COS 4807 Assignment 4

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1 Question 1i

$$\exists x p(x) \rightarrow \exists y (p(y) \wedge \exists x q(x, y)) \tag{1}$$

$$\exists x p(x) \rightarrow \exists y (p(y) \wedge \exists z q(z, y)) \tag{2}$$

Using $A \rightarrow B \equiv \neg A \vee B$

$$\neg \exists x p(x) \rightarrow \exists y (p(y) \wedge \exists z q(z, y)) \tag{3}$$

$$\forall x \neg p(x) \rightarrow \exists y (p(y) \wedge \exists z q(z, y)) \tag{4}$$

$$\forall x (\neg p(x) \rightarrow \exists y (p(y) \wedge \exists z q(z, y))) \tag{5}$$

$$\forall x \exists y (\neg p(x) \rightarrow (p(y) \wedge \exists z q(z, y))) \tag{6}$$

$$\forall x \exists y \exists z (\neg p(x) \rightarrow (p(y) \wedge q(z, y))) \tag{7}$$

Using distributivity of $\vee$ and $\wedge$

$$\forall x \exists y \exists z ((\neg p(x) \vee p(y)) \wedge (\neg (p(x) \vee q(z, y)))) \tag{8}$$

$$\forall x \exists y ((\neg p(x) \vee p(y)) \wedge (\neg (p(x) \vee q(f(x), y)))) \tag{9}$$

$$\forall x ((\neg p(x) \vee p(g(x))) \wedge (\neg (p(x) \vee q(f(x), g(x))))) \tag{10}$$

$$\{\{\neg p(x), p(g(x))\}, \{\neg p(x), q(f(x), g(x))\}\} \tag{11}$$

2 Question 1ii

$$\forall x ((\forall y q(x, y) \vee \forall z q(z, x)) \rightarrow g(x, x)) \tag{12}$$

Using $A \rightarrow B \equiv \neg A \vee B$

$$\forall x (\neg (\forall y q(x, y) \vee \forall z q(z, x)) \vee g(x, x)) \tag{13}$$

$$\forall x ((\neg \forall y q(x, y) \wedge \neg \forall z q(z, x)) \vee g(x, x)) \tag{14}$$

$$\forall x ((\exists y \neg q(x, y) \wedge \exists z \neg q(z, x)) \vee g(x, x)) \tag{15}$$

$$\forall x \exists y ((\neg q(x, y) \wedge \exists z \neg q(z, x)) \vee g(x, x)) \tag{16}$$

$$\forall x \exists y \exists z ((\neg q(x, y) \wedge \neg q(z, x)) \vee g(x, x)) \tag{17}$$

Using distributivity of $\vee$ and $\wedge$

$$\forall x \exists y \exists z((q(x,x) \vee \neg q(x,y)) \wedge (q(x,x) \vee \neg q(z,x))) \tag{18}$$

$$\forall x \exists y((q(x,x) \vee \neg q(x,y)) \wedge (q(x,x) \vee \neg q(f(x),x))) \tag{19}$$

$$\forall x((q(x,x) \vee \neg q(x,g(x))) \wedge (q(x,x) \vee \neg q(f(x),x))) \tag{20}$$

$$\{\{q(x,x), \neg q(x,g(x))\}, \{q(x,x), q(f(x),x)\}\} \tag{21}$$

3 Question 1iii

$$\forall x \forall y(\exists z p(z) \leftrightarrow q(x,y)) \tag{22}$$

Using $A \leftrightarrow B \equiv (A \rightarrow B) \wedge (B \rightarrow A)$

$$\forall x \forall y((\exists z p(z) \rightarrow q(x,y)) \wedge (q(x,y) \rightarrow \exists z p(z))) \tag{23}$$

$$\forall x \forall y((\exists z p(z) \rightarrow q(x,y)) \wedge (q(x,y) \rightarrow \exists k p(k))) \tag{24}$$

Using $A \rightarrow B \equiv \neg A \vee B$

$$\forall x \forall y((\neg \exists z p(z) \vee q(x,y)) \wedge (\neg q(x,y) \vee \exists k p(k))) \tag{25}$$

$$\forall x \forall y((\forall z \neg p(z) \vee q(x,y)) \wedge (\neg q(x,y) \vee \exists k p(k))) \tag{26}$$

$$\forall x \forall y \forall z \exists k((\neg p(z) \vee q(x,y)) \wedge (\neg q(x,y) \vee p(k))) \tag{27}$$

$$\forall x \forall y \forall z((\neg p(z) \vee q(x,y)) \wedge (\neg q(x,y) \vee p(f(x,y,z)))) \tag{28}$$

$$\{\{\neg p(z), q(x,y)\}, \{\neg q(x,y), p(f(x,y,z)\}\} \tag{29}$$

4 Question 2i

Given the formula for Question 1i in clausal form is

$$\{\{\neg p(x), p(g(x))\}, \{\neg p(x), q(f(x), g(x))\}\} \tag{30}$$

The Herbrand universe is:

$$\{a, f(a), g(a), f(g(a)), g(f(a)), f(f(a)), g(g(a)), f(g(f(a))), f(g(g(a))), ...\} \tag{31}$$

The Herbrand base is:

$$\{p(a), q(f(a), g(a)), p(f(a)), q(f(f(a)), g(f(a))), p(f(f(a))), q(f(f(g((a))), g(f(a))), ...\} \tag{32}$$

Then

$$\{v_{I1}(p(a)) = F\} \tag{33}$$

and

$$\{v_{I2}(p(g(a))) = T, v_{I2}(q(f(a), g(a))) = T, v_{I2}(p(g(f(a)))) = T, v_{I2}(q(f(a), g(a))) = T, v_{I2}, ...\} \tag{34}$$

are Herbrand models

5 Question 2ii

Given the formula for Question 1ii in clausal form is

$$\{\{q(x,x), \neg q(x,g(x))\}, \{q(x,x), q(f(x),x)\}\} \tag{35}$$

The Herbrand universe is:

$$\{a, f(a), g(a), f(g(a)), g(f(a)), f(f(a)), g(g(a)), f(g(f(a))), f(g(g(a))), ...\} \tag{36}$$

The Herbrand base is:

$$\{q(a,a,), q(f(a),a), q(g(a),a), q(a,f(a)), q(a,g(a)), q(f(f(a)),f(a)), ...\} \tag{37}$$

2 Herbrand models are:

$$\{v_{I1}(q(a,a) = T)\} \tag{38}$$

and

$$\{v_{I2}(q,a,g(a)) = F, v_{I2}(q(f(a),a))) = T\} \tag{39}$$

6 Question 2iii

Given the formula for Question 1iii in clausal form is

$$\{\{\neg p(z), q(x,y)\}, \{\neg q(x,y), p(f(x,y,z)\}\} \tag{40}$$

$$\{a, f(a,a,a), f(f(a,a,a),a,a), f(a,f(a,a,a),a), f(a,a,f(a,a,a)), f(f(f(a,a,a),a,a),a,a), ...\} \tag{41}$$

$$\{\} \tag{42}$$

$$\{\} \tag{43}$$