Tutorial Letter 101/0/2019

Formal Logic

COS4807

Year module

Department of Computer Science
School of Computing

CONTENTS
General module information and assignments

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CONTENTS

1 INTRODUCTION

Welcome to COS4807. This module provides an introduction to formal logic. The background you need is some discrete mathematics, covered in a module like COS1501. In particular, we assume that you are familiar with set theory and mathematical proof techniques. The study of formal logic requires a high level of mathematical rigour, so if you coped well with mathematics at school or at an undergraduate level, then you should have an aptitude for formal logic too. If you have never encountered formal logic before, you should find the prescribed book for our second-year logic module, COS2661, helpful. (See the book by Barwise and Etchemendy, listed as additional reading in Section 4.3 below.)

2 ABOUT THIS MODULE

2.1 Purpose

Logic is about reasoning, and in particular about the algorithms by which one would reason. It is used to formalise the semantics of programming languages and the specification of programs, to verify the correctness of programs, and in constraint-based reasoning. If you want to devise algorithms that will allow you to automate reasoning, then logic is where you need to start. A background in logic is essential for an understanding of reasoning and inference in artificial intelligence. It also forms the basis of the Web Ontology Language (OWL) used for specifying ontologies for the semantic web.

An exciting application of logic in computer science is logic programming. The best-known logic programming language, namely Prolog, is based on a resolution theorem-proving algorithm. Assignment 4 is devoted to this topic. You won't actually need to use Prolog for this course, but after completing Assignment 4 you should find it relatively easy to learn to use it. If you want more practical Prolog programming experience, you should register for the logic-based reasoning module COS4851.

2.2 Syllabus

The following aspects of propositional and first-order logic logic are covered in COS4807:

Propositional logic: formulas, models, tableaux, deductive systems, clausal form, resolution, binary decision diagrams, SAT solvers

First-order logic: formulas, models, tableaux, deductive systems, clausal form, resolution

2.3 Outcomes

To cover the abovementioned syllabus, you need to master the following techniques of formal logic:

Propositional logic:

- Use truth tables to show the validity or satisfiability of formulas.
- Give semantic arguments to prove the validity or unsatisfiability of formulas.
- Use semantic tableaux to show the satisfiability of formulas.
- Use the Gentzen system $\mathscr{G}$ to prove the validity of formulas.
- Write formulas in clausal form to be able to apply resolution refutation.

- Draw binary decision diagrams to represent formulas, and use the **Apply** and **Reduce** algorithms to combine and simplify them.
- Encode a satisfiability problem as a set of clauses, and use the DPLL algorithm to find a solution.

First-order logic:

- Use semantic tableaux to show the satisfiability of formulas.
- Give semantic arguments to prove the validity or unsatisfiability of formulas.
- Use the Gentzen system $\mathscr{G}$ to prove the validity of formulas.
- Write formulas in clausal form to be able to apply resolution refutation.
- Use SLD-resolution to evaluate queries in logic programs.

3 CONTACT DETAILS

3.1 Lecturer

The lecturer responsible for this module will be specified in the COSALL 301 tutorial letter for 2019.

All queries that are not of a purely administrative nature but are about the content of this module should be directed to your lecturer.

Tutorial Letter 301 will provide an email address and a telephone number to contact the lecturer.

3.2 School

If you are unable to get hold of the lecturer on the phone, you can always leave a message with the school secretary. The secretary's number will also be provided in Tutorial Letter 301.

3.3 University

If you need to contact the University about matters not related to the content of this module, please consult *Study@Unisa* available for download from the Unisa website `http://www.unisa.ac.za/`. This brochure contains information on how to contact the University (e.g. to whom you can write for different queries, important telephone and fax numbers, addresses and details of the times certain facilities are open).

Always have your student number at hand when you contact the University.

4 MODULE-RELATED RESOURCES

COS4807 is presented online. This means that you will not receive tutorial matter for this module in printed form. It will all be available in electronic format on *myUnisa* (see Section 5.1 below).

4.1 Tutorial letters

For COS4807, all communication about teaching, learning and assessment is in the form of tutorial letters which will be made available on *myUnisa* during the year. Tutorial Letter 101 is available in the Official Study Material folder, and all the other tutorial letters will be made available in Additional Resources.

Tutorial Letter 101 (this tutorial letter) contains important information about the syllabus, the prescribed book and other resources for this module. Please study this information carefully and make sure that you obtain the prescribed book as soon as possible. This tutorial letter also specifies the assignments as well as instructions for their submission. We urge you to read this information carefully and to keep it at hand when working through the study material, preparing the assignments, preparing for the examination and addressing questions to your lecturers.

Tutorial Letter 102 contains additional notes on the prescribed book, indicating which parts are important and which parts are for interest only. It also contains a number of worked examples similar to those you can expect in the assignments and the exams.

Tutorial Letter 103 will probably contain examination information, and will be available later in the year.

Solutions to assignments (Tutorial Letters 201, 202, 203 and 204) will be made available about two weeks after the due dates of the corresponding assignments.

4.2 Prescribed book

Ben-Ari, M. *Mathematical Logic for Computer Science*, 3rd edition. Springer-Verlag, 2012.

You need your own copy of this book. Prescribed books can be obtained from Unisa's official booksellers. Consult the list of official booksellers and their addresses listed in the brochure *Study@Unisa*. If you have difficulty in locating this book at these booksellers, please contact the Prescribed Book Section at Tel: 012 429 4152 or email: `vospresc@unisa.ac.za`

The following chapters of the prescribed book cover the syllabus specified in Section 2.2 above:

Propositional logic: Chapters 1, 2, 3, 4, 5 and 6. You can omit Sections 3.3 to 3.9 and 6.6.
First-order logic: Chapters 7, 8, 9, 10 and 11. You can omit Sections 8.2 to 8.5 and 11.4.

Please note that the chapter and section numbers of the prescribed book used in this (and subsequent) tutorial letters refer to the 3rd edition, and differ from previous editions.

4.3 Recommended books

There are **no** recommended books for this module, as it should not be necessary to consult any additional literature in order to master the content of COS4807.

However, if you are interested in reading more on logic, or require more background material, here is a selection of books available from the Unisa library:

Barwise, J & Etchemendy, J. *The Language of First-Order Logic*, 3rd ed. CSLI, 1992.
Burke, E & Foxley, E. *Logic and its Applications.* Prentice Hall, 1996.
Fitting, M. *First-Order Logic and Automated Theorem Proving.* Springer-Verlag, 1996.
Ginsberg, M. *Essentials of Artificial Intelligence.* Morgan Kaufmann, 1993.
Nerode, A & Shore, RA. *Logic for Applications.* Springer-Verlag, 1997.
Russell, S & Norvig, P. *Artificial Intelligence: A Modern Approach*, 3^{rd} ed. Prentice Hall, 2010.
Shinghal, R. *Formal Concepts in Artificial Intelligence.* Chapman & Hall, 1992.

5 STUDENT SUPPORT SERVICES

For information on the various student support services available at Unisa (e.g. student counselling, language support), please consult *Study@Unisa* (see Section 3.3 above).

5.1 *myUnisa*

The *myUnisa* learning management system is Unisa's online campus that will help you to communicate with your lecturers, with other students and with the administrative departments of Unisa by means of a computer and the internet. In particular, you can download tutorial matter and submit your assignments on *myUnisa*. The web address for *myUnisa* is `http://my.unisa.ac.za/`. Please consult *Study@Unisa* for more information on *myUnisa*.

5.2 Discussion forums

myUnisa provides discussion forums for all modules, and we encourage you to make use of the one dedicated to COS4807. The forum provides an excellent means of discussing the work with your fellow students.

5.3 *myLife*

As explained in *Study@Unisa*, every student at Unisa is given a *myLife* email account. We encourage you to make use of your *myLife* account, or at least to forward your *myLife* email to your personal email address (as explained in *Study@Unisa*). From time to time, we send email to the *myLife* accounts of all students registered for CO4807. Also, whenever COS4807 tutorial matter is uploaded to *myUnisa*, an email is automatically sent to all students. If you are not monitoring your *myLife* email, you could miss some important announcements.

6 STUDY PLAN

The learning outcomes of COS4807 specified in Section 2.3 are covered by four assignments.

We recommend that you work through the material for each assignment (in the relevant chapters of the prescribed book and the corresponding sections of Tutorial Letter 102) at least two weeks before the due date of each assignment, to give you enough time to complete and submit the assignment. (See Table 1 below for the assignment due dates.)

After submitting the last assignment, you should spend time revising your work in preparation for the exam. A special tutorial letter about preparation for the exam will be made available at this time.

7 ASSESSMENT

Assessment for COS4807 is on the basis of assignments and an examination. The assignments count towards a year mark, which together with the exam mark contribute to a final mark. (Section 8 contains the assignment questions.)

7.1 Assignment numbers and due dates

There are four assignments that count equally towards your year mark (see Section 7.3):

General number	Prescribed book chapters	Due date	Unique number
1	1, 2 and 3	29 April 2019	855661
2	4, 5 and 6	14 June 2019	899998
3	7 and 8	12 August 2019	749419
4	9, 10 and 11	30 September 2019	885128

Table 1: Assignments

Each assignment has a general number, namely 1, 2, 3 or 4. Please submit each assignment with the correct general number, even if it is not the first, second, third or fourth assignment that you submit.

Each assignment also has a unique number, as specified in Table 1. Please also ensure that you submit your assignments with the correct unique numbers.

Table 1 also gives the due dates of the assignments. There is an automatic extension of two weeks for every assignment. We can give no guarantee that assignments received after this extension will be marked or contribute to your year mark. See Section 7.3 below.

You must submit at least one assignment by its due date to be admitted to the exam. See Section 7.4 below.

7.2 Submission of assignments

You may only submit assignments in PDF format via *myUnisa*. **Please note that assignments should not be submitted by fax or e-mail.** We will place an MS Word document with the necessary logical symbols in Additional Resources for COS4807 on *myUnisa*, so that you can copy them when preparing your assignments. We will also make an application for converting a document to PDF available in Additional Resources. For detailed information and requirements as far as assignments are concerned, refer to the *Study@Unisa* brochure.

Please note that it is **your responsibilty** to check that your assignments are registered correctly on Unisa's assignment database. You can do this on *myUnisa*. Sometimes an assignment is cancelled for some reason (e.g. wrong format), in which case you should be informed by the system in the form of an email sent to your *myLife* account. Even if you do not receive such an email, we recommend that you visit *myUnisa* about two days after you submit an assignment to check whether it has been cancelled.

7.3 Year mark, exam mark and final mark

The four assignments for COS4807 count equally towards the year mark. In other words, your year mark will be calculated as follows:

$$\text{Year mark} = (\text{Assignment } 1 + \text{Assignment } 2 + \text{Assignment } 3 + \text{Assignment } 4)/4$$

The year mark contributes 20% and the exam mark 80% to the final mark. In other words, your final mark for the module as a whole will be calculated according to the following formula:

$$\text{Final mark} = 0.2 \times (\text{year mark}) + 0.8 \times (\text{exam mark})$$

7.4 Examination admission

To be admitted to the examination, you must submit at least one assignment by 29 September 2019.

Please note that lecturers are not responsible for determining examination admission, and **all** enquiries about examination admission should be directed to the Examination Department at Unisa.

7.5 Examination information

There is a 3 hour, written examination for COS4807, which will be held in January/February 2020. Unisa's Examination Department will inform you of the time, date and venue.

To help you in your preparation for the examination, a tutorial letter will be made available at the end of the year, explaining the format of the examination paper, and setting out clearly what material you will need to study for examination purposes.

8 ASSIGNMENTS

ASSIGNMENT 1
Due Date: 29 April 2019

Question 1 ***(20)***

Use truth tables to prove that the following propositional formulas have the corresponding properties. In each case, also explain (in words) why the formula has the relevant property.

(i) $(((p \vee q) \vee r) \wedge (\neg p \wedge \neg q)) \rightarrow r$ is valid
(ii) $(p \rightarrow (q \rightarrow r)) \leftrightarrow ((p \wedge q) \wedge \neg r)$ is unsatisfiable
(iii) $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow r))$ is satisfiable and is falsifiable

Question 2 ***(20)***

Prove that the formulas given in Question 1 (i) and (ii) above have the corresponding properties, by means of semantic arguments in terms of the truth values of formulas.

Question 3 ***(20)***

Section 2.7.2 of the prescribed book provides a proof of the following theorem:

Theorem: *If A is an unsatisfiable formula, then every tableau for A closes.*

Give an outline of the proof of this theorem (in less than 250 words).

Question 4 ***(30)***

Prove that the propositional formulas given in Question 1 (i), (ii) and (iii) above have the corresponding properties, by means of semantic tableaux.

Question 5 ***(10)***

Prove that the formula given in Question 1 (i) above is a theorem of the Gentzen system $\mathscr{G}$.

ASSIGNMENT 2 Due Date: 14 June 2019

Question 1 ***(10)***

Convert the formula $((p \land q) \to r) \to (p \to (q \lor r))$ to CNF. Show your working. Also remove any trivial clauses. Finally, write your answer in (abbreviated) clausal form.

Question 2 ***(15)***

Prove by resolution refutation that the formula $\neg((q \to p) \to r) \land ((p \land \neg r) \to (q \land r))$ is satisfiable.

Hint: First convert the formula to clausal form.

Beware! You may not resolve two clauses that have two or more pairs of complementary literals. To conclude that a set of clauses is satisfiable, you have to show that there are no more pairs of clauses that can be resolved.

Question 3 ***(15)***

Show that $p \to (q \land r)$ is a logical consequence of the set of formulas $\{p \to q, q \to r\}$ by means of resolution refutation.

Hint: Convert each of the formulas in the set as well as the negation of the single formula to clausal form. Then perform resolution refutation on all these clauses.

Question 4 ***(10)***

Draw OBDDs in the form of binary trees for the formula $(p \lor q) \to r$, one using the ordering (p, q, r) and the other using the ordering (r, q, p), and then use the **Reduce** algorithm to convert them to reduced OBDDs.

Question 5 ***(20)***

Use the **Apply** algorithm to determine an OBDD for the formula $(p \leftrightarrow q) \lor (r \to \neg q)$, using the ordering (p, q, r). Show your working.

Hint: First use the **Apply** algorithm to build OBBDs for $p \leftrightarrow q$ and $r \to \neg q$ respectively, and then use it again to combine them to get the final answer.

Question 6 ***(30)***

Use the DPLL algorithm on the clauses for the Simple Sudoku puzzle described in the **Examples** at the end of the additional notes on Chapter 6 in Tut Letter 102, to generate all the solutions to the following puzzle:

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Give the solutions generated by the algorithm.

ASSIGNMENT 3
Due Date: 12 August 2019

Question 1 ***(20)***

Provide semantic arguments (in terms of the definitions of truth, validity and satisfiability) to prove that the following first-order formulas have the corresponding properties:
(i) $\forall x(p(x) \vee q(x)) \rightarrow (\forall x p(x) \vee \exists x q(x))$ is valid.
(ii) $\exists x(p(x) \wedge q(x)) \wedge (\forall x \neg p(x) \vee \forall x \neg q(x))$ is unsatisfiable.

Hint: Use Question 3(ii) below.

Question 2 ***(10)***

(i) Write down a first-order formula which expresses the following property of number theory: A number x is not prime if there is another number y greater than 1 and less than x such that x is divisible by y. Your formula should be satisfied by the interpretation $\{\mathscr{N}, \{prime, greater\_than, divisible\_by\}, \{1\}\}$.
(ii) Give another interpretation of your formula for part (i), also with domain $\mathscr{N}$, which falsifies it.

Question 3 ***(20)***

(i) Prove the following (simplification of the second part of Theorem 7.22):
Let $A(x)$ be a non-closed formula and let $\mathscr{I}_A$ be an interpretation. Then:
- $v_{\sigma_{\mathscr{I}_A}}(A(x)) = T$ *for all assignments* $\sigma_{\mathscr{I}_A}$ *iff* $v_{\mathscr{I}_A}(\forall x A(x)) = T$.

(ii) Provide an argument why the following corollary of the above statement holds:
Let $A(x)$ be a non-closed formula and let $\mathscr{I}_A$ be an interpretation. Then:
- $v_{\sigma_{\mathscr{I}_A}}(A(x)) = F$ *for some assignment* $\sigma_{\mathscr{I}_A}$ *iff* $v_{\mathscr{I}_A}(\forall x A(x)) = F$.

Question 4 ***(20)***

Prove that the formulas given in Question 1 (i) and (ii) above have the corresponding properties by means of semantic tableaux.

Question 5 ***(20)***

Prove that the formula $(\forall x(p(x) \vee q(x)) \rightarrow (\forall x p(x) \vee \forall x q(x))$ is (i) satisfiable and (ii) falsifiable by means of semantic tableaux.

Question 6 ***(10)***

Prove that the formula given in Question 1 (i) above is a theorem in the Gentzen system $\mathscr{G}$. Annotate the steps of your proof with the rule used and the steps it was applied to.

ASSIGNMENT 4 Due Date: 30 September 2019

Question 1 ***(30)***

Convert the following first-order formulas to clausal form:
(i) $\exists xp(x) \rightarrow \exists y(p(y) \wedge \exists xq(x, y))$
(ii) $\forall x((\forall yq(x, y) \vee \forall zq(z, x)) \rightarrow q(x, x))$
(iii) $\forall x \forall y(\exists zp(z) \leftrightarrow q(x, y))$
Use the commutativity of $\wedge$ and $\vee$ (just before extracting quantifiers) to move existential quantifiers as far forwards as possible, so as to minimise the number and arity of skolem functions in the final clausal form.

Question 2 ***(20)***

Give the Herbrand universe, the Herbrand base and two Herbrand models for the answers to Question 1 (i), (ii) and (iii) above.

Question 3 ***(10)***

Do Exercise 10.4 on page 202 of the prescribed book. Give substitutions that will unify the pairs of atoms, or explain why they are not unifiable.

Question 4 ***(20)***

(i) $\forall x(p(x) \rightarrow q(x)) \rightarrow (\exists xp(x) \rightarrow \exists xq(x))$ is valid.
(ii) $\exists xq(x, x)$ is a consequence of the formula $\exists x(p(x) \wedge \forall y(p(y) \rightarrow q(x, y)))$.
Annotate each resolvent with the substitution used and the numbers of parent clauses.

Question 5 ***(20)***

Consider the following logic program intended to express the family relations of $joe, john, jack$ and $jane$:

1. $aunt(x, y) \leftarrow parent(z, y), sister(x, z)$
2. $sibling(x, y) \leftarrow parent(z, x), parent(z, y)$
3. $sister(x, y) \leftarrow sibling(x, y), female(x)$
4. $parent(joe, jack)$
5. $parent(joe, jane)$
6. $parent(jack, john)$
7. $female(jane)$

Draw an SLD-tree for each attempt at SLD-refutation resulting from the queries (i) $\leftarrow aunt(x, john)$ and (ii) $\leftarrow sibling(jane, y)$. Use the selection rule which resolves on the leftmost literal.