COS 4807 Assignment 3

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July 27, 2019

1 Question 1i

Let $\mathscr{I}$ be an arbitrary interpretation such that $v_{\mathscr{I}}(\forall xp(x) \vee \exists xq(x)) = F$. From the truth value of disjunction $v_{\mathscr{I}}(\forall xp(x) = F$ and $v_{\mathscr{I}}\exists xq(x)) = F$. From this using Theorem 7.22 we for all assignments $v_{\sigma\mathscr{I}}(p(x)) = F$ and for some assignments $v_{\sigma\mathscr{I}}(q(x)) = F$. Then by the truth value of disjunction $v_{\sigma\mathscr{I}}(p(x) \vee q(x)) = F$. Then by using Theorem 7.22 $v_I(\forall(p(x) \vee q(x))) = F$. Then if $v_I(\forall xp(x) \vee \exists xq(x)) = F$ then $v_{\mathscr{I}}(\forall x(p(x) \vee q(x) \rightarrow (\forall xp(x) \vee \exists xq(x)) = T))$ by the truth value of implication. And since $\mathscr{I}$ is an arbitrary interpretation, the formula is valid

2 Question 1ii

Let $\mathscr{I}$ ba an arbitrary interpretation such that $v_{\mathscr{I}}(\forall x\neg p(x) \vee \forall x\neg q(x)) = F$. Then from the definition of disjunction $v_{\mathscr{I}}(\forall x\neg p(x)) = F$ and $v_{\mathscr{I}}(\forall x\neg q(x)) = F$. Using the theorem from question 3ii we get for all assignments $v_{\sigma\mathscr{I}}(\neg p(x)) = F$ and for all assignmets $v_{\sigma\mathscr{I}}(\neg q(x)) = F$. Then by the truth values of negation, $v_{\sigma\mathscr{I}}p(x) = T$ and $v_{\sigma\mathscr{I}}q(x) = T$. Then by theorem 7.22 and the definition of conjunction $v_{\mathscr{I}}(\exists xp(x) \wedge q(x)) = T$. Now we have shown that $v_{\mathscr{I}}(\forall x\neg p(x) \vee \forall x\neg q(x)) = F$ and $v_{\mathscr{I}}\exists(xp(x) \wedge q(x)) = T$. Combining these into the original formula we get $v_{\mathscr{I}}(\exists x(p(x) \wedge q(x)) \wedge (\forall x\neg p(x) \vee \forall x\neg q(x))) = F$ by the definition of conjunction. And since $\mathscr{I}$ is an arbitrary interpretation, the formula is unsatifiable.