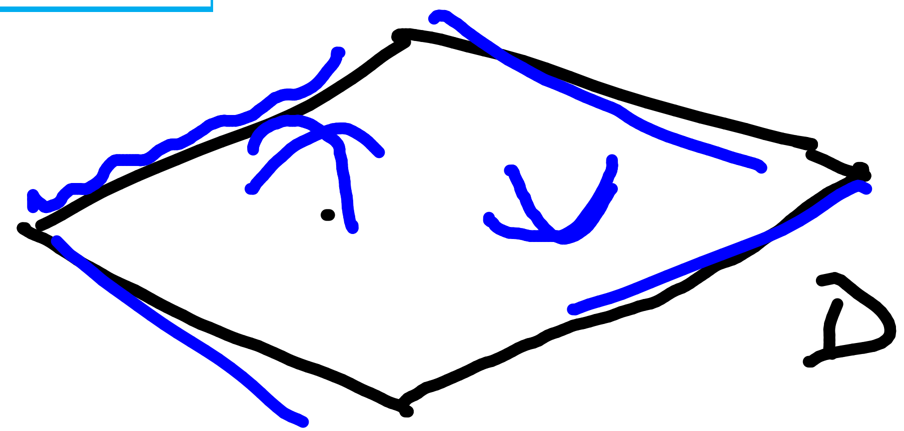
ABSOLUTE MAXIMUM AND MINIMUM VALUES

Section 11.7 continued

5 To find the absolute maximum and minimum values of a continuous function f on a closed, bounded set D:

1. Find the values of f at the critical points of f in D.
2. Find the extreme values of f on the boundary of D.
3. The largest of the values from steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.

$f(x,y)$



now we will maximize & minimize on a bounded domain:

Q: $f(x,y) = x^2+y^2$

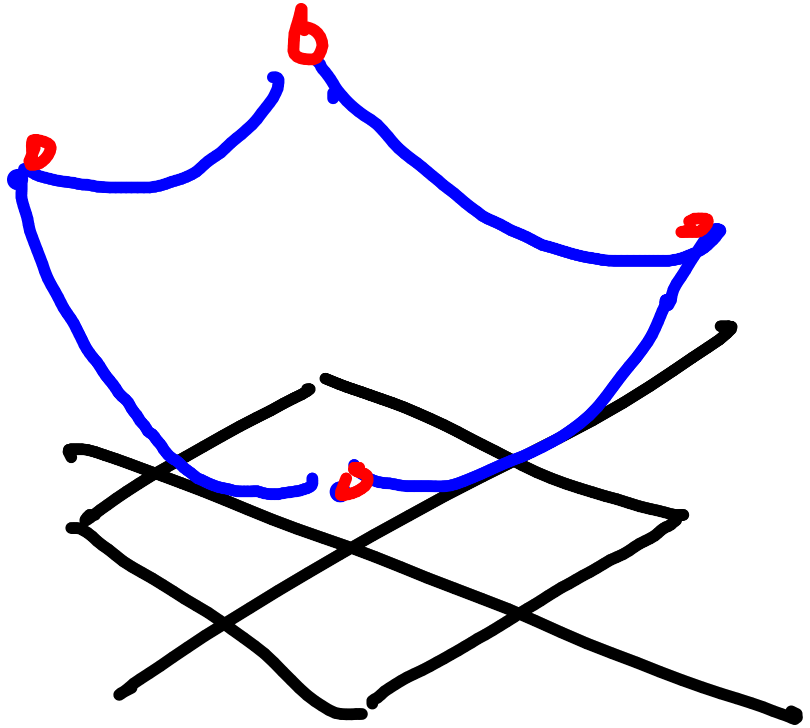
minimum $= 0$

& no maximum

Q: $f(x,y) = x^2 + y^2$, & $-1 \le x \le 1$

$-1 \le y \le 1$

whats max & min of $f(x,y)$

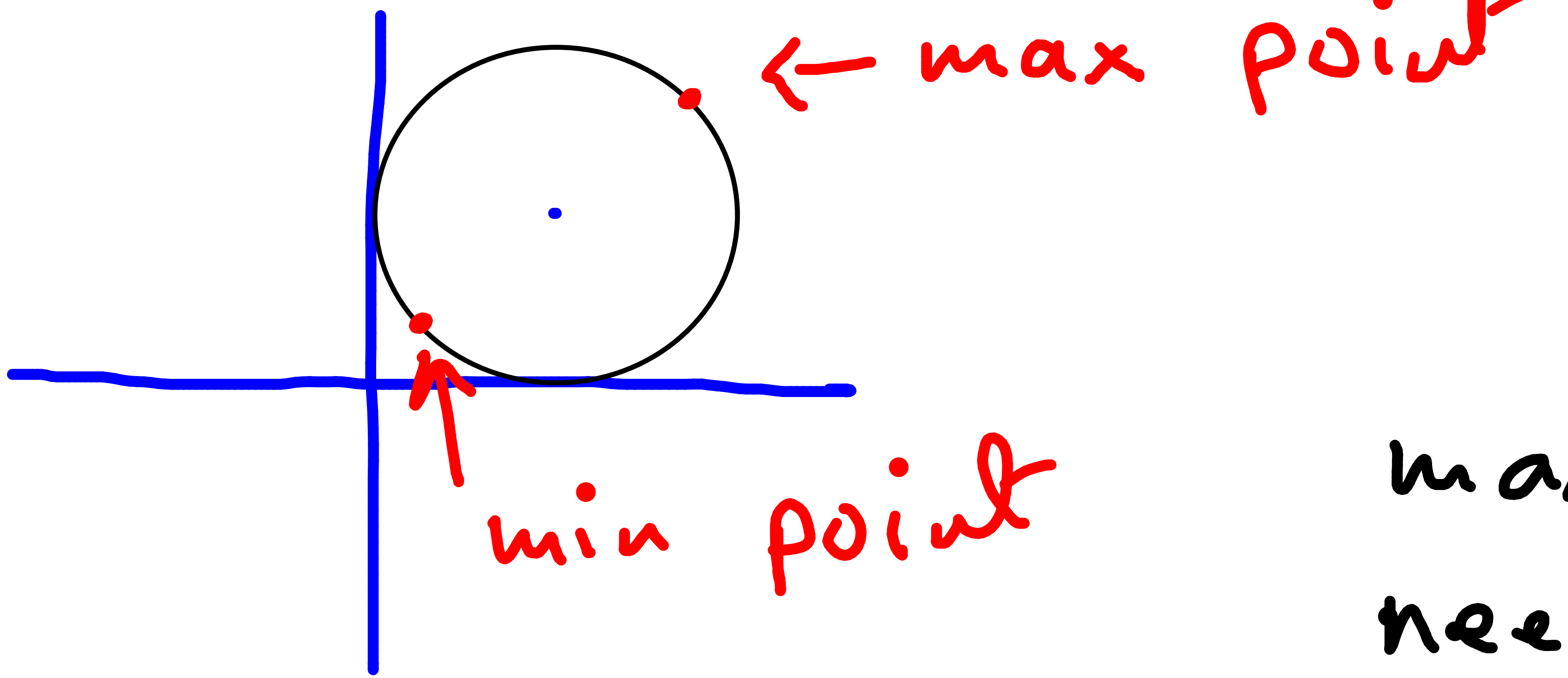


max : 2

& min = 0

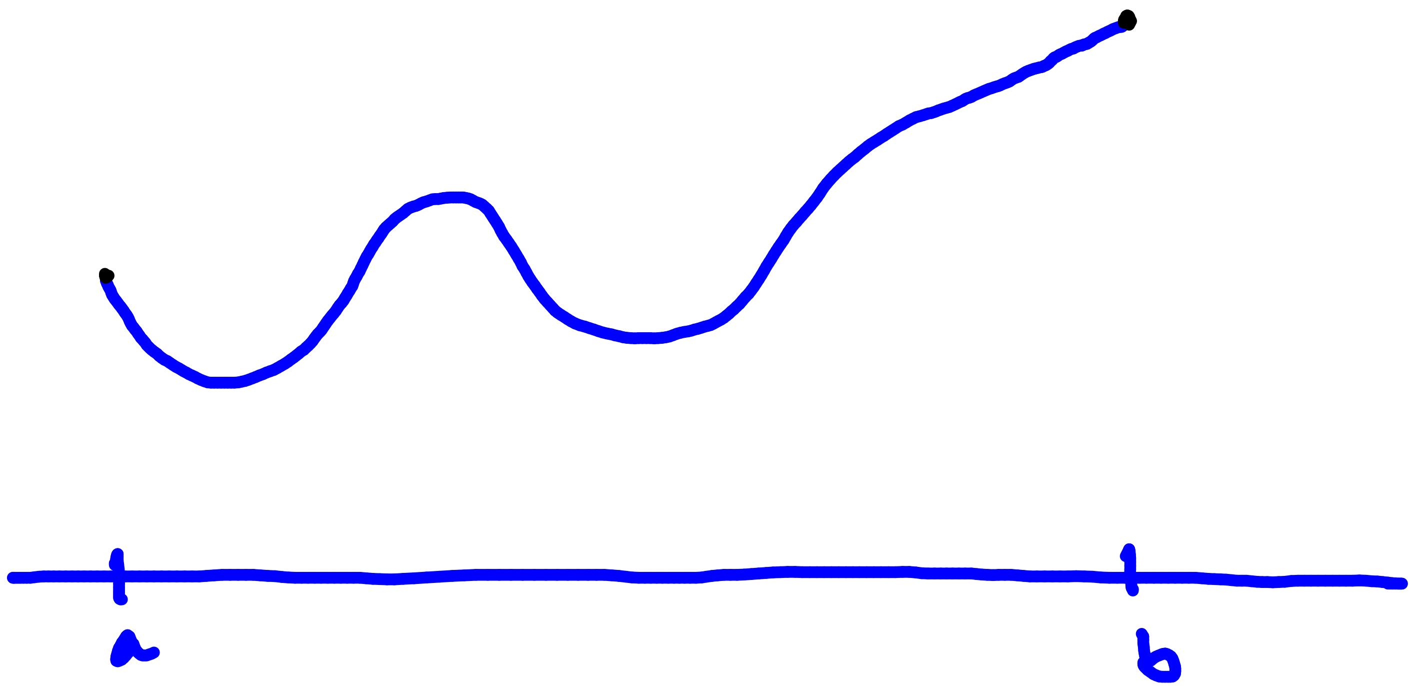
Q. $f(x,y) = x^2 + y^2$

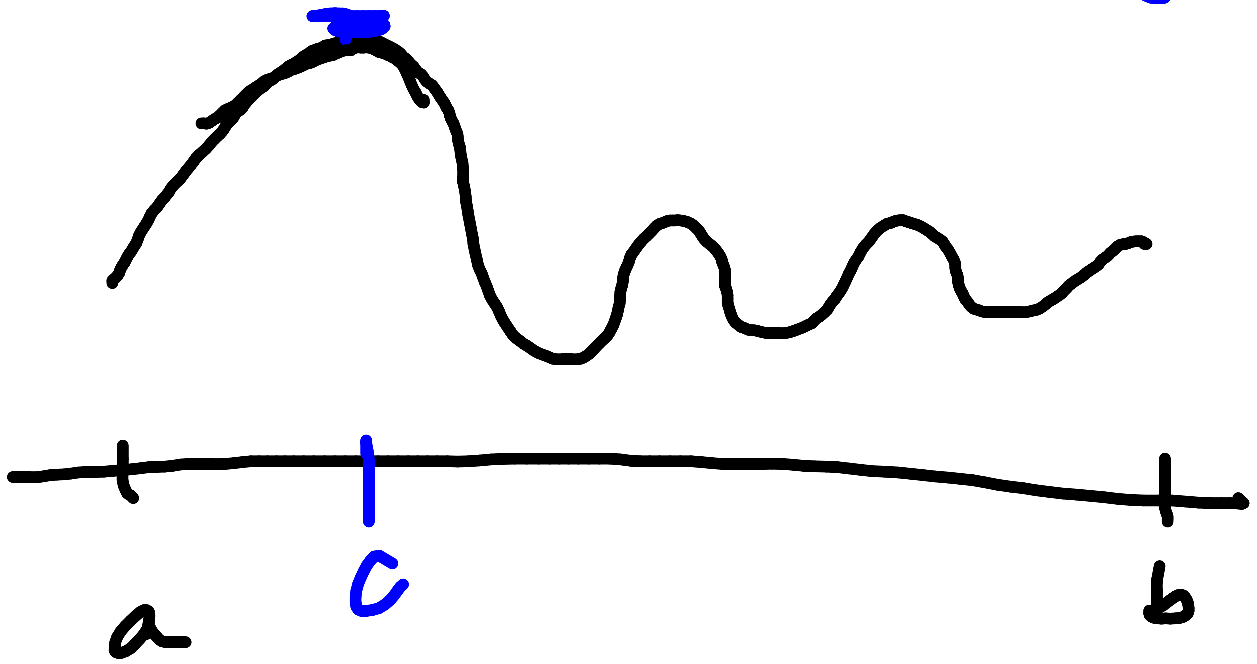
maximize $f(x,y)$ s.t. $\underbrace{(x-1)^2 + (y-1)^2 \le 1}_{\text{what shape is this}}$



max points & min points need not be critical point if domain is bounded.

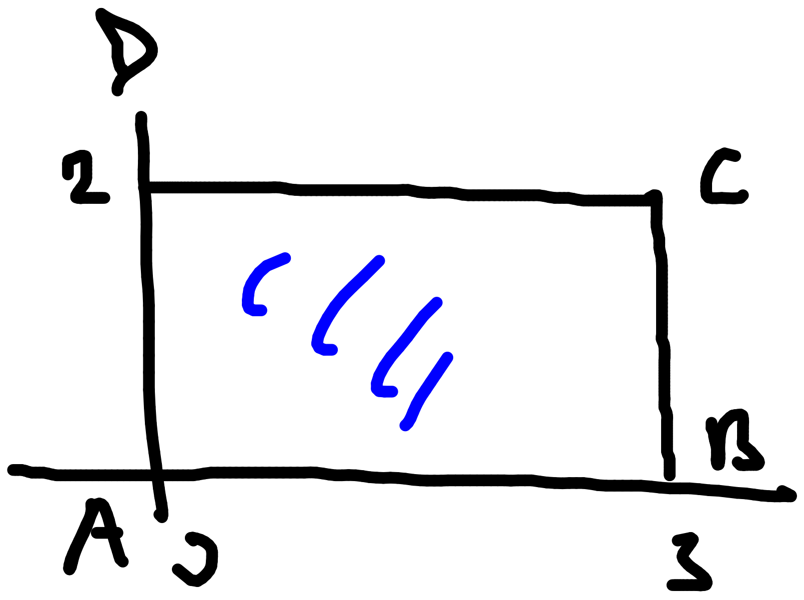
One variable calculus $f(x)$ defined on $[a,b]$





Q: is it possible that a max point in the interior is not a critical point

EXAMPLE 6 Find the absolute maximum and minimum values of the function $f(x, y) = x^2 - 2xy + 2y$ on the rectangle $D = \{(x, y) \mid 0 \leqslant x \leqslant 3, 0 \leqslant y \leqslant 2\}$.

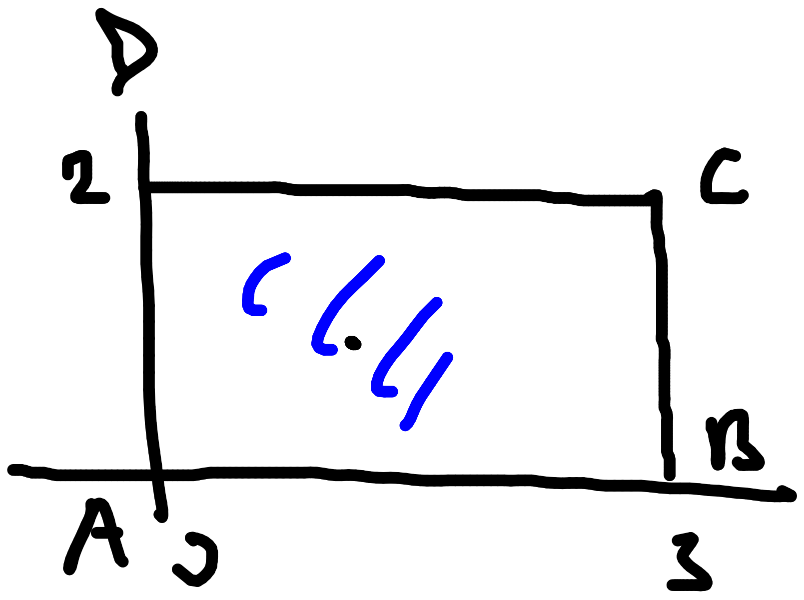


the max can occur in the interior of the rectangle ABCD

$$\frac{\partial f}{\partial x} = 0 \quad \& \quad \frac{\partial f}{\partial y} = 0$$

or the max can occur at any point on the line AB, BC, CD, or DA (not only only in corners)

EXAMPLE 6 Find the absolute maximum and minimum values of the function $f(x, y) = x^2 - 2xy + 2y$ on the rectangle $D = \{(x, y) \mid 0 \leqslant x \leqslant 3, 0 \leqslant y \leqslant 2\}$.



the max can occur in the interior of the rectangle ABCD

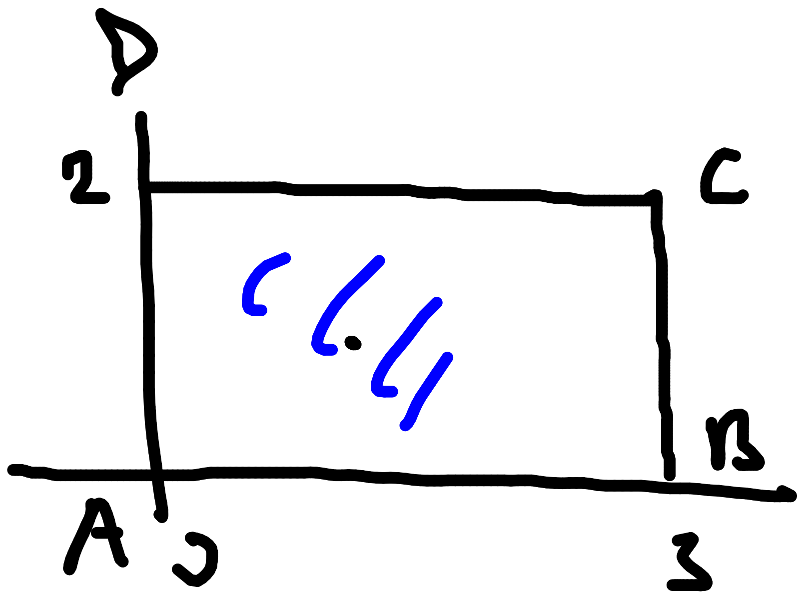
$$\frac{\partial f}{\partial x} = 0 \quad \& \quad \frac{\partial f}{\partial y} = 0$$

critical points:

$$\begin{aligned} 2x - 2y &= 0 \\ -2x + 2 &= 0 \end{aligned} \quad \Bigg| \quad x = 1,\ y = 1$$

if abs max occurs in the interior it will only occur at (1,1)

EXAMPLE 6 Find the absolute maximum and minimum values of the function $f(x, y) = x^2 - 2xy + 2y$ on the rectangle $D = \{(x, y) \mid 0 \leqslant x \leqslant 3, 0 \leqslant y \leqslant 2\}$.



Q: what's max/min of $f(x, y)$ on the line AB

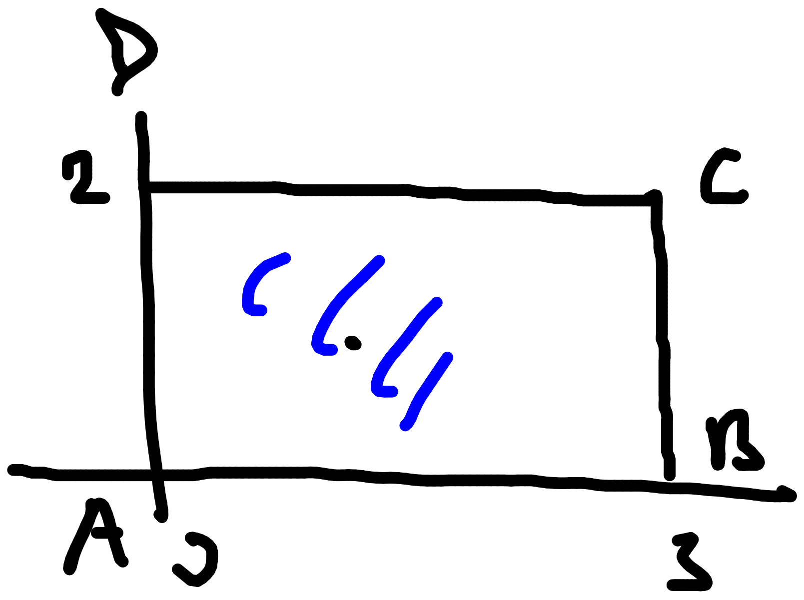
$0 \leq x \leq 3, \quad y = 0$

$f\big|_{AB} = x^2 \longrightarrow$ min at $x = 0$

max at $x = 3$

Repeat this for all line BC, CD, DA

EXAMPLE 6 Find the absolute maximum and minimum values of the function $f(x, y) = x^2 - 2xy + 2y$ on the rectangle $D = \{(x, y) \mid 0 \leqslant x \leqslant 3, 0 \leqslant y \leqslant 2\}$.



Q: what's max/min of $f(x,y)$ on line AD

$x = 0 \qquad 0 \le y \le 2$

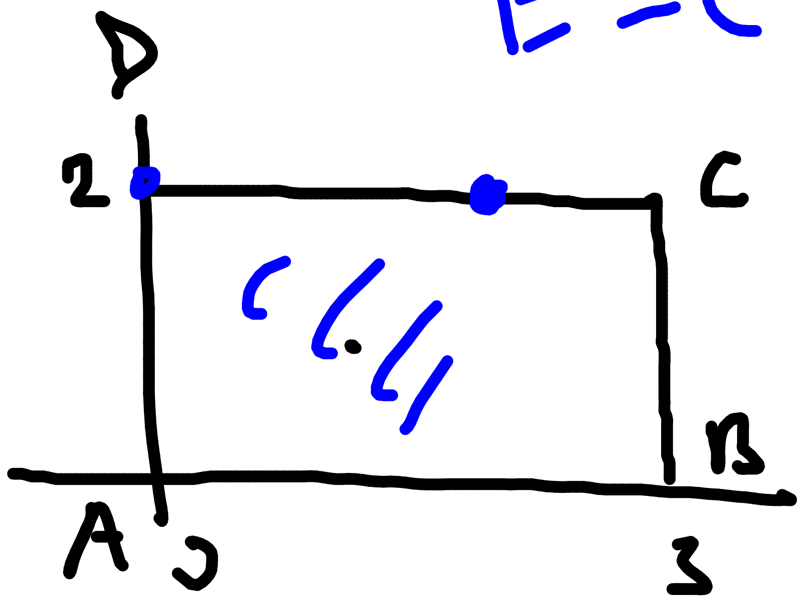
$$f\Big|_{AD} = 2y$$

max $\rightarrow$ $y = 2$

min $\rightarrow$ $y = 0$

EXAMPLE 6 Find the absolute maximum and minimum values of the function $f(x, y) = x^2 - 2xy + 2y$ on the rectangle $D = \{(x, y) \mid 0 \leqslant x \leqslant 3, 0 \leqslant y \leqslant 2\}$.

$E = (2,2)$



Q: what's max/min of $f(x,y)$ on line DC

$0 \le x \le 3$, $y = 2$

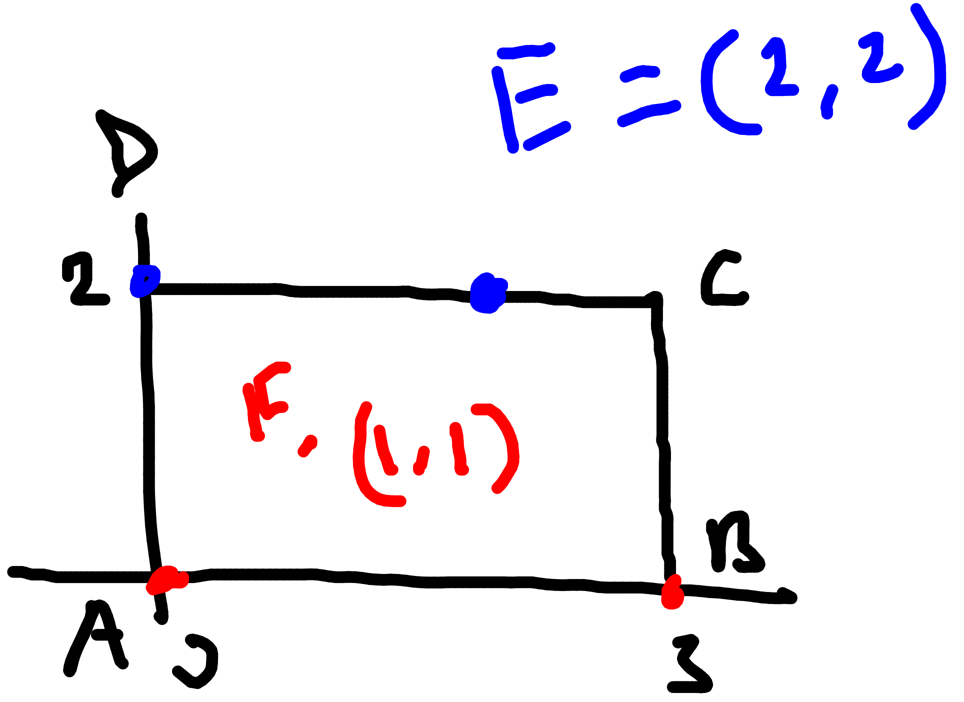
$$f\big|_{DC} = x^2 - 4x + 4$$

Q: what is the max/min of $x^2 - 4x + 4$ when $0 \le x \le 3$

→ min → $x = 2$, $y = 2$

max → $x = 0$, $y = 2$

EXAMPLE 6 Find the absolute maximum and minimum values of the function $f(x, y) = x^2 - 2xy + 2y$ on the rectangle $D = \{(x, y) \mid 0 \leqslant x \leqslant 3, 0 \leqslant y \leqslant 2\}$.



Q: what's max/min of $f(x,y)$ on line BC

→ $x = 3 \quad 0 \leq y \leq 2$

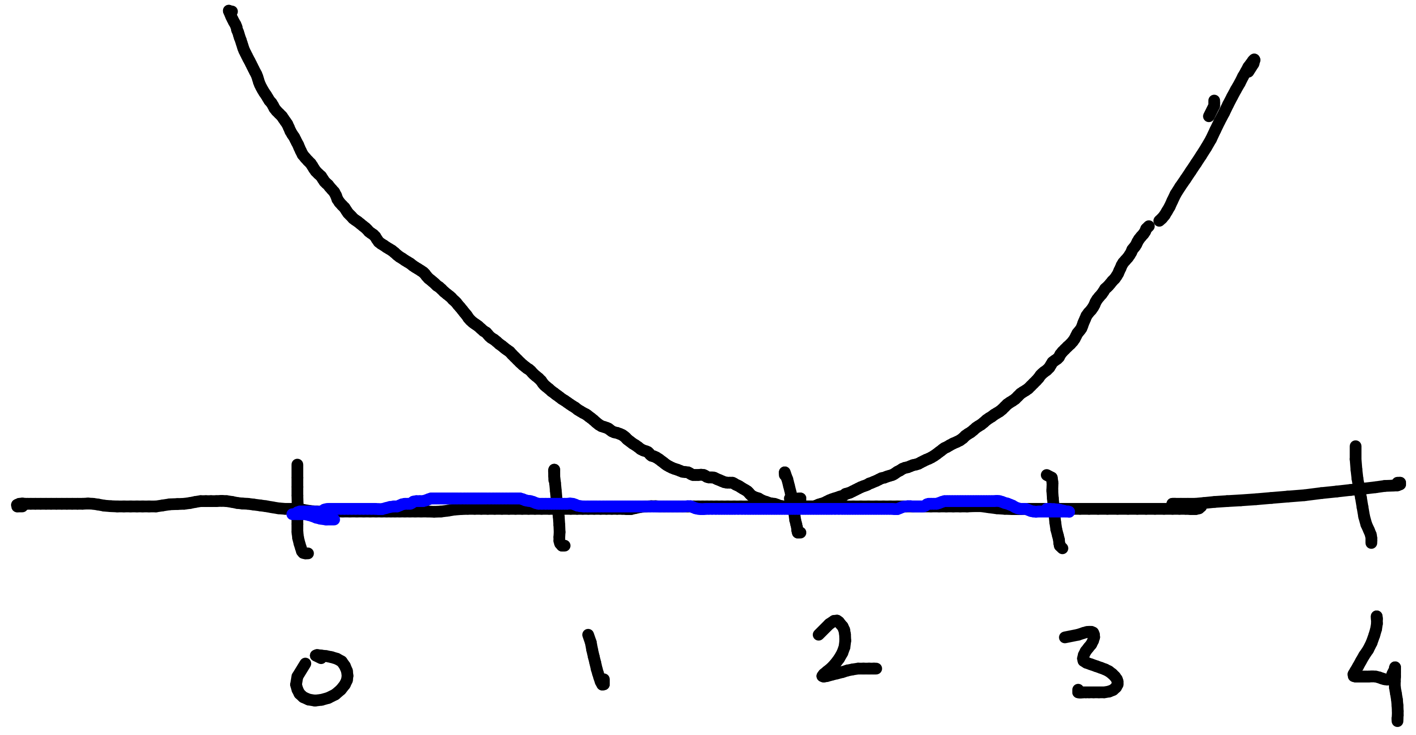
$f\big|_{BC} = 9 - 6y + 2y = 9 - 4y$

max → $y = 0, x = 3$

min at $y = 2, x = 3$

finally absolute max $= \max\{f(A), f(B), f(C), f(D), f(E), f(F)\}$

Q: what is the max / min of x^2-4x+4 when $0 \le x \le 3$

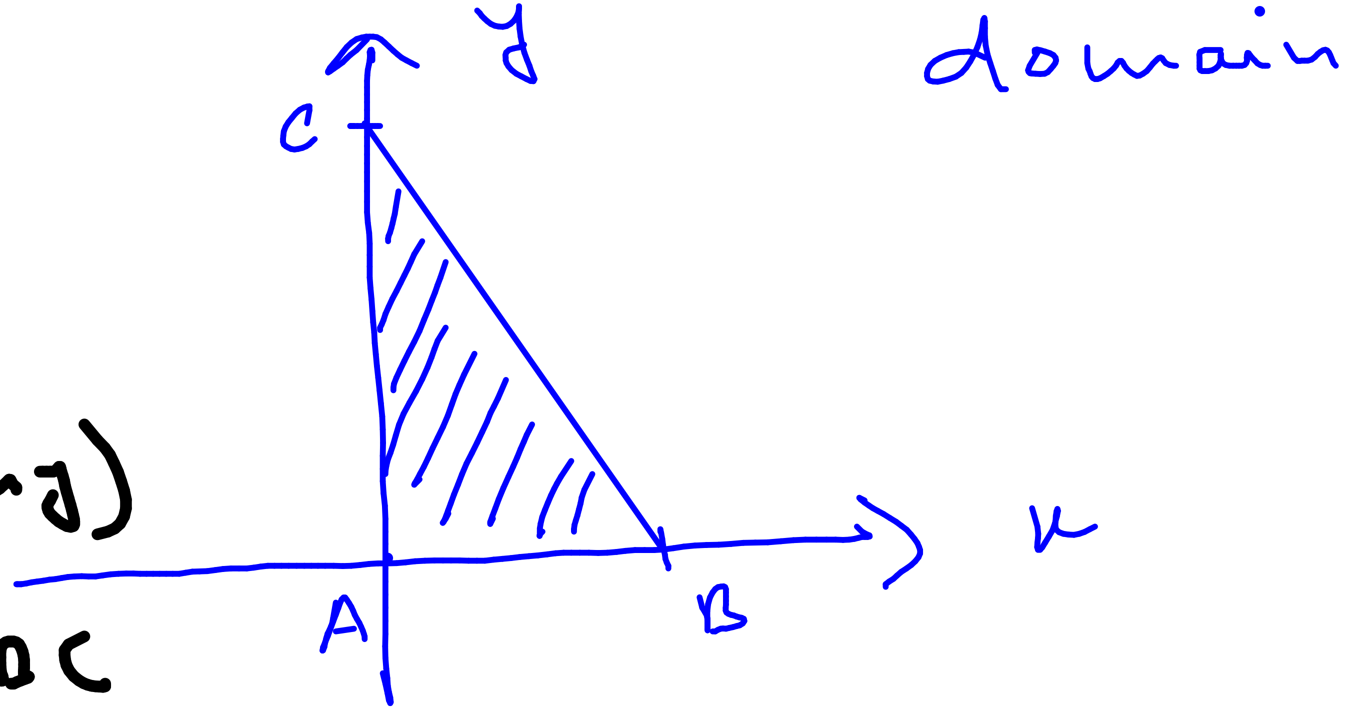


$$x^2-4x+4=(x-2)^2$$

$\rightarrow$ min $x=2$

& max $x=0$

23. $f(x, y) = 1 + 4x - 5y$, D is the closed triangular region with vertices $(0, 0)$, $(2, 0)$, and $(0, 3)$



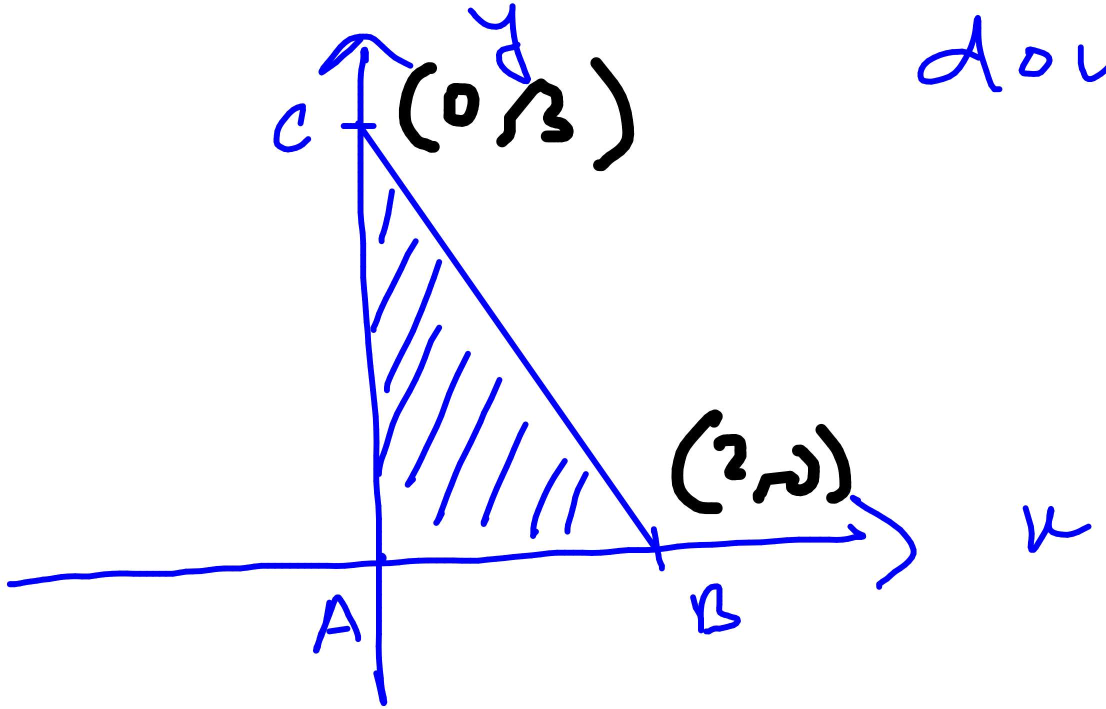
same steps:

→ find critical points (if any) in the interior of ΔABC

→ find max/min of $f(x,y)$ on each boundary segment AB, BC, CA

23. $f(x, y) = 1 + 4x - 5y$, D is the closed triangular region with vertices $(0, 0)$, $(2, 0)$, and $(0, 3)$

domain



→ find possible critical points

$$\frac{\partial f}{\partial x} = 0 \qquad \frac{\partial f}{\partial y} = 0$$

$$4 = 0 \qquad -5 = 0$$

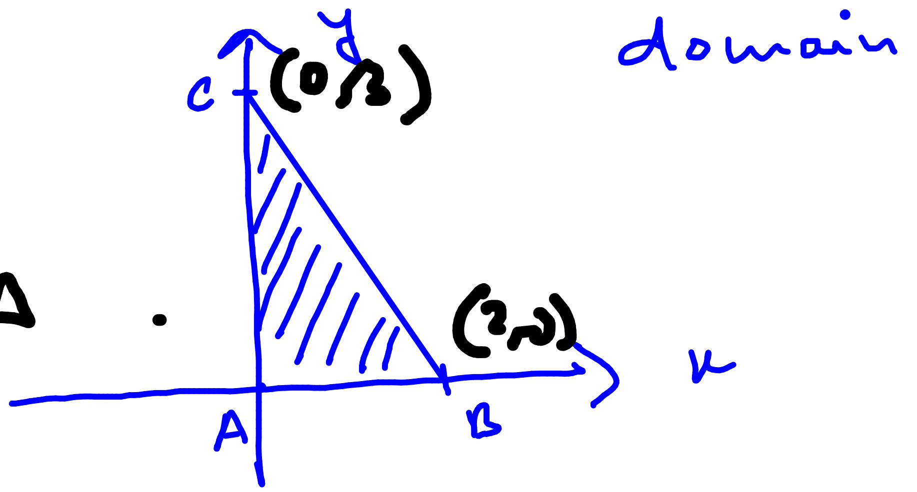
no solution exist for these eq^n.

no critical points

→ max/min can occur at boundaries.

→ at line AB $\quad x$

23. $f(x, y) = 1 + 4x - 5y$, D is the closed triangular region with vertices $(0, 0)$, $(2, 0)$, and $(0, 3)$



→ max/min can occur at boundaries.

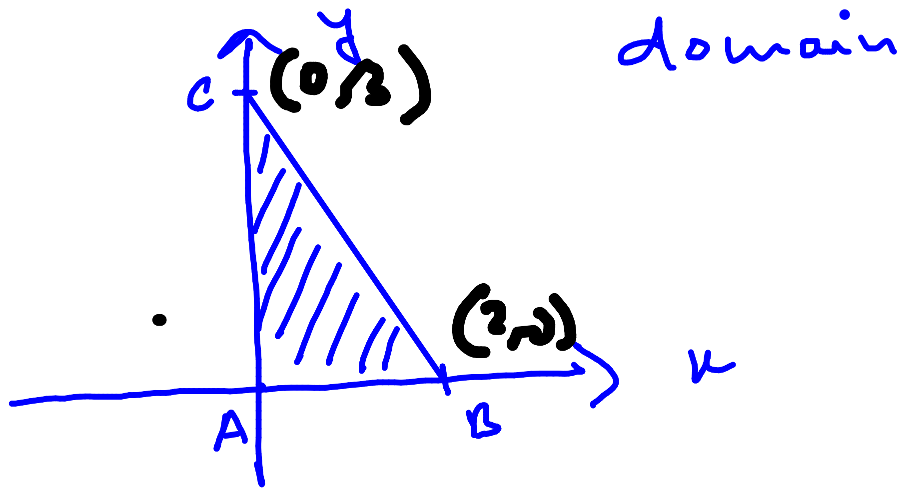
→ at line AB $0 \le x \le 2$, $y = 0$

$$f\big|_{AB} = 1 + 4x \qquad 0 \le x \le 2$$

max at $x = 2$, $y = 0$ B

min $x = 0$, $y = 0$ A

23. $f(x, y) = 1 + 4x - 5y$, D is the closed triangular region with vertices $(0, 0)$, $(2, 0)$, and $(0, 3)$



→ at line BC $\quad (3x+2y=6)$

$$f\big|_{BC} = 1+4x-5\left(\frac{6-3x}{2}\right)$$

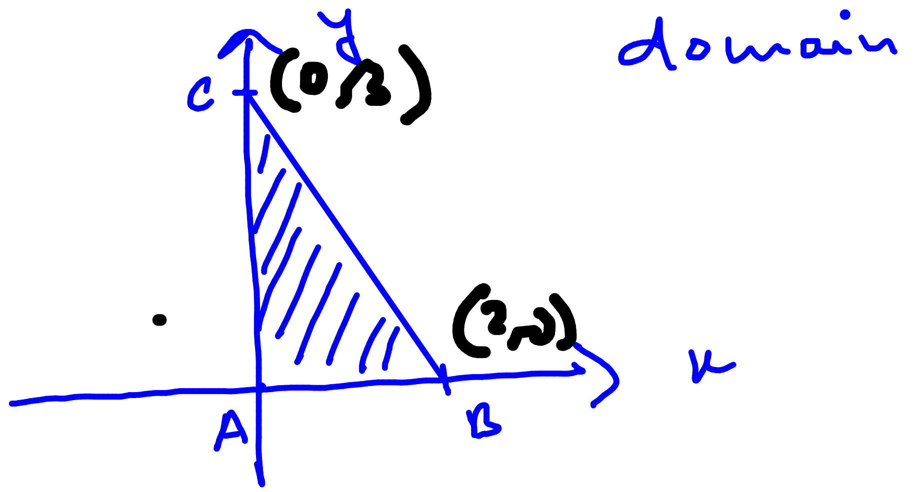
$$= \left(4+\frac{15}{2}\right)x + 1-15$$

$$= \frac{23}{2}x - 14 \quad , \quad 0 \le x \le 2$$

max $\quad x = 2 \quad , \quad y = 0 \quad \to B \quad \big| \quad f(B) = 9$

min $\quad x = 0 \quad , \quad y = 3 \quad \to C \quad \big| \quad f(C) = -14$

23. $f(x, y) = 1 + 4x - 5y$, D is the closed triangular region with vertices $(0, 0)$, $(2, 0)$, and $(0, 3)$



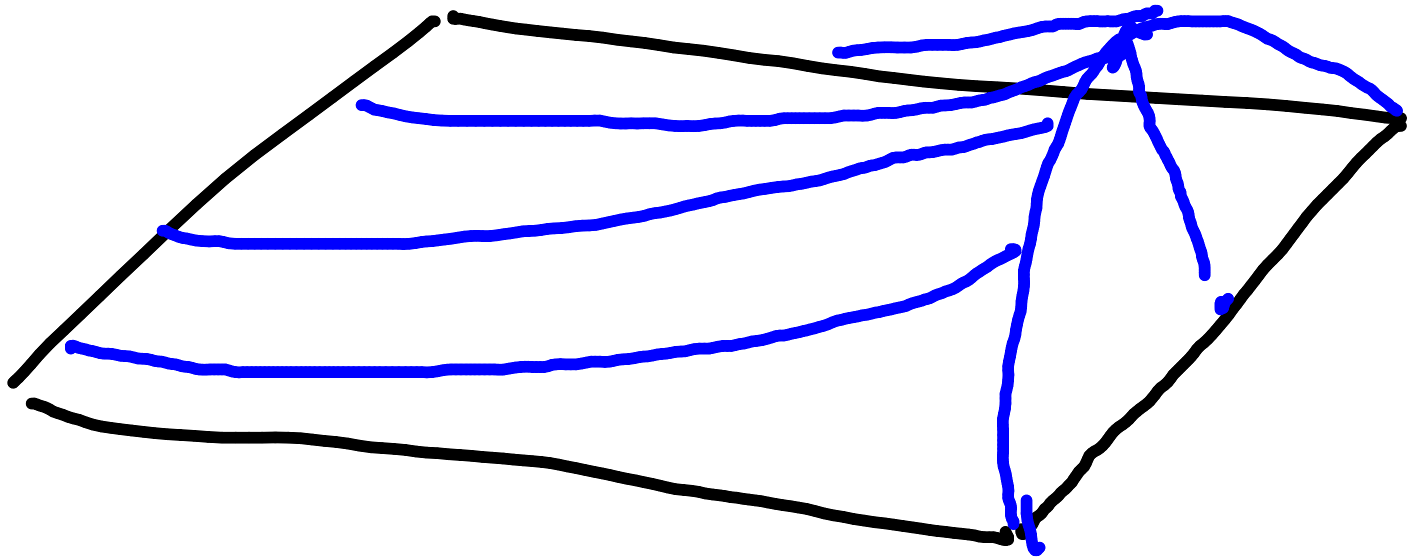
→ at line AC

$x = 0 \qquad 0 \le y \le 3$

$$\boxed{f\big|_{AC} = 1 - 5y \qquad 0 \le y \le 3}$$

max min

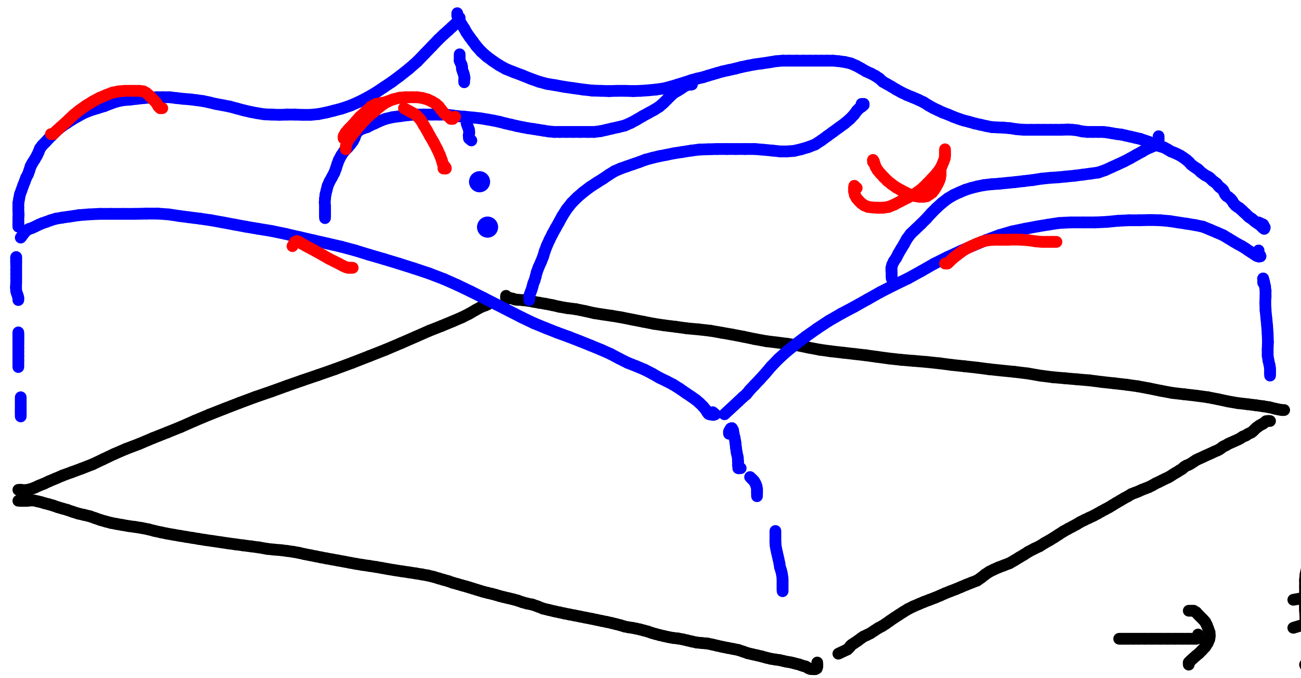
	f	
A	1	
B	9	(abs max)
C	-14	(abs min)



$-x^2+2$

Section 11.7 (continued)

max/min problems



$f(x,y)$ defined on a bounded domain

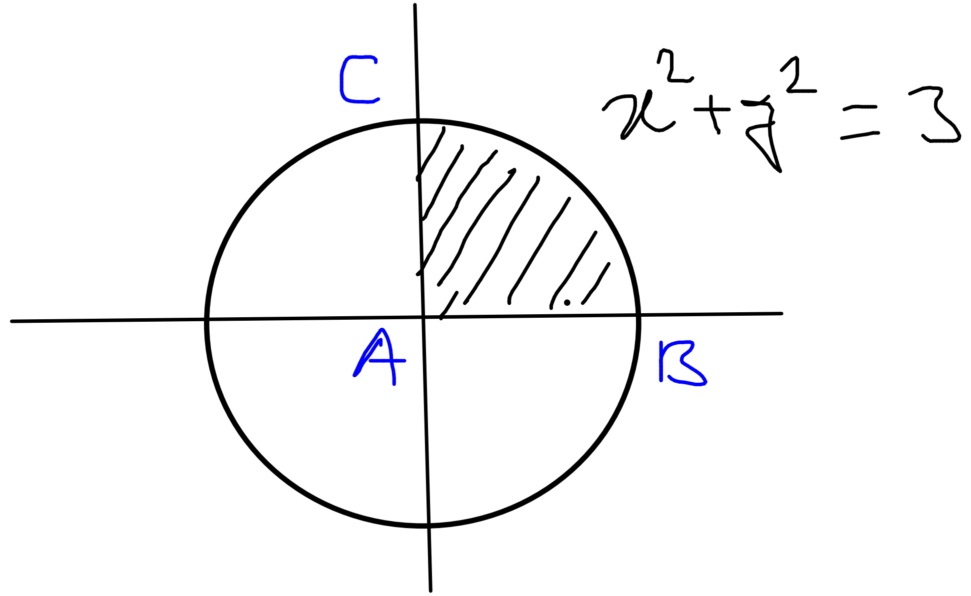
Recall the main steps

→ find (if any) critical points in the domain

$$\frac{\partial f}{\partial x} = 0 , \quad \frac{\partial f}{\partial y} = 0$$

→ find max/min at boundaries

28. $f(x, y) = xy^2, \quad D = \{(x, y) \mid x \geqslant 0,\ y \geqslant 0,\ x^2 + y^2 \leqslant 3\}$



$\underline{\underline{a}}$. sketch the domain

→ critical points

$\frac{\partial f}{\partial x} = 0 \qquad \frac{\partial f}{\partial y} = 0$

$$\boxed{y^2 = 0 \qquad 2xy = 0}$$

$\underline{\underline{\text{solution}}}$ $\quad y^2 = 0 \Rightarrow y = 0$

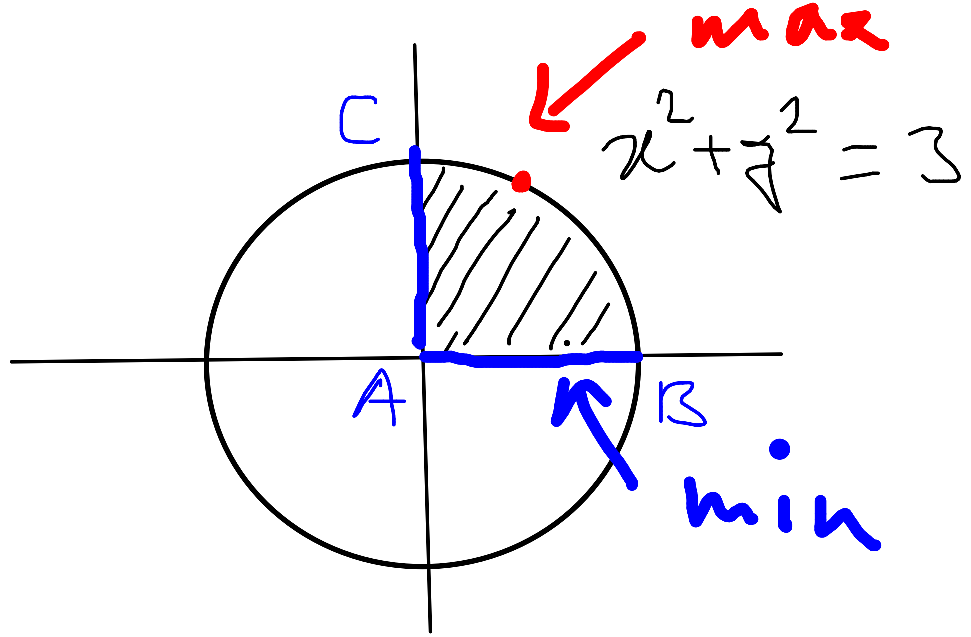
& $\quad 2xy = 0$ [holds for any x since $y=0$]

$\Rightarrow x$ can be anything in the domain

$\Rightarrow$ any point on line AB ($0 \le x \le \sqrt{3}$) is a critical point

→ note: no isolated critical point in the interior

28. $f(x, y) = xy^2, \quad D = \{(x, y) \mid x \geqslant 0,\ y \geqslant 0, x^2 + y^2 \leqslant 3\}$



$$y^2 = 3 - x^2$$

$$f|_{\widehat{BC}} = x(3-x^2) = g(x) \quad \& \ 0 \leq x \leq \sqrt{3}$$

max, min ??

max/min of $g(x)$ will happen at $\{0, \sqrt{3}\}$, or

or at any point in $(0, \sqrt{3})$ where $g'(x) = 0$.

$$g'(x) = 3 - 3x^2 = 0$$

$$x = 1$$

y	x	g
$\sqrt{3}$	0	0
$\sqrt{2}$	1	2
0	$\sqrt{3}$	0

abs max

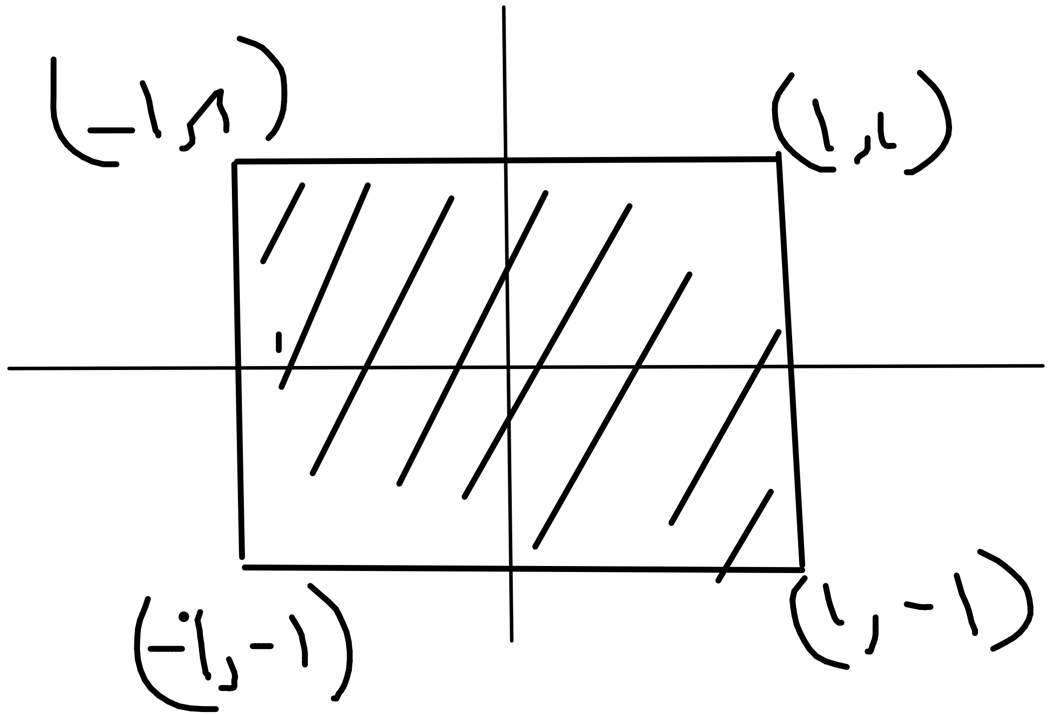
a. $f(x,y) = x^2 + y^2 + x^2 y + 4$

$D = \{ (x,y) \mid |x| \le 1, |y| \le 1 \}$

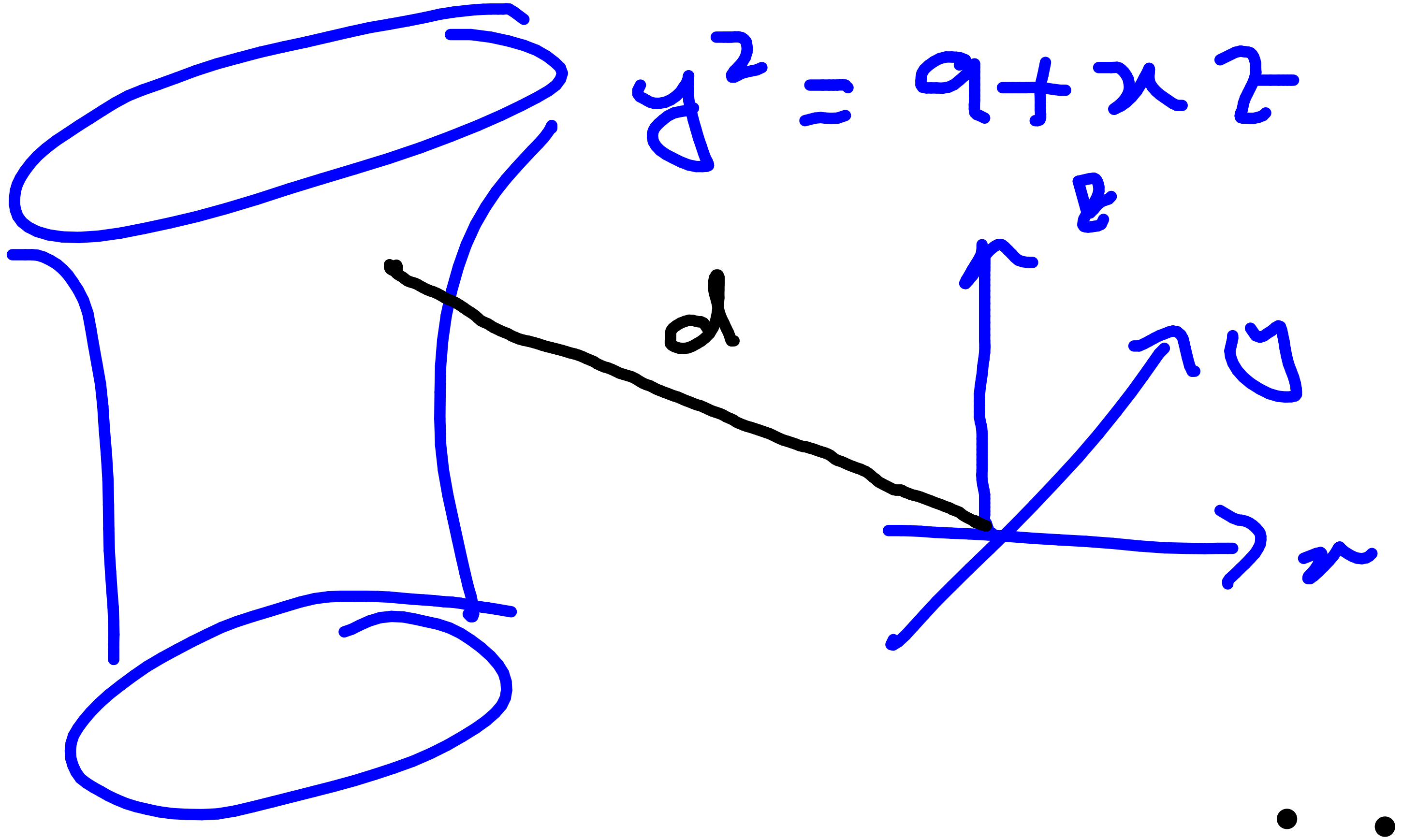
sketch the domain:

$|x| \le 1 \Rightarrow -1 \le x \le 1$

$|y| \le 1 \Rightarrow -1 \le y \le 1$



Q. find the point on the surface $y^2 = 9 + xz$ closest to the origin.



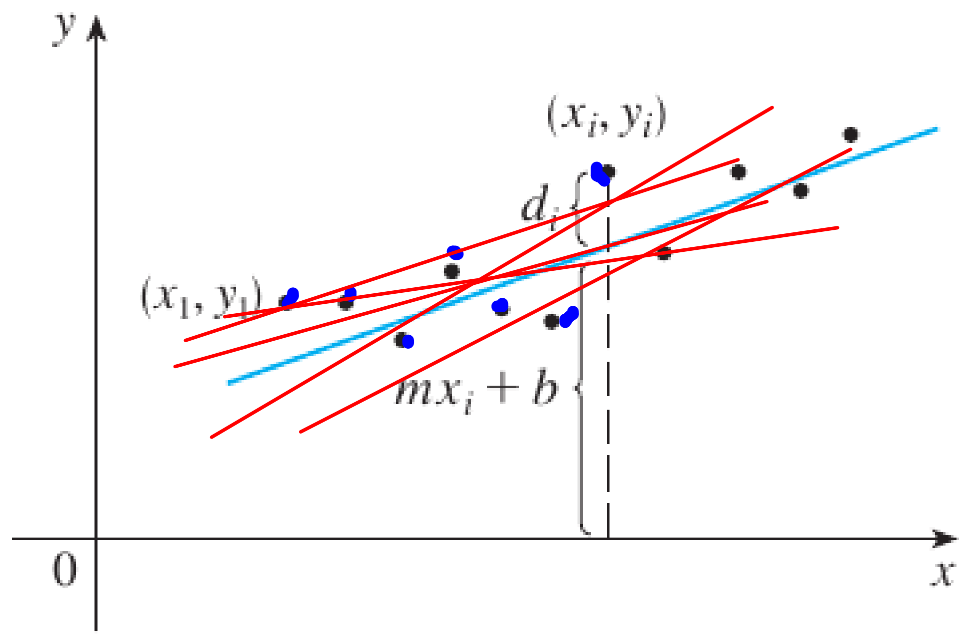
minimize

$$d = x^2 + y^2 + z^2$$

(use $y^2 = 9 + xz$ to eliminate z)

minimize $f(x, z) = x^2 + 9 + xz + z^2$

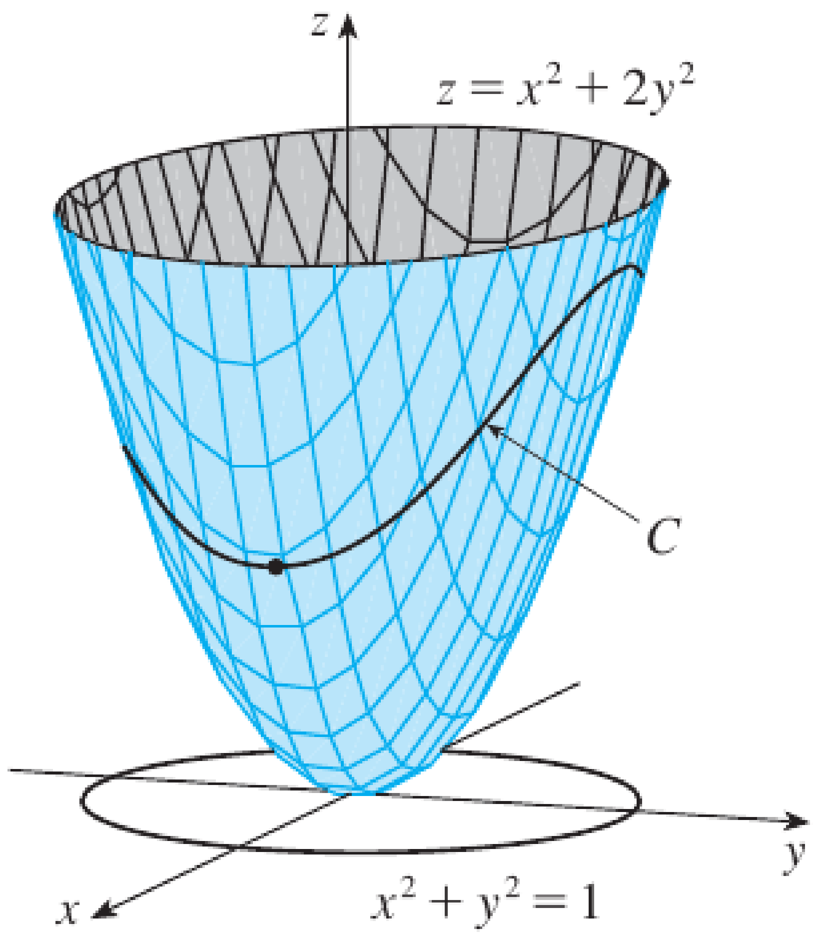
47. Suppose that a scientist has reason to believe that two quantities x and y are related linearly, that is, $y = mx + b$, at least approximately, for some values of m and b. The scientist performs an experiment and collects data in the form of points $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$, and then plots these points. The points don't lie exactly on a straight line, so the scientist wants to find constants m and b so that the line $y = mx + b$ "fits" the points as well as possible. (See the figure.)



Aim: find a line which passes through the data points as closely as possible

11.8 LAGRANGE MULTIPLIERS

V EXAMPLE 2 Find the extreme values of the function $f(x, y) = x^2 + 2y^2$ on the circle $x^2 + y^2 = 1$.



$$f(x, y) = x^2y; \quad x^2 + 2y^2 = 6$$