Problem 8:

Pdf of an exponential distribution

$$= f_T(t) = \lambda e^{-\lambda t} \quad t \geq 0$$

$$\lambda = \frac{1}{\text{mean}}$$

Given mean = 500 hours

a) $P[400 < t < 600]$

$$= \int_{400}^{600} \left(\frac{1}{500}\right) e^{-(1/500)t} \, dt = 0.148$$

b) $P\left[t > \text{another } 250 \,\middle|\, \text{Running for 300 hrs without failure}\right]$

Say $A = t >$ another 250 hrs

Even though system has worked for past 300 hrs but it should not affect the next 250 hrs

Hence $P\left[\begin{array}{c}\text{system works} \\ \text{for another} \\ \text{250 hrs}\end{array}\right] = P[T > 250]$

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$F_T(t) = CDF = P[T \leq t] = 1 - e^{-\lambda t}$

To ~~stove~~ solve, $P[T > 250 \text{ hrs}]$ is nothing but $= 1 - P[T \leq 250]$

$$P[T > 250 \text{ hrs}] = [1 - (1 - e^{-\lambda t})]$$

$$= e^{-\lambda t}$$

$$= e^{-\lambda 250} = e^{-0.5}$$

$$= 0.606$$

Problem 9:

Normal Gaussian distribution $= X \sim N(0, 0.5)$

a) $P[-0.2 < X < 0.3]$

To use Z-score formula/table

$$Z = \frac{X - \mu}{\sigma} \Rightarrow Z_1 = \frac{-0.2 - 0}{0.5} = -0.4$$

$$Z_2 = \frac{0.3 - 0}{0.5} = 0.6$$

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From standard normal table

$F(0.6) = 0.725$

$F(0.4) = 0.6554$

$P(-0.4 < Z < 0.6)$

$= F(0.6) - F(-0.4)$

$= F(0.6) - (1 - F(0.4))$

$= 0.381$

ⓑ Noise definition

$X > 1.5\sigma + \mu$ or $X < -1.5\sigma + \mu$

P(random reading is an outlier)

$= P(X > 1.5\sigma) + P(X < -1.5\sigma)$

Since $\mu = 0$

$= 2P(X < -1.5\sigma)$

define $Z = \frac{X - \mu}{\sigma} = \frac{1.5\sigma}{\sigma} = 1.5$

$= 2P[Z < -1.5]$

$= 2(0.066) = 0.133$

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Problem 10:

Given $X \sim N(150, 20^2)$

$\mu = 150$ msec , $\sigma = 20$ msec

(a) P(datapacket experiences latency > 180 msec)

$= 1 - P(\text{latency} \le 180)$

Define $Z = \frac{X-\mu}{\sigma}$

$= 1 - P\left(Z \le \frac{180-150}{20}\right)$

$= 1 - P(Z \le 1.5)$

$= 1 - 0.933 = 0.067$

(b) Say the threshold is t msec

Given There is 90% chance that latency is ≤ threshold t msec

$\Rightarrow \quad P(T \le t) = 0.90$

$P\left(Z \le \frac{t-\mu}{\sigma}\right) = 0.90$

say $z = \frac{t-\mu}{\sigma}$

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$x = \mu + z \cdot \sigma$ ③

From z-table, $z = 1.2816$

$x = 150 + 1.2816 \times 20$

$= 175.6$ msec

ⓒ Critical Latency $= 2.5\sigma + \mu$

P(critical latency) = ?

Say Latency = T msec

$P(\text{critical latency}) = P(T > 2.5\sigma + \mu)$

$\Rightarrow P(Z > 2.5) \Rightarrow 1 - P(Z \leq 2.5)$

From z-table

$\Rightarrow 1 - 0.9938 = 0.0062$

Problem 11:

Given $X \sim N(2500, 300^2)$

ⓐ $P(X \geq 2000 \text{ hrs})$

$\Rightarrow 1 - P(X < 2000 \text{ hrs})$

$\Rightarrow 1 - P\left(Z < \frac{2000 - 2500}{300}\right)$

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$\Rightarrow 1 - P(Z < -1.67)$

$\Rightarrow 0.952$

(b) Say warranty period w hrs

$P(X \leq w) = 0.01$

$P\left(Z \leq \frac{w-\mu}{\sigma}\right) = 0.01$

$\Rightarrow$ From Z table

$\frac{w-\mu}{\sigma} = -2.33$

$w = -2.33\sigma + \mu$

$w = 1801$ hrs

(c) Total components $= 10,000$

$P(2000 \leq X \leq 2800)$ or $P(2000 < X < 2800)$

$\Rightarrow P\left(\frac{2000-\mu}{\sigma} < X < \frac{2800-\mu}{\sigma}\right)$

$\Rightarrow P(-1 < Z < 1) \Rightarrow F(1) - F(-1)$

$\Rightarrow 0.841 - 0.158$

$\Rightarrow 0.682$

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This means that 68.2% of 10,000 components will last between 2200 to 2800 hrs

$$\frac{\text{number of components}}{10000} = 68.2\%$$

number of components with lifespan between 2200 to 2800 hrs = 682

Problem 15:-

Given $A = \begin{bmatrix} 5 & 2 \\ 2 & 2 \end{bmatrix}$

$$|A - \lambda I| = 0$$

$$\Rightarrow A - \lambda I = \begin{bmatrix} 5-\lambda & 2 \\ 2 & 2-\lambda \end{bmatrix}$$

$$\Rightarrow |A - \lambda I| = (5-\lambda)(2-\lambda) - 4$$

$$\lambda^2 - 7\lambda + 6 = 0$$

Solutions of above equation

$\lambda_1 = 6, \lambda_2 = 1$ are eigen values

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Now, From the definition of eigen values and eigen vectors

$\Rightarrow \quad A\vec{v} = \lambda \vec{v}$

$(A - \lambda I)\vec{v} = 0$

For $\lambda_1 = 6$ $\left(\begin{bmatrix} 5 & 2 \\ 2 & 2 \end{bmatrix} - \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix} \right) \vec{v} = 0$

$\Rightarrow \left(\begin{bmatrix} -1 & 2 \\ 2 & -4 \end{bmatrix} \right) \vec{v} = 0$

$\Rightarrow$ if $\vec{v} = \begin{bmatrix} \vec{x} \\ \vec{y} \end{bmatrix}$

$\Rightarrow \left(\begin{bmatrix} -1 & 2 \\ 2 & -4 \end{bmatrix} \right) \begin{bmatrix} \vec{x} \\ \vec{y} \end{bmatrix} = 0$

$\Rightarrow \quad -\vec{x} + 2\vec{y} = 0 \quad$ or $\quad 2\vec{x} - 4\vec{y} = 0$

Redundant eqns

Hence $\quad \vec{x} = 2\vec{y} \quad$ or $\quad x = 2\vec{y}$

so one set of eigen vector

$\vec{v}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

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For $\lambda_2 = 1$ $\left(\begin{bmatrix} 5 & 2 \\ 2 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)\vec{v} = 0$ (5)

$\Rightarrow \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}\vec{v} = 0$

$\Rightarrow 4x + 2y = 0$ or $2x + y = 0$

$\Rightarrow x = -\frac{1}{2}y$

One set of eigen vector $\vec{v}_2 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

(b) **Given** $B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ -1 & 1 & 3 \end{bmatrix}$

For eigen values

$$\det(B - \lambda I) = 0$$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 1 & 0 \\ 0 & 2-\lambda & 0 \\ -1 & 1 & 3-\lambda \end{vmatrix} = 0$$

Using cramer's rule

$$(1-\lambda)\begin{vmatrix} 2-\lambda & 0 \\ 1 & 3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda)(2-\lambda)(3-\lambda) = 0$$

$$\Rightarrow \lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 3$$

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For $\lambda_1 = 1$

$(B - \lambda I)\vec{v} = 0$

$$\left(\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ -1 & 1 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \vec{v} = 0$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ -1 & 1 & 2 \end{bmatrix} \vec{v} = 0$$

Say $\vec{v} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$\Rightarrow y = 0$; $-x + y + 2z = 0$

$\Rightarrow +z = +\frac{x}{2}$

One set of eigen vector

$$\vec{v} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

For $\lambda_2 = 2$

$$\left(\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ -1 & 1 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \right) \vec{v} = 0$$

$$\left(\begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 0 \\ -1 & 1 & 1 \end{bmatrix} \right) \vec{v} = 0 \Rightarrow \begin{aligned} -x + y &= 0 \\ -x + y + z &= 0 \end{aligned}$$

$\Rightarrow z = 0$ and $x = y$

$$\vec{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

For $\lambda_3 = 3$

$$\left(\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ -1 & 1 & 3 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \right) \vec{v} = 0$$

$$\Rightarrow \begin{bmatrix} -2 & 1 & 0 \\ 0 & -1 & 0 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\Rightarrow \quad -2x + y = 0, \quad y = 0, \quad -x + y = 0$$

Say $z = 1$ $\quad \vec{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

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