

COS 4807 Assignment 2

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1 Question 1

$$((p \wedge q) \rightarrow r) \rightarrow (p \rightarrow (q \vee r)) \quad (1)$$

$$\neg((p \wedge q) \rightarrow r) \vee (p \rightarrow (q \vee r)) \quad (2)$$

$$\neg(\neg(p \wedge q) \vee r) \vee (\neg p \vee (q \vee r)) \quad (3)$$

$$((p \wedge q) \wedge (\neg r)) \vee (\neg p \vee q \vee r) \quad (4)$$

$$((\neg p \vee q \vee r) \vee (p \wedge q)) \wedge ((\neg p \vee q \vee r) \vee \neg r) \quad (5)$$

$$((\neg p \vee q \vee r) \vee (p \wedge q)) \wedge (true) \quad (6)$$

$$((\neg[\vee q \vee r] \vee p) \wedge ((\neg p \vee q \vee r) \vee q)) \quad (7)$$

$$(true) \wedge ((\neg p \vee q \vee r) \vee q) \quad (8)$$

$$((\neg p \vee q \vee r) \vee q) \quad (9)$$

$$\{\{\neg p, q, r\}\} \quad (10)$$

Which in abbreviated clausal form is $\{\bar{p}qr\}$.

2 Question 2

$$\neg((q \rightarrow p) \rightarrow r) \wedge ((p \wedge \neg r) \rightarrow (q \wedge r)) \quad (11)$$

$$\neg(\neg(p \rightarrow p) \vee r) \wedge (\neg(p \wedge \neg r) \vee (q \wedge r)) \quad (12)$$

$$\neg(\neg(\neg q \vee p) \vee r) \wedge ((\neg p \vee r) \vee (q \wedge r)) \quad (13)$$

$$(\neg q \vee p) \wedge (\neg r) \wedge (((\neg p \vee r) \vee q) \wedge ((\neg p \vee r) \vee r)) \quad (14)$$

$$(\neg q \vee p) \wedge (\neg r) \wedge (\neg p \vee q \vee r) \wedge (\neg p \vee r) \quad (15)$$

Which is $\{\{p, \neg q\}, \{\neg r\}, \{\neg p, q, r\}, \{\neg p, r\}\}$ in clausal form

1. $p\bar{q}$
2. \bar{r}
3. $\bar{p}qr$
4. $\bar{p}r$
5. \bar{p} 2 and 4
6. \bar{q} 1 and 5
7. $\bar{p}r$ 3 and 6
8. \bar{p} 2 and 7
9. $\bar{q}r$ 1 and 4
10. \bar{q} 2 and 9

No boxes have been found so this formula must be satisfiable

3 Question 3

Formula A:

$$p \rightarrow q \quad (16)$$

$$\neg p \vee q \quad (17)$$

$\{\bar{p}q\}$

Formula B:

$$q \rightarrow r \quad (18)$$

$$\neg q \vee r \quad (19)$$

$\{\bar{q}r\}$

Negation of the single formula

$$\neg(p \rightarrow (q \wedge r)) \quad (20)$$

$$\neg(\neg p \vee (q \wedge r)) \quad (21)$$

$$p \wedge (\neg q \vee r) \quad (22)$$

$$(p \wedge \neg q) \vee (p \wedge r) \quad (23)$$

$$((p \wedge \neg q) \vee p) \wedge ((p \wedge r) \vee \neg r) \quad (24)$$

$$(p \vee p) \wedge (p \vee \neg q) \wedge (\neg r \vee p) \wedge (\neg r \vee \neg q) \quad (25)$$

Which is $\{p, p\bar{q}, p\bar{r}, \bar{q}\bar{r}\}$

Now for the resolution refutation

1. p
2. $p\bar{q}$
3. $p\bar{r}$
4. $\bar{q}\bar{r}$
5. $\bar{p}q$
6. $\bar{q}r$
7. q 1 and 5
8. $q\bar{r}$ 3 and 5
9. \bar{r} 4 and 7
10. \bar{r} 4 and 8
11. r 6 and 7
12. \square 10 and 11

Because resolution refutation shows that the set of clauses is unsatisfiable we can conclude that $\{p \rightarrow q, q \rightarrow r\} \models p \rightarrow (q \wedge r)$

4 Question 4

For

$$(p \vee q) \rightarrow r$$

(26)

using ordering (p,q,r)

