COS 4807 Assignment 4

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1 Question 1i

$$\exists x p(x) \rightarrow \exists y (p(y) \wedge \exists x q(x, y)) \tag{1}$$

$$\exists x p(x) \rightarrow \exists y (p(y) \wedge \exists z q(z, y)) \tag{2}$$

Using $A \rightarrow B \equiv \neg A \vee B$

$$\neg \exists x p(x) \rightarrow \exists y (p(y) \wedge \exists z q(z, y)) \tag{3}$$

$$\forall x \neg p(x) \rightarrow \exists y (p(y) \wedge \exists z q(z, y)) \tag{4}$$

$$\forall x (\neg p(x) \rightarrow \exists y (p(y) \wedge \exists z q(z, y))) \tag{5}$$

$$\forall x \exists y (\neg p(x) \rightarrow (p(y) \wedge \exists z q(z, y))) \tag{6}$$

$$\forall x \exists y \exists z (\neg p(x) \rightarrow (p(y) \wedge q(z, y))) \tag{7}$$

Using distributivity of $\vee$ and $\wedge$

$$\forall x \exists y \exists z ((\neg p(x) \vee p(y)) \wedge (\neg (p(x) \vee q(z, y)))) \tag{8}$$

$$\forall x \exists y ((\neg p(x) \vee p(y)) \wedge (\neg (p(x) \vee q(f(x), y)))) \tag{9}$$

$$\forall x ((\neg p(x) \vee p(g(x))) \wedge (\neg (p(x) \vee q(f(x), g(x))))) \tag{10}$$

$$\{\{\neg p(x), p(g(x))\}, \{\neg p(x), q(f(x), g(x))\}\} \tag{11}$$

2 Question 1ii

$$\forall x ((\forall y q(x, y) \vee \forall z q(z, x)) \rightarrow g(x, x)) \tag{12}$$

Using $A \rightarrow B \equiv \neg A \vee B$

$$\forall x (\neg (\forall y q(x, y) \vee \forall z q(z, x)) \vee g(x, x)) \tag{13}$$

$$\forall x ((\neg \forall y q(x, y) \wedge \neg \forall z q(z, x)) \vee g(x, x)) \tag{14}$$

$$\forall x ((\exists y \neg q(x, y) \wedge \exists z \neg q(z, x)) \vee g(x, x)) \tag{15}$$

$$\forall x \exists y ((\neg q(x, y) \wedge \exists z \neg q(z, x)) \vee g(x, x)) \tag{16}$$

$$\forall x \exists y \exists z ((\neg q(x, y) \wedge \neg q(z, x)) \vee g(x, x)) \tag{17}$$

Using distributivity of $\vee$ and $\wedge$

$$\forall x \exists y \exists z((q(x,x) \vee \neg q(x,y)) \wedge (q(x,x) \vee \neg q(z,x))) \tag{18}$$

$$\forall x \exists y((q(x,x) \vee \neg q(x,y)) \wedge (q(x,x) \vee \neg q(f(x),x))) \tag{19}$$

$$\forall x((q(x,x) \vee \neg q(x,g(x))) \wedge (q(x,x) \vee \neg q(f(x),x))) \tag{20}$$

$$\{\{q(x,x), \neg q(x,g(x))\}, \{q(x,x), q(f(x),x)\}\} \tag{21}$$