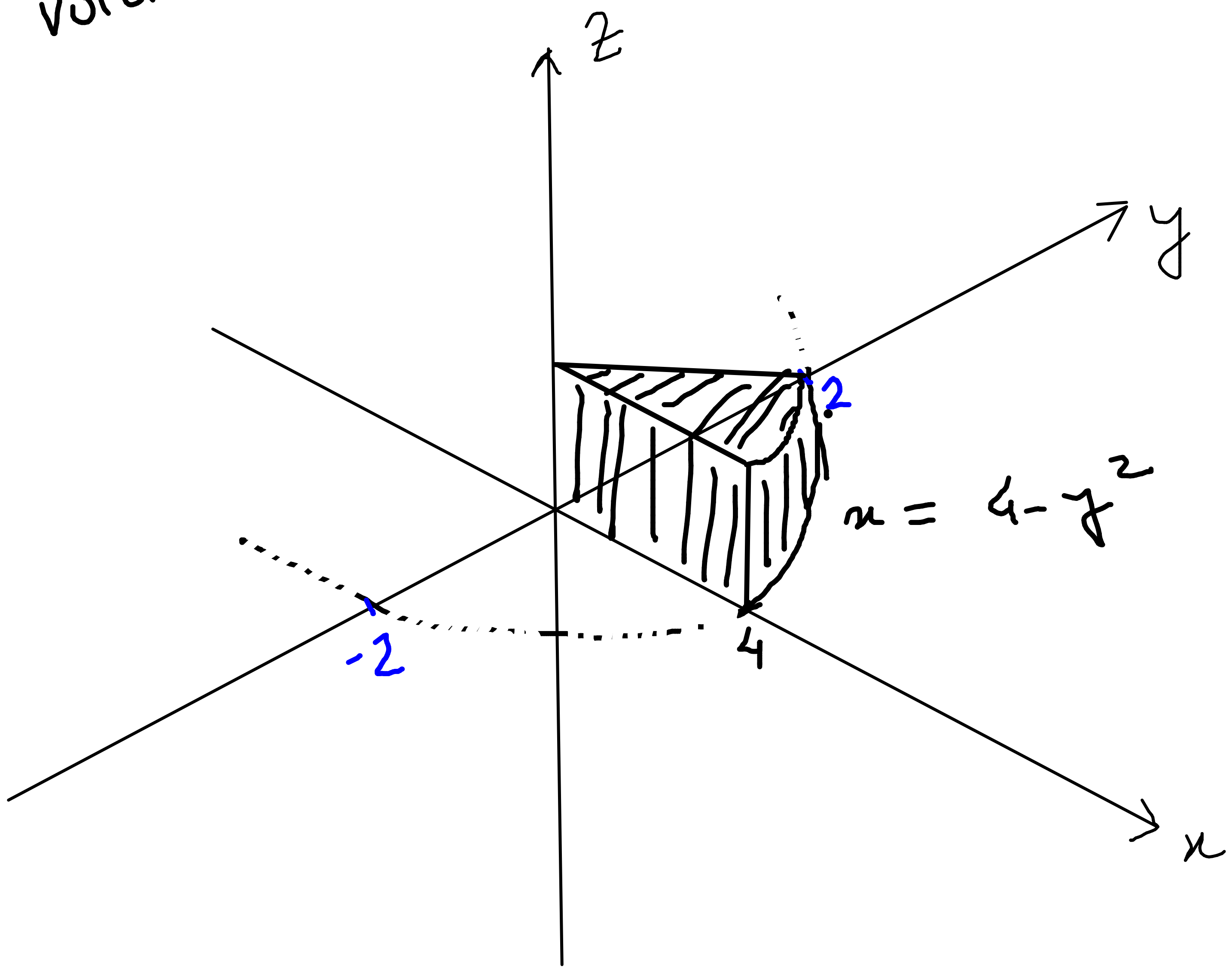
Sketch the solid whose volume is given by the iterated integral.

volume $= \int_0^2 \int_0^{2-y} \int_0^{4-y^2} 1 \, dx \, dz \, dy$

$z = 2 - y$

$z + y = 2$



$= ??$

symbolab.com/solver/triple-integrals-calculator/%5Cint_%7B0%7D%5E%7B2%7D%5Cint_%7B0%7D%5E%7B2-y%7D%5C

- Indefinite Integrals
- Definite Integrals
- Specific-Method
- Improper Integrals
- Antiderivatives
- Double Integrals
- Triple Integrals
- Multiple Integrals
- Integral Applications
- Riemann Sum (new)
- Series
- ODE
- Multivariable Calculus (new)
- Laplace Transform
- Taylor/Maclaurin Series
- Fourier Series

$\int_0^2 \int_0^{2-y} \int_0^{4-y^2} dxdzdy$

Go

Examples »

Solution

Keep Practicing >

Show Steps

$\int_0^2 \int_0^{2-y} \int_0^{4-y^2} 1dxdzdy = \frac{20}{3}$ (Decimal: $6.66666\ldots$)

Steps

$\int_0^2 \int_0^{2-y} \int_0^{4-y^2} 1dxdzdy$

$\int_0^{4-y^2} 1dx = -y^2 + 4$ *Show Steps*

$= \int_0^2 \int_0^{2-y} \left(-y^2 + 4\right)dzdy$

$\int_0^{2-y} \left(-y^2 + 4\right)dz = (-y + 2)\left(-y^2 + 4\right)$ *Show Steps*

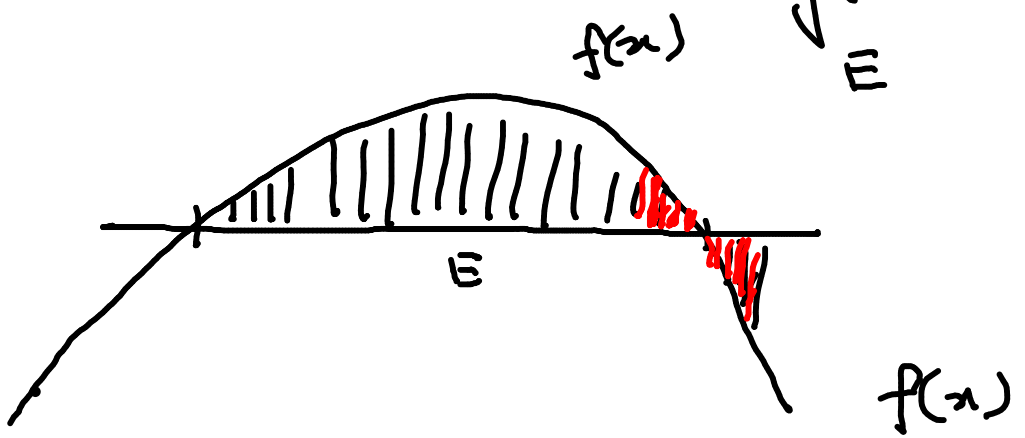
$= \int_0^2 (-y + 2)\left(-y^2 + 4\right)dy$

$= \frac{20}{3}$

47. Find the region E for which the triple integral

$$\iiint_E (1 - x^2 - 2y^2 - 3z^2)\, dV$$

is a maximum.



find E s.t.

$\int_E f\,dx$ is as large as possible.

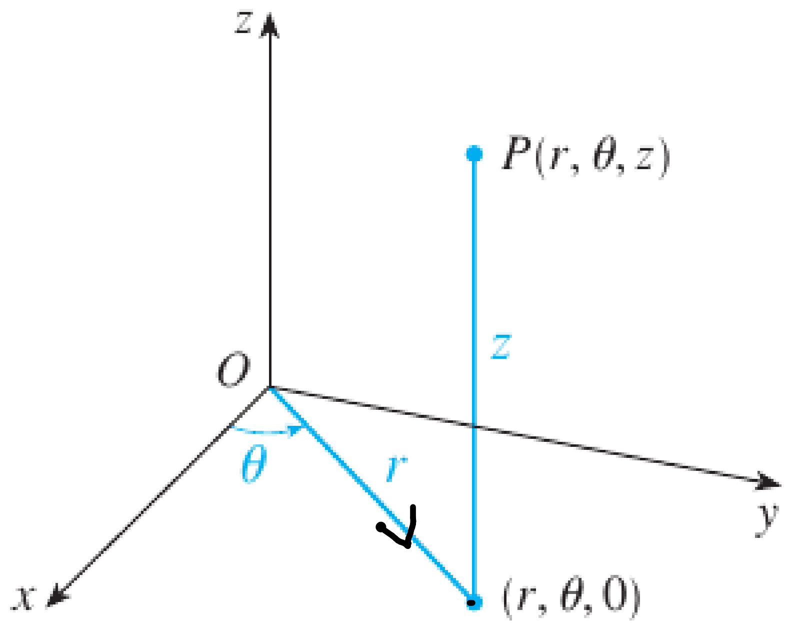
E:

$$1 - x^2 - 2y^2 - 3z^2 \geq 0$$

$$x^2 + 2y^2 + 3z^2 \leq 1$$

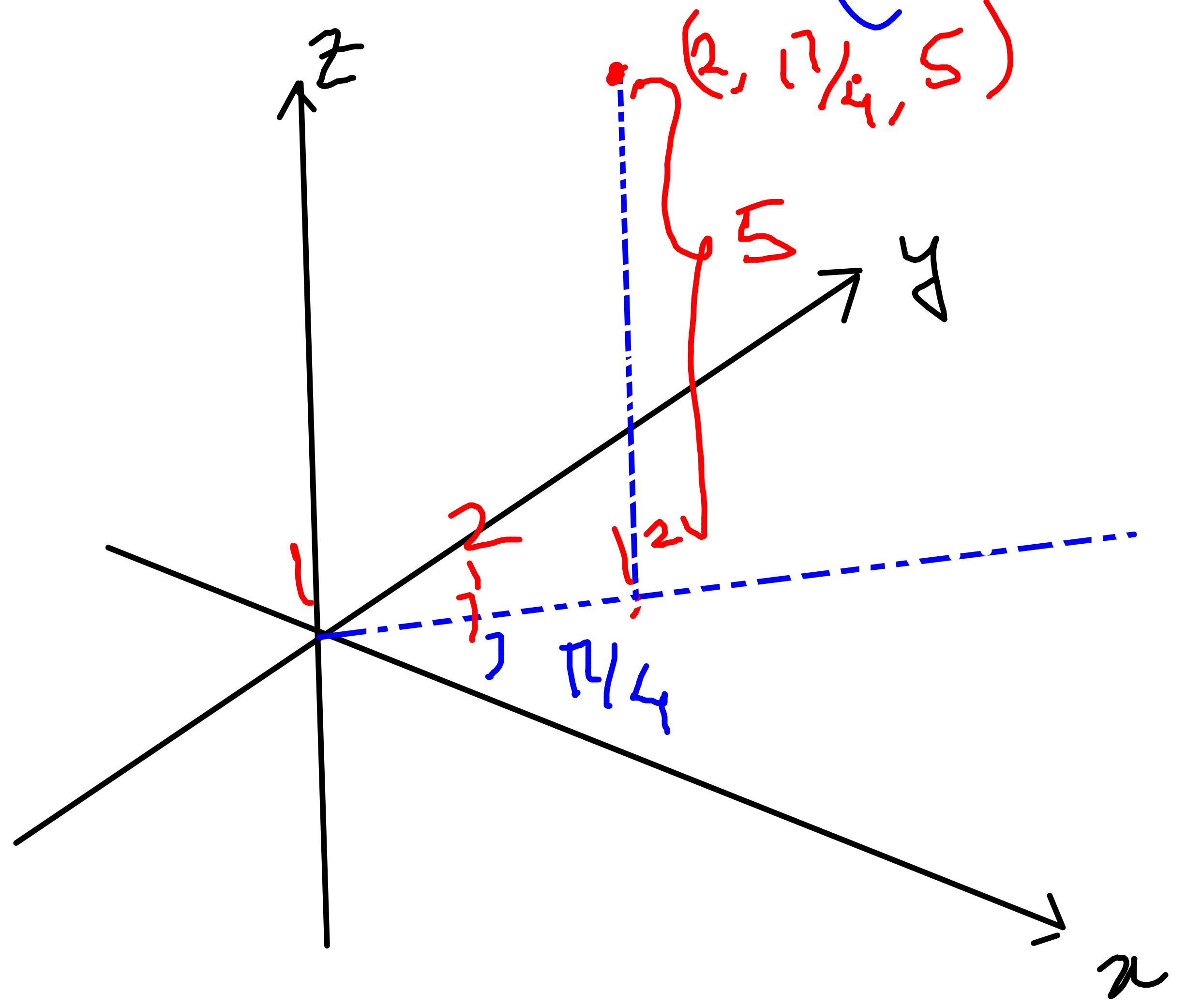
$$x^2 + \frac{y^2}{(1/\sqrt{2})^2} + \frac{z^2}{(1/\sqrt{3})^2} \leq 1$$

12.6 TRIPLE INTEGRALS IN CYLINDRICAL COORDINATES



$$x = r\cos\theta \qquad y = r\sin\theta \qquad z = z$$

Q. Locate the point whose cylindrical coordinates are $(r, \theta, z) = \left(2, \pi/4, 5\right)$.

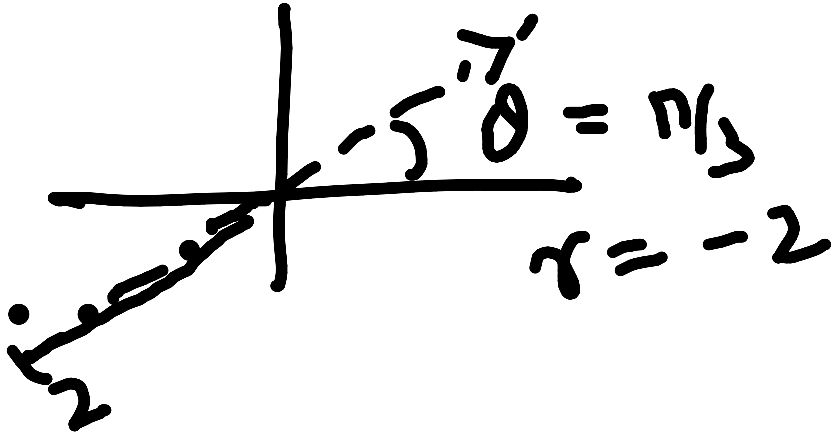


EXAMPLE 2 Describe the surface whose equation in cylindrical coordinates is $z = r$.

what surface is this ??

$r = \sqrt{x^2+y^2}$

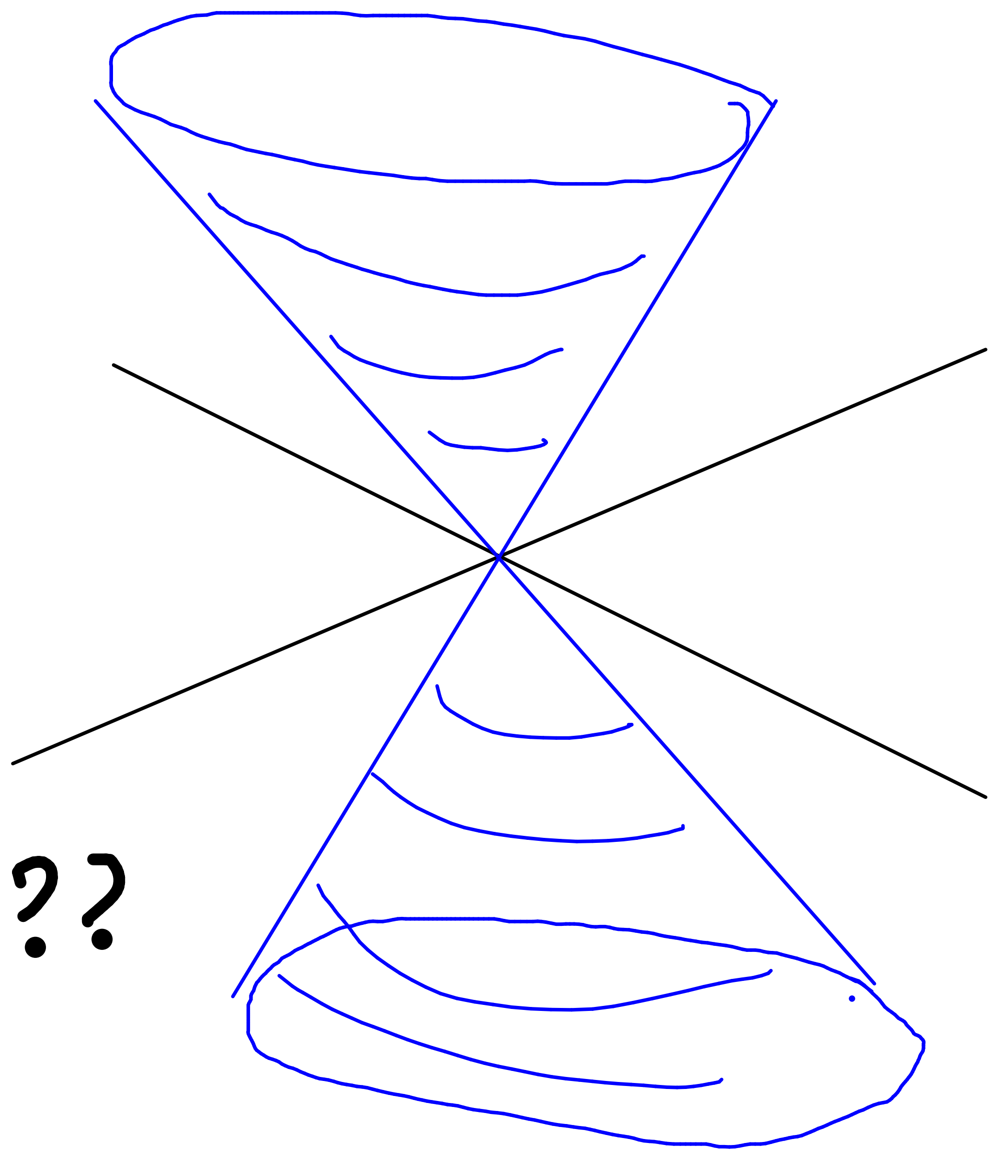
$z = \pm\sqrt{x^2+y^2}$



$y = 0$

$z = \sqrt{x^2}$

$= |x|$



why bottom cone as well??

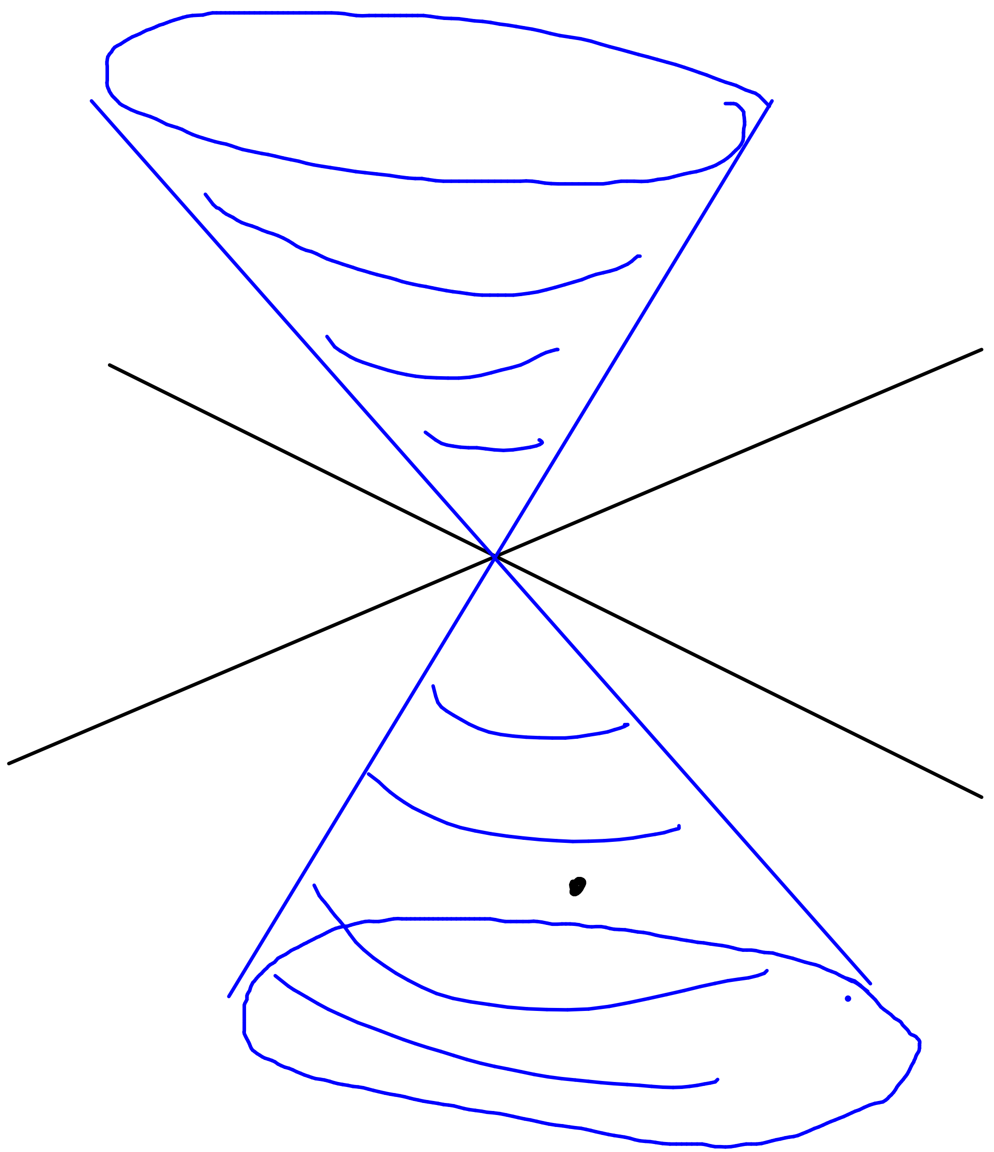
$z = r$

EXAMPLE 2 Describe the surface whose equation in cylindrical coordinates is $z = r$.

$r = \sqrt{x^2 + y^2}$

surface ??

collection of all points (r, θ, z) which satisfy the equation $z = r$



Recall :

$$\iint \text{—}\, dx\,dy \longleftrightarrow \iint \text{—}\, du\,dv$$

$$dx\,dy = ??\; du\,dv$$

Jacobian

$dx\,dy\,dz \longrightarrow ???\ dr\,d\theta\,dz$

Jacobian of switching from (x, y, z) to cylindrical coordinates

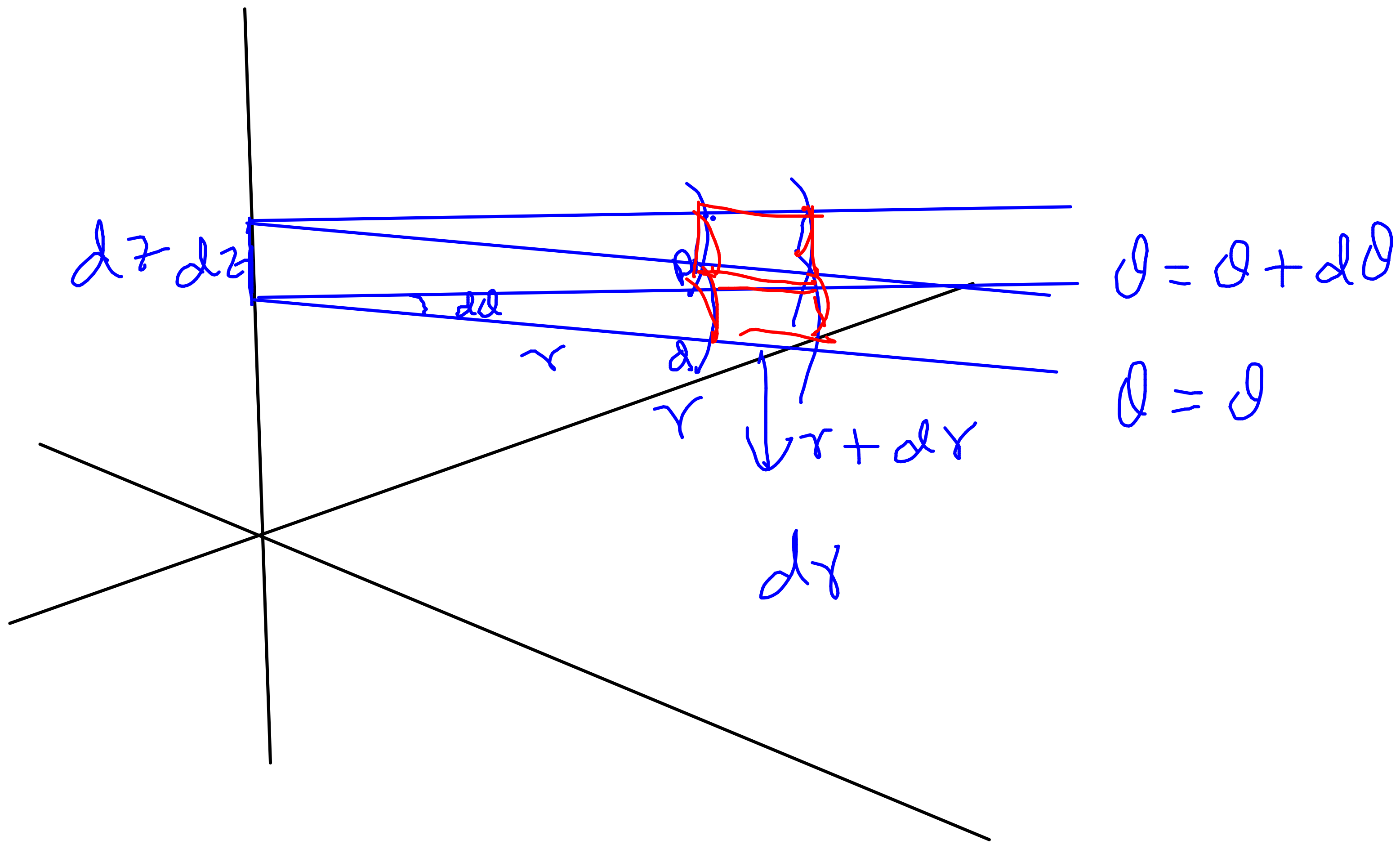
$x = r\cos\theta$

$y = r\sin\theta$

$z = z$

$$J = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} & \frac{\partial x}{\partial z} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} & \frac{\partial y}{\partial z} \\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial \theta} & \frac{\partial z}{\partial z} \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\sin\theta & 0 \\ \sin\theta & r\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = r$$

$$\therefore \quad dx\,dy\,dz = r\,dr\,d\theta\,dz$$



$$\widehat{PQ} = r\,d\theta$$

$$r\,dr\,d\theta\,dz = (r\,d\theta)(dr)(dz)$$

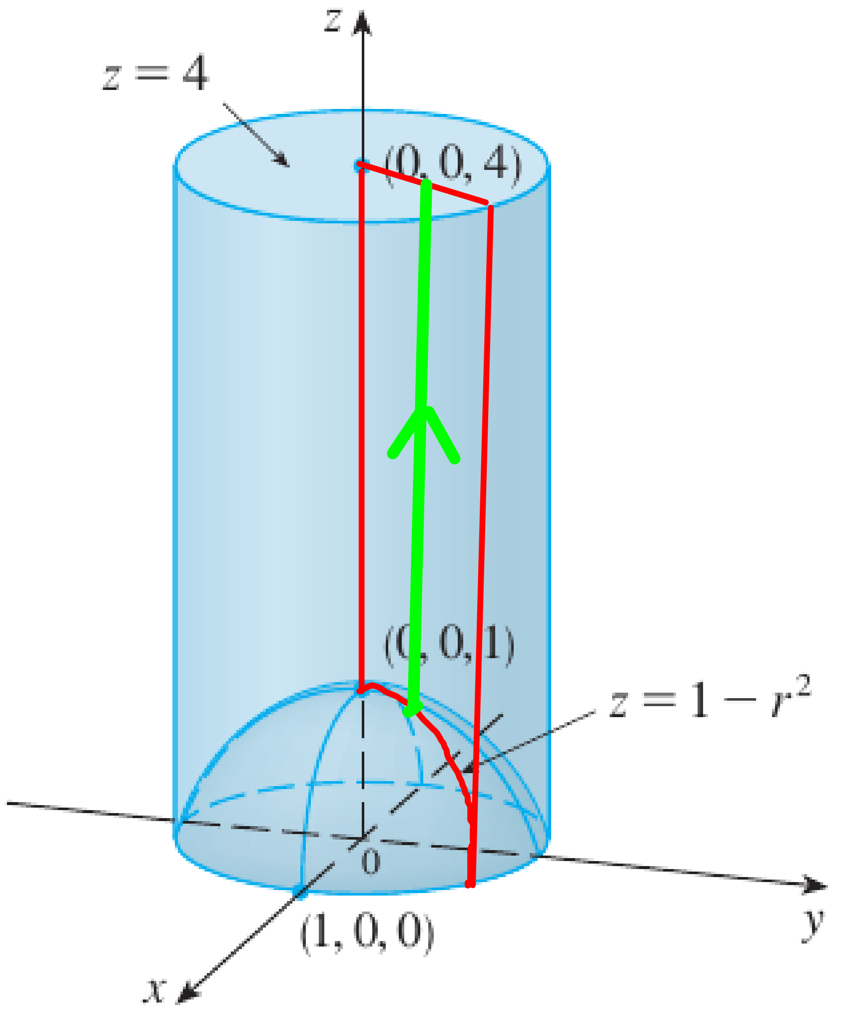
volume swiped
for small change
$d\theta$, dr, dz

$$dV = r\,dr\,d\theta\,dz$$

$r^2=1 \qquad r=1$

EXAMPLE 3 A solid E lies within the cylinder $x^2 + y^2 = 1$, below the plane $z = 4$, and above the paraboloid $z = 1 - x^2 - y^2$. (See Figure 8.) The density at any point is proportional to its distance from the axis of the cylinder. Find the mass of E.

K: some constant not given



$$\rho(r,\theta,z) = K r$$

$$z = 1 - x^2 - y^2 = 1 - r^2$$

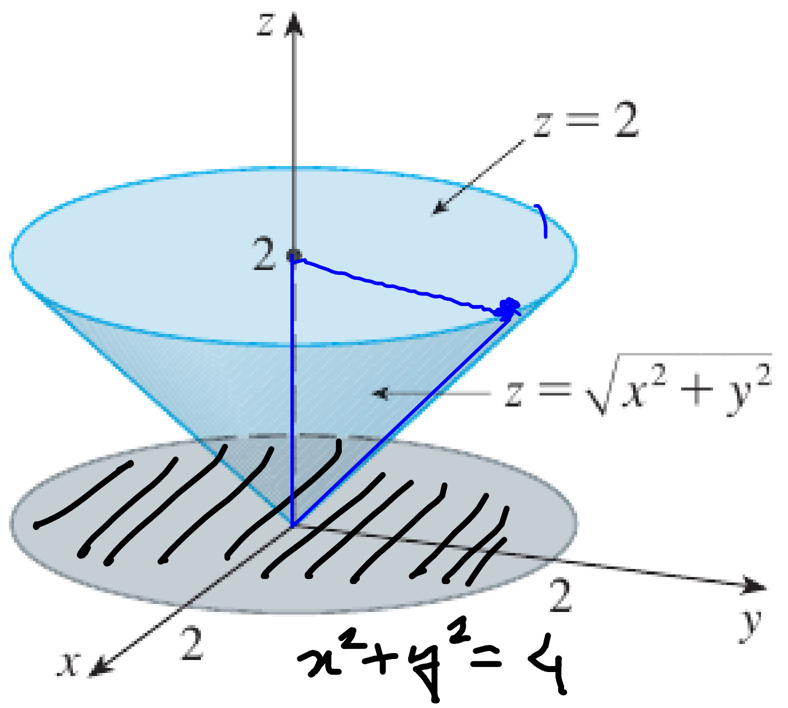
$$\text{mass} = \iiint_E \rho \, dV$$

$$= \int_0^{2\pi} \int_0^1 \int_{1-r^2}^4 (Kr)\, r \, dz\, dr\, d\theta = K \frac{24}{10} \pi$$

EXAMPLE 4 Evaluate $\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{2}(x^2+y^2)\,dz\,dy\,dx$.

Rewrite this integration in cylindrical coordinates

$$\int_0^{2\pi}\int_0^{2}\int_r^{2} r^2 \; r \; dz\,dr\,d\theta$$



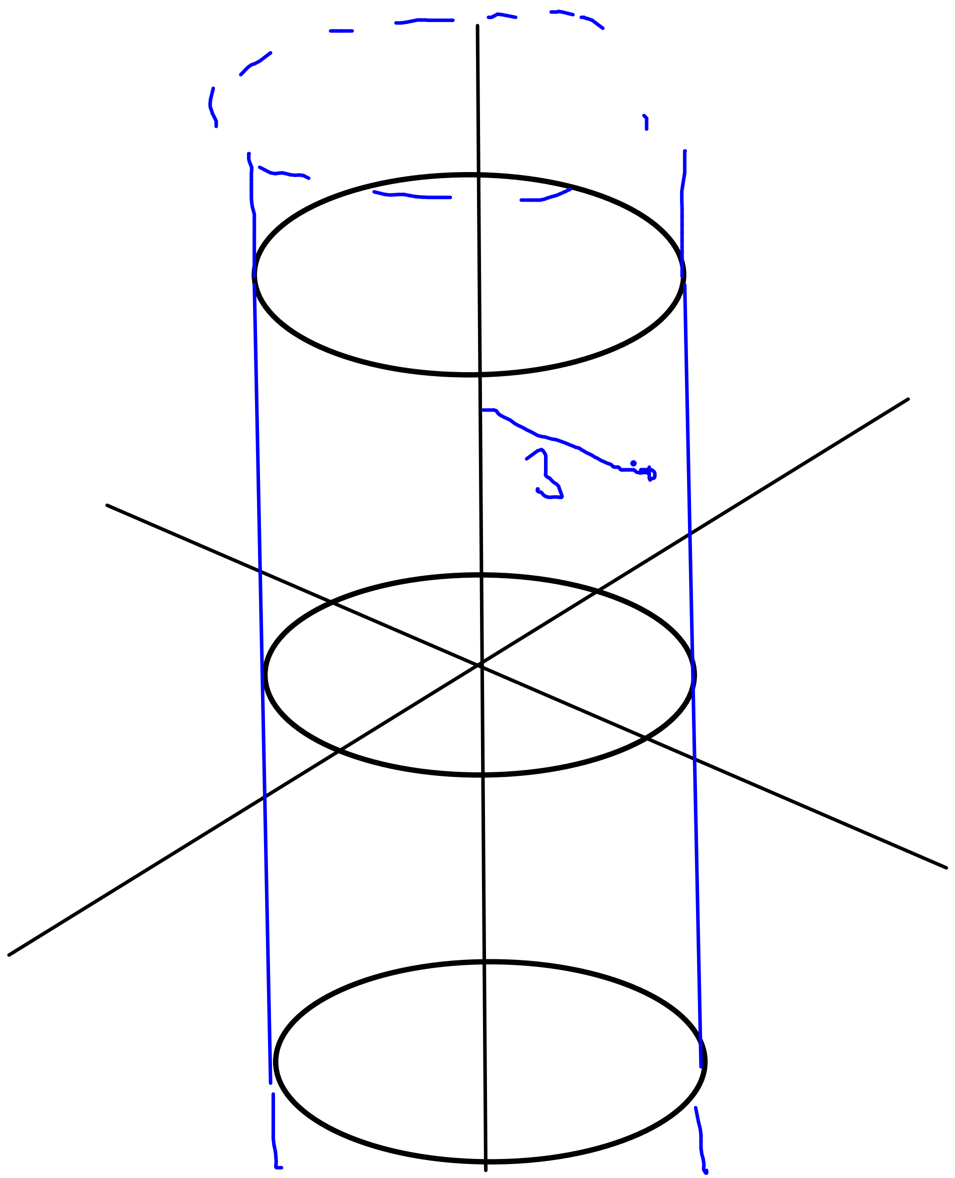
$r^2=4$

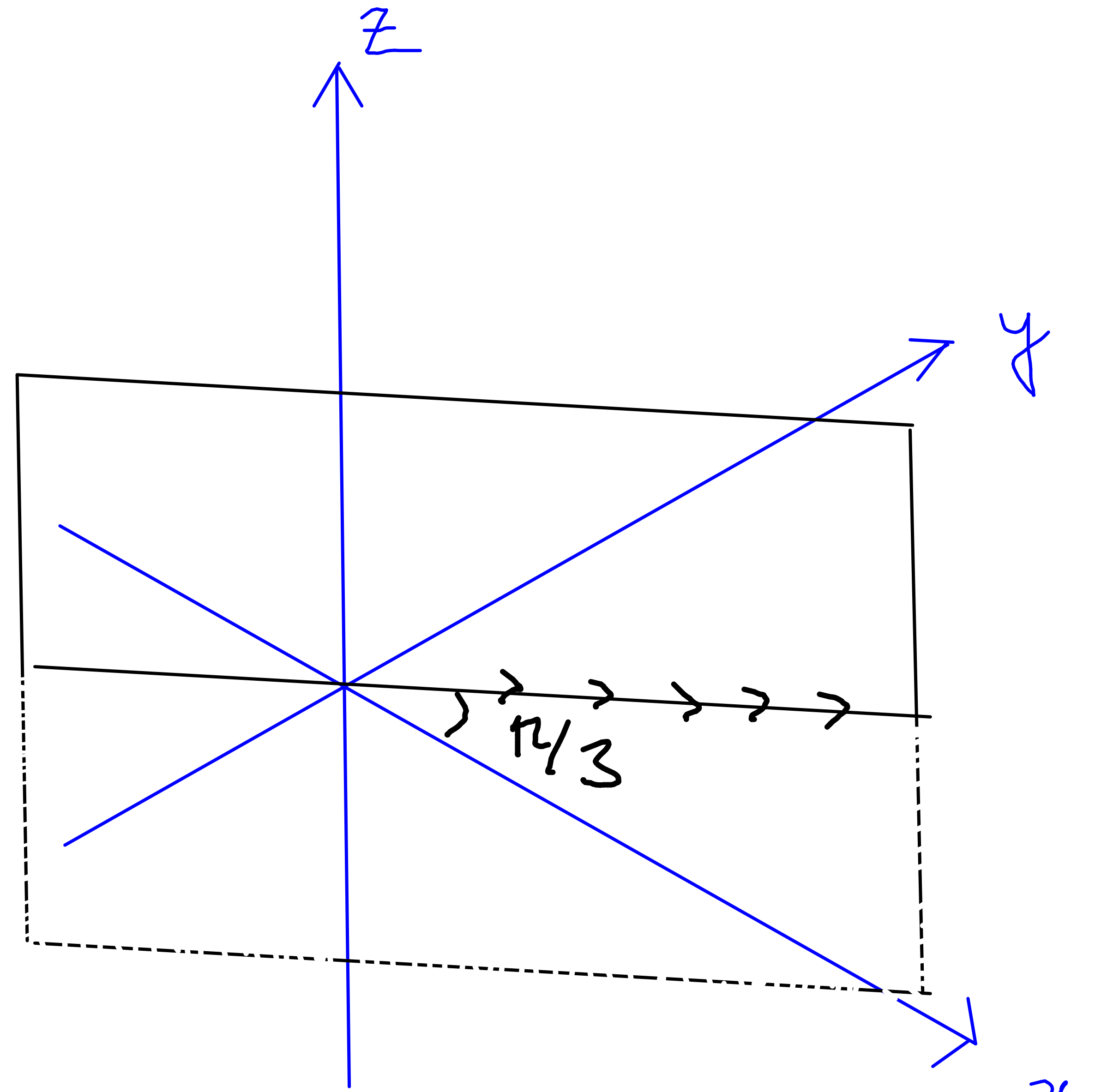
$r=2$

5–6 ▪ Describe in words the surface whose equation is given.

5. $r = 3$

6. $\theta = \pi/3$





Recall: r can be negative

9–10 ▪ Write the equations in cylindrical coordinates.

9. (a) $z = x^2 + y^2$ (b) $x^2 + y^2 = 2y$

a) $x^2+y^2 = r^2$

$z = x^2+y^2$

in cylindrical coordinates

$$\boxed{z = r^2}$$

b) $r^2 = 2r\sin\theta$

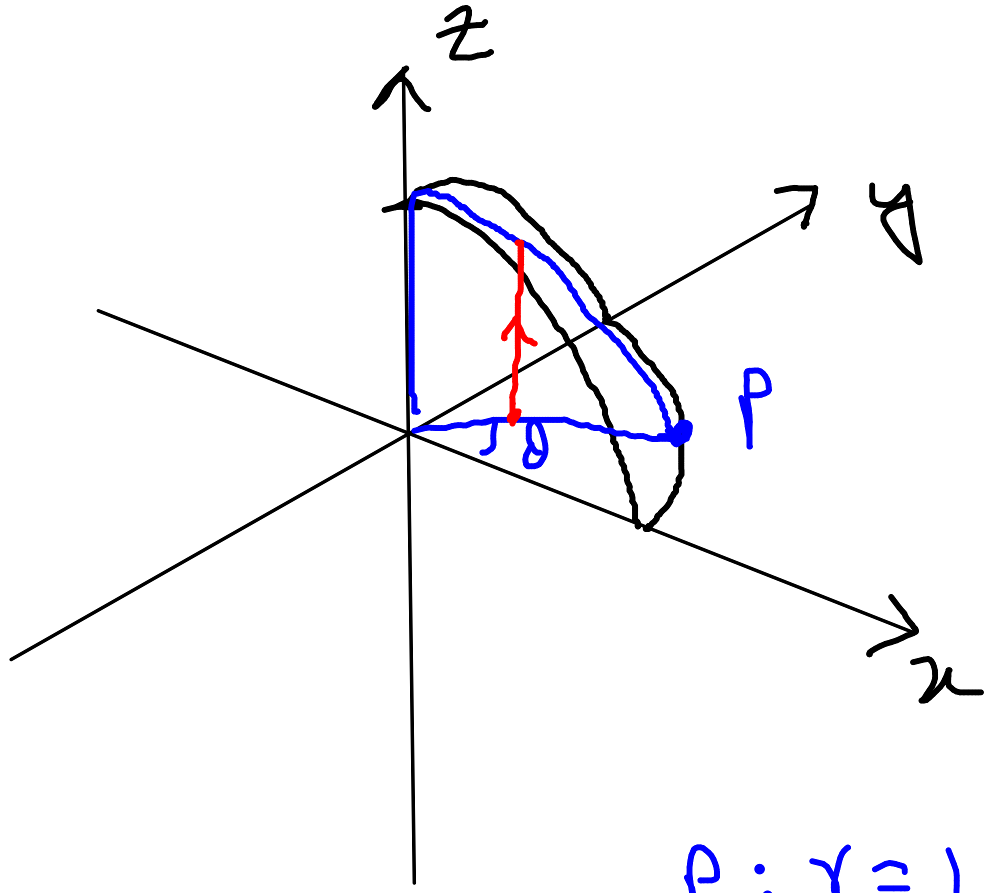
$$\boxed{r = 2\sin\theta}$$

Evaluate $\iiint_E (x^3 + xy^2)\, dV$, where E is the solid in the first octant that lies beneath the paraboloid $z = 1 - x^2 - y^2$.

$z = 1 - r^2$

$1 - r^2 = 0$

$r = 1$



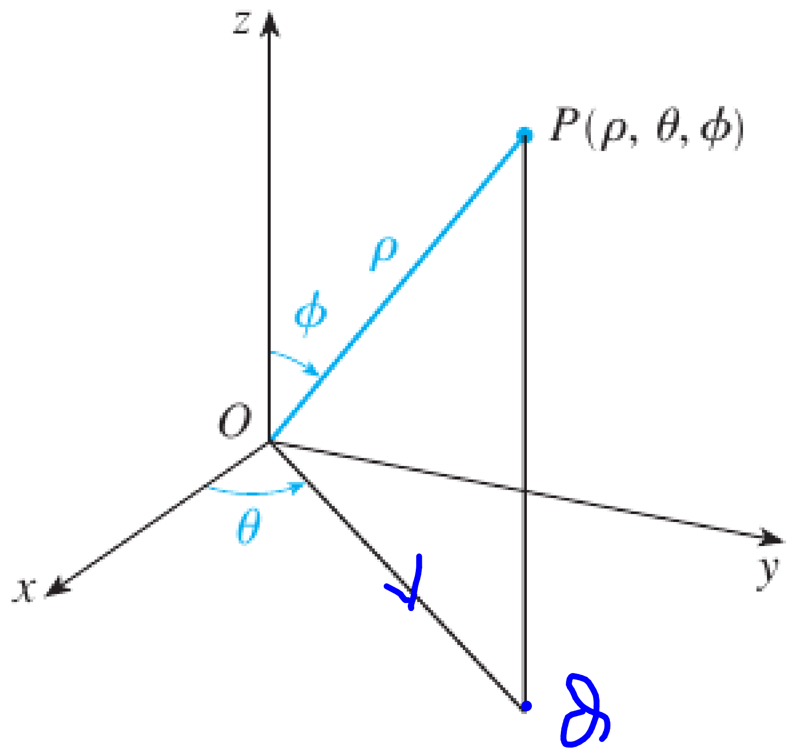
$P : r = 1$

Jacobian

$$\int_0^{\pi/2} \int_0^1 \int_0^{1-r^2} \left((r\cos\theta)^3 + r\cos\theta\,(r\sin\theta)^2 \right) r\, dz\, dr\, d\theta$$

$= \text{whatever} = \dfrac{2}{35}$

12.7 TRIPLE INTEGRALS IN SPHERICAL COORDINATES



ρ : distance from origin

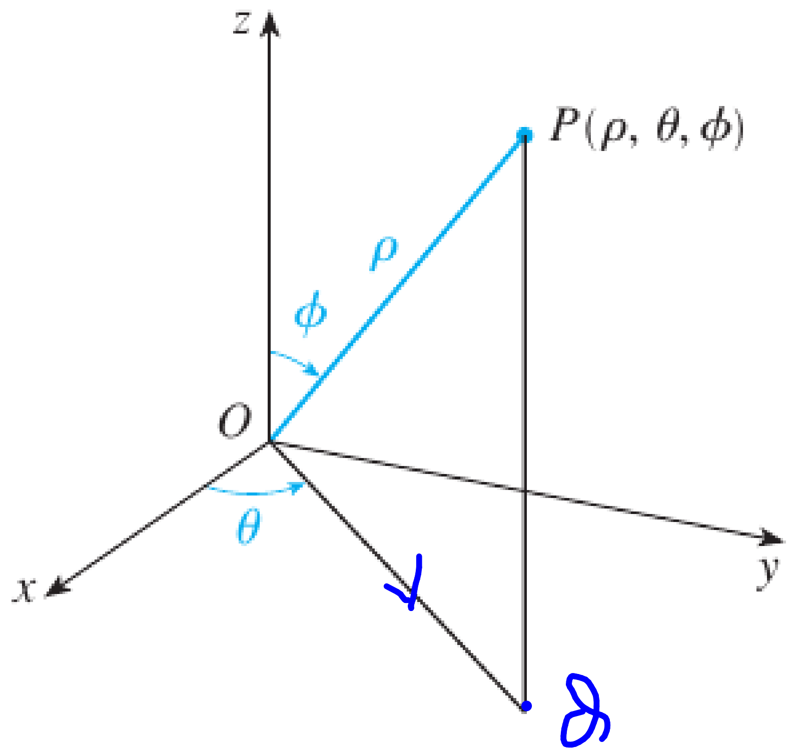
ϕ : angle between positive z axis & vector $\overrightarrow{OP}$

$$0 \le \phi \le \pi$$

ϑ: projection of P on the xy plane.

θ : angle between positive x axis & the projected vector $\overrightarrow{O\vartheta}$

12.7 TRIPLE INTEGRALS IN SPHERICAL COORDINATES



ρ : distance from origin | $\rho \geq 0$

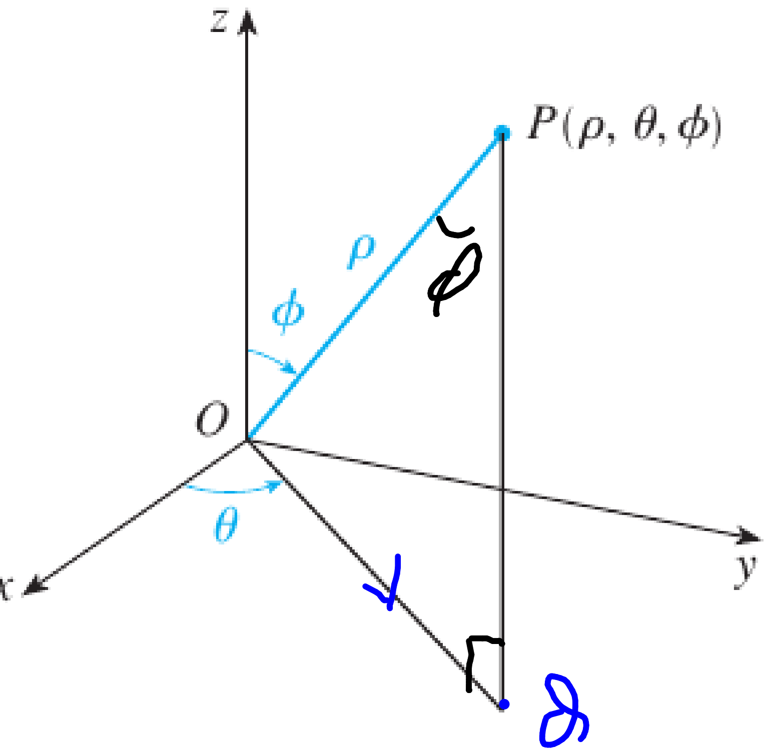
ϕ : angle between positive z axis & vector $\vec{OP}$

$$0 \leq \phi \leq \pi$$

Q: projection of P on the xy plane.

θ : angle between positive x axis & the projected vector $\vec{OQ}$

$$0 \leq \theta \leq 2\pi$$



$$x = OQ \cos\theta = \rho \sin\phi \cos\theta$$

$$y = OQ \sin\theta = \rho \sin\phi \sin\theta$$

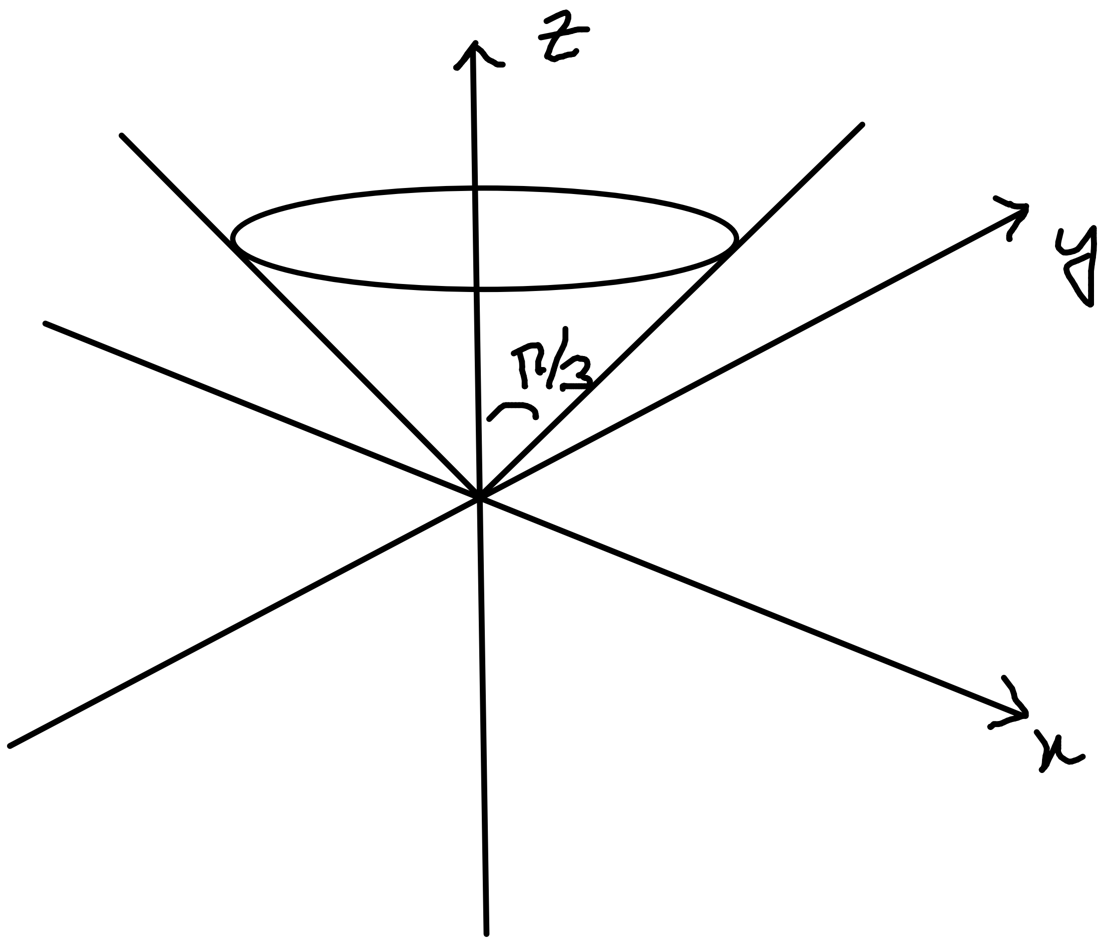
$$z = \rho \cos\phi = \rho \cos\phi$$

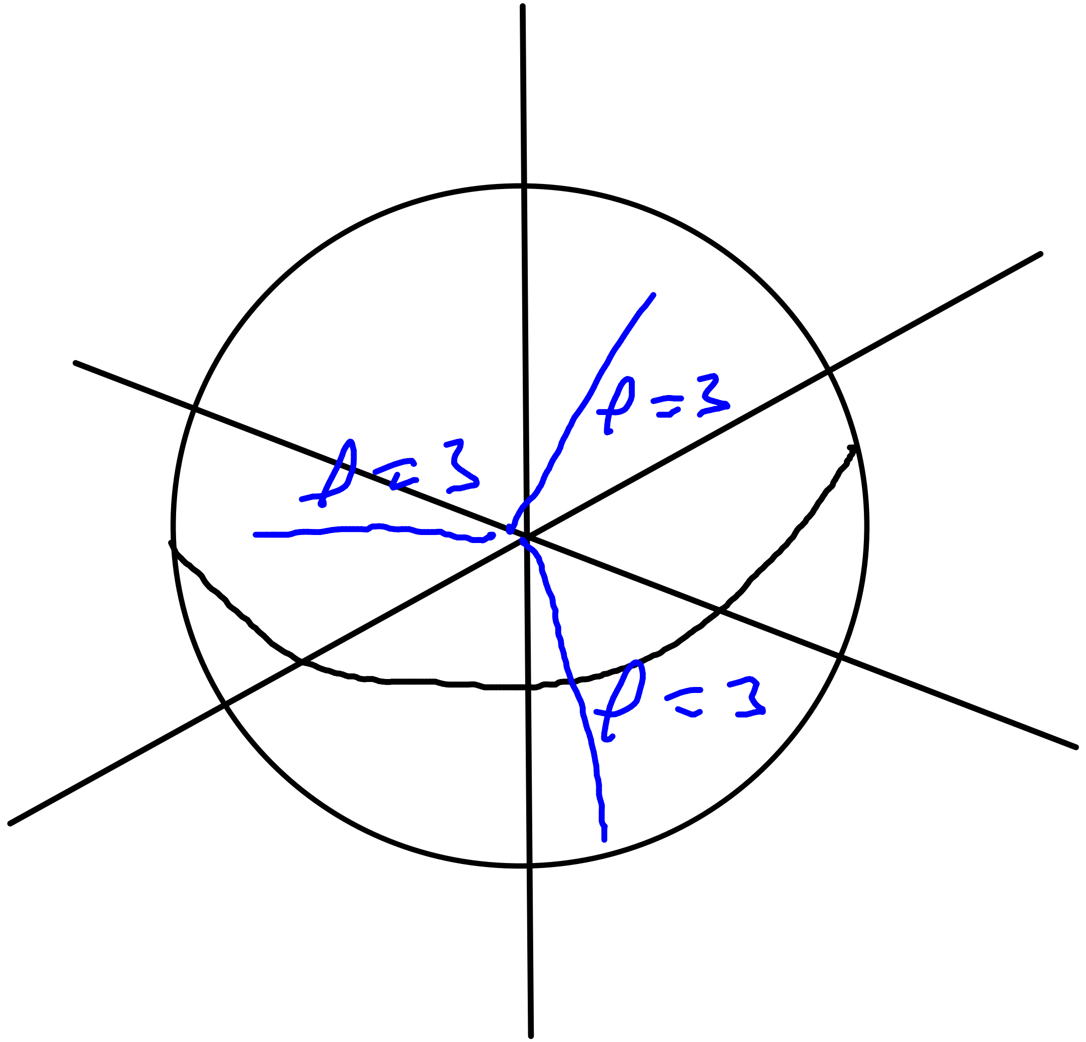
$$OQ = \rho \sin\phi$$

5–6 ▪ Describe in words the surface whose equation is given.

5. $\phi = \pi/3$

6. $\rho = 3$

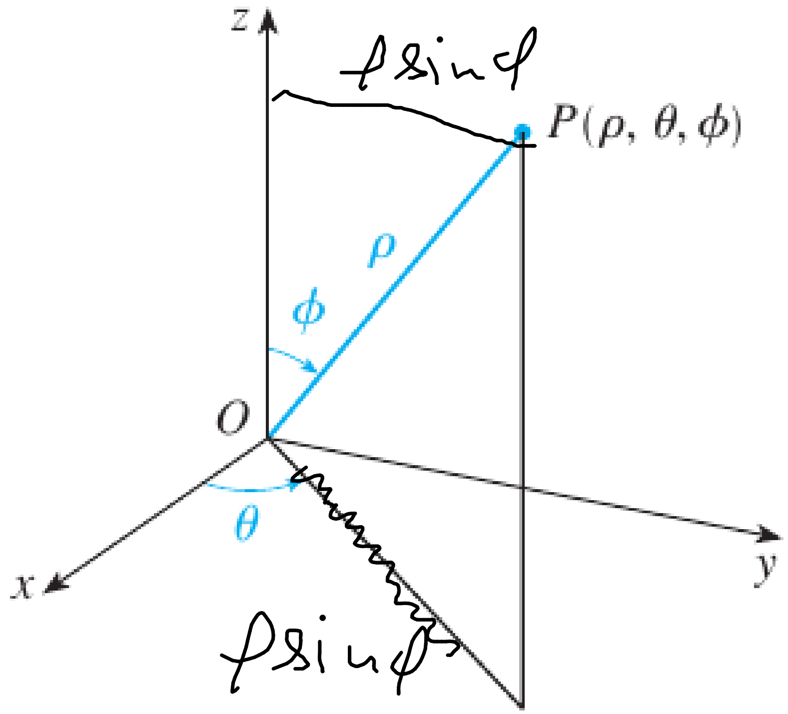




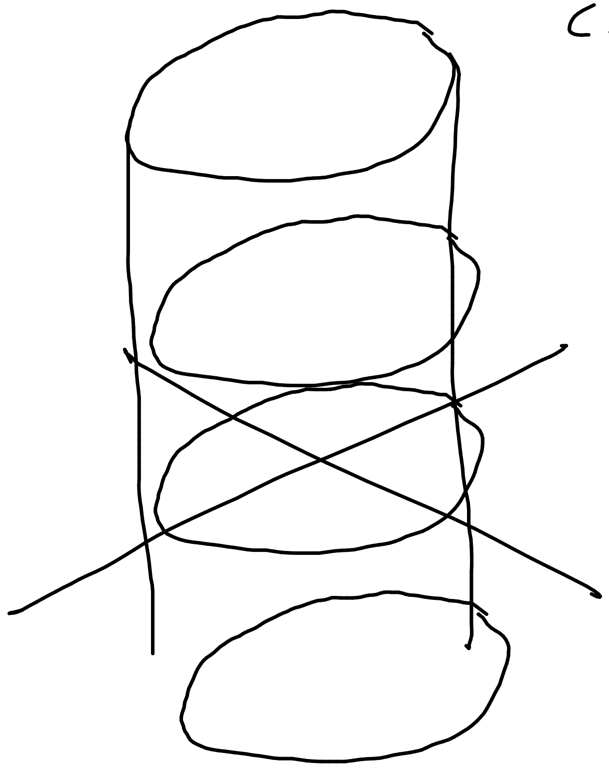
7–8 ■ Identify the surface whose equation is given.

7. $\rho \sin \phi = 2$

8. $\rho = 2 \cos \phi$



$\boxed{r = 2}$ in cylindrical coordinats



$\rho^2 = 2\rho\cos\phi$

$x^2+y^2+z^2 = 2z$??

$x^2+y^2+(z-1)^2 = 1$

sphere

V EXAMPLE 3 Evaluate $\iiint_B e^{(x^2+y^2+z^2)^{3/2}}\, dV$, where B is the unit ball:

$$B = \{(x, y, z) \mid x^2 + y^2 + z^2 \leqslant 1\}$$

V EXAMPLE 4 Use spherical coordinates to find the volume of the solid that lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = z$. (See Figure 9.)