



ERWIN KREYSZIG

ADVANCED ENGINEERING MATHEMATICS

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GUIDES AND MANUALS	
Maple Computer Guide Mathematica Computer Guide	
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same
topics
next
sem

PART A Ordinary Differential Equations (ODEs)

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What's ODE?

Ordinary Differential Equation

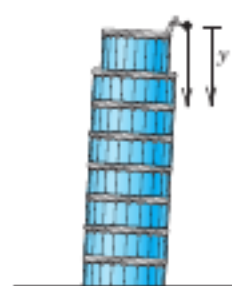
Idea: we know something about $\frac{df}{dx}$ or $\frac{d^2f}{dx^2}$

& we wish to find out about f .

$$\left. \begin{array}{l} \frac{df}{dx} = 2, \text{ find } f(x) \\ \frac{df}{dx} + f = 5 \end{array} \right\}$$

$$\left. \begin{array}{l} \frac{d^2f}{dx^2} + \frac{df}{dx} + f = e^x \\ \text{find } f(x) \end{array} \right\}$$

Why care about knowing to solve ODEs:



Falling stone

$$y'' = g = \text{const.}$$

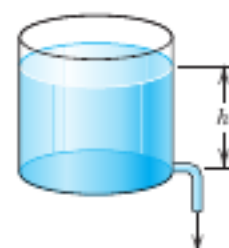
(Sec. 1.1)



Parachutist

$$mv' = mg - bv^2$$

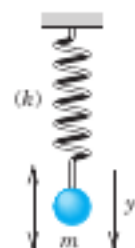
(Sec. 1.2)



Water level h

$$h' = -k\sqrt{h}$$

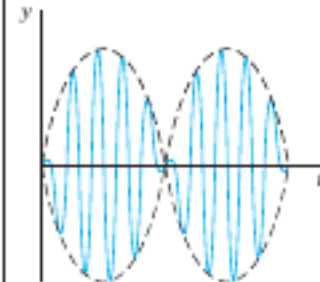
(Sec. 1.3)



Displacement y

$$my'' + ky = 0$$

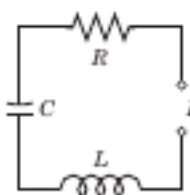
(Secs. 2.4, 2.8)



Beats of a vibrating system

$$y'' + \omega_0^2 y = \cos \omega t, \quad \omega_0 \approx \omega$$

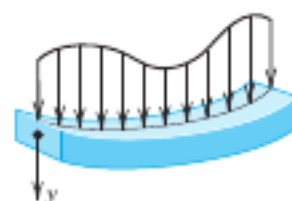
(Sec. 2.8)



Current I in an RLC circuit

$$LI'' + RI' + \frac{1}{C}I = E'$$

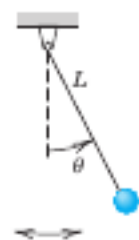
(Sec. 2.9)



Deformation of a beam

$$EIy'''' = f(x)$$

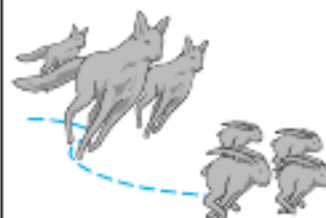
(Secs. 3.2, 3.3)



Pendulum

$$L\theta'' + g \sin \theta = 0$$

(Secs. 4.1, 4.2)



Lotka-Volterra predator-prey model

$$\begin{aligned} y_1' &= ay_1 - by_1y_2 \\ y_2' &= ky_1y_2 - ly_2 \end{aligned}$$

(Secs. 5.1, 5.2)

H.W. → Just read section 1.1 & 1.2.

→ Don't solve exercise problems.

mathematical modelling: converting real world scenarios
into mathematical equations.
↳ in particular into ODE eqns.

1.3 Separable ODEs

1st order
& separable ODEs

$$\left[\frac{dy}{dx} + y^2 = \sin(x) \right], \text{ solve for } y$$

$$\left[\frac{d^2 y}{dx^2} + y^2 = \sin(x) \right] \text{ 2nd order ODE}$$

$$\left[\frac{dy}{dx} + y = \frac{d^2}{dx^2}(e^x) \right] \text{ 1st order ODE}$$

Q. //

$$y' = (x+1)e^{-x}y^2$$

$$\frac{dy}{dx} = (1+x)e^{-x}y^2$$

$$\frac{1}{y^2} dy = (1+x)e^{-x} dx$$

$$\int \frac{1}{y^2} dy = \int (1+x)e^{-x} dx$$

$$\frac{1}{y} = -(x+2)e^{-x} + C$$

&

$$y = \frac{1}{(x+2)e^{-x} + C}$$

Use separation of variables
 \hookrightarrow try to move all x in one side
 & all y in other side
 & integrate

C is an arbitrary constant

d.

Solve $y' = -2xy$, $y(0) = 1.8$.

initial condition

$$\frac{dy}{dx} = -2xy$$

$$\frac{1}{y} dy = -2x dx$$

$$\ln y = -x^2 + C$$

$y(0) = 1.8$

$$\ln 1.8 = -0^2 + C$$

$$C = \ln 1.8$$

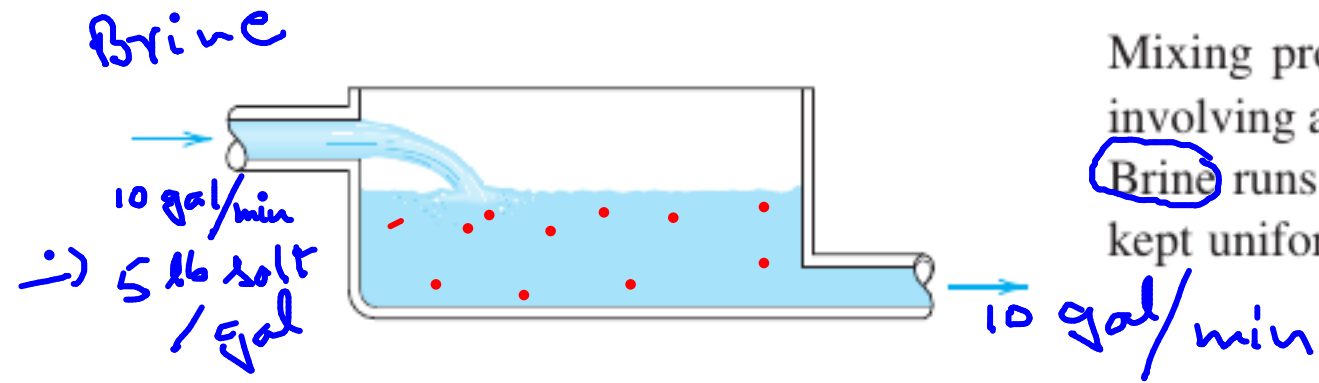
Solve using separation of variables
use the extra condition to eliminate the constant C

$$\ln y = -x^2 + \ln 1.8$$

$$\ln\left(\frac{y}{1.8}\right) = -x^2$$
$$\frac{y}{1.8} = e^{-x^2}$$

$$y = 1.8 e^{-x^2}$$

EXAMPLE 5 Mixing Problem



Mixing problems occur quite frequently in chemical industry. We explain here how to solve the basic model involving a single tank. The tank in Fig. 11 contains 1000 gal of water in which initially 100 lb of salt is dissolved. Brine runs in at a rate of 10 gal/min, and each gallon contains 5 lb of dissolved salt. The mixture in the tank is kept uniform by stirring. Brine runs out at 10 gal/min. Find the amount of salt in the tank at any time t .

$y(t)$: amount of salt in the tank at time t

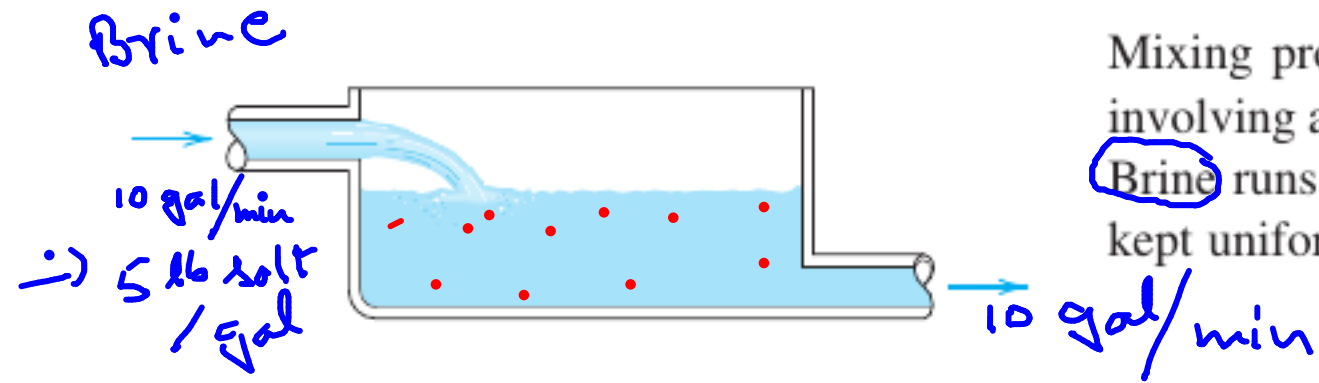
$$y(t) = ??$$

$$y(0) = 100$$

initial salt density ??
0.1 lb / gal

$$\lim_{t \rightarrow \infty} y(t) = \text{guess} = 5000 \text{ lb}$$

EXAMPLE 5 Mixing Problem



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$y(t)$: amount of salt in the tank at time t

$$y(t) = ??$$

$$y(0) = 100$$

$$\begin{aligned} \frac{dy}{dt} &= \text{rate of change of salt mass} \\ &= \text{inflow rate} - \text{outflow rate} \\ &= 50 - \frac{10}{1000} y \end{aligned}$$

Brine \leftrightarrow salt + water

initial salt density ??

$$0.1 \text{ lb / gal}$$

Solve y from

$$\frac{dy}{dt} = 50 - \frac{y}{100}, \quad y(0) = 100$$

Solve $\frac{dy}{dt} = 50 - \frac{y}{100}$ using separation of variables.

$$\frac{dy}{dt} = \frac{5000 - y}{100}$$

$$\frac{1}{5000 - y} dy = \frac{1}{100} dt$$

$$\int \frac{1}{5000 - y} dy = \int \frac{1}{100} dt$$

$$-\ln(5000 - y) = \frac{t}{100} + C$$

$$t = 0$$

$$y = 100$$

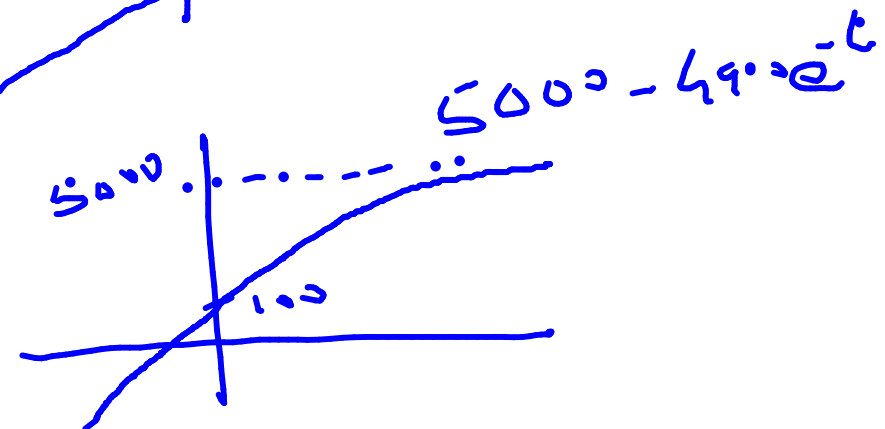
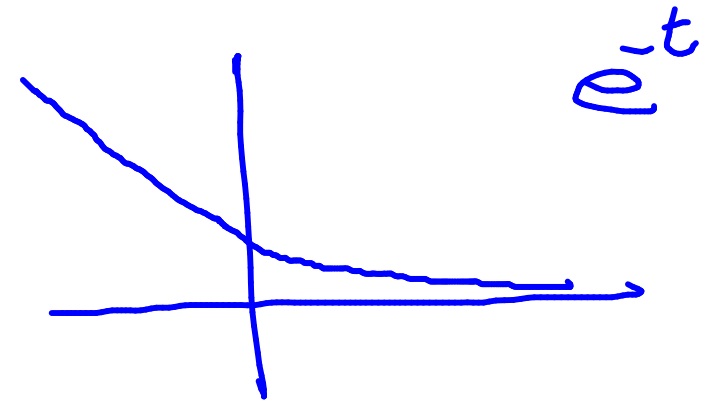
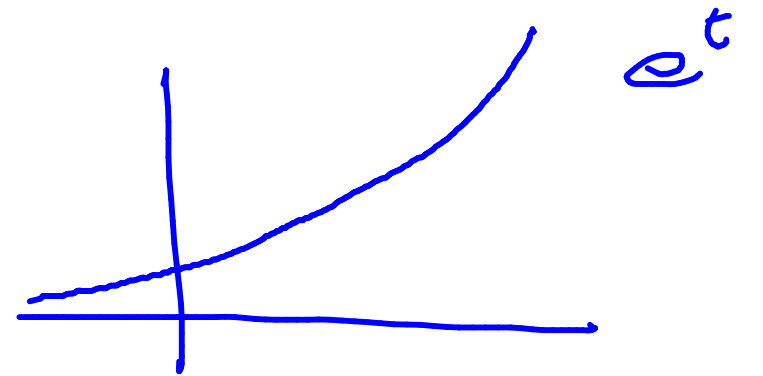
$$C = -\ln 4900$$

$$\ln\left(\frac{5000 - y}{4900}\right) = -\frac{t}{100}$$

$$y = ??$$

$$5000 - y = 4900 e^{-t/100}$$

$$y = 5000 - 4900 e^{-t/100}$$



Recall: what did we start ?? ODE

In differential equations, we solve for a function:

↳ we have some information about derivatives

e.g. $\frac{dx}{dt} = 2$, find $x(t)$.

$2 \frac{d^2x}{dt^2} = 7$, find $x(t)$

→ We learnt last time: the first thing we should try when solving an ODE is variable separable

$$\boxed{x \frac{dy}{dx} = 5x^2 y}$$

is this an ODE ??

$$\frac{1}{y} dy = \frac{5x^2}{x} dx$$

$$\int \frac{1}{y} dy = \int 5x dx$$

$$\sin\left(\frac{dy}{dx}\right) = x$$
$$\frac{dy}{dx} = \sin^{-1}(x)$$

Extended Method: Reduction to Separable Form

$$y' = f\left(\frac{y}{x}\right)$$

$$y_x = v$$

$$2xyy' = y^2 - x^2.$$

$$2xy \frac{dy}{dx} = y^2 - x^2 \quad \text{try to move } x \text{ \& } y \text{ in two sides}$$

$$\downarrow ??$$
$$(\text{only } y \text{ terms}) dy = (\text{only } x \text{ terms}) dx$$

$$2 \frac{dy}{dx} = \frac{y^2 - x^2}{xy}$$

$$\frac{dy}{dx} = \frac{1}{2} \left[\left(\frac{y}{x} \right) - \left(\frac{x}{y} \right) \right]$$

& eliminate y

$$v = \frac{y}{x}$$

$$\frac{dy}{dx} = \frac{1}{2} \left[v - \frac{1}{v} \right]$$

↙

$$y = vx$$

$$\frac{dy}{dx} = \frac{d}{dx}(vx)$$

$$= v + x \frac{dv}{dx}$$

$$v + x \frac{dv}{dx} = \frac{1}{2} \left[v - \frac{1}{v} \right]$$

↳ is this separable??

$$x \frac{dv}{dx} = -\frac{1}{2} \left[\frac{v^2 + 1}{v} \right]$$

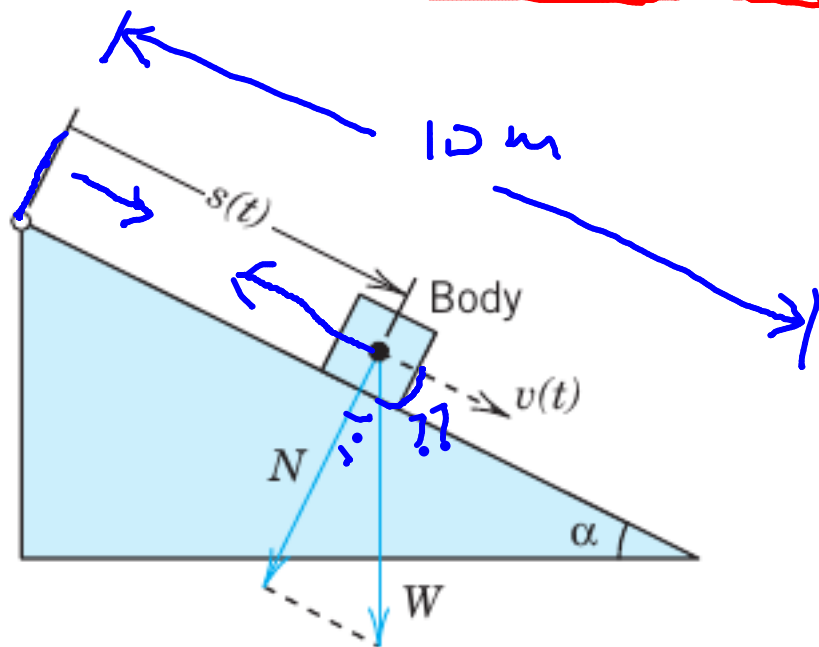
$$\frac{2v}{v^2 + 1} dv = -\frac{dx}{x} \quad \rightarrow \quad \ln(v^2 + 1) = -\ln x + \ln C$$

$$v^2 + 1 = \frac{C}{x^2}$$

$$\boxed{\frac{y^2}{x^2} + 1 = \frac{c}{x}}$$

y is given implicitly by this eqⁿ

32. Friction. If a body slides on a surface, it experiences friction F (a force against the direction of motion). Experiments show that $|F| = \mu|N|$ (Coulomb's⁶ law of kinetic friction without lubrication), where N is the normal force (force that holds the two surfaces together; see Fig. 15) and the constant of proportionality μ is called the *coefficient of kinetic friction*. In Fig. 15 assume that the body weighs 45 nt (about 10 lb; see front cover for conversion). $\mu = 0.20$ (corresponding to steel on steel), $\alpha = 30^\circ$, the slide is 10 m long, the initial velocity is zero and air resistance is negligible. Find the velocity of the body at the end of the slide.



$$W = 45 \text{ N} \quad \mu = 0.2$$

$$\alpha = 30^\circ$$

$s(t)$: distance from the peak of the slide

→ we use $F = ma$ to get an eqⁿ for $s(t)$

$$m \frac{d^2 s}{dt^2} = W \cos(60^\circ) - \mu W \cos(30^\circ)$$

$$\cancel{\frac{W}{g}} \frac{d^2 s}{dt^2} = \cancel{W} \cos(60^\circ) - \mu \cancel{W} \cos(30^\circ)$$

$$\frac{d^2 s}{dt^2} = g \left[\frac{1}{2} - 0.2 \frac{\sqrt{3}}{2} \right]$$

$$= \underbrace{\quad}_A \approx 3.202$$

$$\frac{d^2 s}{dt^2} = A,$$

$$s(0) = 0$$

$$s'(0) = 0$$

Q: what $\frac{ds}{dt}$ when $s = 10$??

Can you complete this ??

$$\frac{d^2 s}{dt^2} = A$$

$$\frac{ds}{dt} = At + C$$

$$v(t) = \boxed{\frac{ds}{dt} = At}$$

★

$$s(t) = \frac{At^2}{2} + D$$

$$\boxed{s(t) = \frac{At^2}{2}}$$

$$\begin{cases} C = ?? \\ \frac{ds}{dt}(0) = 0 \\ C = 0 \end{cases}$$

$$\begin{cases} D = ?? \\ s(0) = 0 \\ D = 0 \end{cases}$$

$$\begin{aligned} \frac{At^2}{2} &= 10 \\ t &= \sqrt{\frac{20}{A}} \\ v\left(\sqrt{\frac{20}{A}}\right) &= A \sqrt{\frac{20}{A}} = \sqrt{20A} \end{aligned}$$

1.4 Exact ODEs. Integrating Factors

$$M(x, y) + N(x, y)y' = 0,$$

next time

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}.$$

$$\cos (x + y) \, dx + (3y^2 + 2y + \cos (x + y)) \, dy = 0.$$

$$(\cos y \sinh x + 1) \, dx - \sin y \cosh x \, dy = 0, \quad y(1) = 2.$$

Reduction to Exact Form. Integrating Factors

• Reduction to Exact Form

• Integrating Factors

• Exact Equations

• Homogeneous Equations

• Bernoulli Equations

• Riccati Equations

• Linear Equations

• Separable Equations

• Exact Equations

• Homogeneous Equations

• Bernoulli Equations

• Riccati Equations

• Linear Equations

• Integrating Factors

$$-y \, dx + x \, dy = 0.$$

$$(e^{x+y} + ye^y) dx + (xe^y - 1) dy = 0$$