

COS 4892 Assignment 2

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1 Question 1

We will define

$$Q = ((Q_1 \wedge B) \vee (Q_2 \wedge \neg B)) \quad (1)$$

Now we calculate from Equation 1

$$\begin{aligned} Q \wedge B &= ((Q_1 \wedge B) \vee (Q_2 \wedge \neg B)) \wedge B \\ &= (Q_1 \wedge B \wedge B) \vee (Q_2 \wedge \neg B \wedge B) \\ &= Q_1 \wedge B \end{aligned} \quad (2)$$

and

$$\begin{aligned} Q \wedge B &= ((Q_1 \wedge B) \vee (Q_2 \wedge \neg B)) \wedge B \\ &= (Q_1 \wedge B \wedge \neg B) \vee (Q_2 \wedge \neg B \wedge \neg B) \\ &= Q_2 \wedge \neg B \end{aligned} \quad (3)$$

2 Question 2.1

$$\{?\}x := 3.2z\{wy - 2w^2 < z\} \quad (4)$$

$$\{wy - 2w^2 < z\}x := 3.2z\{wy - 2w^2 < z\} \quad (5)$$

The post condition is not dependant on x . Thus there is nothing to replace by using the assignment Axiom

3 Question 2.2

$$\{?\}x := x - 1; y := y - 1\{z - 1 \leq y < x \leq w\} \quad (6)$$

$$\{z - 1 \leq y - 1 < x - 1 \leq w\}x := x - 1; y := y - 1\{z - 1 \leq y < x \leq w\} \quad (7)$$

$$\{z \leq y < x \leq w + 1\}x := x - 1; y := y - 1\{z - 1 \leq y < x \leq w\} \quad (8)$$

4 Question 2.3

$$\{?\}if even(x) \rightarrow x := x - 1 else odd(x) \rightarrow z := z + yx\{x \geq 0 \wedge z + yx = ab\} \quad (9)$$

Using the retrogressive theorem for the if statement:

Taking the case as of $S1 = even(x) \rightarrow x := x - 1$

$$\{?\}x : x - 1\{x \geq 0 \wedge z + yx = ab\} \quad (10)$$

$$\{x \geq 0 \wedge z + y(x - 1) = ab\}x : x - 1\{x \geq 0 \wedge z + yx = ab\} \quad (11)$$

$$\{x \geq 0 \wedge z + yx - y = ab\}x : x - 1\{x \geq 0 \wedge z + yx = ab\} \quad (12)$$

Taking the case as of $S2 = \text{odd}(x) \rightarrow z := z + yx$

$$\{?\}z : z + yx\{x \geq 0 \wedge z + yx = ab\} \quad (13)$$

$$\{x \geq 0 \wedge (z + yx) + yx = ab\}z : z + yx\{x \geq 0 \wedge z + yx = ab\} \quad (14)$$

$$\{x \geq 0 \wedge z + 2yx = ab\}z : z + yx\{x \geq 0 \wedge z + yx = ab\} \quad (15)$$

Now applying the retrogressive theorem for the if statement

$$\begin{aligned} & \{(x \geq 0 \wedge z + yx - y = ab \wedge \text{even}(x)) \vee (x \geq 0 \wedge z + 2yx = ab \wedge \text{odd}(x))\} \\ & \quad \text{if even}(x) \rightarrow x := x - 1 \text{ else odd}(x) \rightarrow z := z + yx \\ & \quad \{x \geq 0 \wedge z + yx = ab\} \end{aligned} \quad (16)$$

5 Question 2.4

$$\{?\} \text{while } 0 \geq c \geq -2 \text{ do } x := x - 1 \text{ endwhile } \{x = -3\} \quad (17)$$

$$B = 0 \geq c \geq -2 \quad (18)$$

$$S = x := x - 1 \quad (19)$$

Starting with

$$\begin{aligned} & \{Z_0\}S^0\{\neg B\} \\ & \{B \wedge C_0\}x := x - 1\{x = -3\} \\ & \{0 \geq x \geq -2 \wedge x - 1 = -3\}x := x - 1\{x = -3\} \\ & \{0 \geq x \geq -2 \wedge x = -2\}x := x - 1\{x = -3\} \end{aligned} \quad (20)$$

$$\begin{aligned} & \{Z_1\}S\{Z_0\} \\ & \{0 \geq x \geq -2 \wedge x - 1 = -2\}x := x - 1\{0 \geq c \geq -2 \wedge x = -2\} \\ & \{0 \geq x \geq -2 \wedge x = -1\}x := x - 1\{0 \geq c \geq -2 \wedge x = -2\} \end{aligned} \quad (21)$$

$$\begin{aligned} & \{Z_2\}S\{Z_1\} \\ & \{0 \geq x \geq -2 \wedge x - 1 = -1\}x := x - 1\{1 \geq x \geq -1 \wedge x = -1\} \\ & \{0 \geq x \geq -2 \wedge x = 0\}x := x - 1\{1 \geq x \geq -1 \wedge x = -1\} \end{aligned} \quad (22)$$

$$\begin{aligned} & \{Z_3\}S\{Z_2\} \\ & \{0 \geq x \geq -2 \wedge x - 1 = 0\}x := x - 1\{2 \geq x \geq 0 \wedge x = 0\} \\ & \{0 \geq x \geq -2 \wedge x = 1\}x := x - 1\{2 \geq x \geq 0 \wedge x = 0\} \end{aligned} \quad (23)$$

Z_3 is false thus the preconditions of S are

$$\begin{aligned} & \{Z_0 \vee Z_1 \vee Z_2\} \\ & \{(0 \geq x \geq -2 \wedge x = -2) \vee (0 \geq x \geq -2 \wedge x = -1) \vee (0 \geq x \geq -2 \wedge x = 0)\} \end{aligned} \quad (24)$$

6 Question 3

A Hoare triple $\{P\}S\{Q\}$ is valid if before execution statement S , P is valid and directly after executing S , Q is valid. Therefore for each possible value of P , statement S has to be executed and determined whether the result causes Q to be valid or not.

7 Question 4

In this example P can have value of $x = 3$, $x = -3$, $x = 4$ or $x = -4$. In the first case when $x = 3$, after applying the statement we get $y = 3$. For the case of $x = -3$ then $y = 3$. When $x = 4$ the results in $y = 4$ and for $x = -4$, after applying the if statement, $y = 4$. In each of the 4 cases the value of y is within the range defined by the postcondition. Thus the statement is valid.

8 Question 5