COS 4807 Assignment 1

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1 Question 1.1

p	q	r	$\neg p$	$\neg q$	$p \vee q$	$(p \vee q) \vee r$	$\neg p \wedge \neg q$	$(((p \vee q) \vee r) \wedge (\neg p \wedge \neg q))$	$(((p \vee q) \vee r) \wedge (\neg p \wedge \neg q)) \rightarrow r$
0	0	0	1	1	0	0	1	0	1
0	0	1	1	1	0	1	1	1	1
0	1	0	1	0	1	1	0	0	1
0	1	1	1	0	1	1	0	0	1
1	0	0	0	1	1	1	0	0	1
1	0	1	0	1	1	1	0	0	1
1	1	0	0	0	1	1	0	0	1
1	1	1	0	0	1	1	0	0	1

2 Question1.2

p	q	r
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

3 Question 4.1

For formula

$$(((p \vee q) \vee r) \wedge (\neg p \wedge \neg q)) \rightarrow r \tag{1}$$

Applying the β formula for implication gives

$$\neg(((p \vee q) \vee r) \wedge (\neg p \wedge \neg q)) \tag{2}$$

and

$$r \tag{3}$$

Where Formula 3 is satisfyable.
Continueing for Formula 2 by applying the β formula for $\neg(B_1 \wedge B_2)$ we get

$$\neg((p \vee q) \vee r) \tag{4}$$

and

$$\neg(\neg p \wedge \neg q) \tag{5}$$

Continueing for Formula 4 and applying α formula $\neg(A_1 \vee A_2)$ we get

$$\neg(p \vee q), \neg r \tag{6}$$

Then applying formula $\neg(A_1 \vee A_2)$ again we get

$$\neg p, \neg q, \neg r \tag{7}$$

Which is satisfyable.
Going back to Formula 5 which can by simplified to

$$p \vee q \tag{8}$$

Then applying β formula $B_1 \vee B_2$ we get formulas

$$p \tag{9}$$

and

$$q \tag{10}$$

Both of which are satifyable. Now looking at all the leaf nodes namely Formalas 3, 7, 9 and 10 we can see that they are all satisfyable. Thus Formula 1 is valid.

4 Question 4.2

$$(p \rightarrow (q \rightarrow r)) \leftrightarrow ((p \wedge q) \wedge \neg r) \tag{11}$$

By substituting for double implication operator:

$$(p \rightarrow (q \rightarrow r)) \rightarrow ((p \wedge q) \wedge \neg r), ((p \wedge q) \wedge \neg r) \rightarrow (p \rightarrow (q \rightarrow r)) \tag{12}$$

Substituting for the implication operator in the first term gives:

$$\neg(p \rightarrow (q \rightarrow r)), ((p \wedge q) \wedge \neg r) \rightarrow (p \rightarrow (q \rightarrow r)) \tag{13}$$

and

$$((p \wedge q) \wedge \neg r), ((p \wedge q) \wedge \neg r) \rightarrow (p \rightarrow (q \rightarrow r)) \tag{14}$$

Equation 13 becomes

$$p, \neg(q \rightarrow r)), ((p \wedge q) \wedge \neg r) \rightarrow (p \rightarrow (q \rightarrow r)) \tag{15}$$

then

$$p, q, \neg r, ((p \wedge q) \wedge \neg r) \rightarrow (p \rightarrow (q \rightarrow r)) \tag{16}$$

Substituting for the implication in the above formula gives:

$$p, q, \neg r, \neg((p \wedge q) \wedge \neg r) \tag{17}$$

and

$$p, q, \neg r, (p \rightarrow (q \rightarrow r)) \tag{18}$$

Formula 17 becomes

$$p, q, \neg r, \neg(p \wedge q) \tag{19}$$

and

$$p, q, \neg r, r \tag{20}$$

We can see that Formula 20 is unsatifyable. Then Formula 19 becomes

$$p, q, \neg r, \neg p \tag{21}$$

and

$$p, q, \neg r, \neg q \tag{22}$$

Both of which are unsatifyable. Now having done all the leaves under Formula 17, we continue with the leaves under Equation 18.

Equation 18 becomes

$$p, q, \neg r, \neg p \tag{23}$$

which is unsatifyable and

$$p, q, \neg r, q \to r \tag{24}$$

which becomes

$$p, q, \neg r, \neg q, r \tag{25}$$

Having finished all the leaves under Formula 13 we continue with Formula 14 which becomes

$$p, q, \neg r, ((p \wedge q) \wedge \neg r) \to (p \to (q \to r)) \tag{26}$$

But Formula 26 is the same as Formula 16. Formula 16 will have the same leaf nodes as Formula 26.

Thus we have all the leaf nodes namely Formulas 20, 21, 22, 23 and 25. All of these Formulas are unsatisfyable, therefore the original Formula 11 is unsatisfyable.