Problem 8:-

Given Info:

Pro Life Sample Info:

$\bar{x}_{pro} = 120$ hours $\quad s_{pro} = 10$ hours

Standard Sample Info

$\bar{x}_{std} = 115$ hours $\quad s_{std} = 12$ hours

a) Null hypothesis

Say μ_{pro} & μ_{std} are the mean lifes of pro and standard batteries respectively

$H_0: \mu_{pro} \leq \mu_{std}$

$H_a: \mu_{pro} > \mu_{std}$

b) To compute z-statistic we first need to compute the standard error of 2 independent means.

Z-statistic, $z = \dfrac{\bar{x}_{Pro} - \bar{x}_{Std}}{SE}$

where SE = standard error

For any 2 Random Variables X and Y

$$Var(X-Y) = Var(X) + Var(Y) - 2\,Cov(X,Y)$$

if X & Y are independent

then $Var(X-Y) = Var(X) + Var(Y)$

hence $SE = \sqrt{\dfrac{s_{Pro}^2}{40} + \dfrac{s_{Std}^2}{50}}$

$= 2.32$

$z = \dfrac{120-115}{2.32} = 2.15$

Considering right-sided z-distribution

$$Z_\alpha = Z_{0.02} = 2.05$$

$Z > Z_\alpha \Rightarrow$ Reject Null hyopthesis

(c) there is enough statistical evidence at the 2% significance level that Pro batteries will last longer than standard ones.

Problem-9:

Given

Sample size = 30

$\bar{x}$ = ?

s = ?

a) $\bar{x} = 24.5$, $s = 3.59$

b,c) Null hypothesis, $H_0: \mu \leq 20$

$H_a: \mu > 20$

Using t-statistic

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{24.5 - 20}{3.59/\sqrt{30}}$$

$$= 6.40$$

Degrees of freedom = $n-1 = 29$

$t_{critical}(\alpha = 0.01) = 2.462$

$t_{critical} < t$.

we can reject Null hypothesis

d) There is strong statistical evidence that waiting time is longer than 20 minutes

Problem 10:

a) Null hypothesis H_0: Age group and subscription are independent

H_a: Age group and subscription are dependent

(b) Expected frequencies

$$E_{ij} = \frac{\text{Row total} \times \text{Column total}}{\text{Grand total}}$$

Row Totals

Age group

18 - 29 : 120

30 - 49 : 100

50+ : 80

Column Total

Basic : 100

Premium = 150

Elite = 50

Expected values

Age grp	Basic	Premium	Elite	Row total
18-29	$\frac{120 \times 100}{300}$	$\frac{120 \times 150}{300}$	$\frac{50 \times 120}{300}$	120
30-49	$\frac{100 \times 100}{300}$	$\frac{150 \times 100}{300}$	$\frac{50 \times 100}{300}$	100
50+	$\frac{80 \times 100}{300}$	$\frac{80 \times 150}{300}$	$\frac{80 \times 50}{300}$	80

(c) Using chi-square test statistic

$$X^2 = \sum \frac{(O-E)^2}{E}$$

$O \Rightarrow$ observed $\quad E =$ Expected

$X^2 = 0.625 + 0 + 1.25 + 0.33 + 0 + 0.66 + 0.105 + 0 + 0.20 = 3.18$

d) Degrees of freedom

$= (3-1)(3-1) = 4$

$X^2_{critical} (\alpha = 0.05) = 9.48$

$X^2_{critical} > X^2$

We fail to reject Null hypothesis

(e) There is not enough statistical infor evidence to confirm if age group and subscription category are dependent.

Bonus Problem

Given Info

Design A

Number of days $= n = 30$

$\bar{x}_A = 2.85\%$

$S_A = 0.45\%$

Design B

$n = 30$

$\bar{x}_B = 3.05\%$

$S_B = 0.60\%$

a) Null hypothesis $H_0: \mu_A = \mu_B$

$H_a = \mu_A \neq \mu_B$

μ_A, μ_B being true mean daily conversion rate for Designs A and B respectively

(c) Pooled variance

$$S_p^2 = \frac{(n_A - 1)S_A^2 + (n_B - 1)S_B^2}{n_A + n_B - 2}$$

$$S_p^2 = 0.281 \Rightarrow S_p = 0.5303$$

Standar Error SE

$$= S_p \sqrt{\frac{1}{n_A} + \frac{1}{n_B}}$$

$$\Rightarrow 0.5303 \sqrt{2/30}$$

$$\Rightarrow 0.1370$$

T-test statistic

$$t = \frac{\bar{x}_B - \bar{x}_A}{SE}$$

$$= 1.4606$$

$$t_{critical}(\alpha = 0.05/2, df = 58)$$

$$= \pm 2.001$$

$|t\ statistic| \leq t_{critical}$

↳ we fail to reject Null hypothesis

tstatistic = 1.46. lies within non-regection region -2.001 to 2.001)

There is no statistical evidence that there is significant difference betwee the average daily conversion rate of Design A & Design B

(9) $CI = (\bar{x}_B - \bar{x}_A) \pm$ Margin of Error

Margin of Error $= t \times SE$

$= 2.001 \times 0.13$

$= 0.274$

$[-0.074 , 0.474]$

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