Regression Discontinuity

EC420 MSU

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- Diff-in-diff
  - Made an assumption (parallel trends)
  - That let us say we know $E[Y_0|D=1]$
  - Had to use the Parallel Trends assumption
  - Could estimate as a regression (simple!)
- Synthetic
  - Concept: pre-treatment match gives us post-treatment counterfactual

Let's return to the endogeneity issue:

$$y_i = \beta_0 + \beta_1 D_i + \beta_2 x_i + u_i$$

And we have the usual endogeneity problem that: D_i may be correlated with u_i, even once we condition on exogenous x_i.

Let's set up an example

- y_i is semesters of college attended
- D_i is "getting a scholarship"
- x_i is parental income
- u_i is potentially endogenous error

We might think that *ability*

1. Affects semesters of college attended
2. Unobserved (and thus in u_i)
3. Correlated with getting a scholarship

$$\Rightarrow (Y_{i0}, Y_{i1}) \not\perp D_i | x_i$$

Let's return to the endogeneity issue:

$$y_i = \beta_0 + \beta_1 D_i + \beta_2 x_i + u_i$$

What if we had a scholarship that was awarded to all students with a 1200 SAT score or above?

- $D_i = 1(SAT_i > 1200)$

Does this solve our endogeneity problem?

- Is SAT_i, the student's SAT score, exogenous?
- What do we learn about scholarships in general from understanding the effect of *this* particular scholarship? We'll return to this in a little bit.

Does this solve our endogeneity problem?

- Is SAT_i, the student's SAT score, exogenous?

What if I compared a student with an 800 to a student with a 1600 (and with the exact same x_i)

- Would you be concerned about that unobserved *ability* in u_i?

What about comparing a 900 student to a 1500 student?

What about comparing a 1000 student to a 1400 student?

What about comparing a 1100 student to a 1300 student?

What about comparing a 1190 student to a 1210 student?

What about comparing a 1199 student to a 1201 student?

The intuition behind RD is as follows:

If we compare students within a *small enough window* around a threshold, then treatment D_i is **as good as randomly assigned**

And *as good as randomly assigned* lets us treat the treatment as **exogenous**

The *unobserved* endogenous problem in u_i is no longer a problem.

What do we need for an RD?

I. A **threshold**

- And it has to be exogenous
- Usually from a policy
- Arbitrary policies are...great!

What do we need for an RD?

II. A **running variable**

- We need some variable that crosses the threshold
  - It cannot be binary
- It has to be *observed*
- It has to determine the treatment of interest †

> "Although treatment isn't randomly assigned, we know where it comes from"

— Mastering Metrics, Ch. 4

† A *fuzzy* RD relaxes this assumption

The **running variable** in our example is SAT score

- Crosses the threshold
- Non-binary
- Observed
- Determines scholarship (treatment) discontinuously
  - A "jump"

What do we need for an RD?

III. A **window**

- We need somewhere to "draw the line"
- In our SAT example, some of us would have been OK with 1150 and 1250.
  - Some would want 1190 to 1210
  - Some would say 1195 to 1205

A **threshold**

- Set exogenously

A **running variable**

- Determines treatment
- Continuous
- Observable

A **window**

- How close is close enough

The identifying assumption:

Within a small enough **window**, the treatment (as determined by the **running variable** and the **threshold**) is as good as randomly assigned

Specification of an RD

- Let's call the running variable x_r
- Let's call the threshold τ

Our RD equation would be:

$$y_i = \beta_0 + \beta_1 D_i + \beta_2 x_{i,r} + \beta_3 x_{exogenous} + u_i$$

- y_i is outcome
- $D_i = 1(x_{i,r} > \tau)$
- τ is the threshold
- $x_{i,r}$ is the *running variable*
- $x_{i,exogenous}$ is an exogenous covariate (a control)
- u_i is the error

Our RD equation would be:

$$y_i = \beta_0 + \beta_1 D_i + \beta_2 x_{i,r} + \beta_3 x_{exogenous} + u_i$$

and

$$D_i = 1(x_{i,r} > \tau)$$

Note that we have a continuous running variable $x_{i,r}$ with a linear coefficient

- We can still control for the effect of the running variable
- That's why we need the τ threshold where treatment "jumps"

And we can still control for other things

- $x_{exogenous}$

Our RD equation would be:

$$y_i = \beta_0 + \beta_1 D_i + \beta_2 x_{i,r} + \beta_3 x_{exogenous} + u_i$$

But where is the window?

- We *could* just use a sample within the window
- Define the window as being (a,b)
  - R: `myData[myData$xir >=a & myData$xir <=b,]`
- Another option is to use *local linear regresion*, which can be implemented as a *weighted regression*

A quick aside to establish weighted regression:

We know that OLS minimizes the sum of the squared error term u_i

$$\sum(y_i - \beta_0 - \beta_1 x_1)^2 = \sum(u_i)^2$$

Weighted regression just puts weights on that sum:

$$\sum w_i(u_i)^2$$

and

$$\sum w_i = 1$$

Weights can set the estimation sample

If we said that $w_i = 0$ if i is not "in the window", and constant otherwise, we would just be setting the estimating sample.

Weights can be continuous

What if we set w_i to be some function of *how far away from the threshold* an observation $x_{i,r}$ is?

- If $x_{i,r}$ is far away from τ, then it gets a low weight
- If $x_{i,r}$ is practically equal to τ, then it gets a high weight

$$\tilde{w_i} = \frac{1}{|x_{i,r} - \tau|}$$

Of course, we might have to re-scale all the w_i's to make sure they add to 1:

Weighted Regression

This "local linear regression" because it puts more weight on those observations where $x_{i,r}$ is very close to τ

- You still have to determine the form of the weights
- We call the function that generates weights the *kernel*
- Since we are not making an assumption about how the distance away from τ enters the main regression, this is **non-parametric**.

Other kernels include:

- Rectangular Kernel
  - Puts equal weight on all observations between a and b
- Triangle Kernel
  - Puts weight on those closer to the center

Applying weights to OLS is easy:

```
myOLS = lm(Y ~ X1 + D + X2, data=myData, weights = 1/abs(myData$X1 - tau))
```

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Example 1

Maimonides Rule

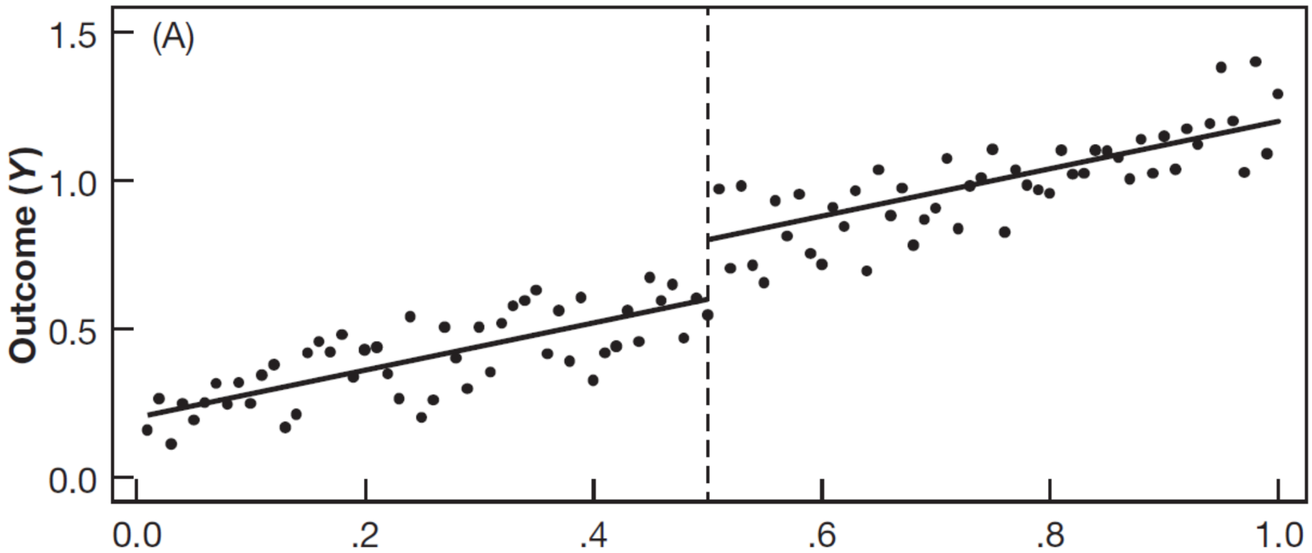
- 12th century rabbinic scholar who determined that 40 students was the max. class size.
- Anything larger had to be cut into two 20/21-person classes
- Angrist and Lavy (1999) used threshold to look at effect of class size on student outcomes
  - What is the endogeneity problem between student outcomes and *assignment* of class sizes?
  - What is the threshold and is it exogenously set?
  - What is running variable?
  - Evaluate whether or not we have an as-good-as-random treatment assignment

Example 2

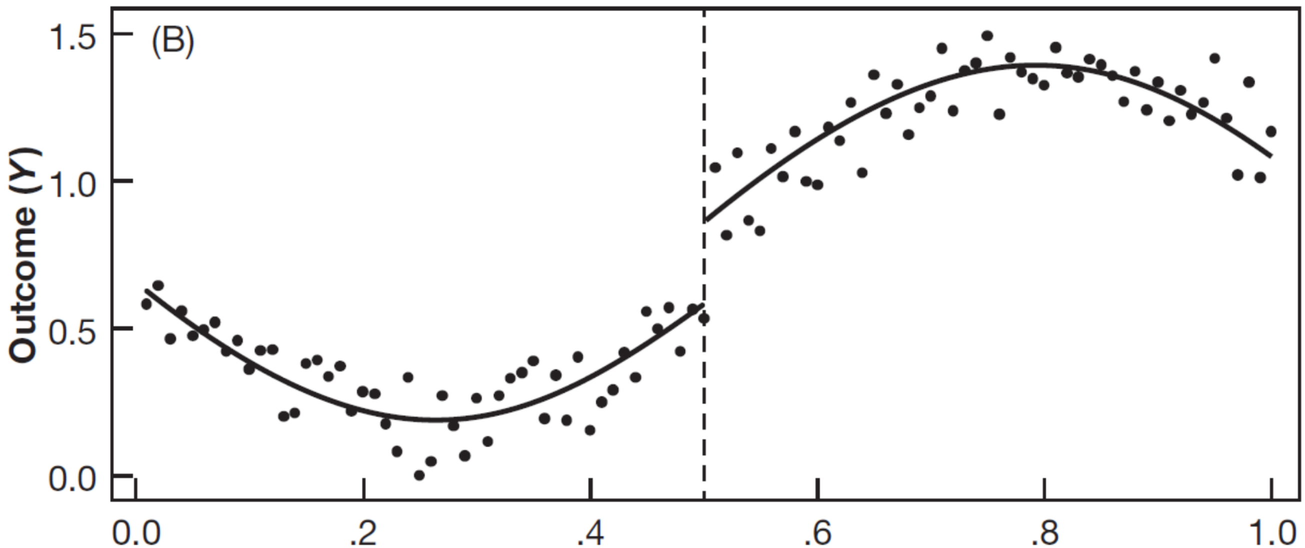
Superfund cleanup

- All superfund sites have a risk score
- Congress determines how far down the
- Greenstone and Gallagher (2008) and Gamper-Rabindran and Timmins (2011) used threshold to look at effect of cleanup on housing prices
    - What is the endogeneity problem between home values and *assignment* of superfund cleanup?
    - What is the threshold and is it exogenously set?
    - What is the running variable?
    - Evaluate whether or not we have an as-good-as-random treatment assignment

$$y_i = \beta_0 + \beta_1 D_i + \beta_2 x_{i,r} + u_i$$



$$y_i = \beta_0 + \beta_1 D_i + \beta_2 x_{i,r} + \beta_3 x_{i,r}^2 + \beta_4 x_{i,r}^3 + u_i$$



Running Variables

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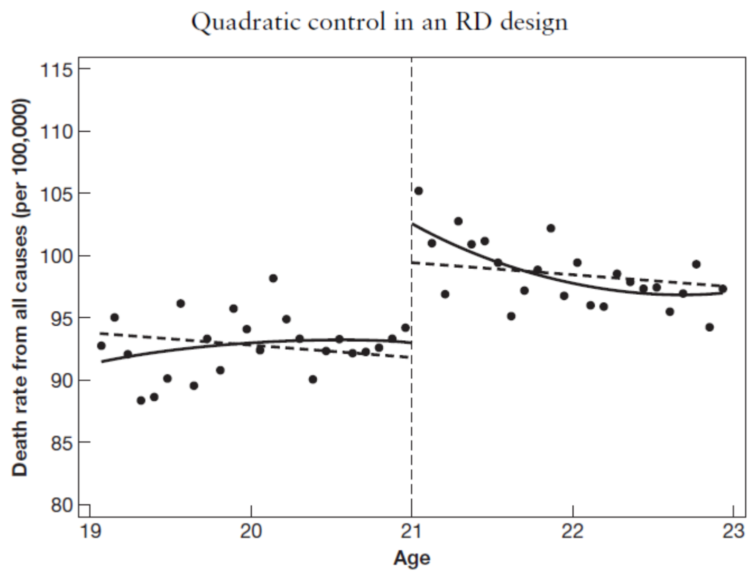
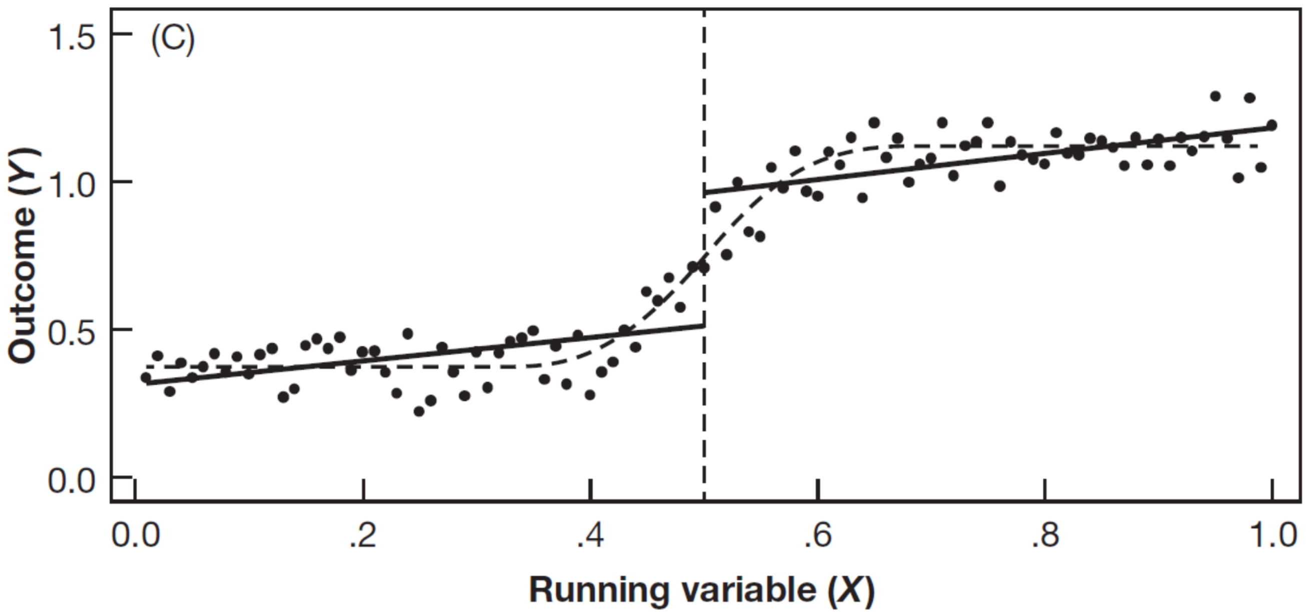
$$y_i = \beta_0 + \beta_1 D_i + \beta_2(x_{i,r} - \tau) + \beta_3(x_{i,r} - \tau) * D_i + u_i$$

Here, we let the *slope* of the coefficient on the running variable change when it crosses the threshold. To the left of the threshold, $\frac{dy}{dx_{i,r}} = \beta_2$, and to the right, it is $\beta_2 + \beta_3$ (dashed line).





This one is not so good - the dashed line fits a possible non-linear trend.

- Note that there is an upward trend just before the threshold
- That trend is more consistent with a polynomial trend, and the linear trend + threshold gives a false effect.

The implicit assumption in our RD "window"

is that everything else *around* the threshold τ varies *smoothly*

- Then, we can control for it parametrically.

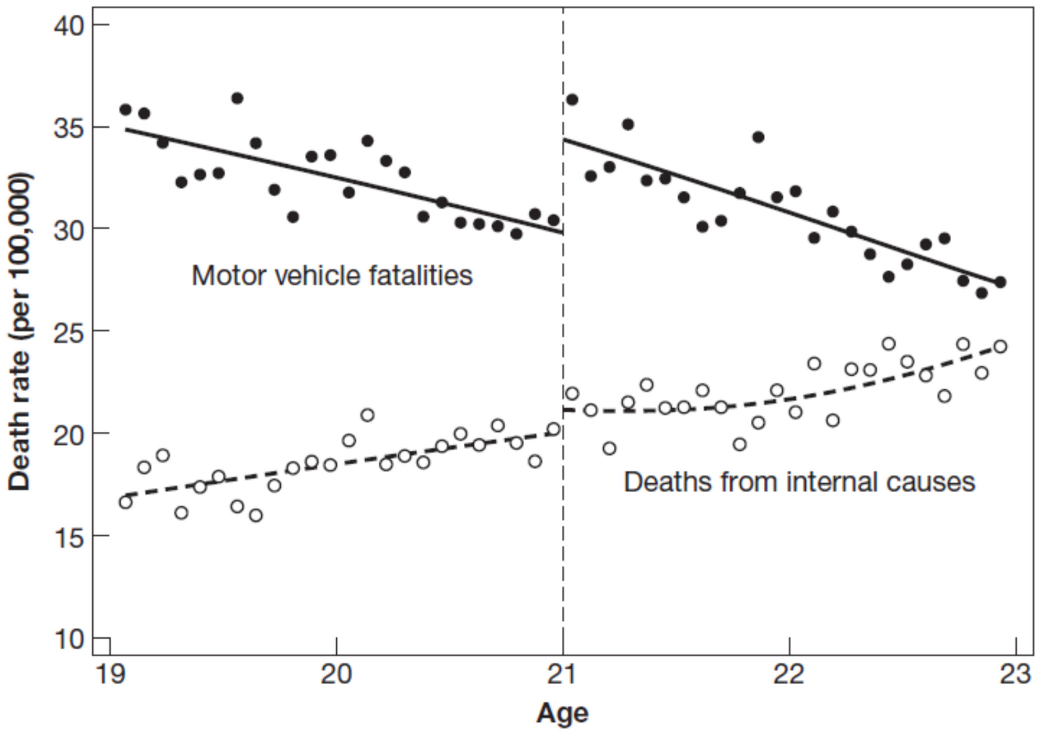
Which means that *other variables* not affected by the treatment should vary smoothly over the threshold

- After all, if some unrelated variable is "jumping" at the threshold, then we shouldn't have much confidence in that threshold being exogenous
  - And this would ruin our assumption.

> (Z)ero effects on outcomes that should be unchanged by treatment raise our confidence in the causal effects we are after

`(-)` Mastering Metrics

RD Diagnostics



What if the threshold doesn't perfectly determine treatment, but the other assumptions hold?

In this case, we would have:

- Something exogenous
- That has an affect on treatment
- But doesn't affect outcome except through treatment

It's an instrument!

Back to our original example, except maybe we don't have perfect awarding of scholarships to SAT>1200

First Stage:

$$SCHOLARSHIP_i = \alpha_0 + \alpha_1 SAT_i + \alpha_2 1(SAT_i > 1200) + \alpha_3 x_i + v_i$$

This is just like our original equation, but with $SCHOLARSHIP_i$ on the LHS.

Second Stage:

$$y_i = \beta_0 + \beta_1 \widehat{SCHOLARSHIP}_i + \beta_2 SAT_i + \beta_3 x_i + u_i$$

β_1 is our treatment of interest.

Note that SAT_i is still in the second stage **but** the threshold dummy $\mathbf{1}(SAT_i > 1200)$ is *not*.

This just follows the IV method, but gets the first-stage exogeneity from the RD specification! The *crossing of the threshold* is the exogenous event.

In practice...

We rarely get a threshold that is perfectly "sharp".

- Age of drinking is pretty sharp
- But many other things don't have a perfect relationship

So fuzzy is more common in the literature

- And remember: when you have an instrument that is almost perfectly correlated with the endogenous variable, IV turns into OLS.
  - If the first stage Z has a perfect prediction of D, then $\hat{D} = D = Z$