COS 4892 Assignment 2

Adriaan Louw (53031377)

July 7, 2019

1 Question 1

We will define

$$Q = ((Q_1 \wedge B) \vee (Q_2 \wedge \neg B)) \tag{1}$$

Now we calculate from Equation 1

$$\begin{aligned} Q \wedge B &= ((Q_1 \wedge B) \vee (Q_2 \wedge B)) \wedge B \\ &= (Q_1 \wedge B \wedge B) \vee (Q_2 \wedge \neg B \wedge B) \\ &= Q_1 \wedge B \end{aligned} \tag{2}$$

and

$$\begin{aligned} Q \wedge B &= ((Q_1 \wedge B) \vee (Q_2 \wedge B)) \wedge B \\ &= (Q_1 \wedge B \wedge \neg B) \vee (Q_2 \wedge \neg B \wedge \neg B) \\ &= Q_2 \wedge \neg B \end{aligned} \tag{3}$$

2 Question 2.1

$$\{?\}x := 3.2z\{wy - 2w^2 < z\} \tag{4}$$

$$\{wy - 2w^2 < z\}x := 3.2z\{wy - 2w^2 < z\} \tag{5}$$

The post condition is not dependant on x. Thus there is notihing to replace by using the assignment Axiom

3 Question 2.2

$$\{?\}x := x - 1; y := y - 1\{z - 1 \leq y < x \leq w\} \tag{6}$$

$$\{z - 1 \leq y - 1 < x - 1 \leq w\}x := x - 1; y := y - 1\{z - 1 \leq y < x \leq w\} \tag{7}$$

$$\{z \leq y < x \leq w + 1\}x := x - 1; y := y - 1\{z - 1 \leq y < x \leq w\} \tag{8}$$

4 Question 2.3

$$\{?\}if even(x) \rightarrow x := x - 1 else odd(x) \rightarrow z := z + yx\{x \geq 0 \wedge z + yx = ab\} \tag{9}$$

Using the retrogressive theorem for the if statement:
Taking the case as of $S1 = even(x) \rightarrow x := x - 1$

$$\{?\}x : x - 1\{x \geq 0 \wedge z + yx = ab\} \tag{10}$$

$$\{x \geq 0 \wedge z + y(x - 1) = ab\}x : x - 1\{x \geq 0 \wedge z + yx = ab\} \tag{11}$$

$$\{x \geq 0 \wedge z + yx - y = ab\}x : x - 1\{x \geq 0 \wedge z + yx = ab\} \tag{12}$$

Taking the case as of $S2 = odd(x) \rightarrow z := z + yx$

$$\{?\}z : z + yx\{x \geq 0 \wedge z + yx = ab\} \tag{13}$$

$$\{x \geq 0 \wedge (z + yx) + yx = ab\}z : z + yx\{x \geq 0 \wedge z + yx = ab\} \tag{14}$$

$$\{x \geq 0 \wedge z + 2yx = ab\}z : z + yx\{x \geq 0 \wedge z + yx = ab\} \tag{15}$$

Now applying the retrogressive theorem for the if statement

$$\begin{gathered}\{(x \geq 0 \wedge z + yx - y = ab \wedge even(x)) \vee (x \geq 0 \wedge z + 2yx = ab \wedge odd(x))\} \\ if even(x) \rightarrow x := x - 1 else odd(x) \rightarrow z := z + yx \\ \{x \geq 0 \wedge z + yx = ab\}\end{gathered} \tag{16}$$

5 Question 2.4

$$\{?\}while 0 \geq c \geq -2 do x := x - 1 endwhile\{x = -3\} \tag{17}$$

$$B = 0 \geq c \geq -2 \tag{18}$$

$$S = x := x - 1 \tag{19}$$

Starting with

$$\begin{gathered}\{Z_0\}S^0\{\neg B\} \\ \{B \wedge C_0\}x := x - 1\{x = -3\} \\ \{0 \geq x \geq -2 \wedge x - 1 = -3\}x := x - 1\{x = -3\} \\ \{0 \geq x \geq -2 \wedge x = -2\}x := x - 1\{x = -3\}\end{gathered} \tag{20}$$

$$\begin{gathered}\{Z_1\}S\{Z_0\} \\ \{0 \geq x \geq -2 \wedge x - 1 = -2\}x := x - 1\{0 \geq c \geq -2 \wedge x = -2\} \\ \{0 \geq x \geq -2 \wedge x = -1\}x := x - 1\{0 \geq c \geq -2 \wedge x = -2\}\end{gathered} \tag{21}$$

$$\begin{gathered}\{Z_2\}S\{Z_1\} \\ \{0 \geq x \geq -2 \wedge x - 1 = -1\}x := x - 1\{1 \geq x \geq -1 \wedge x = -1\} \\ \{0 \geq x \geq -2 \wedge x = 0\}x := x - 1\{1 \geq x \geq -1 \wedge x = -1\}\end{gathered} \tag{22}$$

$$\begin{gathered}\{Z_3\}S\{Z_2\} \\ \{0 \geq x \geq -2 \wedge x - 1 = 0\}x := x - 1\{2 \geq x \geq 0 \wedge x = 0\} \\ \{0 \geq x \geq -2 \wedge x = 1\}x := x - 1\{2 \geq x \geq 0 \wedge x = 0\}\end{gathered} \tag{23}$$

Z_3 is false thus the preconditions of S are

$$\begin{gathered}\{Z_0 \vee Z_1 \vee Z_2\} \\ \{(0 \geq x \geq -2 \wedge x = -2) \vee (0 \geq x \geq -2 \wedge x = -1) \vee (0 \geq x \geq -2 \wedge x = 0)\}\end{gathered} \tag{24}$$