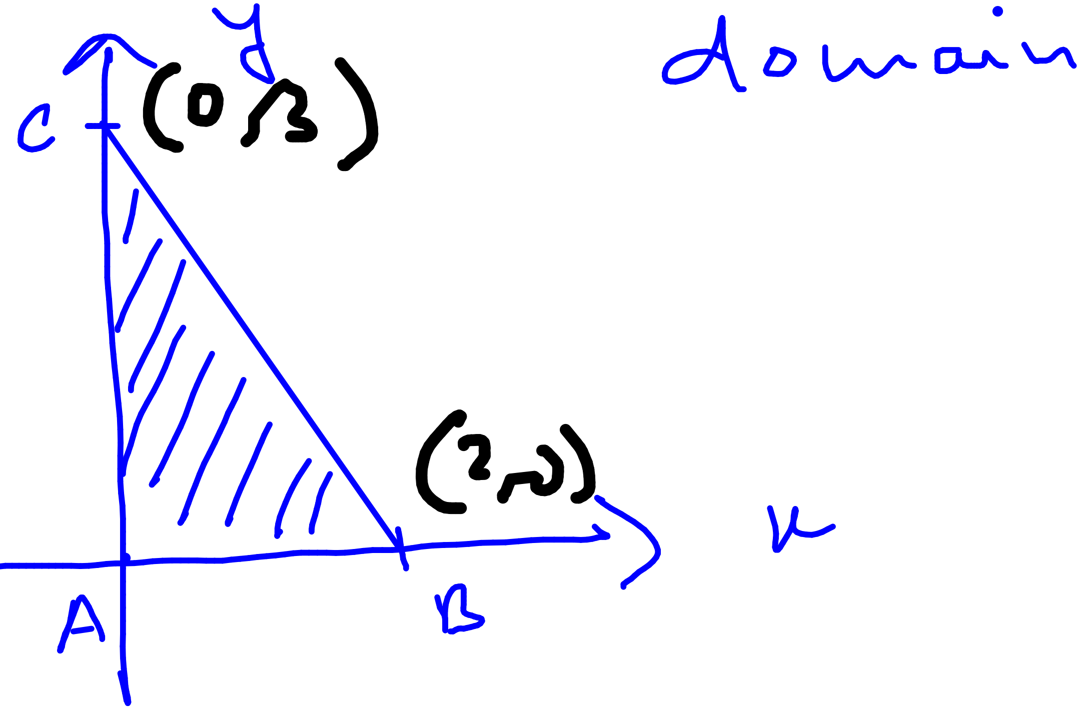


same steps:

→ find critical points (if any) in the interior of ΔABC

→ find max/min of $f(x,y)$ on each boundary segment AB, BC, CA

23. $f(x, y) = 1 + 4x - 5y$, D is the closed triangular region with vertices $(0, 0)$, $(2, 0)$, and $(0, 3)$



eqn of BC

$$\boxed{\frac{x}{2} + \frac{y}{3} = 1}$$

$$3x + 2y = 6$$

$$\boxed{y = \frac{6 - 3x}{2}}$$

$$f\Big|_{BC} = 1 + 4x - 5y$$

$$= 1 + 4x - 5\left(\frac{6 - 3x}{2}\right)$$

$$0 \leq x \leq 2$$

28. $f(x, y) = xy^2, \quad D = \{(x, y) \mid x \geqslant 0,\ y \geqslant 0, x^2 + y^2 \leqslant 3\}$

47. Suppose that a scientist has reason to believe that two quantities x and y are related linearly, that is, $y = mx + b$, at least approximately, for some values of m and b. The scientist performs an experiment and collects data in the form of points $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$, and then plots these points. The points don't lie exactly on a straight line, so the scientist wants to find constants m and b so that the line $y = mx + b$ "fits" the points as well as possible. (See the figure.)