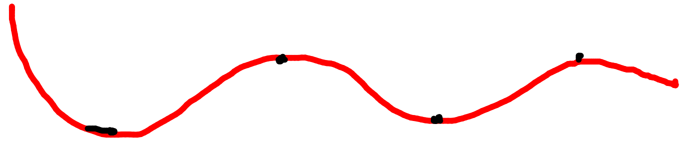
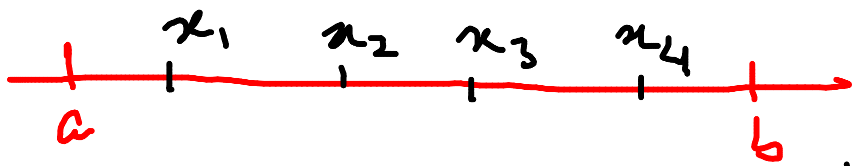
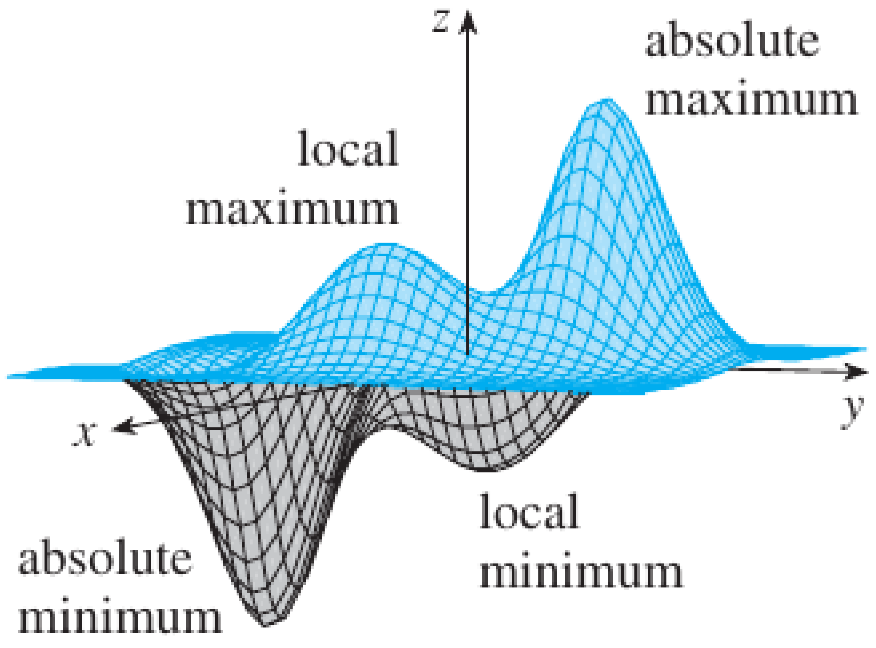
Aim: find max/min of g in $[a,b]$

11.7 MAXIMUM AND MINIMUM VALUES

$g(x)$







find critical points

$g'(x) = 0$

Then max of g on the interval $[a, b]$

$= \max\{g(x_1), g(x_2), g(x_3), g(x_4), g(a), g(b)\}$

A point (a, b) is called a **critical point** (or *stationary point*) of f if $f_x(a, b) = 0$ and $f_y(a, b) = 0$, or if one of these partial derivatives does not exist.

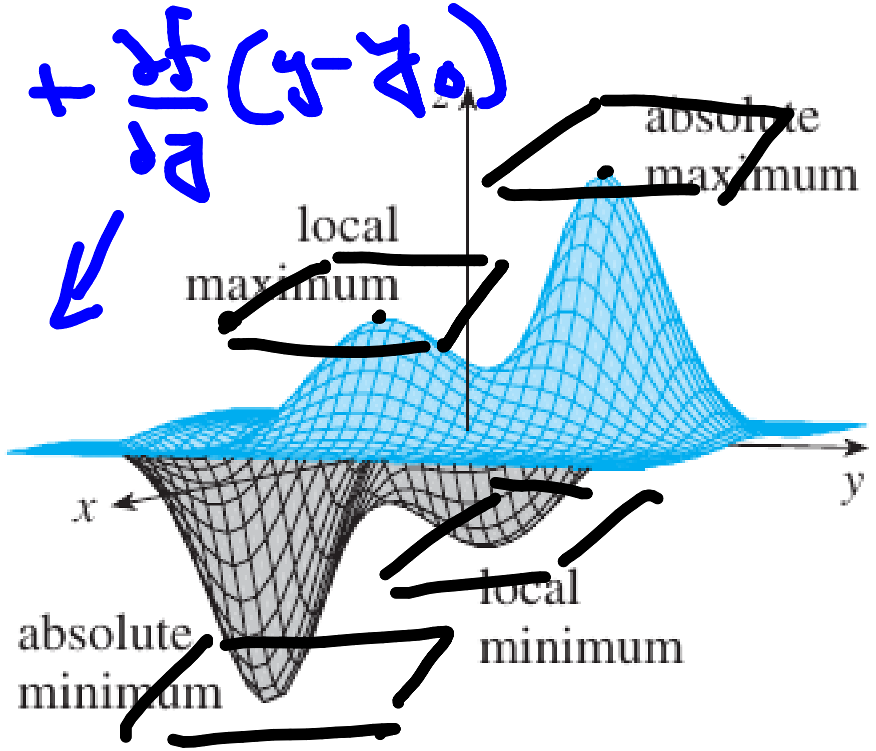
11.7 MAXIMUM AND MINIMUM VALUES

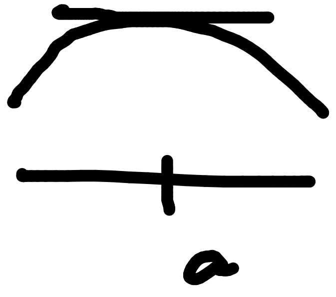
$f(x,y)$: temperature at point (x,y)

$$z - z_0 = \frac{\partial f}{\partial x}(x - x_0) + \frac{\partial f}{\partial y}(y - y_0)$$

$$\frac{\partial f}{\partial x} = 0$$

$$\frac{\partial f}{\partial y} = 0$$



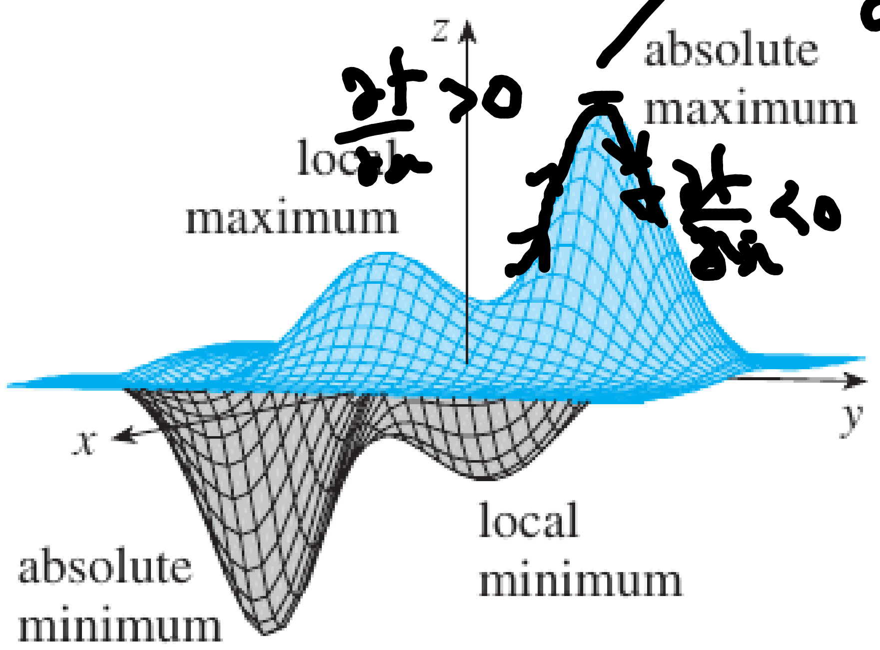


$g'(a) = 0$

horizontal tangent line

A point (a, b) is called a **critical point** (or *stationary point*) of f if $f_x(a, b) = 0$ and $f_y(a, b) = 0$, or if one of these partial derivatives does not exist.

$\frac{\partial f}{\partial x} = 0$



black curve: only x is changing
y : constant

local max / local min

A point (a, b) is called a **critical point** (or *stationary point*) of f if $f_x(a, b) = 0$ and $f_y(a, b) = 0$, or if one of these partial derivatives does not exist.

11.7 MAXIMUM AND MINIMUM VALUES

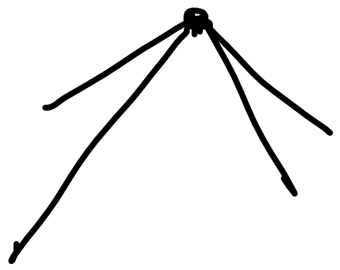
P (a,b)

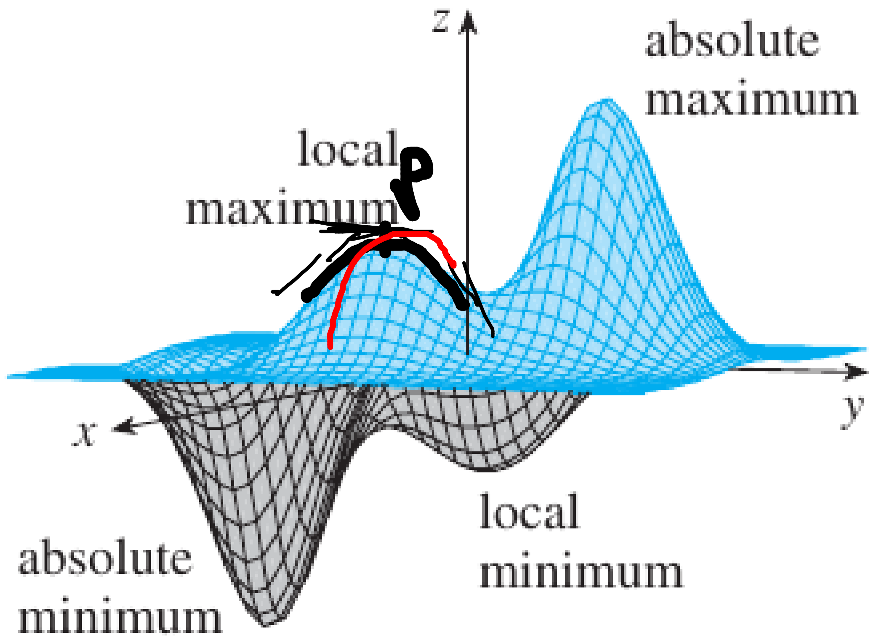
q: at P,

$\frac{\partial f}{\partial x} > 0$, or $\frac{\partial f}{\partial x} < 0$,

$\frac{\partial f}{\partial x} = 0$

q. at P $\frac{\partial f}{\partial y} = 0$



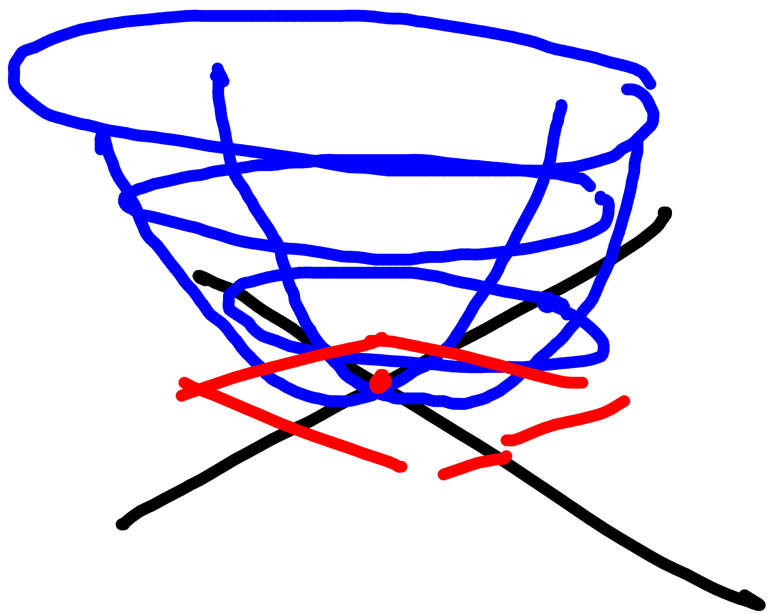


$\frac{\sim}{x-1}$

A point (a, b) is called a **critical point** (or *stationary point*) of f if $f_x(a, b) = 0$ and $f_y(a, b) = 0$, or if one of these partial derivatives does not exist.

↳ pointy graph.

Q. find critical points of $f(x,y) = x^2+y^2$



simply solve: $\frac{\partial f}{\partial x} = 0$, $\frac{\partial f}{\partial y} = 0$

$2x = 0$, $2y = 0$

critical point $= (0,0)$

EXAMPLE 1 Let $f(x, y) = x^2 + y^2 - 2x - 6y + 14$. $=$

graph of $(x-1)^2 + (y-3)^2 + 4$
related to graph of x^2+y^2

$\rightarrow$ 1 unit in x dir
$\rightarrow$ 3 unit in y dir
$\rightarrow$ 4 units in z dir

find the critical points:

$\frac{\partial f}{\partial x} = 0$ $\quad$ $\frac{\partial f}{\partial y} = 0$

$2x - 2 = 0$ $\quad$ $2y - 6 = 0$

$x = 1$ $\quad$ $y = 3$

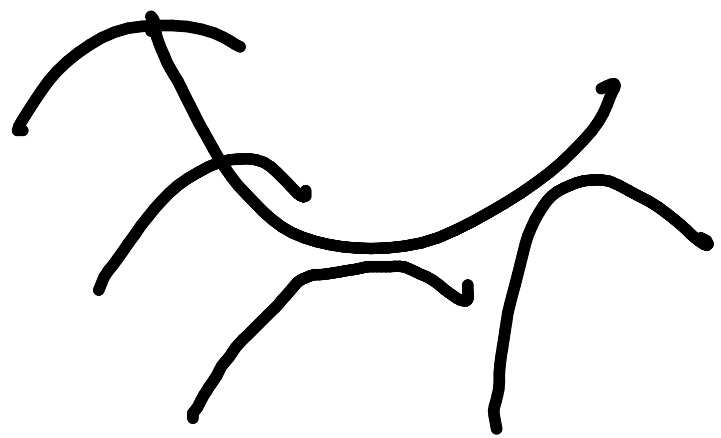
Q. $f(x,y) = x^2 - y^2$

find critical points.

$$\frac{\partial f}{\partial x} = 0 \qquad \frac{\partial f}{\partial y} = 0$$

$$2x = 0 \qquad -2y = 0$$

$$0 \qquad 0$$



neither max or min

$\rightarrow$ saddle point

EXAMPLE 2 Find the extreme values of $f(x, y) = y^2 - x^2$.

3 SECOND DERIVATIVES TEST Suppose the second partial derivatives of f are continuous on a disk with center (a, b), and suppose that $f_x(a, b) = 0$ and $f_y(a, b) = 0$ [that is, (a, b) is a critical point of f]. Let

$$D = D(a, b) = f_{xx}(a, b) f_{yy}(a, b) - [f_{xy}(a, b)]^2$$

(a) If $D > 0$ and $f_{xx}(a, b) > 0$, then $f(a, b)$ is a local minimum.

(b) If $D > 0$ and $f_{xx}(a, b) < 0$, then $f(a, b)$ is a local maximum.

(c) If $D < 0$, then $f(a, b)$ is not a local maximum or minimum.

$f(x,y) = x^2 \sin(y)$

next time: classification of critical points into local max / min / saddle point

V EXAMPLE 3 Find the local maximum and minimum values and saddle points of $f(x, y) = x^4 + y^4 - 4xy + 1$.

V EXAMPLE 4 Find the shortest distance from the point $(1, 0, -2)$ to the plane $x + 2y + z = 4$.