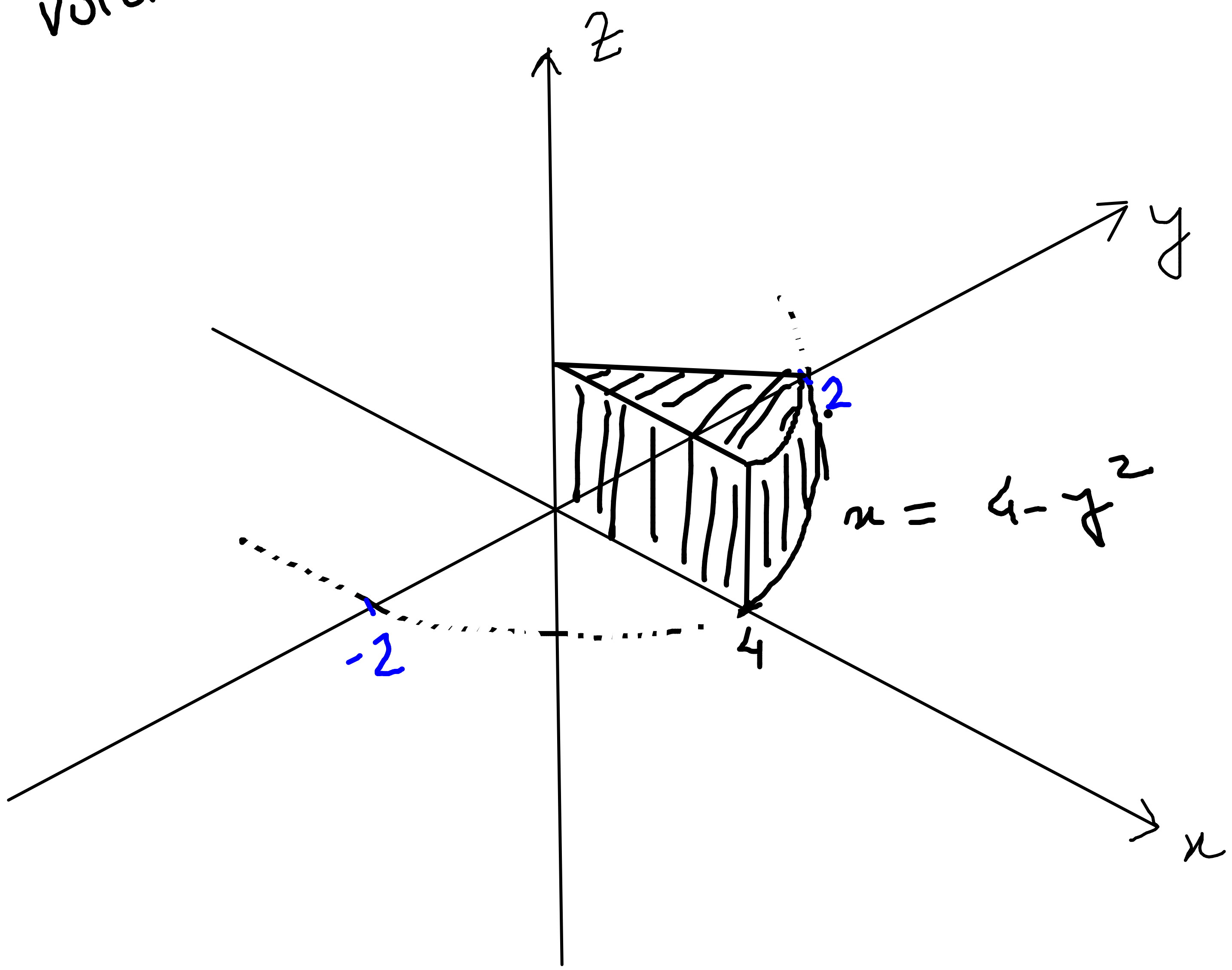
Sketch the solid whose volume is given by the iterated integral.

volume $= \int_0^2 \int_0^{2-y} \int_0^{4-y^2} 1 \, dx \, dz \, dy$

$z = 2 - y$

$z + y = 2$



$= ??$

symbolab.com/solver/triple-integrals-calculator/%5Cint_%7B0%7D%5E%7B2%7D%5Cint_%7B0%7D%5E%7B2-y%7D%5C

Indefinite Integrals
Definite Integrals
Specific-Method
Improper Integrals
Antiderivatives
Double Integrals
Triple Integrals
Multiple Integrals

Integral Applications

Riemann Sum (new)

Series

ODE

Multivariable Calculus (new)

Laplace Transform

Taylor/Maclaurin Series

Fourier Series

$\int_0^2\int_0^{2-y}\int_0^{4-y^2} dxdzdy$

Go

Examples »

Solution

Keep Practicing >

Show Steps

$\int_0^2\int_0^{2-y}\int_0^{4-y^2} 1dxdzdy=\frac{20}{3}$ (Decimal: $6.66666\ldots$)

Steps

$\int_0^2\int_0^{2-y}\int_0^{4-y^2} 1dxdzdy$

$\int_0^{4-y^2} 1dx=-y^2+4$ Show Steps

$=\int_0^2\int_0^{2-y}\left(-y^2+4\right)dzdy$

$\int_0^{2-y}\left(-y^2+4\right)dz=(-y+2)\left(-y^2+4\right)$ Show Steps

$=\int_0^2(-y+2)\left(-y^2+4\right)dy$

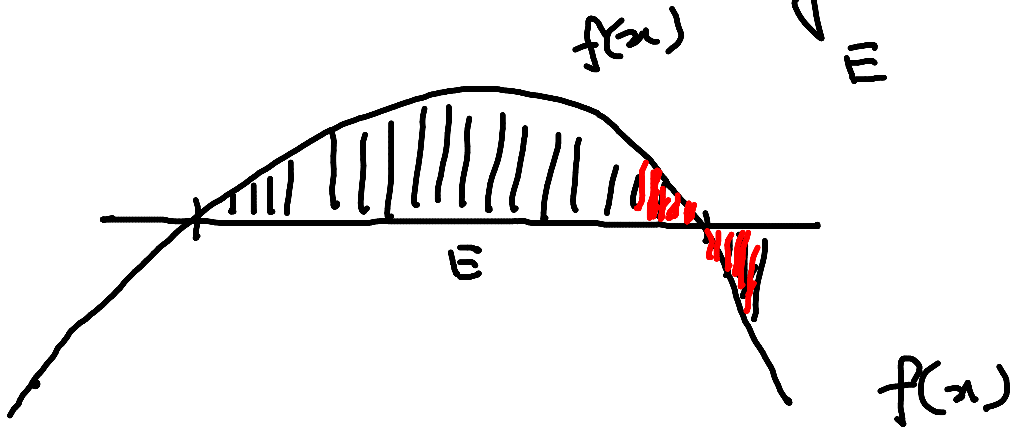
$$=\frac{20}{3}$$

47. Find the region E for which the triple integral

$$\iiint_E (1 - x^2 - 2y^2 - 3z^2)\, dV$$

is a maximum.

$$\int_E f\,dx$$



find E s.t.

$\int_E f\,du$ is as large as possible.

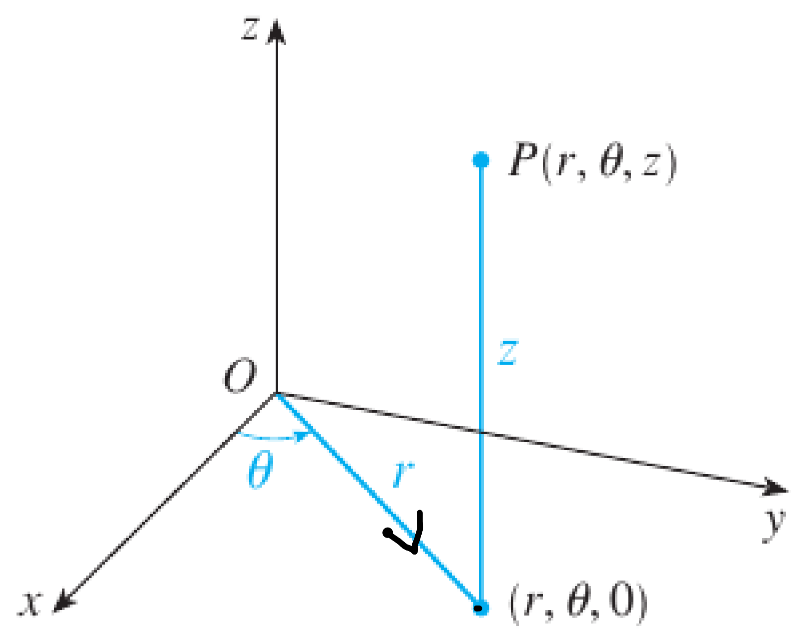
E:

$$1 - x^2 - 2y^2 - 3z^2 \geq 0$$

$$x^2 + 2y^2 + 3z^2 \leq 1$$

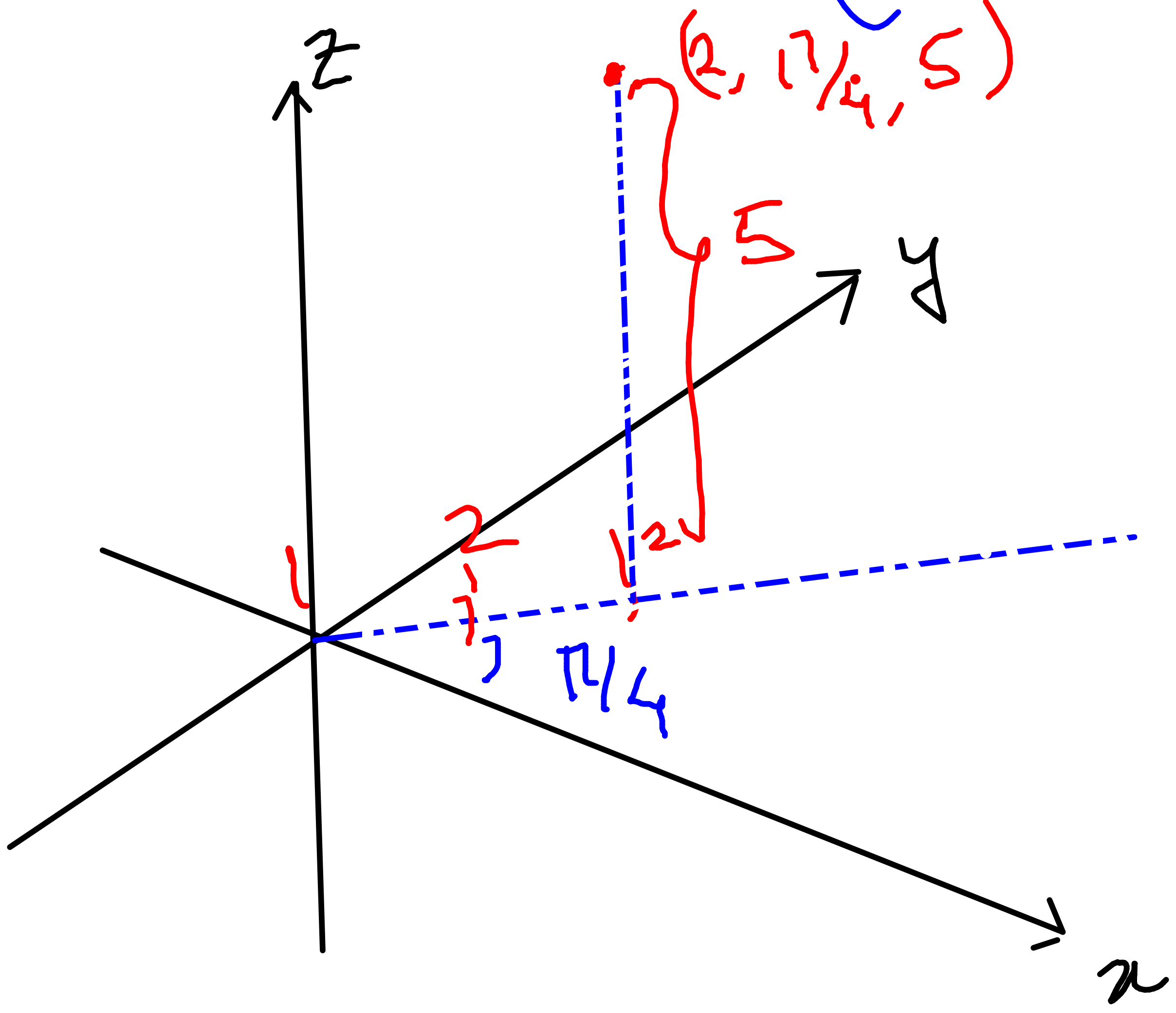
$$x^2 + \frac{y^2}{(1/\sqrt{2})^2} + \frac{z^2}{(1/\sqrt{3})^2} \leq 1$$

12.6 TRIPLE INTEGRALS IN CYLINDRICAL COORDINATES



$$x = r\cos\theta \qquad y = r\sin\theta \qquad z = z$$

Q. Locate the point whose cylindrical coordinates are $(r, \theta, z) = \left(2, \pi/4, 5\right)$.

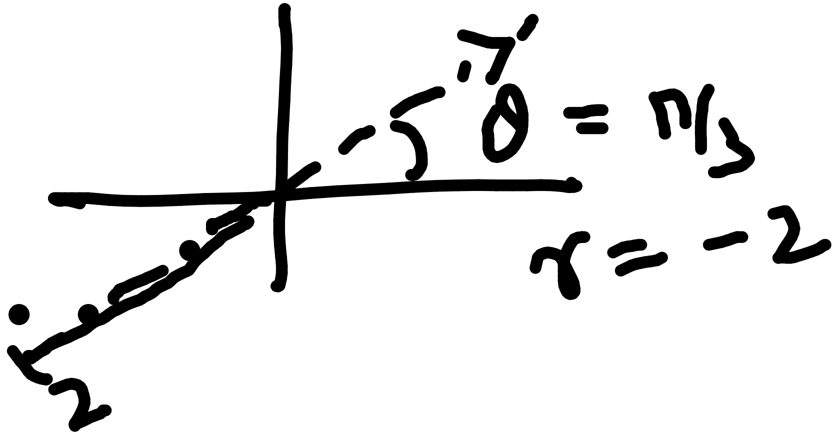


EXAMPLE 2 Describe the surface whose equation in cylindrical coordinates is $z = r$.

what surface is this ??

$r = \sqrt{x^2+y^2}$

$z = \pm\sqrt{x^2+y^2}$



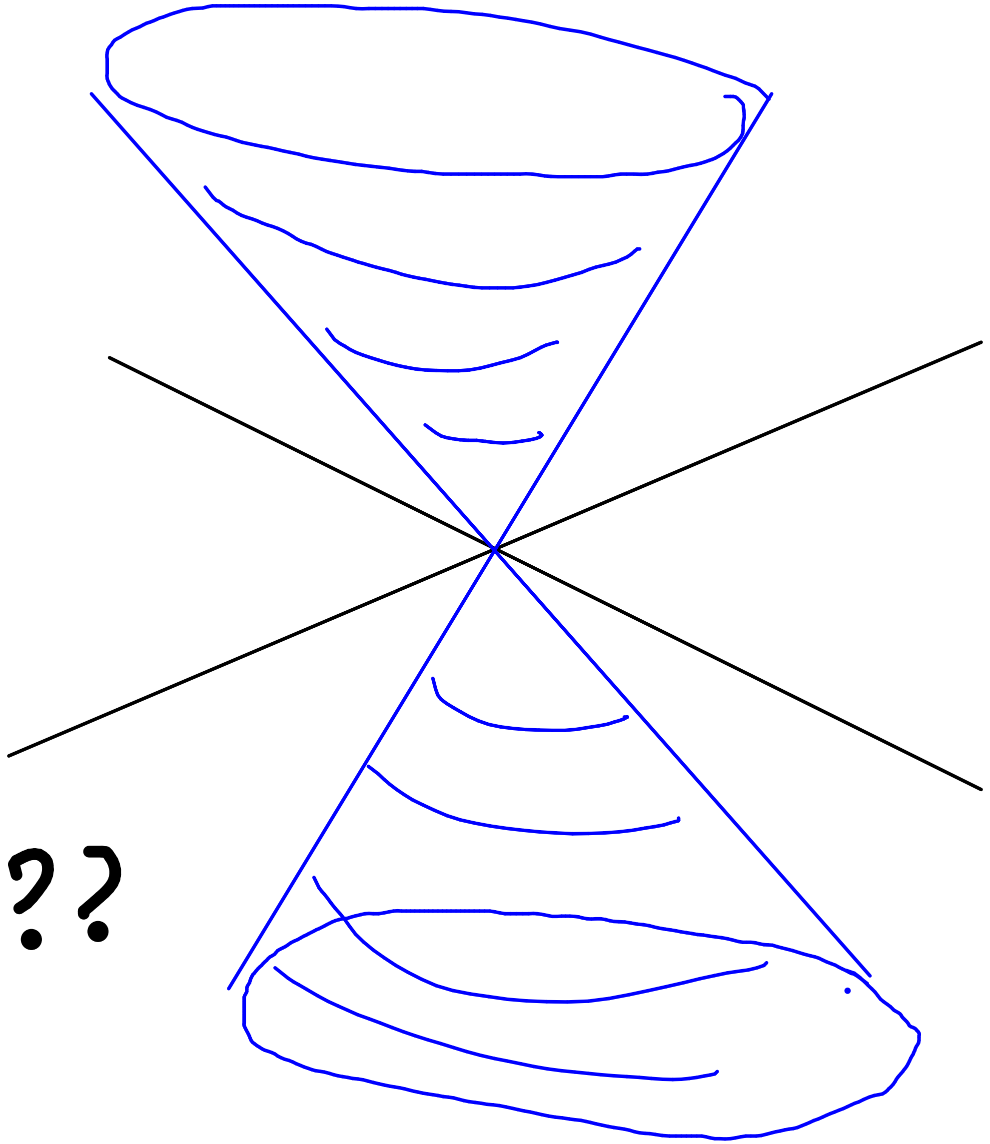
$y = 0$

$z = \sqrt{x^2}$

$= |x|$

why bottom cone as well??

$z = r$

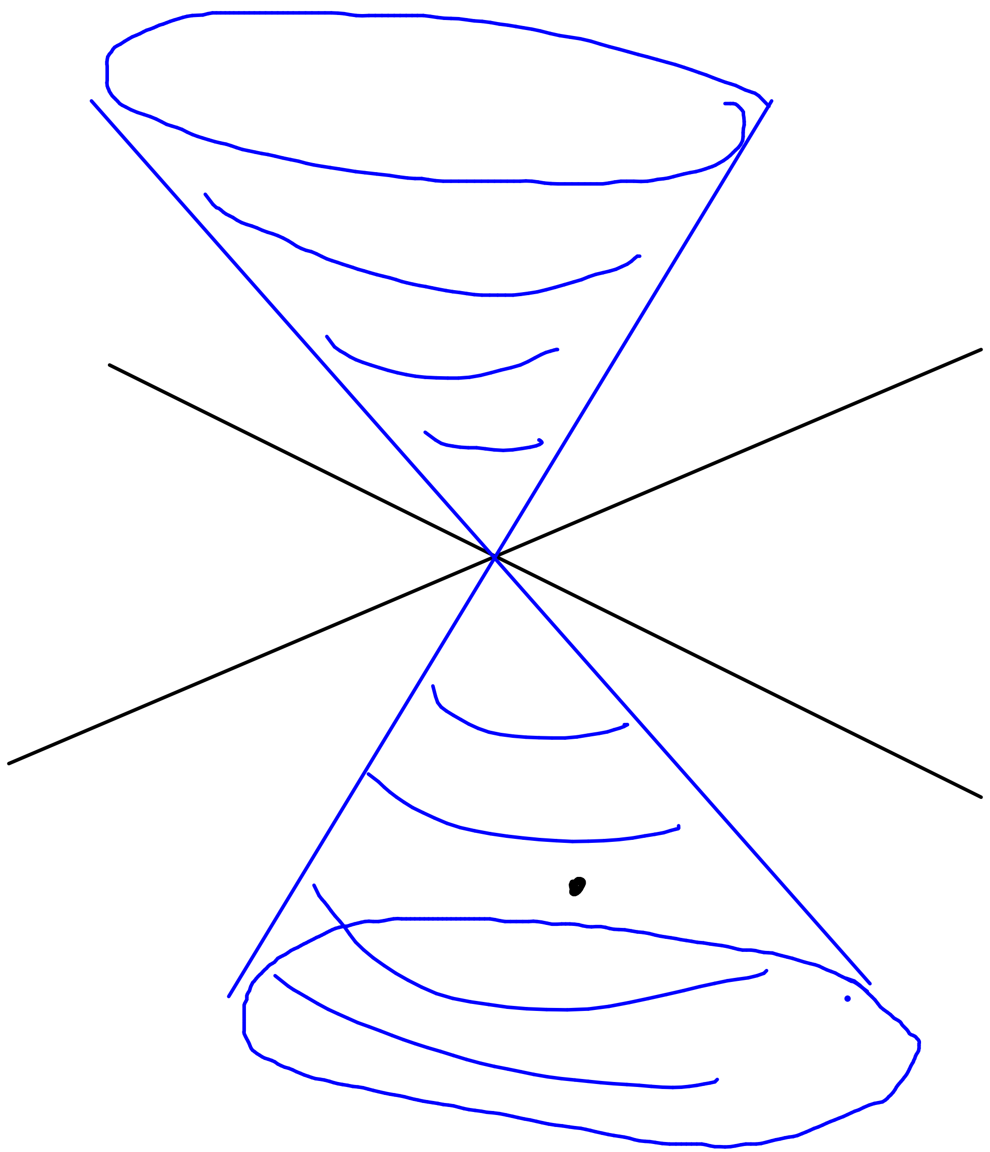


EXAMPLE 2 Describe the surface whose equation in cylindrical coordinates is $z = r$.

$r = \sqrt{x^2+y^2}$

surface ??

collection of all points (r, θ, z) which satisfy the equation $z = r$



Recall :

$$\iint \text{---}\, dx\,dy \longleftrightarrow \iint \text{---}\, du\,dv$$

$$dx\,dy = ??\; du\,dv$$

Jacobian

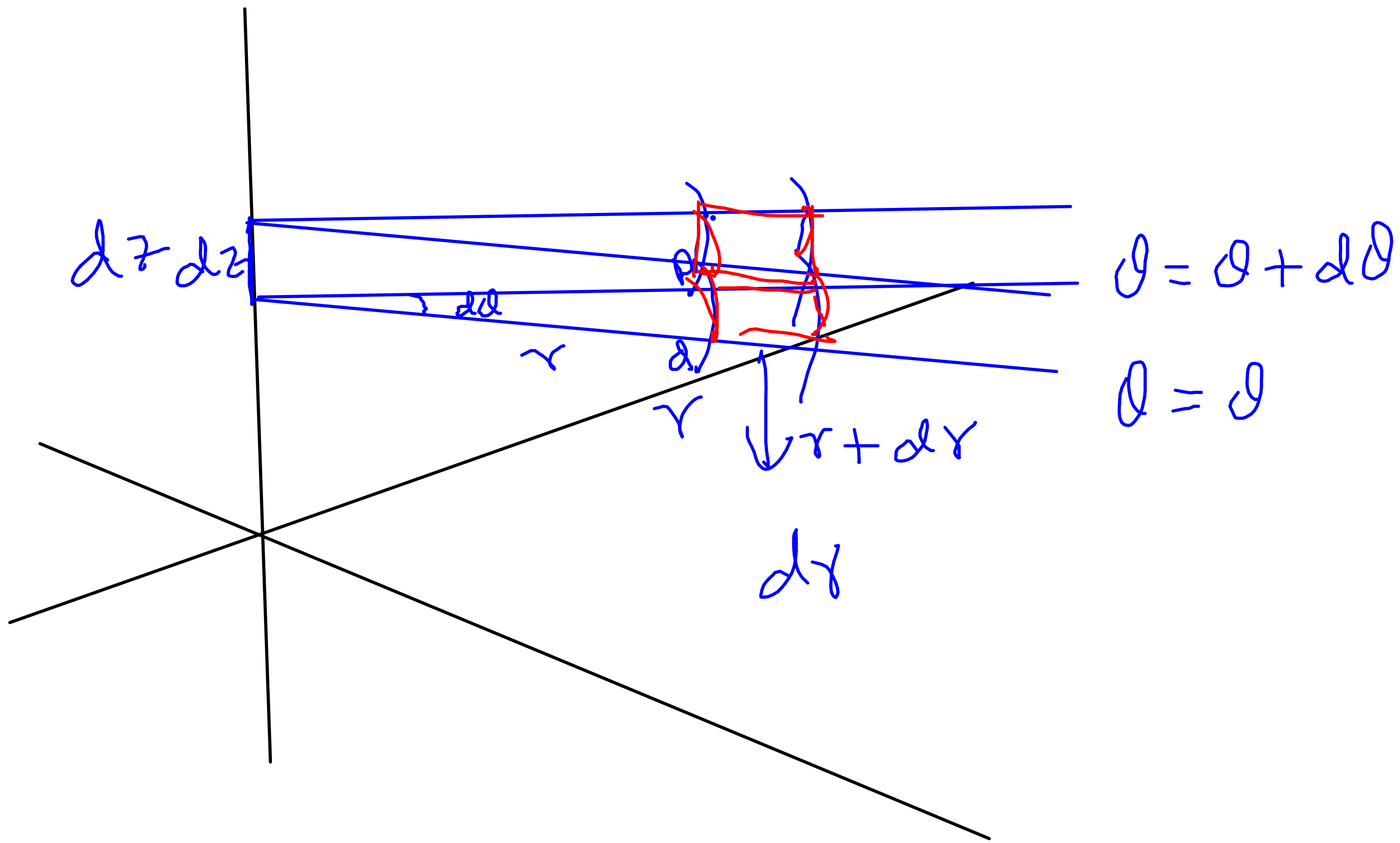
$dx\,dy\,dz \longrightarrow ???\; dr\,d\theta\,dz$

Jacobian of switching from (x, y, z) to cylindrical coordinates

$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$z = z$$

$$J = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} & \frac{\partial x}{\partial z} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} & \frac{\partial y}{\partial z} \\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial \theta} & \frac{\partial z}{\partial z} \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\sin\theta & 0 \\ \sin\theta & r\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = r$$

$$\therefore \quad dx\,dy\,dz = r\,dr\,d\theta\,dz$$



$\widehat{pq} = r\,d\theta$

$\underbrace{r}\,dr\,d\theta\,dz = (r\,d\theta)(dr)(dz)$

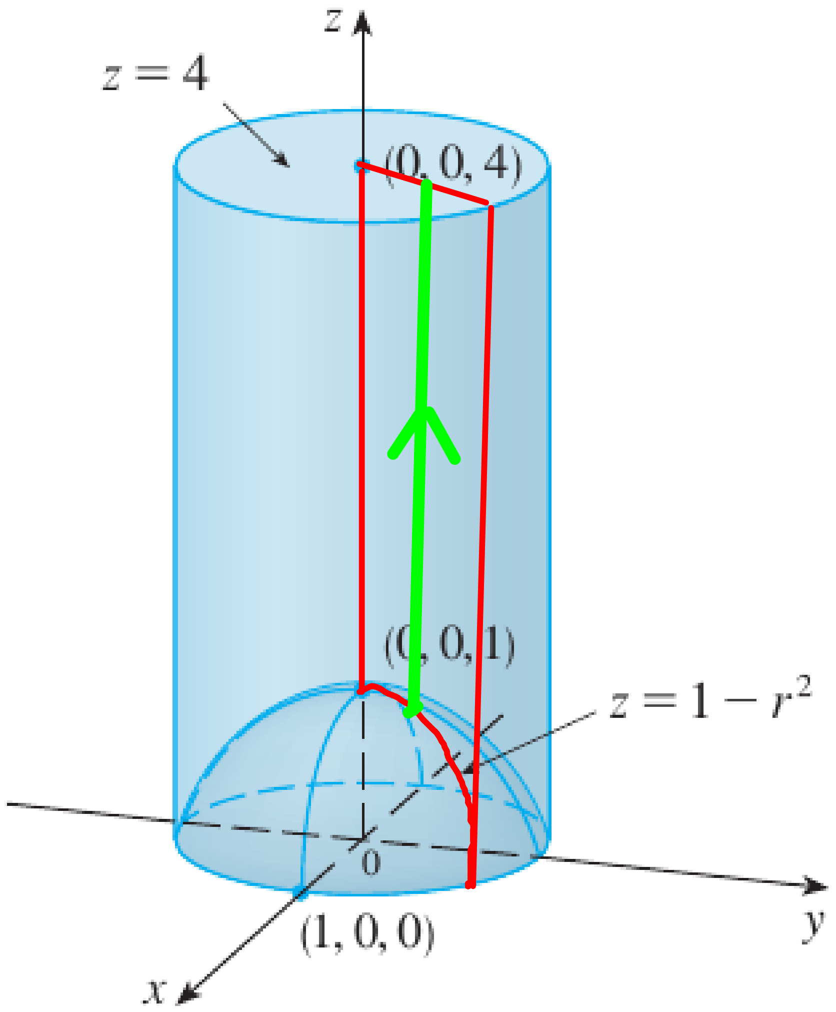
volume swiped for small change $d\theta$, dr, dz

$dV = r\,dr\,d\theta\,dz$

$r^2 = 1 \qquad r = 1$

EXAMPLE 3 A solid E lies within the cylinder $x^2 + y^2 = 1$, below the plane $z = 4$, and above the paraboloid $z = 1 - x^2 - y^2$. (See Figure 8.) The density at any point is proportional to its distance from the axis of the cylinder. Find the mass of E.

K: some constant not given



$$\rho(r, \theta, z) = K r$$

$$\left| \begin{aligned} z &= 1 - x^2 - y^2 \\ &= 1 - r^2 \end{aligned} \right|$$

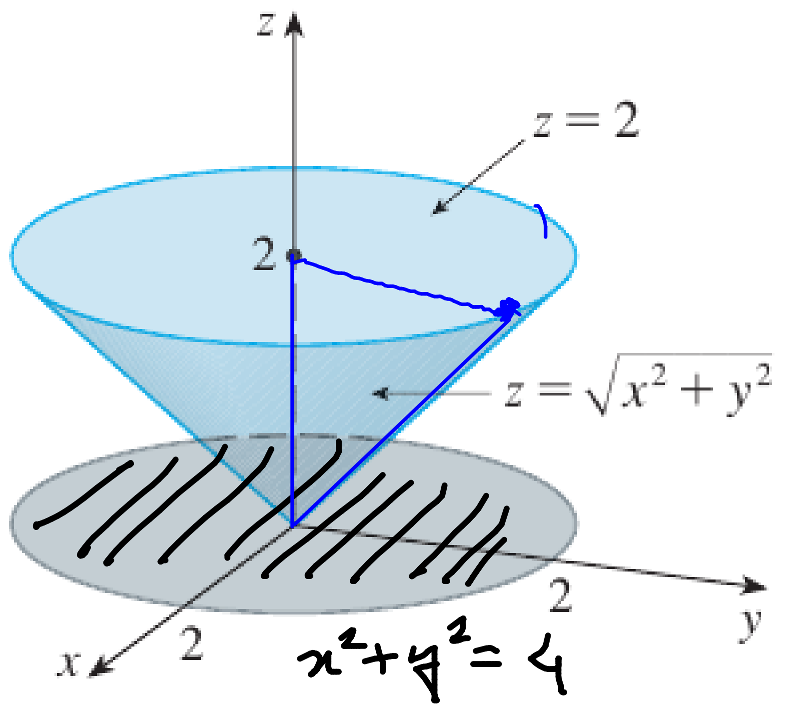
$$\text{mass} = \iiint_E \rho \, dV$$

$$= \int_0^{2\pi} \int_0^1 \int_{1-r^2}^4 (K r) \underbrace{r} \, dz \, dr \, d\theta = K \frac{24}{10} \pi$$

EXAMPLE 4 Evaluate $\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{2} (x^2 + y^2)\, dz\, dy\, dx.$

Rewrite this integration in cylindrical coordinates

$$\int_0^{2\pi} \int_0^2 \int_r^2 r^2 \, r \, dz\, dr\, d\theta$$



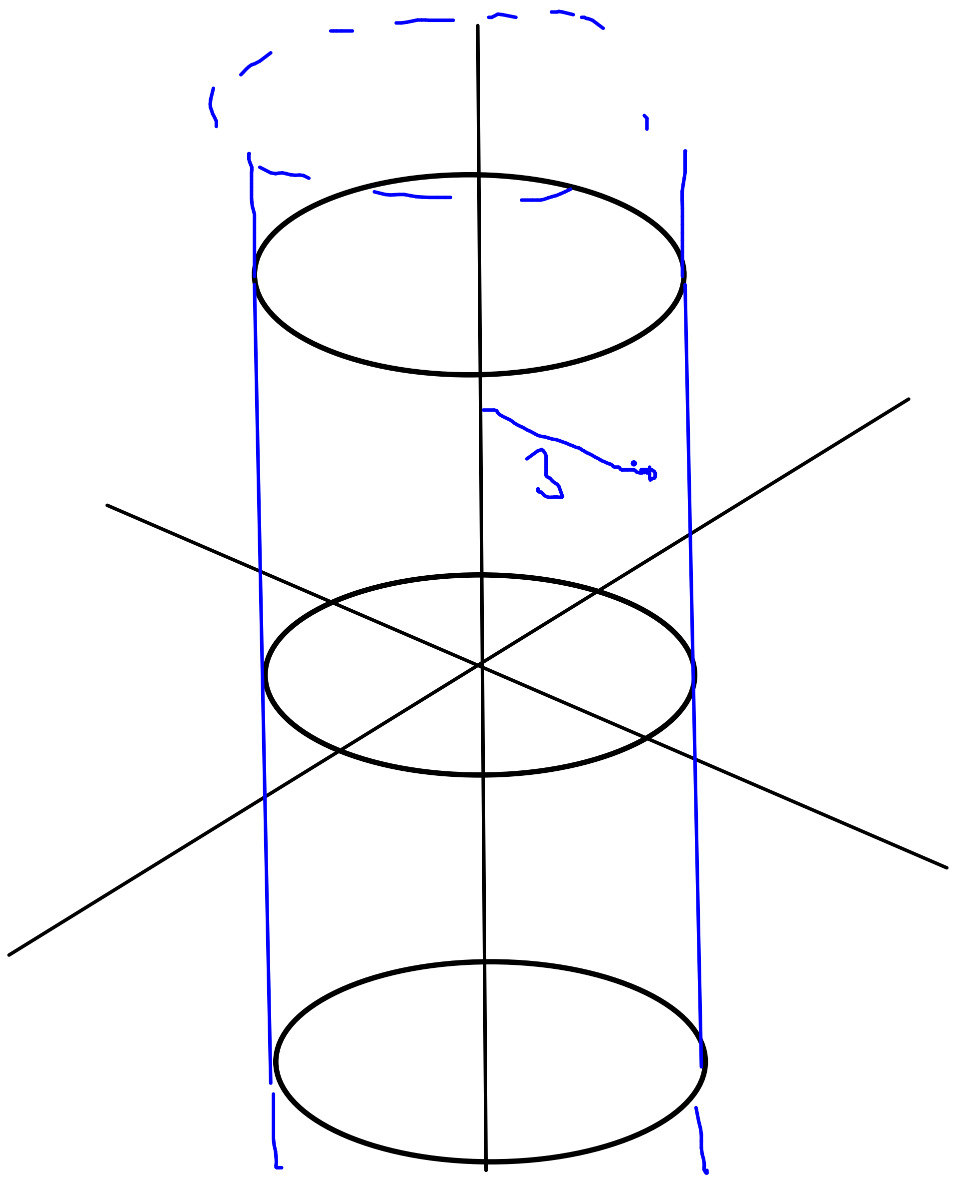
$r^2 = 4$

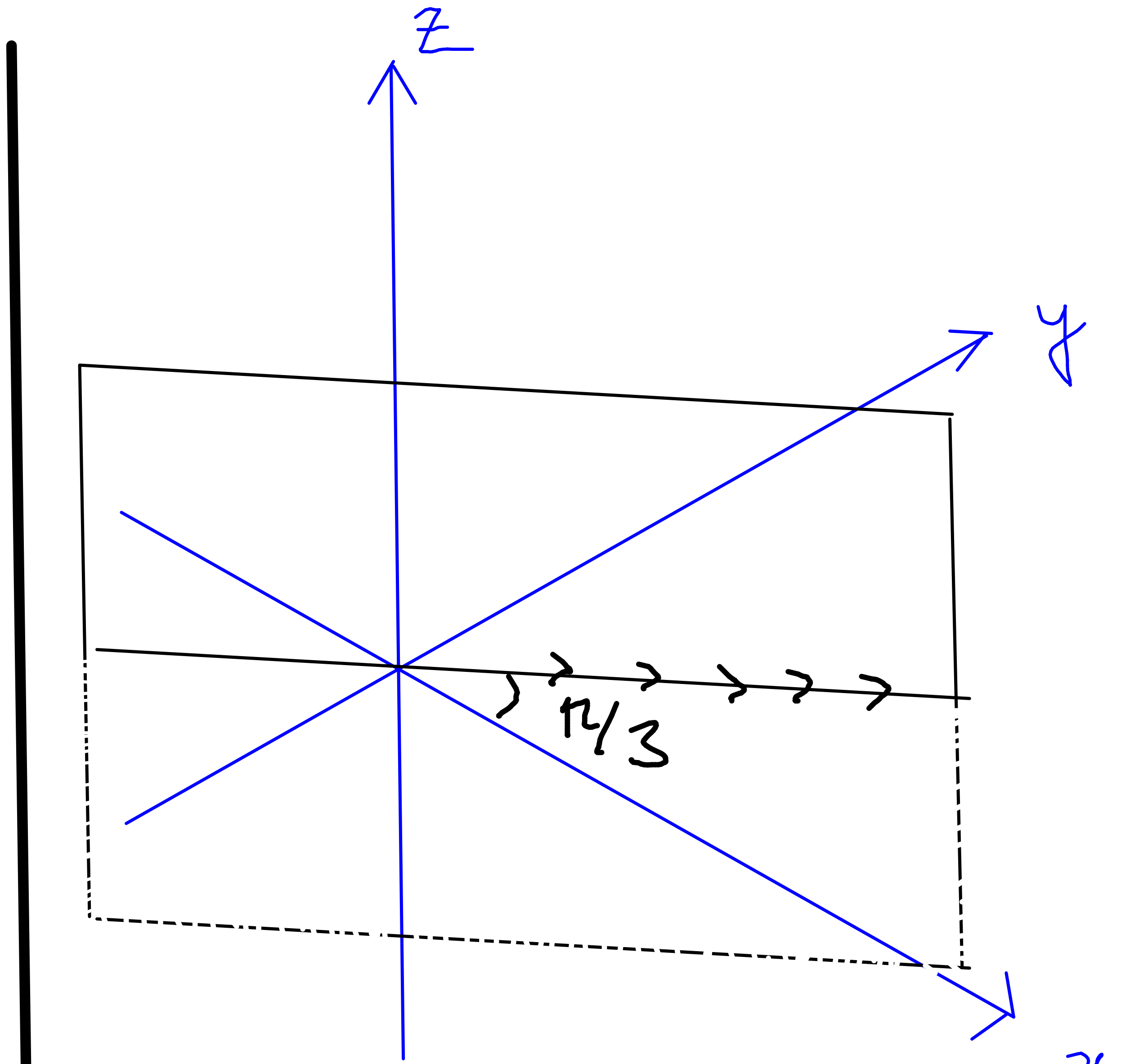
$r = 2$

5–6 ▪ Describe in words the surface whose equation is given.

5. $r = 3$

6. $\theta = \pi/3$





Recall: r can be negative

9–10 ▪ Write the equations in cylindrical coordinates.

9. (a) $z = x^2 + y^2$ (b) $x^2 + y^2 = 2y$

a) $x^2+y^2 = r^2$

$z = x^2+y^2$

in cylindrical coordinates

$\boxed{z = r^2}$

b) $r^2 = 2r\sin\theta$

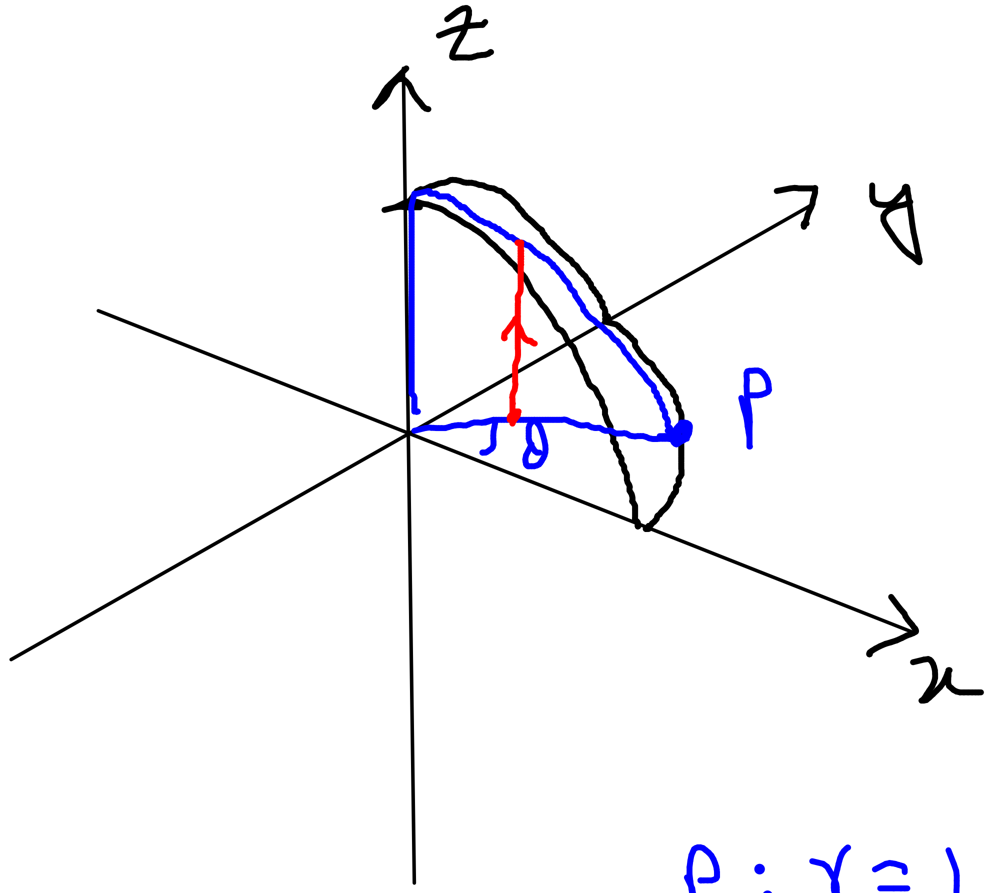
$\boxed{r = 2\sin\theta}$

Evaluate $\iiint_E (x^3 + xy^2)\, dV$, where E is the solid in the first octant that lies beneath the paraboloid $z = 1 - x^2 - y^2$.

$z = 1 - r^2$

$1 - r^2 = 0$

$r = 1$

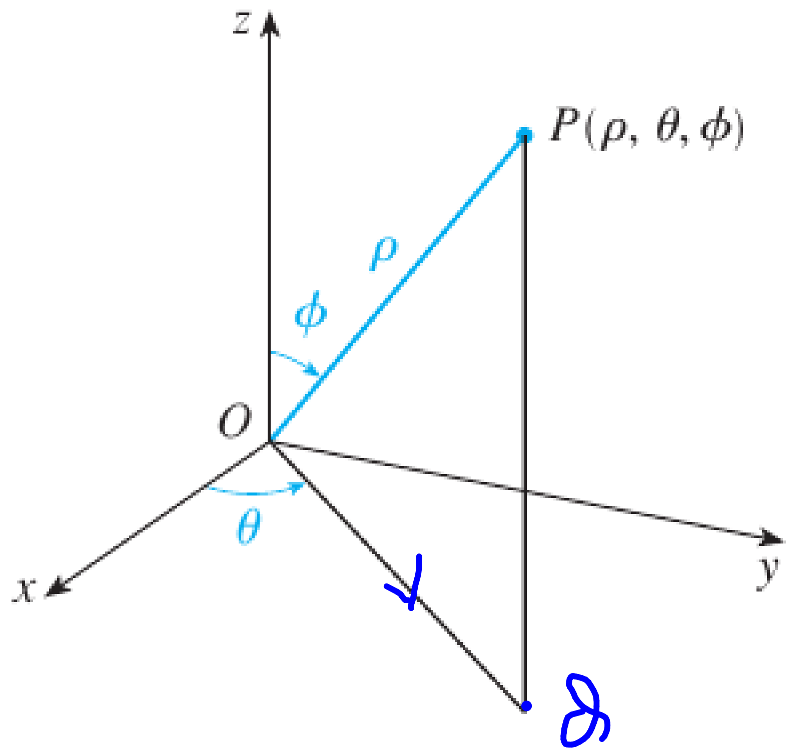


$P : r = 1$

$$\int_0^{\pi/2} \int_0^1 \int_0^{1-r^2} (r\cos\theta)^3 + r\cos\theta\,(r\sin\theta)^2 \; r\, dz\, dr\, d\theta$$

Jacobian

$= \text{whatever} = \dfrac{2}{35}$



ρ : distance from origin

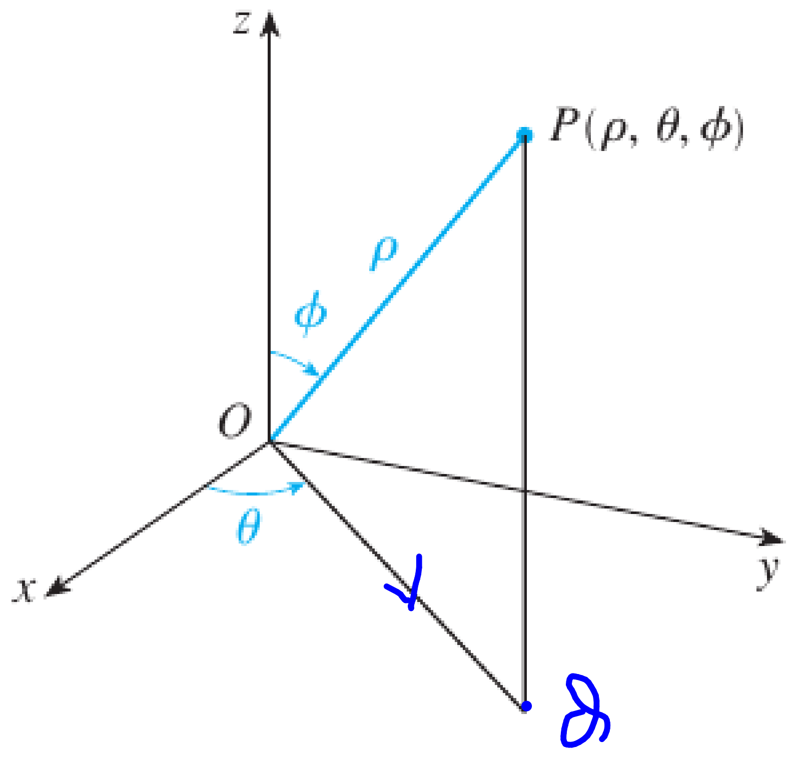
ϕ : angle between positive z axis & vector $\overrightarrow{OP}$

$$0 \le \phi \le \pi$$

$\mathcal{Q}$: projection of P on the xy plane.

θ : angle between positive x axis & the projected vector $\overrightarrow{O\mathcal{Q}}$

12.7 TRIPLE INTEGRALS IN SPHERICAL COORDINATES



ρ : distance from origin | $\rho \geq 0$

ϕ : angle between positive z axis & vector $\vec{OP}$

$$0 \le \phi \le \pi$$

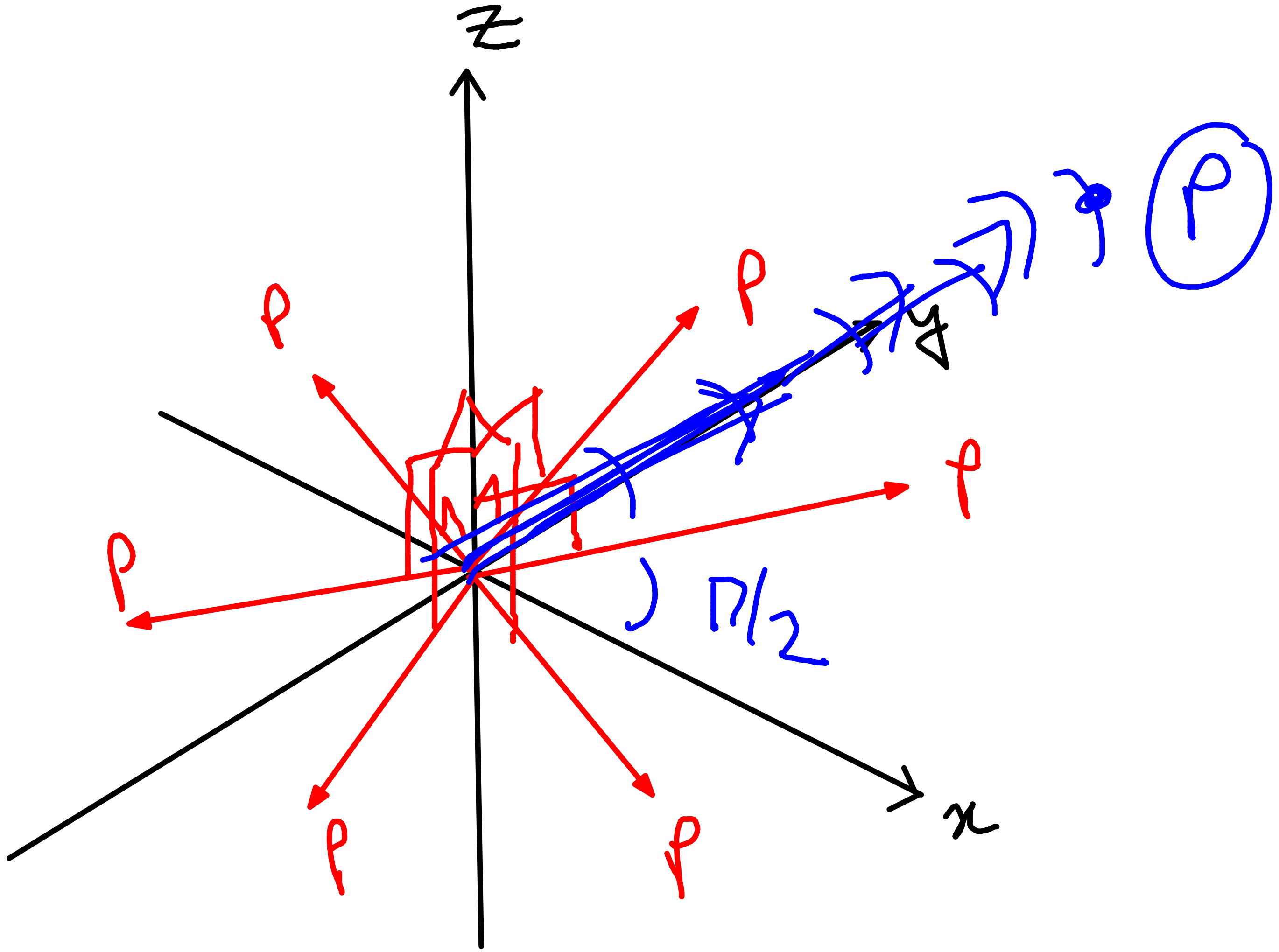
Q: projection of P on the xy plane.

θ : angle between positive x axis & the projected vector $\vec{OQ}$

$$0 \le \theta \le 2\pi$$

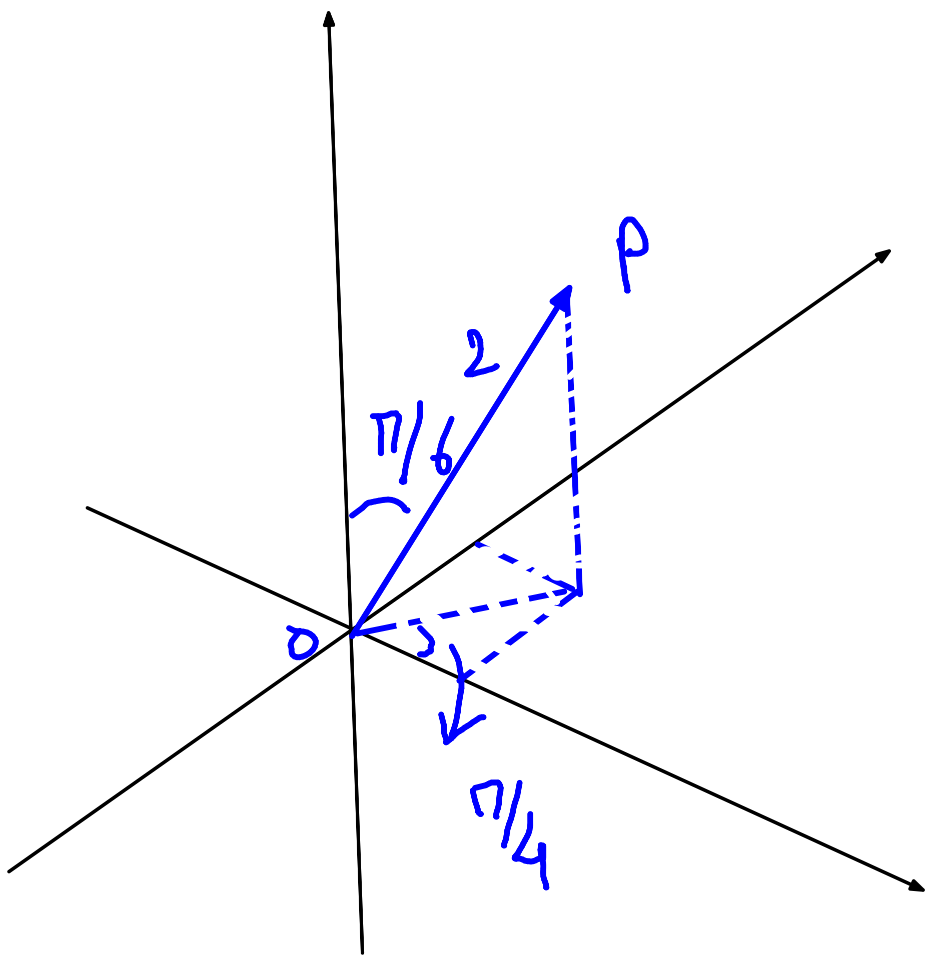
Q: $(\rho, \theta, \phi) = (5, \pi/2, \pi/2) = P$

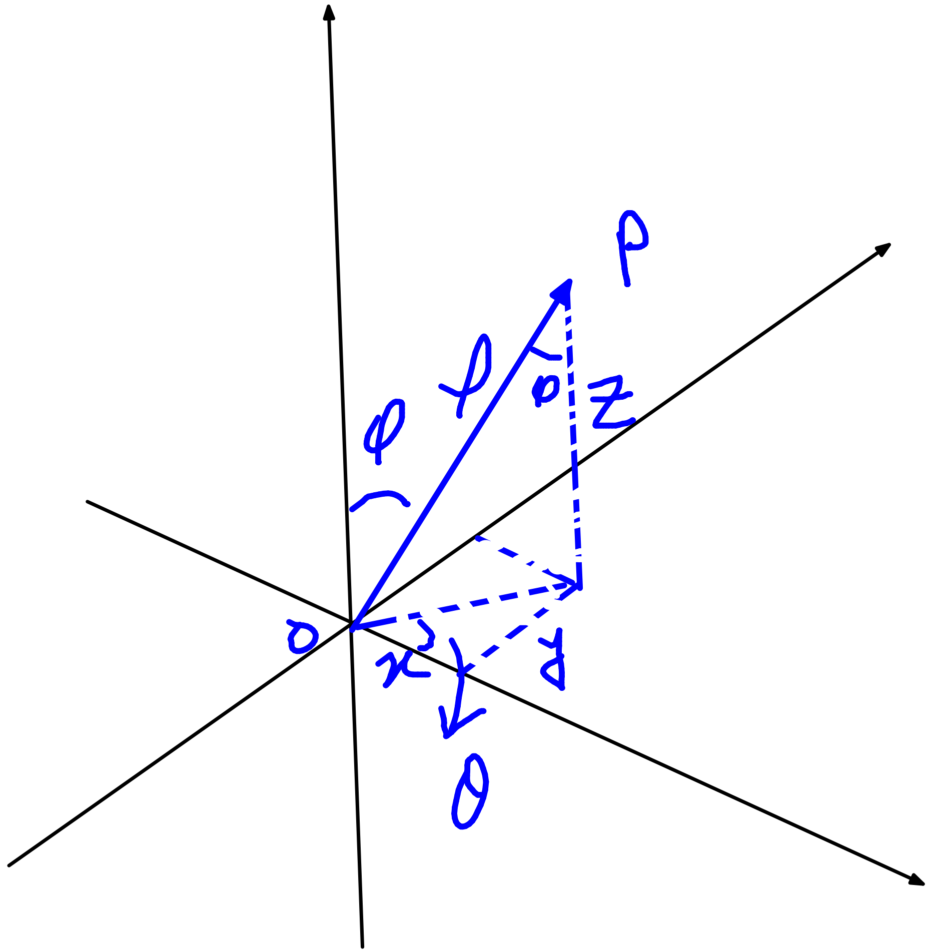
Can you mark this point in 3d ??



Q. $(r, \theta, \phi) = (2, \pi/4, \pi/6) = P$

mark this point in 3d space





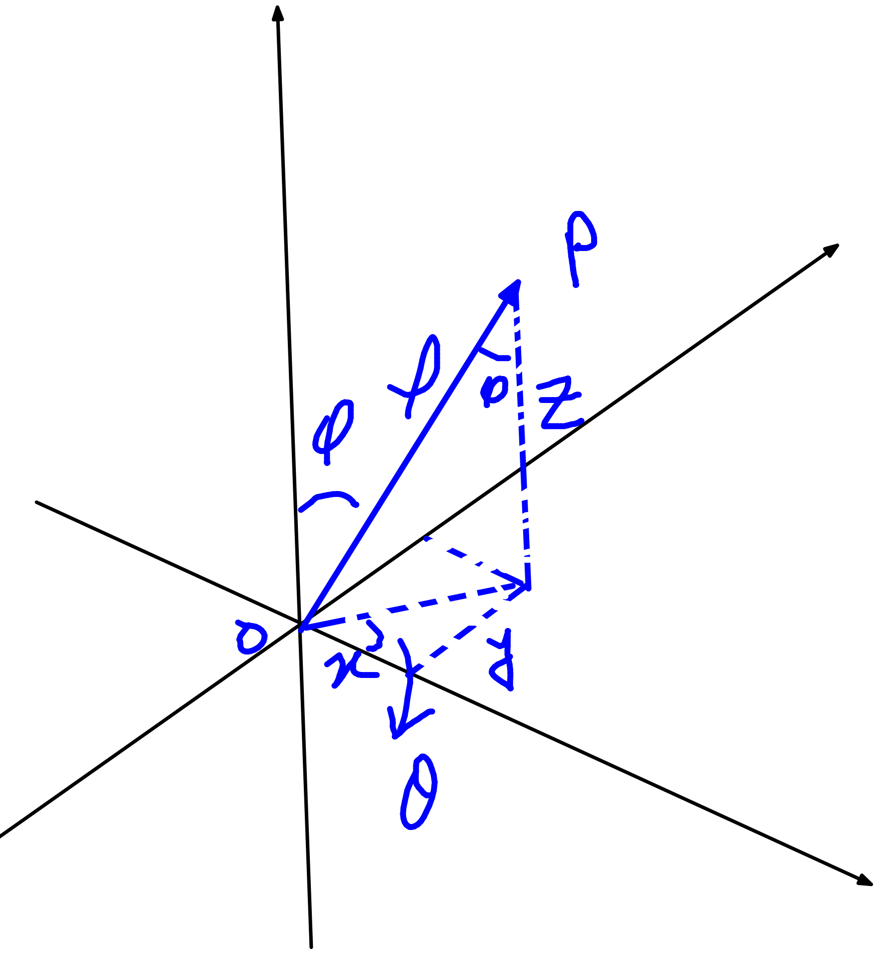
$$x = \rho \cos\theta \sin\phi$$

$$y = \rho \sin\theta \sin\phi$$

$$z = \rho \cos\phi$$

Suppose therse is some integration

$$\iiint_{V} f(x,y,z)\, dx\, dy\, dz \longrightarrow \iiint_{\tilde{V}} \tilde{f}(\rho,\theta,\phi)\ ???\ d\rho\, d\theta\, d\phi$$



$x = \rho \cos\theta \sin\phi$

$y = \rho \sin\theta \sin\phi$

$z = \rho \cos\phi$

$$\boxed{\begin{array}{l}\text{Recall} \\ 0 \le \phi \le \pi \\ \sin\phi \ge 0\end{array}}$$

$$??? = \text{Jacobian} = \frac{\partial(x,y,z)}{\partial(\rho,\theta,\phi)} = \begin{vmatrix} \frac{\partial x}{\partial \rho} & \frac{\partial x}{\partial \theta} & \frac{\partial x}{\partial \phi} \\ \frac{\partial y}{\partial \rho} & \frac{\partial y}{\partial \theta} & \frac{\partial y}{\partial \phi} \\ \frac{\partial z}{\partial \rho} & \frac{\partial z}{\partial \theta} & \frac{\partial z}{\partial \phi} \end{vmatrix}$$

$$= \rho^2 \sin\phi$$

$$\text{Jacobian} = |\rho^2 \sin\phi| = \rho^2 \sin\phi$$

Theorem: when doing integration in spherical coordinates,

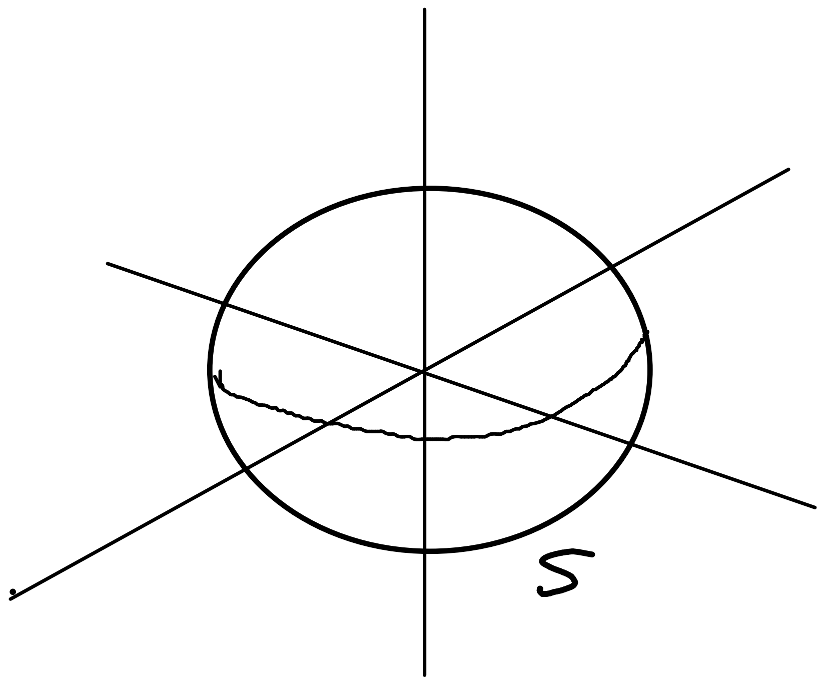
$$\boxed{dV = \underbrace{\rho^2 \sin\phi}\ d\rho\ d\theta\ d\phi}$$

i.e. dV = volume swiped for small change $d\rho$, $d\theta$, $d\phi$

we will discuss on this more.

Q:

Sphere of radius r

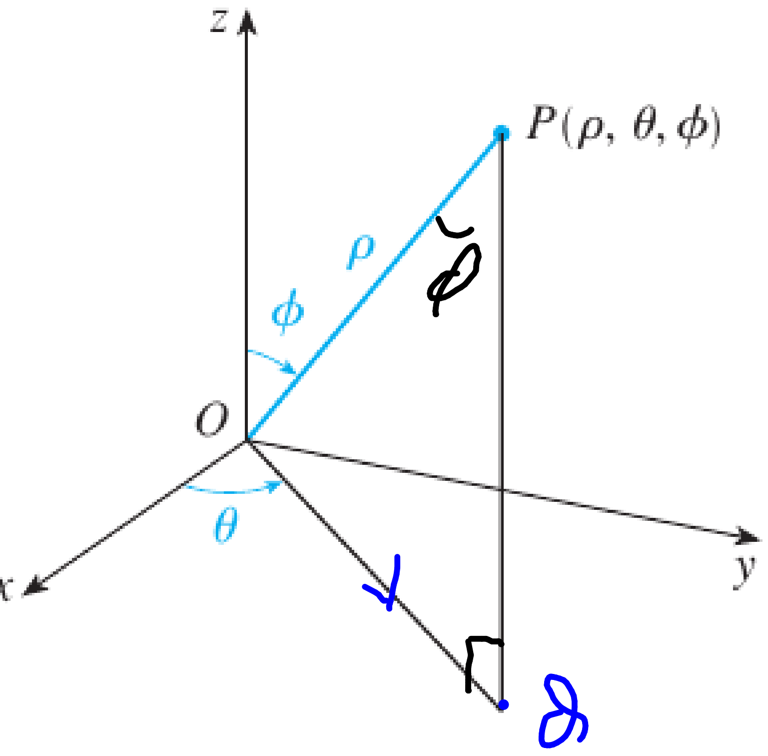


$$\text{Volume} = ?? = \frac{4}{3}\pi r^3$$

$$0 \le \rho \le r$$
$$0 \le \theta \le 2\pi$$
$$0 \le \phi \le \pi$$

$V = \iiint_S dV$ = set up the triple integration in spherical coordinates $= \int_0^r \int_0^{2\pi} \int_0^{\pi} \rho^2 \sin\phi \, d\phi \, d\theta \, d\rho$

$$= \frac{4}{3}\pi r^3$$



$$x = OQ\cos\theta = \rho\sin\phi\cos\theta$$

$$y = OQ\sin\theta = \rho\sin\phi\sin\theta$$

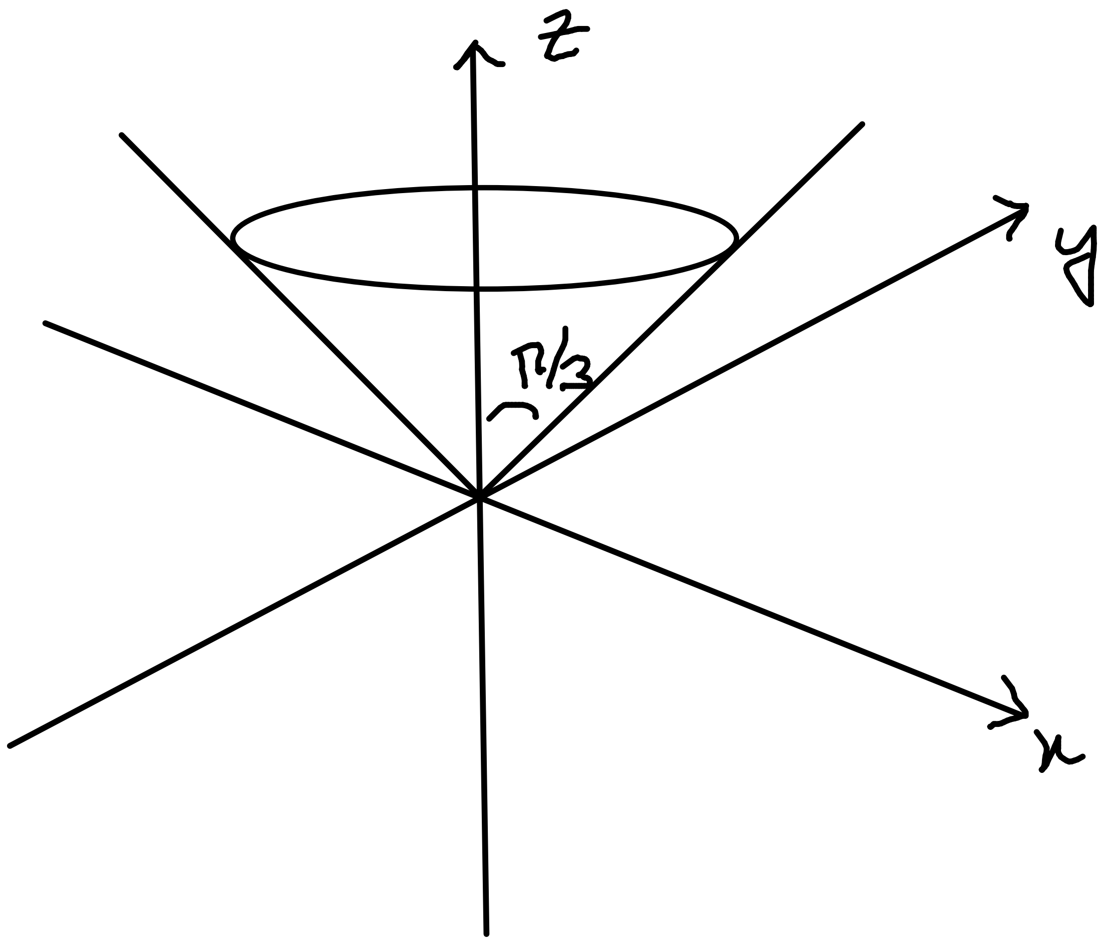
$$z = \rho\cos\phi = \rho\cos\phi$$

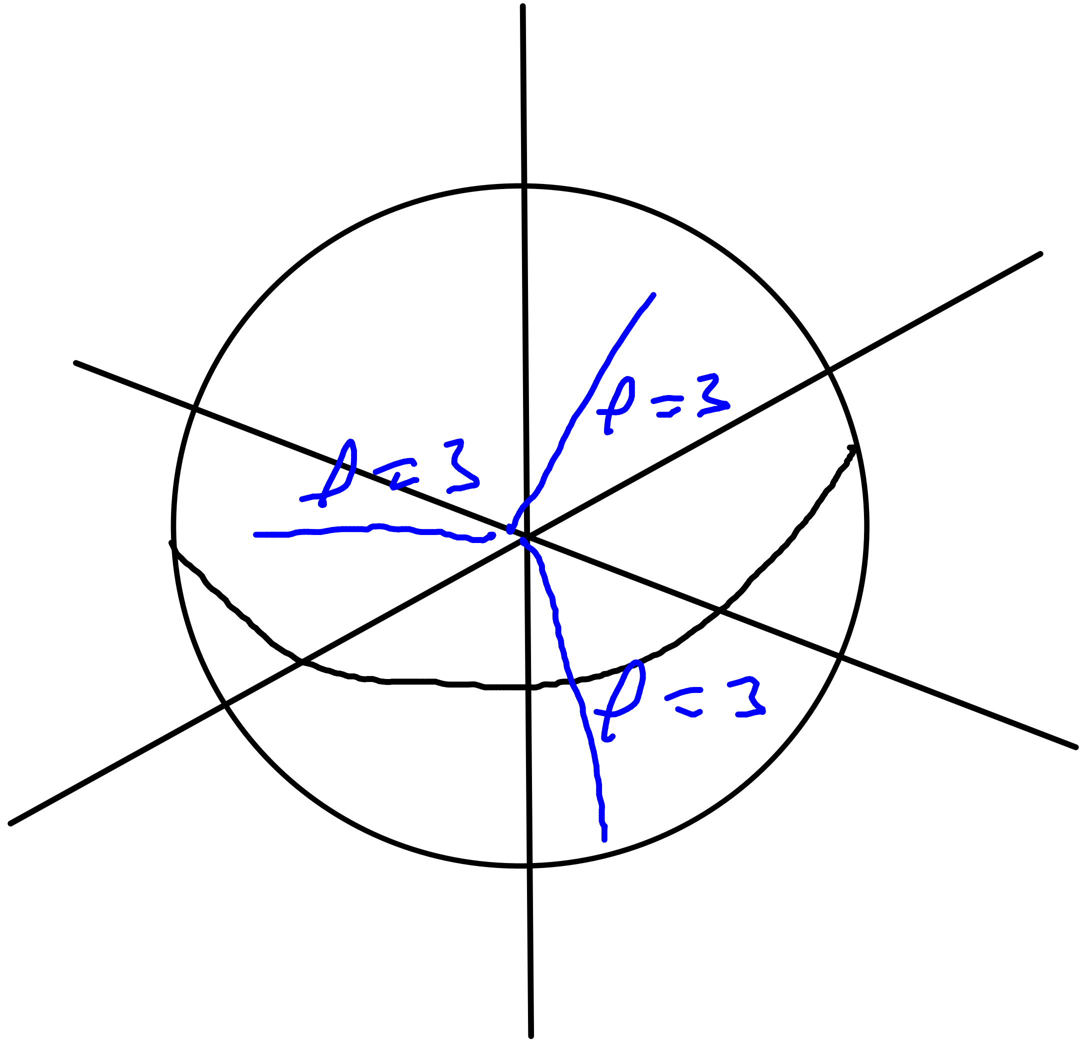
$$OQ = \rho\sin\phi$$

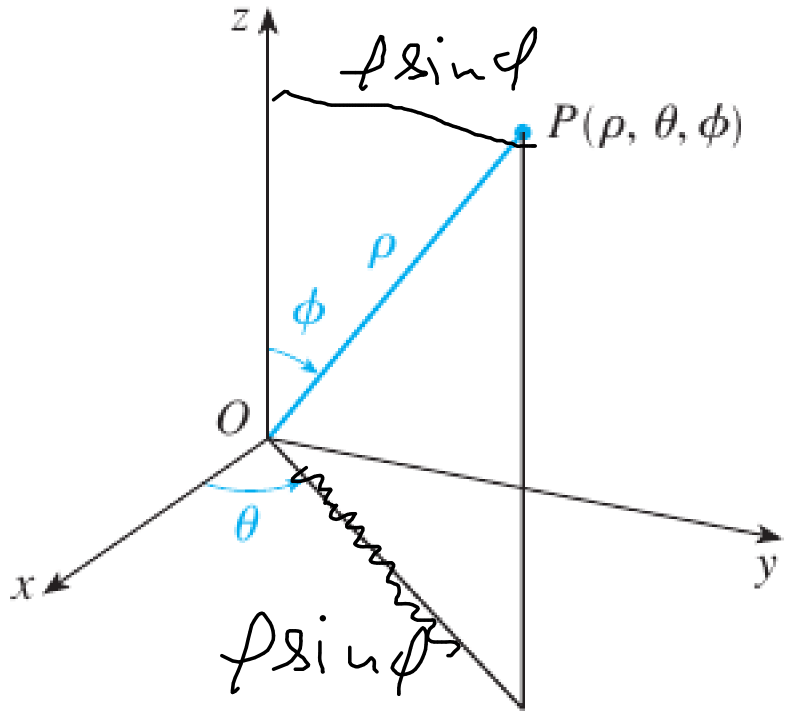
5–6 ▪ Describe in words the surface whose equation is given.

5. $\phi = \pi/3$

6. $\rho = 3$



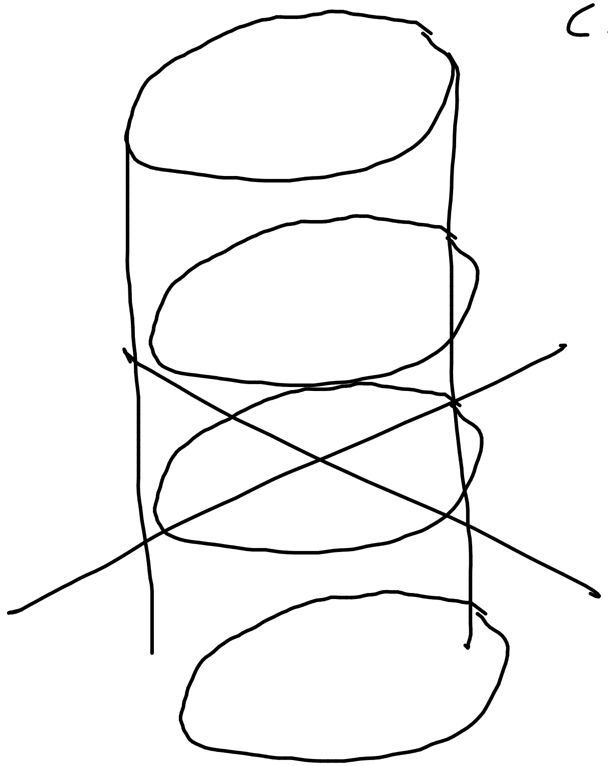




7–8 ▪ Identify the surface whose equation is given.

7. $\rho \sin \phi = 2$

$\boxed{r = 2}$ in cylindrical coordinats



8. $\rho = 2 \cos \phi$

$$\rho^2 = 2\rho\cos\phi$$

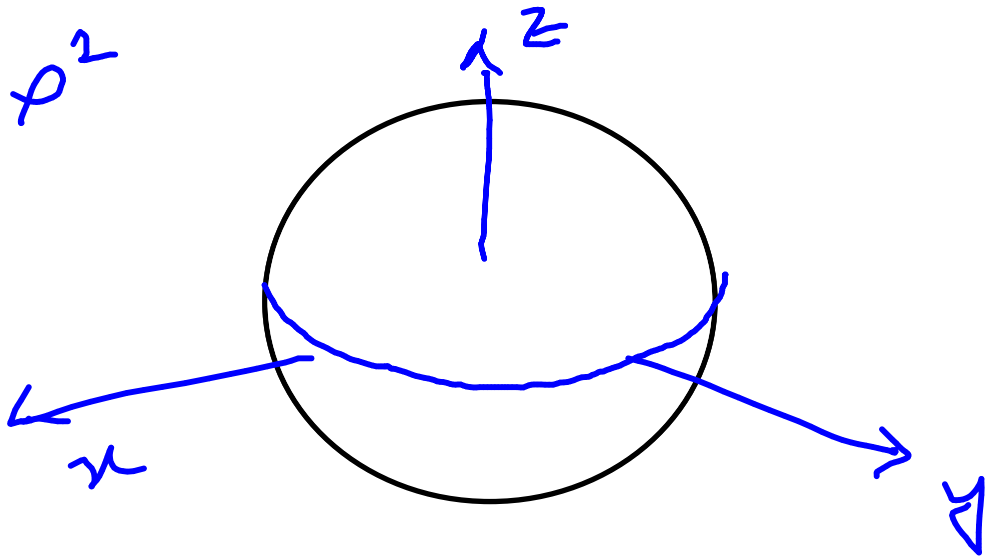
$$x^2 + y^2 + z^2 = 2z \quad ??$$

$$x^2 + y^2 + (z-1)^2 = 1$$

sphere

V EXAMPLE 3 Evaluate $\iiint_B e^{(x^2+y^2+z^2)^{3/2}}\, dV$, where B is the unit ball: | $x^2+y^2+z^2=\rho^2$

$$B = \{(x, y, z) \mid x^2 + y^2 + z^2 \leqslant 1\}$$



→ sketch the domain

→ set up the limits of ρ, θ, ϕ

$0 \le \rho \le 1, \quad 0 \le \theta \le 2\pi, \quad 0 \le \phi \le \pi$

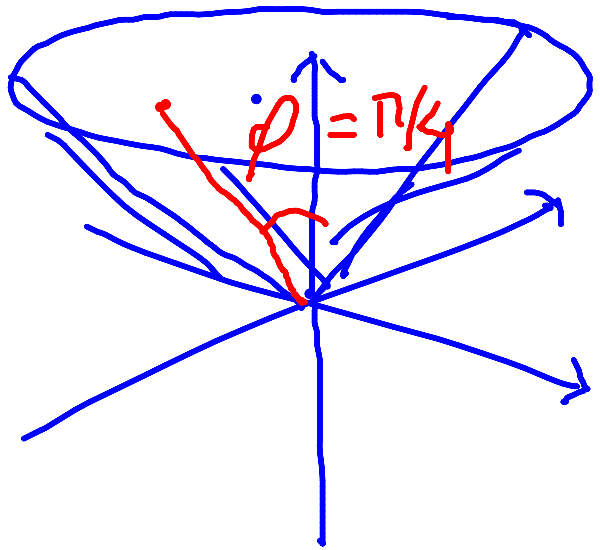
→ rewrite the integration in spherical coordinates

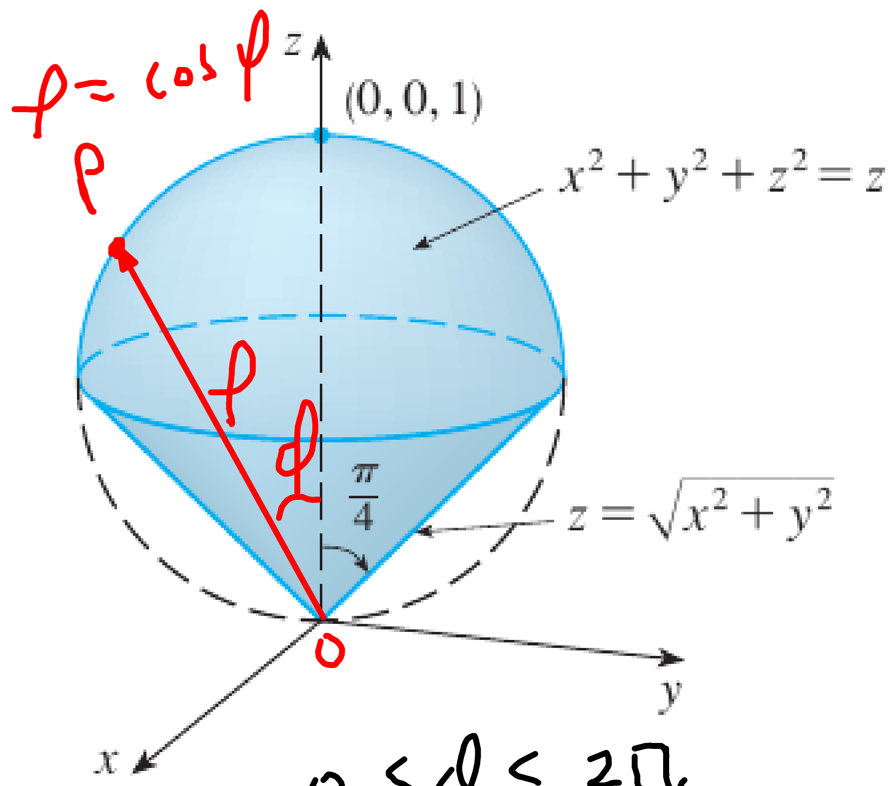
$$\int_0^1 \int_0^{2\pi} \int_0^{\pi} e^{\rho^3} \rho^2 \sin\phi \, d\phi \, d\theta \, d\rho = \frac{4}{3}\pi(e-1)$$

EXAMPLE 4 Use spherical coordinates to find the volume of the solid that lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = z$. (See Figure 9.)

(ρ, θ, ϕ)

$$\boxed{\begin{aligned} z &= \sqrt{x^2+y^2} \\ \phi &= \pi/4 \end{aligned}}$$





$$x^2+y^2+z^2 = z$$

$$x^2+y^2+\left(z-\tfrac{1}{2}\right)^2 = \left(\tfrac{1}{2}\right)^2$$

to spherical coordinates

$$\rho^2 = \rho\cos\phi$$

$$\rho = \cos\phi$$

$$0 \le \theta \le 2\pi$$
$$0 \le \phi \le \pi/4$$
$$0 \le \rho \le \cos\phi$$

$$\int_0^{2\pi}\int_0^{\pi/4}\int_0^{\cos\phi} \rho^2 \sin\phi \; d\rho\, d\phi\, d\theta$$

$$= \frac{\pi}{8} \quad ??$$

H.W. **Theorem:** when doing integration in spherical coordinates,

$$\boxed{dV = \underbrace{\rho^2 \sin\phi}\, d\rho\, d\theta\, d\phi}$$

i.e. dV = volume swiped for small change $d\rho, d\theta, d\phi$

we will discuss on this more.

Read up on this. Find a visual demo of why $dV = \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$

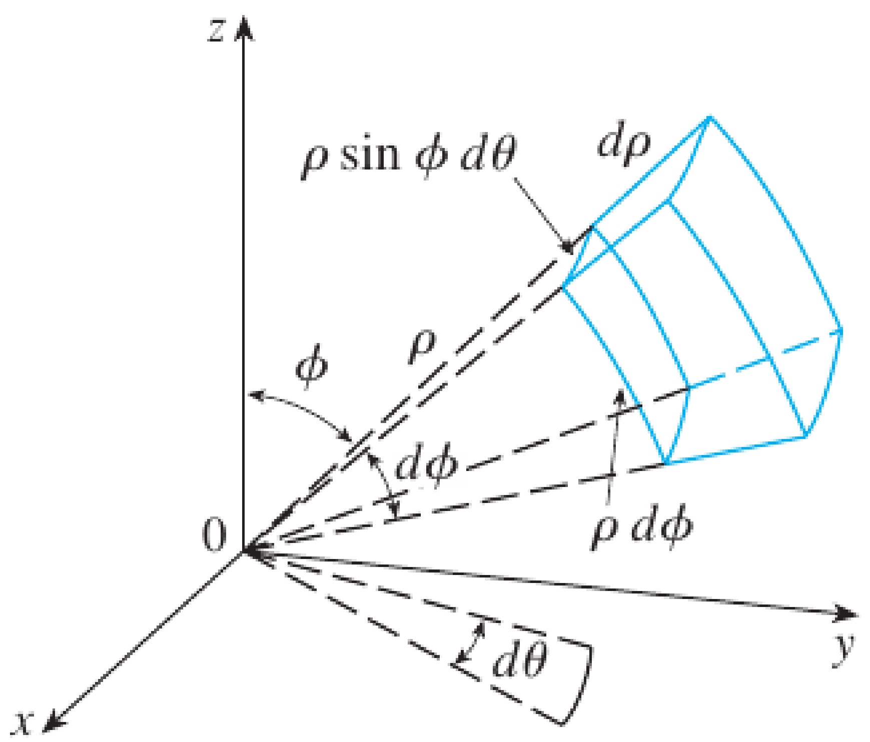


FIGURE 8

Read explanation of Fig 8 in section 12.7