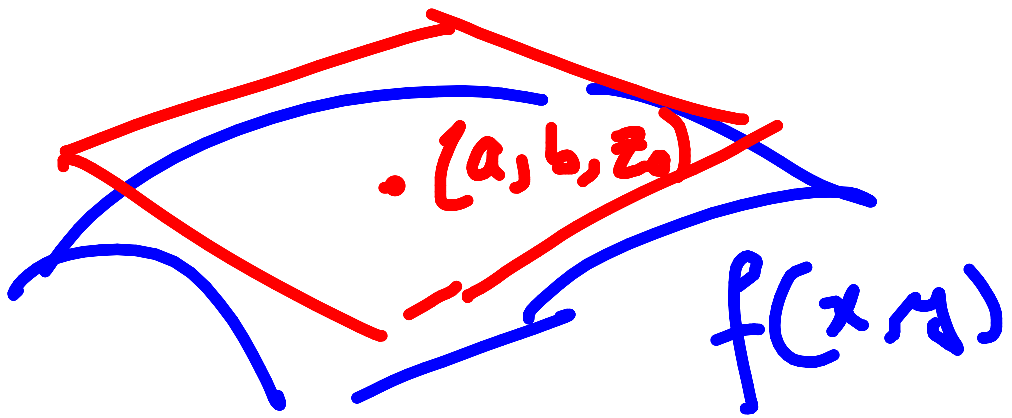
Recall:

11.1 : function of two variables
- graphs, domains, level curves

11.2 : limits & continuity] skipped for now

11.3 : partial derivatives
↪ Clairaut's theorem
↪ $f_{xy} = f_{yx}$

11.4: Tangent Plane



$$z_0 = f(a,b)$$

$$z - z_0 = A(x-a) + B(y-b)$$

where, $A = \frac{\partial f}{\partial x}(a,b)$

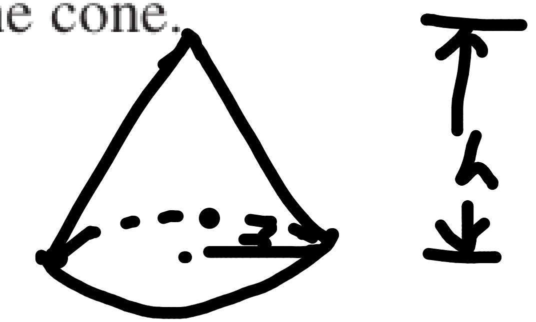
$$B = \frac{\partial f}{\partial y}(a,b)$$

derivation is still pending

$$\underbrace{z - z_0}_{dz} = \frac{\partial f}{\partial x}\underbrace{(x-a)}_{dx} + \frac{\partial f}{\partial y}\underbrace{(y-b)}_{dy}$$

differentials: $dz = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$

EXAMPLE 4 The base radius and height of a right circular cone are measured as 10 cm and 25 cm, respectively, with a possible error in measurement of as much as 0.1 cm in each. Use differentials to estimate the maximum error in the calculated volume of the cone.



$$V = \frac{1}{3}\pi r^2 h$$

$$dr = 0.1$$
$$dh = 0.1$$

$$dV = \frac{\partial V}{\partial r}dr + \frac{\partial V}{\partial h}dh$$
$$= \frac{2\pi r h}{3}dr + \frac{1}{3}\pi r^2 dh$$
$$= \frac{2\pi}{3}10\cdot 25\,(0.1) + \frac{1}{3}\pi\, 10^2(0.1)$$
$$= 20\pi \approx 63\ \text{cm}^3$$

Plan today

→ 2-3 review problem

⇒ 11.5 chain rule

$r = 10$
$h = 25$
$dr = 0.1$
$dh = 0.1$

1–6 ▪ Find an equation of the tangent plane to the given surface at the specified point.

$z = y\cos(x-y), \quad (2, 2, 2)$

Q: is the point $(2,2,2)$ on the graph??

$2 = 2\cos(0)$

Q: $z-2 = A\,(x-2) + B\,(y-2)$

$A = \frac{\partial z}{\partial x} = -y\sin(x-y)$

$A(2,2) = 0$

$B = \frac{\partial z}{\partial y}$

$\cos(x-y) + y\sin(x-y)$

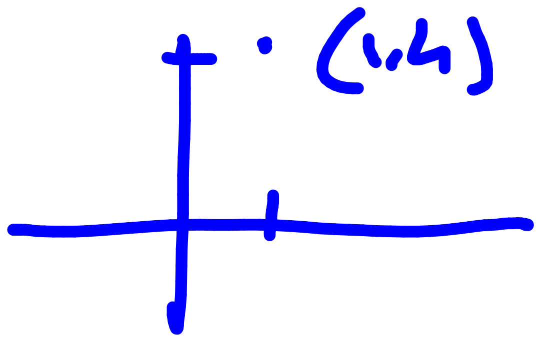
$B = 1$

$z-2 = y-2$

$\boxed{z = y}$

11–14 ■ Explain why the function is differentiable at the given point. Then find the linearization $L(x, y)$ of the function at that point.

$$f(x, y) = x\sqrt{y}, \quad (1, 4)$$



$$f_x = \sqrt{y}$$

$$f_y = \frac{x}{2\sqrt{y}}$$

both f_x & f_y are smooth functions at $(1,4)$ therefore f is differentiable at $(1,4)$

Recall:

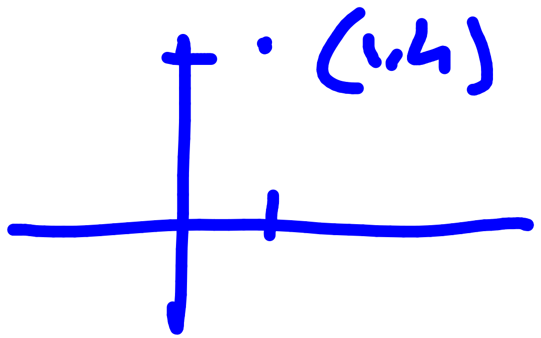
f is differentiable at (a,b) if f_x & f_y both exist & are continuous at the point.

Q: is f differentiable at $(1,0)$

→ NO. ∵ f_y does not exist

11–14 ▪ Explain why the function is differentiable at the given point. Then find the linearization $L(x, y)$ of the function at that point.

$$f(x, y) = x\sqrt{y}, \quad (1, 4)$$



$f_x = \sqrt{y}$

$f_y = \dfrac{x}{2\sqrt{y}}$

$A = 2$

$B = \dfrac{1}{4}$

$$z - z_0 = A(x-a) + B(y-b)$$

$$z = z_0 + A(x-a) + B(y-b)$$

$$\boxed{L(x,y) = z_0 + A(x-a) + B(y-b)}$$

$z_0 = 2$

$$z - 2 = 2(x-1) + \frac{1}{4}(y-4)$$

$$L(x,y) = 2 + 2(x-1) + \frac{1}{4}(y-4)$$

30. The pressure, volume, and temperature of a mole of an ideal gas are related by the equation $PV = 8.31T$, where P is measured in kilopascals, V in liters, and T in kelvins. Use differentials to find the approximate change in the pressure if the volume increases from 12 L to 12.3 L and the temperature decreases from 310 K to 305 K.

$$f(x,y)$$

$$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$$

q: find dP, given $\boxed{dV = 0.3,\ dT = -5 \quad V = 12,\ T = 310}$

$$P = 8.31\ T/V$$

$$dP = \frac{\partial P}{\partial T}dT + \frac{\partial P}{\partial V}dV$$

$$= \frac{8.31}{V}dT + \left(-\frac{8.31\,T}{V^2}\right)dV$$

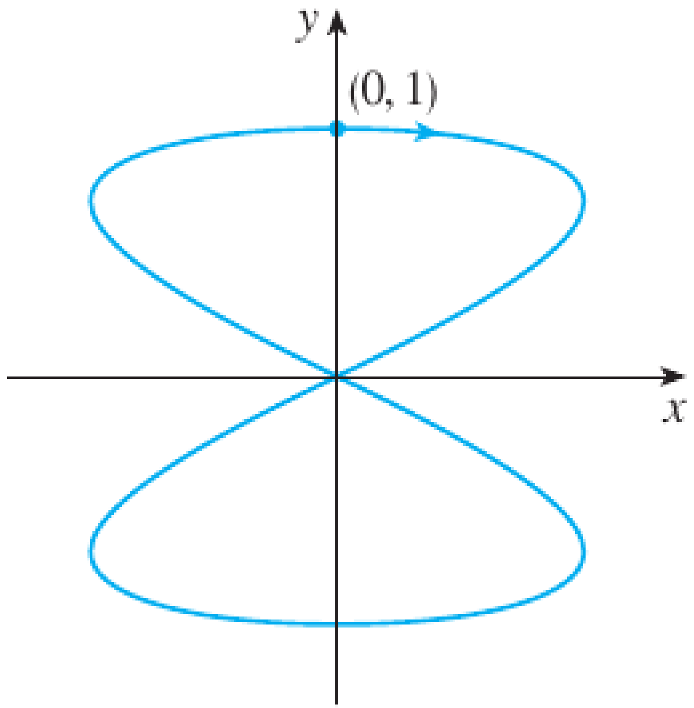
$$dP = -8.83 \quad kPa$$

11.5 THE CHAIN RULE

$$\begin{tikzcd} f \arrow[d, dash] \\ x \arrow[d, dash] \\ u \arrow[d, dash] \\ v \end{tikzcd}$$

$$\frac{df}{dv} = \frac{df}{dx}\,\frac{dx}{du}\,\frac{du}{dv}$$

EXAMPLE 1 If $z = x^2y + 3xy^4$, where $x = \sin 2t$ and $y = \cos t$, find dz/dt when $t = 0$.



$\frac{dz}{dt}$ = well defined??

$$= \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

$$= (2xy + 3y^4)(2\cos 2t) + \underbrace{(x^2 + 12xy^3)(-\sin t)}_{=0}$$

$t = 0$

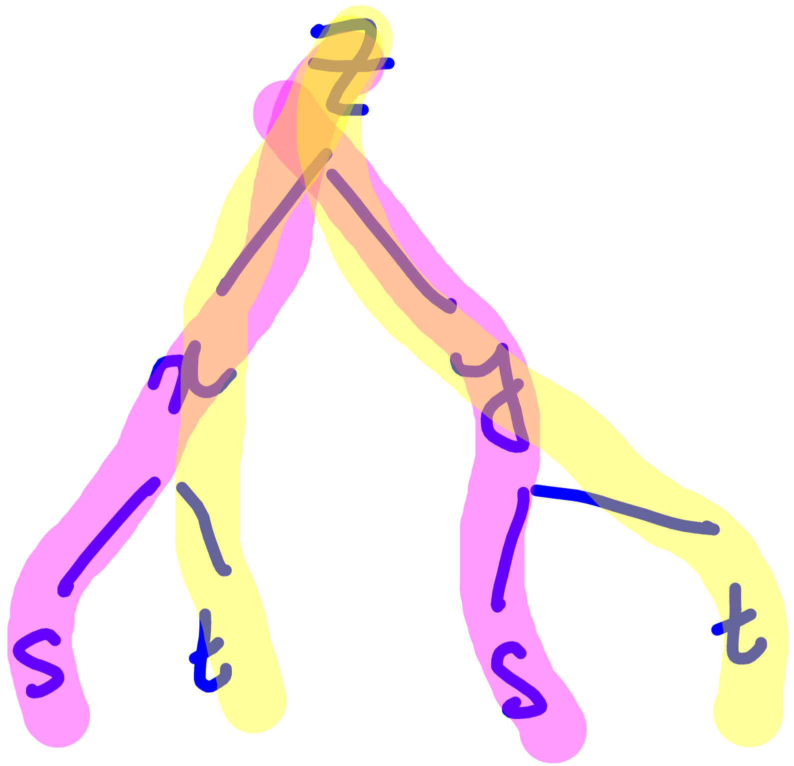
$x = \sin(2t) = 0$

$y = \cos(t) = 1$

$$= 3.2.\cos(0)$$

$$= 6$$

EXAMPLE 3 If $z = e^x \sin y$, where $x = st^2$ and $y = s^2t$, find $\partial z/\partial s$ and $\partial z/\partial t$.



$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$$

$$= ?? \qquad \text{H.W.}$$

leave x, y in the formula

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$

$$= ?? \qquad \text{H.W.}$$

V EXAMPLE 5 If $u = x^4y + y^2z^3$, where $x = rse^t$, $y = rs^2e^{-t}$, and $z = r^2s\sin t$, find the value of $\partial u/\partial s$ when $r = 2$, $s = 1$, $t = 0$.

32. The radius of a right circular cone is increasing at a rate of 1.8 in/s while its height is decreasing at a rate of 2.5 in/s. At what rate is the volume of the cone changing when the radius is 120 in. and the height is 140 in.?

29. The temperature at a point (x, y) is $T(x, y)$, measured in degrees Celsius. A bug crawls so that its position after t seconds is given by $x = \sqrt{1 + t}$, $y = 2 + \frac{1}{3}t$, where x and y are measured in centimeters. The temperature function satisfies $T_x(2, 3) = 4$ and $T_y(2, 3) = 3$. How fast is the temperature rising on the bug's path after 3 seconds?