End Sem
on 8th
May

Syllabus: Everything after midsem
+ selected section from
midsem
(email on this)

Last time:

$$y'' + ky = \sin \omega t$$

we have so far:

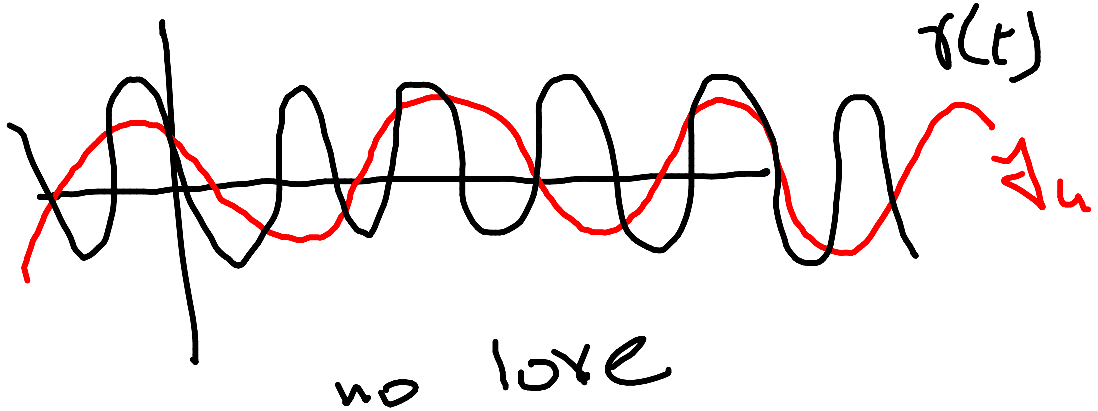
→ solve homogenous sol^n y_h

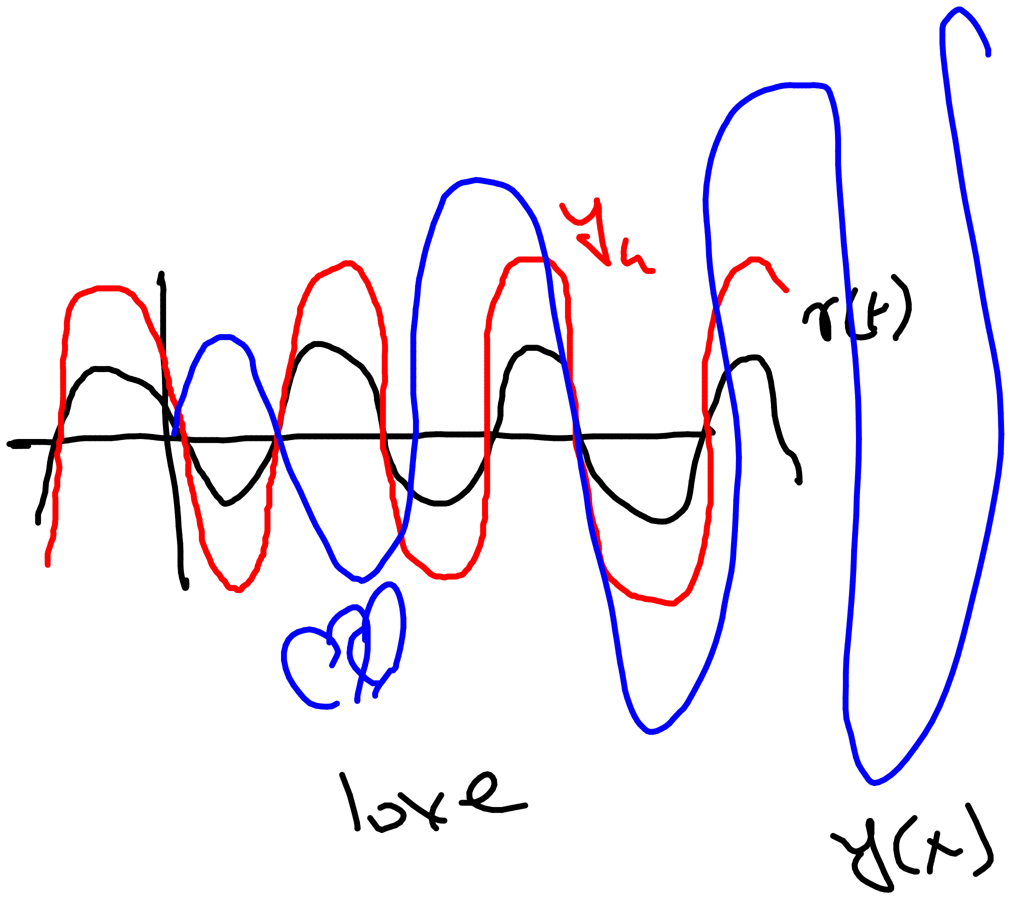
→ solve for y_p:

e.g. $y_h = 5 \sin 2t$

$r(t) = 3 \sin 5t$

resonance ??





2.9 Modeling: Electric Circuits

sum of voltage drop accross each component = voltage supplied

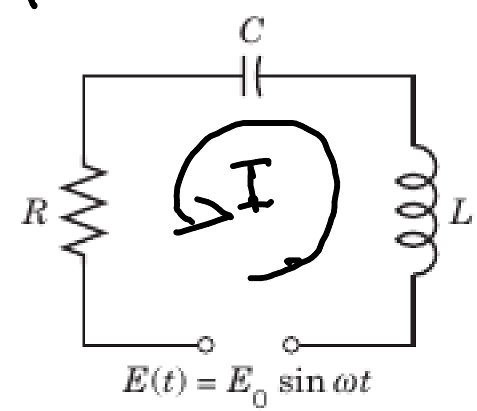
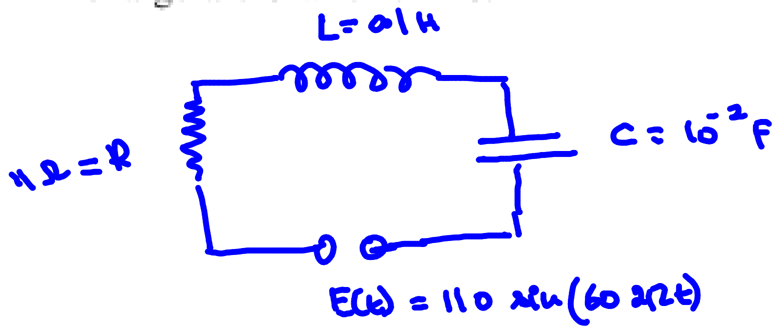


Fig. 61. *RLC*-circuit

Name	Symbol	Notation	Unit	Voltage Drop
Ohm's Resistor		R Ohm's Resistance	ohms (Ω)	RI
Inductor		L Inductance	henrys (H)	$L \frac{dI}{dt}$
Capacitor		C Capacitance	farads (F)	Q/C

RLC-Circuit

Find the current $I(t)$ in an *RLC*-circuit with $R = 11\ \Omega$ (ohms), $L = 0.1$ H (henry), $C = 10^{-2}$ F (farad), which is connected to a source of EMF $E(t) = 110 \sin(60 \cdot 2\pi t) = 110 \sin 377\, t$ (hence 60 Hz = 60 cycles/sec, the usual in the U.S. and Canada; in Europe it would be 220 V and 50 Hz). Assume that current and capacitor charge are 0 when $t = 0$.



$I(0) = 0$, $q(0) = 0$

$$\begin{pmatrix}\text{voltage drop}\\\text{over}\\R\end{pmatrix} + \begin{pmatrix}\text{voltage drop}\\\text{over}\\L\end{pmatrix} + \begin{pmatrix}\text{voltage drop}\\\text{over}\\C\end{pmatrix} = 110 \sin 377 t$$

$$\left[11 I + 0.1 \frac{dI}{dt} + 100 q = 110 \sin 377 t \right]$$

problem! we don't know q.

$\left(I = \frac{dq}{dt} \right)$ so what??

differentiate the eqn

$$0.1 \frac{d^2 I}{dt^2} + 11 \frac{dI}{dt} + 100\, I = (110)(377) \cos(377 t)$$

easily solvable

8–14 Find the **steady-state current** in the *RLC*-circuit in Fig. 61 for the given data. Show the details of your work.

$$R = 4\ \Omega,\ L = 0.5\ \text{H},\ C = 0.1\ \text{F},\ E = 500 \sin 2t\ \text{V}$$

8–14 Find the **steady-state current** in the *RLC*-circuit in Fig. 61 for the given data. Show the details of your work.

$$R = 4\ \Omega,\ L = 0.1\ \mathrm{H},\ C = 0.05\ \mathrm{F},\ E = 110\ \mathrm{V}$$

2.10 Solution by Variation of Parameters

$y'' + ay' + by = r(x)$

Recall the process:

→ solve homogeneous eqⁿ

$y_h = c_1 y_1 + c_2 y_2$

→ $y = y_h + y_p$

how to find y_p

variation of parameters

undetermined coeffs

(Wronskian) $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$

$$u = -\int \frac{y_2 r}{W}\,dx, \qquad v = \int \frac{y_1 r}{W}\,dx.$$

$$y_p(x) = u(x)y_1(x) + v(x)y_2(x)$$

Solve the nonhomogeneous ODE

$$y'' + y = \sec x$$

homogenous part

$$y'' + y = 0$$

$$y_h = c_1 \underbrace{\cos x}_{y_1} + c_2 \underbrace{\sin(x)}_{y_2}$$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = 1$$

$$u = -\int \frac{y_2 r}{W} dx = -\int \frac{\sin x \sec x}{1} dx = \log|\cos x|$$

$$v = \int \frac{y_1 r}{W} dx = \int \frac{\cos x \sec x}{1} dx = x$$

$$y_p = u y_1 + v y_2 = \left(\log|\cos x|\right) \cos x + x \sin x$$

→ solve homogenous part get $y_h = c_1 y_1 + c_2 y_2$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$r = \sec x$

$$u = -\int \frac{y_2 r}{W} dx, \qquad v = \int \frac{y_1 r}{W} dx.$$

$$y_p(x) = u(x) y_1(x) + v(x) y_2(x)$$

Q:

$$y'' + 9y = \csc 3x$$

$$y_h = A \underbrace{\cos 3x}_{y_1} + B \underbrace{\sin 3x}_{y_2}$$

$$W = \begin{vmatrix} \cos 3x & \sin 3x \\ -3\sin 3x & 3\cos 3x \end{vmatrix} = 3$$

$$u = -\int \frac{y_2 r}{W} dx = -\int \frac{\cancel{\sin 3x}\ \cancel{\csc} 3x}{3} dx = \frac{-x}{3}$$

$$v = \int \frac{y_1 r}{W} dx = \frac{1}{3}\int \frac{\cos 3x \cdot \csc 3x}{} dx$$

$$= \frac{1}{9} \log|\sin 3x|$$

$$y_p = \frac{-x}{3}\cos 3x + \frac{1}{9}\log|\sin 3x| \sin 3x$$

→ solve homogenous part get $y_h = c_1 y_1 + c_2 y_2$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$r = \sec x$

$$u = -\int \frac{y_2 r}{W} dx, \qquad v = \int \frac{y_1 r}{W} dx.$$

Idea of the Method.

next time :