2nd order ODEs

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = r(x)$$

why??

applications

mechanical vibration

circuits

Can we solve this ??

not yet

We can only solve if $r(x) = 0$

called ??

homogenous

8. Archimedian principle. This principle states that the buoyancy force equals the weight of the water displaced by the body (partly or totally submerged).

Another application of 2nd order ODE

$y(t)$ = height of cylinder inside water

The cylindrical buoy of diameter 60 cm in Fig. 43 is floating in water with its axis vertical. When depressed downward in the water and released, it vibrates with period 2 sec. What is its weight?

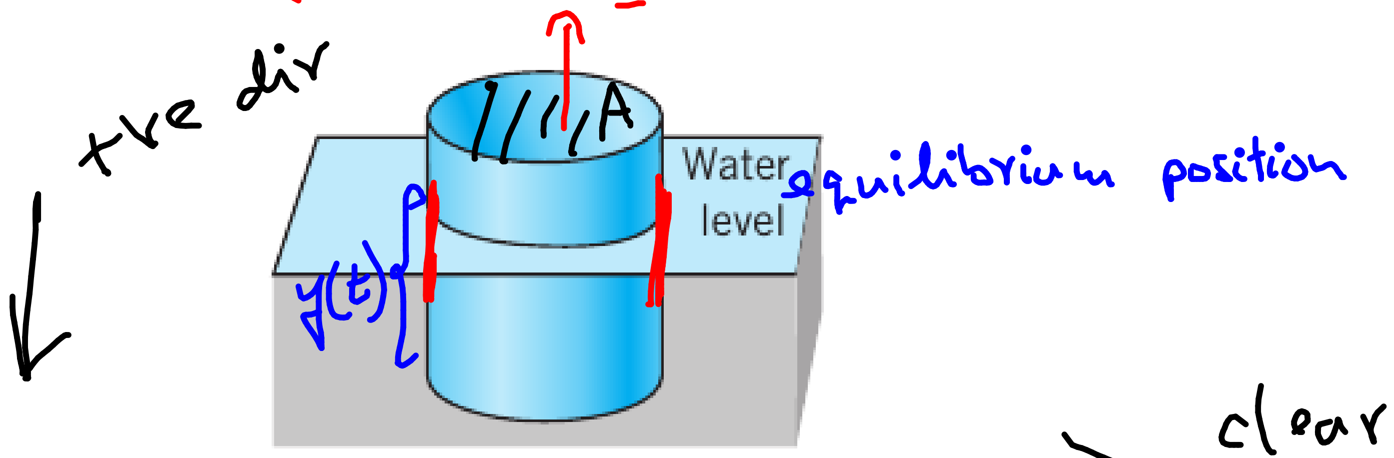


Fig. 43. Buoy (Problem 8)

ρ : density of water

$$m a = \text{net force on the buoy}$$

$$m \frac{d^2y}{dt^2} = \text{gravity} - \text{buoyant force}$$

$$m \frac{d^2y}{dt^2} = mg - \rho A y g$$

clear??

$$\boxed{m y'' + (\rho A g) y = mg}$$

y could have been chosen in such a way that resulting eq^n is a homogenous eq^n

→ Suppose: governing eq^n is

$$\boxed{m y'' + (\rho A g) y = 0}$$

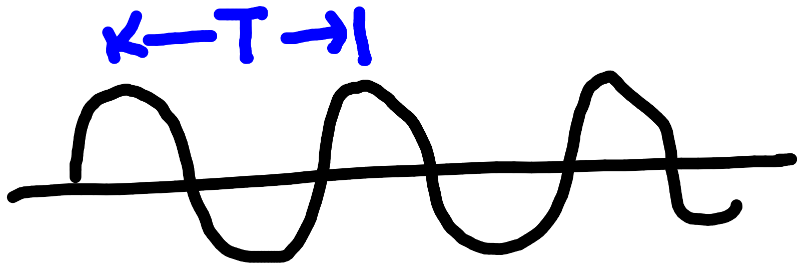
$$m\alpha^2 + (\rho A g) = 0$$

→ $$\alpha = \pm i \sqrt{\frac{\rho A g}{m}}$$

$$y(t) = A\cos\left(\sqrt{\frac{\rho A g}{m}}\, t\right) + B\sin\left(\sqrt{\frac{\rho A g}{m}}\, t\right)$$

	Period	freq
$\sin(x)$	2π	$1/2\pi$
$\sin(2x)$	π	$1/\pi$
$\sin(3x)$	$2\pi/3$	
$\sin(2\pi x)$	1	
$\sin(kx)$	$2\pi/k$	$k/2\pi$

→ periodic



$$\text{freq} = \frac{1}{2\pi}\sqrt{\frac{\rho A g}{m}} = \frac{1}{2}$$] solve for m

Do you really want to be an engineer??

Do this experiment at home

→ a bucket of water

→ a cylindrical body that floats

→ stopwatch

→ a weighing scale

innovation

Aim:

calculated weight $\overset{???}{=}$ actual weight

2.7 Nonhomogeneous ODEs

$$y'' + p(x)y' + q(x)y = r(x)$$

undetermined coefficients

- easier
- range of r(x) is small

variation of parameters

- slightly tedious
- more general

2.7 Nonhomogeneous ODEs

$$y'' + p(x)y' + q(x)y = r(x) \qquad \text{———} \quad (*)$$

↳ step ① solve a corresponding homogenous eqⁿ

$$y'' + py' + qy = 0$$

→ let solution y_h

↳ step ② find a particular solution y_p

$$y'' + py' + qy = r$$

either by
→ undetermined coefficients
→ variation of parameter

↳ final step : $y = y_h + y_p$ [general solution of Eq (*)]

Recall the ODE :

$$y'' + py' + qy = r$$

$$y_h'' + py_h' + qy_h = 0$$

$$y_p'' + py_p' + qy_p = r$$

$$(y_h+y_p)'' + p(y_h+y_p)' + q(y_h+y_p) = r$$

Method of Undetermined Coefficients

how to find y_p

Table 2.1 Method of Undetermined Coefficients

Term in $r(x)$	Choice for $y_p(x)$
$ke^{\gamma x}$	$Ce^{\gamma x}$
$kx^n \ (n = 0, 1, \cdots)$	$K_n x^n + K_{n-1}x^{n-1} + \cdots + K_1 x + K_0$
$k \cos \omega x$	$K \cos \omega x + M \sin \omega x$
$k \sin \omega x$	$K \cos \omega x + M \sin \omega x$
$ke^{\alpha x} \cos \omega x$	$e^{\alpha x}(K \cos \omega x + M \sin \omega x)$
$ke^{\alpha x} \sin \omega x$	$e^{\alpha x}(K \cos \omega x + M \sin \omega x)$

OK will do this next time

e.g. $r(x) = e^{-5x}$ / Try $y_p = Ae^{-5x}$

EXAMPLE 1

Solve the initial value problem

(5) $$y'' + y = 0.001x^2, \qquad y(0) = 0, \qquad y'(0) = 1.5.$$

EXAMPLE 2

Solve the initial value problem

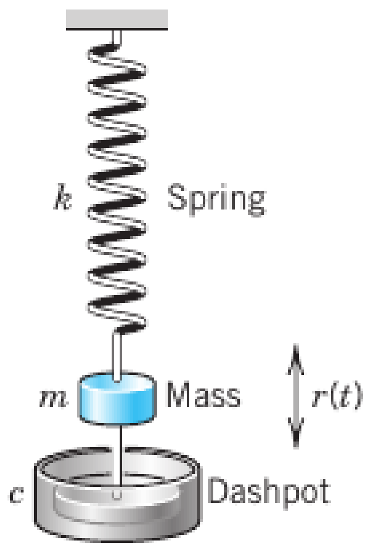
$$y'' + 3y' + 2.25y = -10e^{-1.5x}, \qquad y(0) = 1, \qquad y'(0) = 0. \tag{6}$$

EXAMPLE 3

Solve the initial value problem

$$(7) \qquad y'' + 2y' + 0.75y = 2\cos x - 0.25\sin x + 0.09x, \qquad y(0) = 2.78, \qquad y'(0) = -0.43.$$

2.8 Modeling: Forced Oscillations. Resonance



$$my'' + cy' + ky = F_0 \cos \omega t.$$

https://www.youtube.com/watch?v=XwIZBJIp1AA