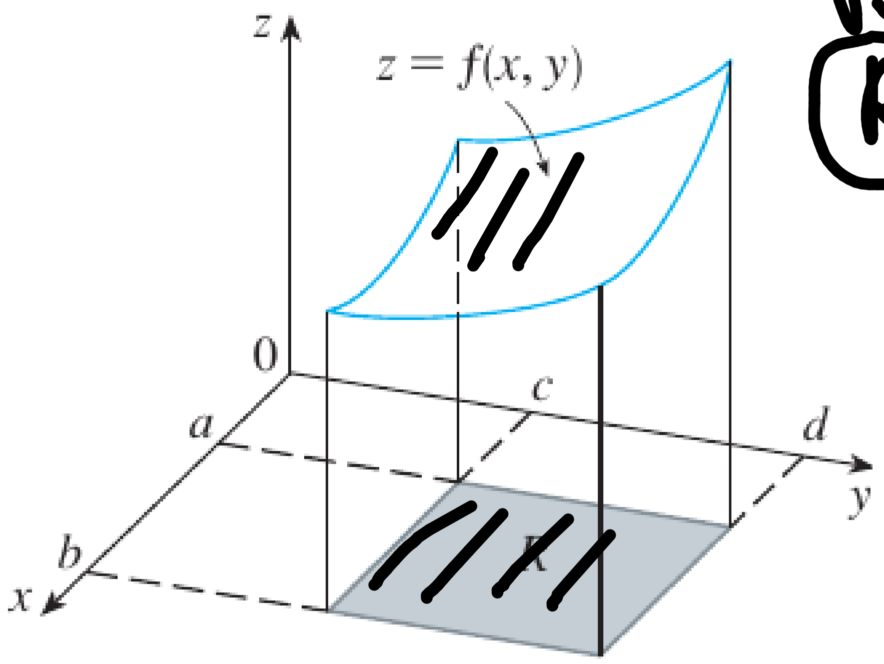
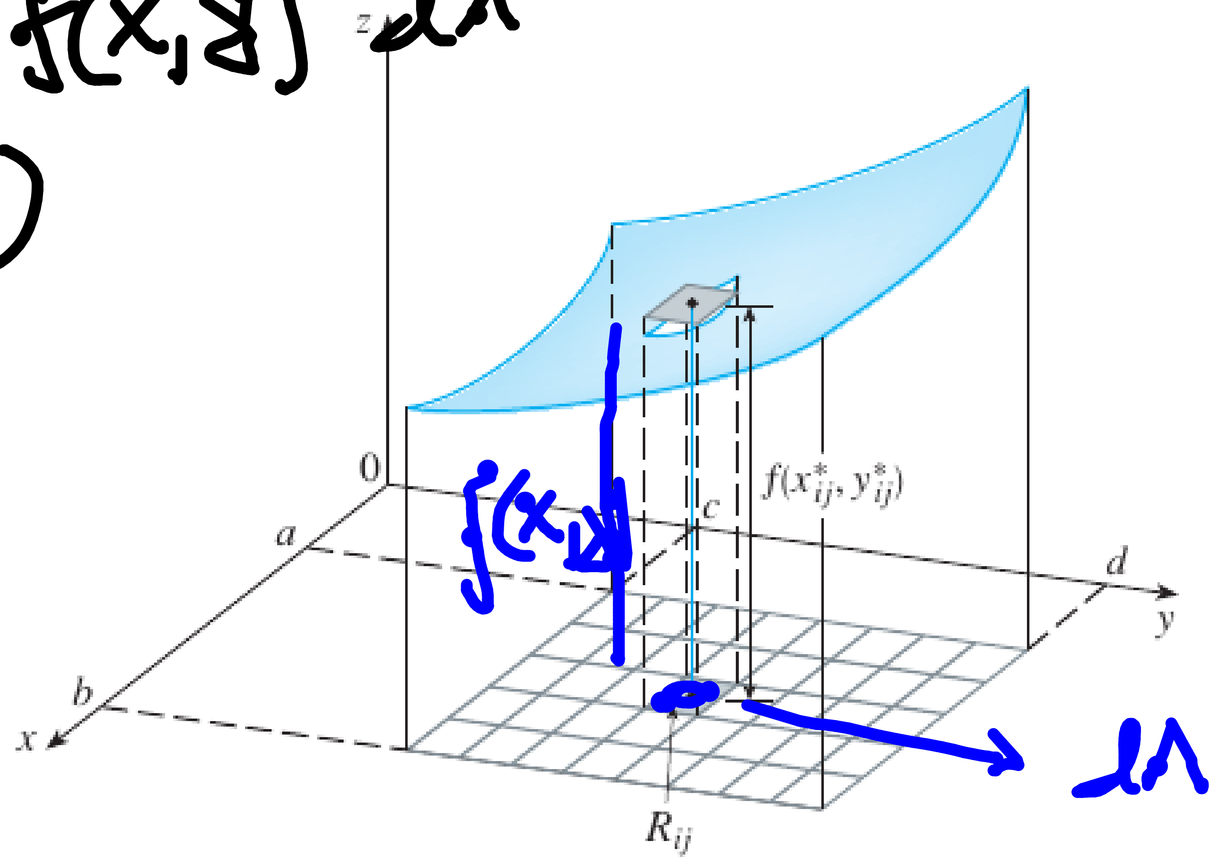
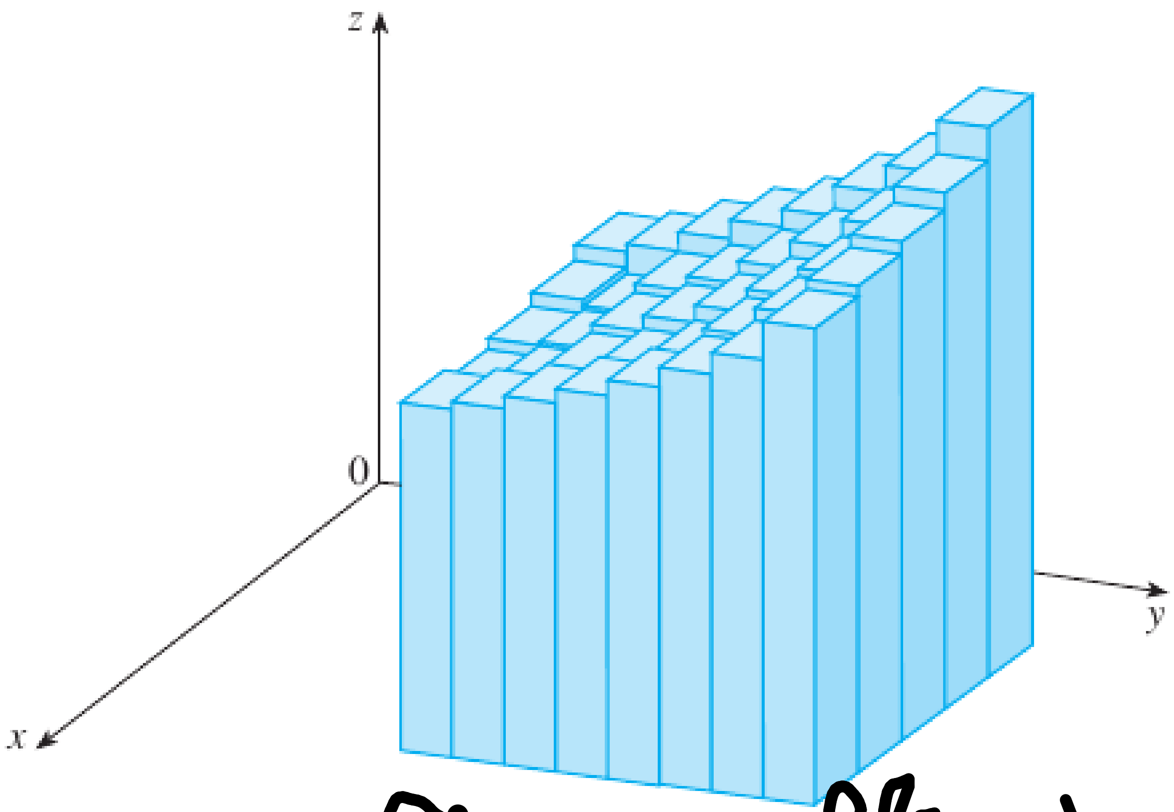
12 MULTIPLE INTEGRALS

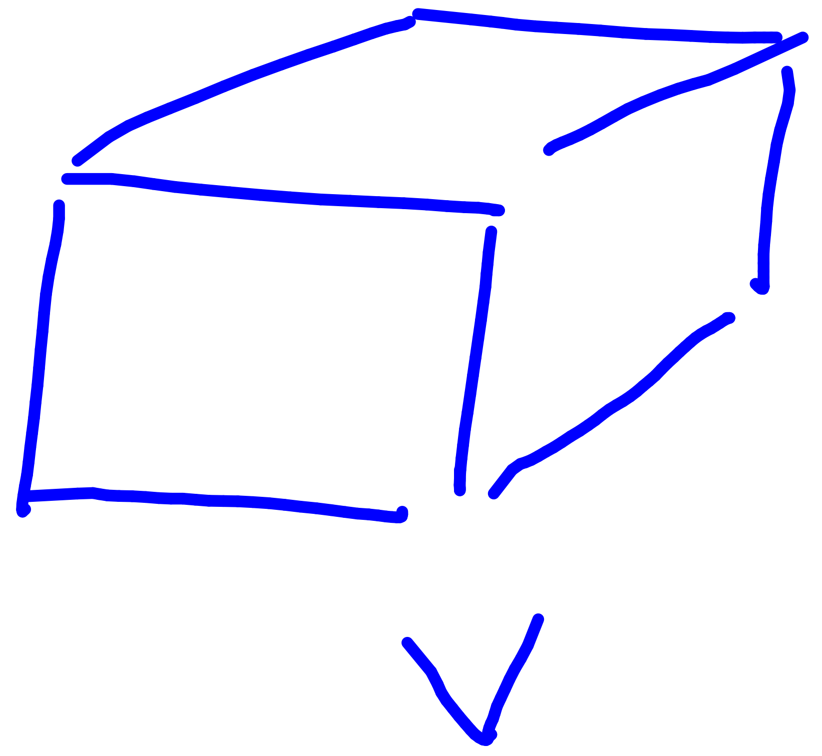
$$\iint_{R} f(x,y)\, dA$$



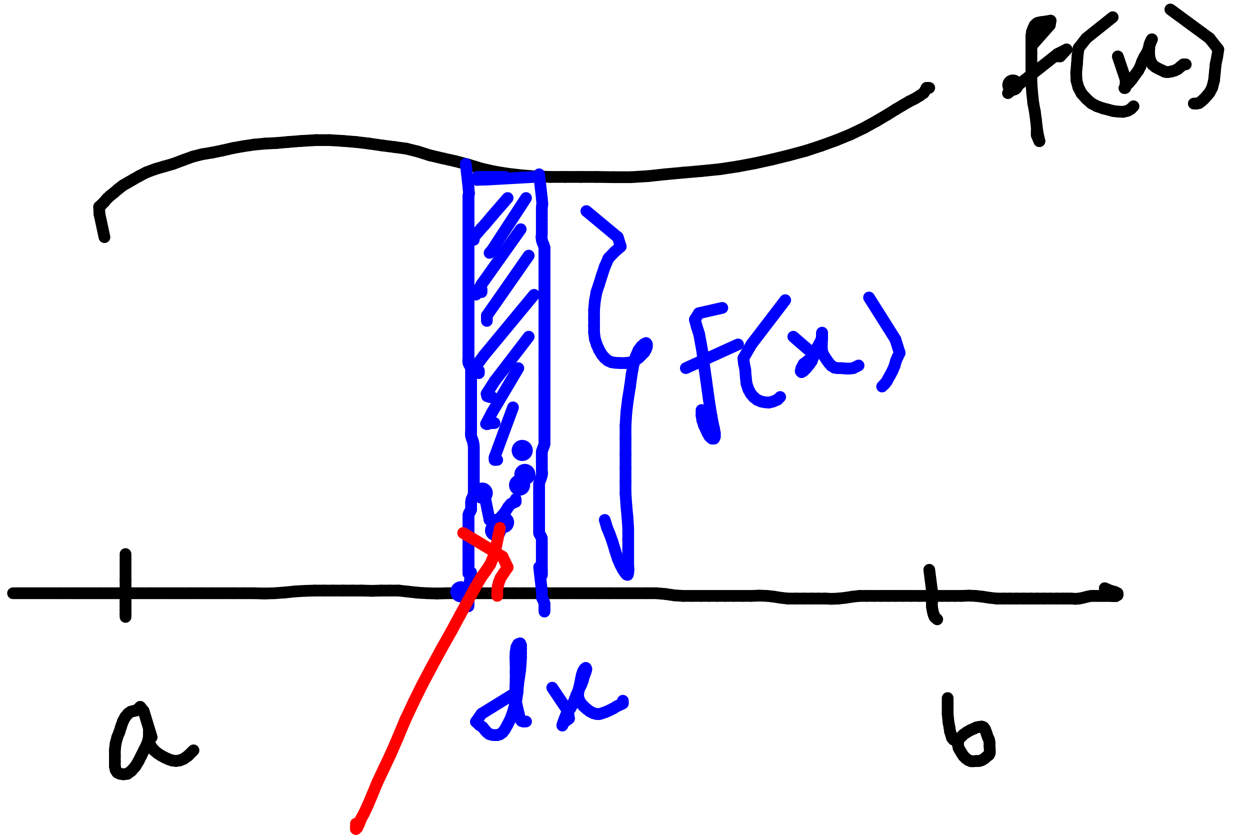




$$dV = f(x,y)\, dA \quad \Big| \quad V = \iint dV = \iint f(x,y)\, dA$$



$$\iiint_V f(x, y, z)\, dV$$



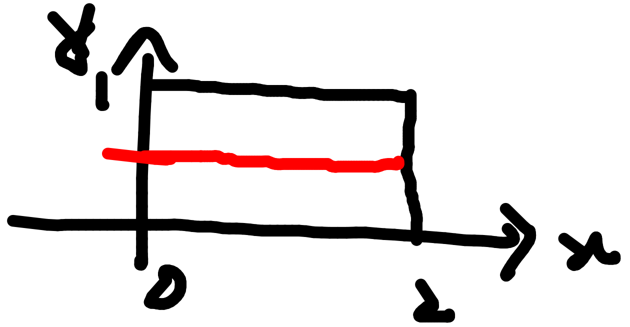
area = $f(x)\,dx$

$$\int_a^b f(x)\,dx$$

$f(x)\,dx$

$\int f(x)\,dx$ = infinite sum of $f(x)\,dx$

d. $f(x,y) = x$

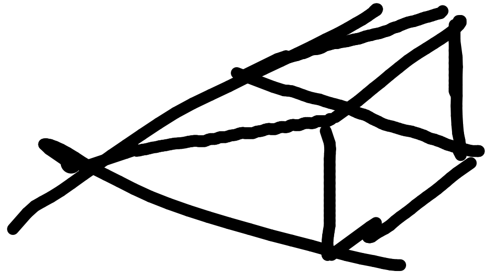


R

$$\iint_R x\,dA = ?? = \int_0^2 \left(\int_0^1 x \, dy \right) dx$$

$$= \int_0^2 \left[xy \Big|_{y=0}^{y=1} \right] dx = \int_0^2 x\,dx$$

$$= 2$$

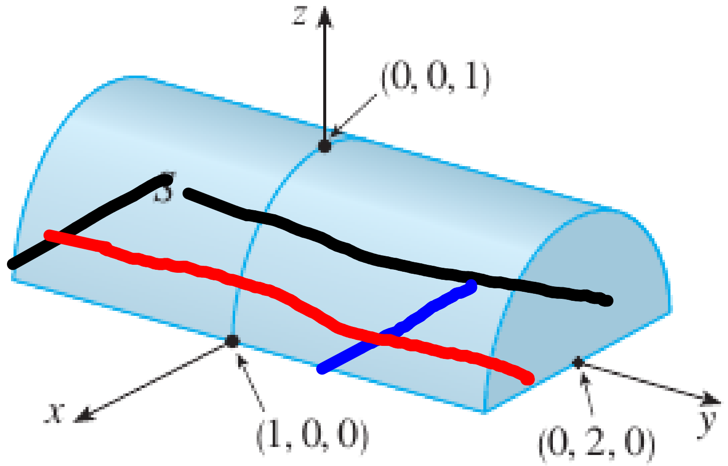


$$= \int_0^1 \left(\int_0^2 x \, dx \right) dy$$

$$= \int_0^1 2\,dy = 2$$

EXAMPLE 2 If $R = \{(x, y) \mid -1 \leqslant x \leqslant 1, -2 \leqslant y \leqslant 2\}$, evaluate the integral

$$\iint_R \sqrt{1 - x^2}\, dA$$



$f(x, y) = \sqrt{1-x^2}$

$z = \sqrt{1-x^2}$

$z^2 + x^2 = 1$

$$= \int_{-2}^{2} \int_{-1}^{1} \sqrt{1-x^2}\, dx\, dy$$

$$= \int_{-1}^{1} \int_{-2}^{2} \sqrt{1-x^2}\, dy\, dx$$

$$= \int_{-1}^{1} 4\sqrt{1-x^2}\, dx = 2\pi$$

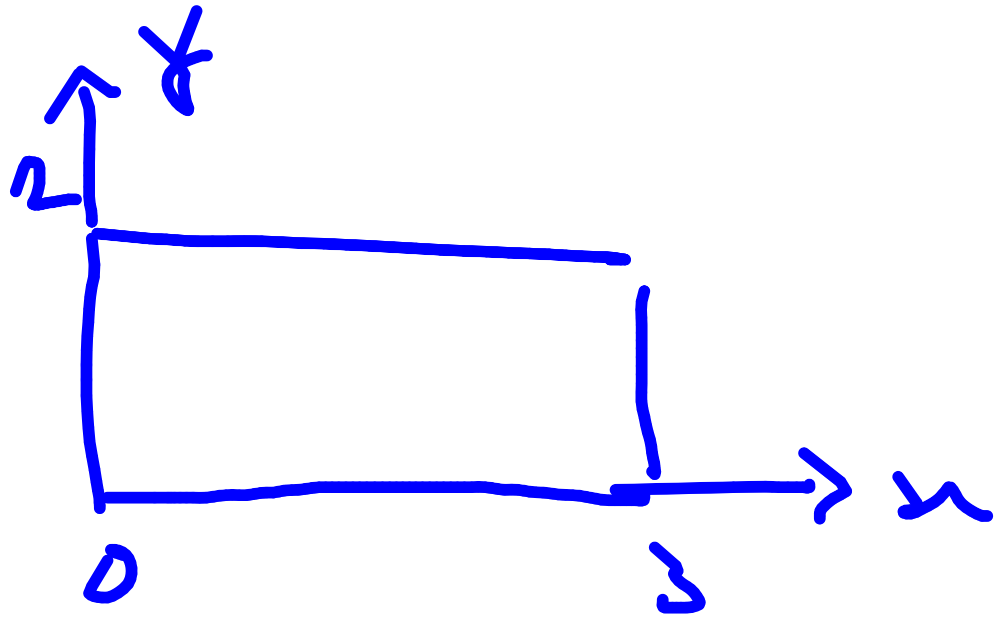
EXAMPLE 4 Evaluate the iterated integrals.

(a) $\int_0^3 \int_1^2 x^2 y \, dy \, dx$

$\frac{27}{2}$

(b) $\int_1^2 \int_0^3 x^2 y \, dx \, dy$

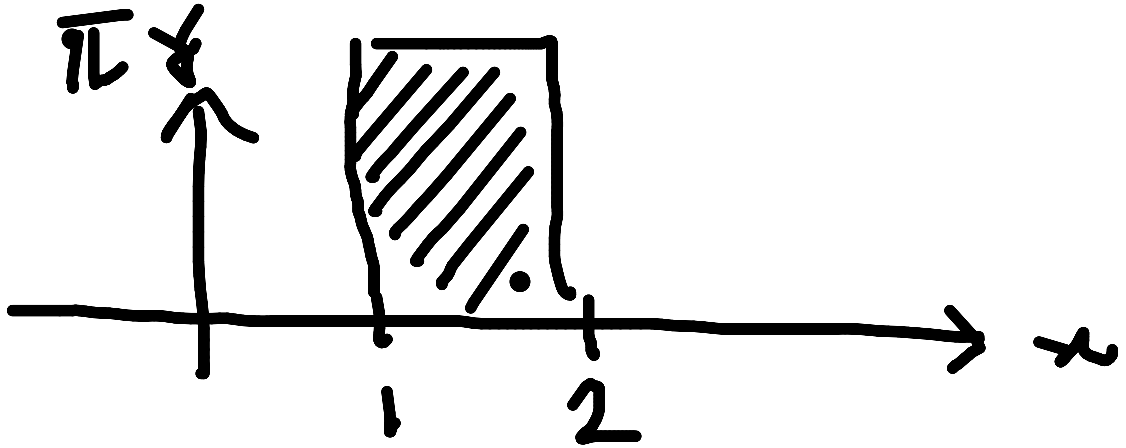
$\frac{27}{2}$



10 FUBINI'S THEOREM If f is continuous on the rectangle $R = \{(x, y) \mid a \leqslant x \leqslant b,\ c \leqslant y \leqslant d\}$, then

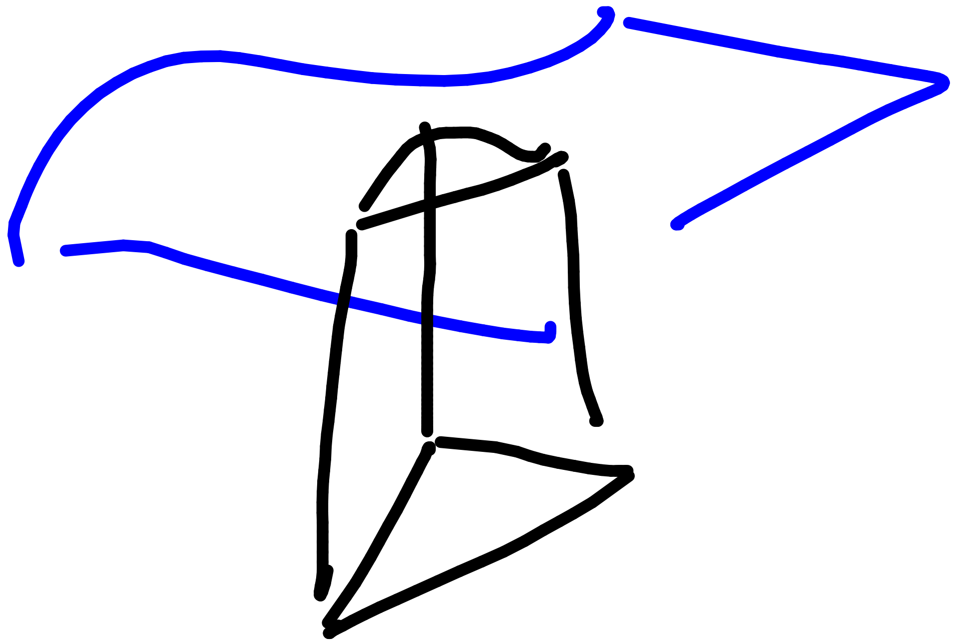
$$\iint_R f(x, y)\, dA = \int_a^b \int_c^d f(x, y)\, dy\, dx = \int_c^d \int_a^b f(x, y)\, dx\, dy$$

Evaluate $\iint_R y \sin(xy)\, dA$, where $R = [1, 2] \times [0, \pi]$.



H.W.

EXAMPLE 7 Find the volume of the solid S that is bounded by the elliptic paraboloid $x^2 + 2y^2 + z = 16$, the planes $x = 2$ and $y = 2$, and the three coordinate planes.



PROPERTIES OF DOUBLE INTEGRALS

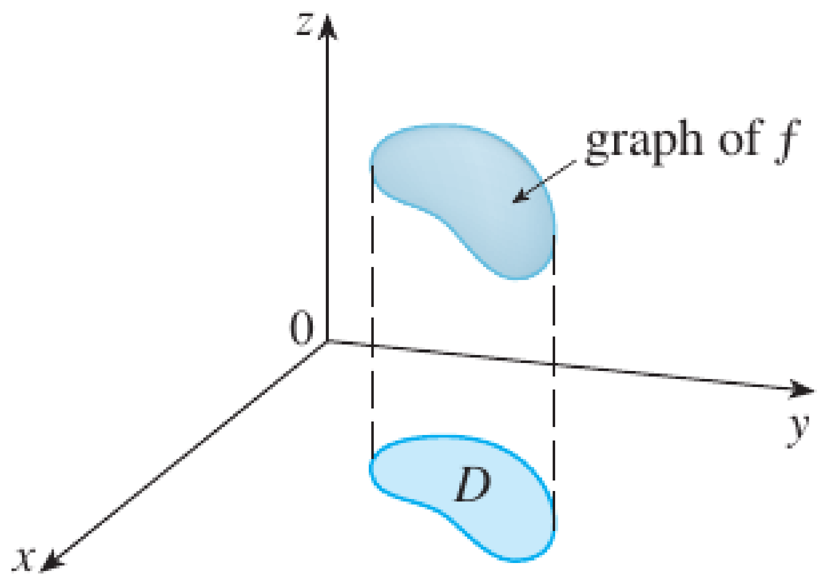
12 $$\iint_R [f(x, y) + g(x, y)]\, dA = \iint_R f(x, y)\, dA + \iint_R g(x, y)\, dA$$

13 $$\iint_R c f(x, y)\, dA = c \iint_R f(x, y)\, dA \qquad \text{where } c \text{ is a constant}$$

If $f(x, y) \geqslant g(x, y)$ for all (x, y) in R, then

14 $$\iint_R f(x, y)\, dA \geqslant \iint_R g(x, y)\, dA$$

12.2 DOUBLE INTEGRALS OVER GENERAL REGIONS



V EXAMPLE 1 Evaluate $\iint_D (x + 2y)\, dA$, where D is the region bounded by the parabolas $y = 2x^2$ and $y = 1 + x^2$.

EXAMPLE 4 Find the volume of the tetrahedron bounded by the planes $x + 2y + z = 2$, $x = 2y$, $x = 0$, and $z = 0$.