COS 4807 Assignment 2

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1 Question 1

$$((p \wedge q) \rightarrow r) \rightarrow (p \rightarrow (q \vee r)) \tag{1}$$

$$\neg((p \wedge q) \rightarrow r) \vee (p \rightarrow (q \vee r)) \tag{2}$$

$$\neg(\neg(p \wedge q) \vee r) \vee (\neg p \vee (q \vee r)) \tag{3}$$

$$((p \wedge q) \wedge (\neg r)) \vee (\neg p \vee q \vee r) \tag{4}$$

$$((\neg p \vee q \vee r) \vee (p \wedge q)) \wedge ((\neg p \vee q \vee r) \vee \neg r) \tag{5}$$

$$((\neg p \vee q \vee r) \vee (p \wedge q)) \wedge (true) \tag{6}$$

$$((\neg[\vee q \vee r) \vee p) \wedge ((\neg p \vee q \vee r) \vee q) \tag{7}$$

$$(true) \wedge ((\neg p \vee q \vee r) \vee q) \tag{8}$$

$$((\neg p \vee q \vee r) \vee q) \tag{9}$$

$$\{\{\neg p, q, r\}\} \tag{10}$$

Which in abreviated clausal form is $\{\bar{p}qr\}$.

2 Question 2

$$\neg((q \rightarrow p) \rightarrow r) \wedge ((p \wedge \neg r) \rightarrow (q \wedge r)) \tag{11}$$

$$\neg(\neg(p \rightarrow p) \vee r) \wedge (\neg(p \wedge \neg r) \vee (q \wedge r)) \tag{12}$$

$$\neg(\neg(\neg q \vee p) \vee r) \wedge ((\neg p \vee r) \vee (q \wedge r)) \tag{13}$$

$$(\neg q \vee p) \wedge (\neg r) \wedge (((\neg p \vee r) \vee q) \wedge ((\neg p \vee r) \vee r)) \tag{14}$$

$$(\neg q \vee p) \wedge (\neg r) \wedge (\neg p \vee q \vee r) \wedge (\neg p \vee r) \tag{15}$$

Which is $\{\{p, \neg q\}, \{\neg r\}, \{\neg p, q, r\}, \{\neg p, r\}\}$ in clausal form

1. $p\bar{q}$
2. $\bar{r}$
3. $\bar{p}qr$
4. $\bar{p}r$
5. $\bar{p}$ 2 and 4
6. $\bar{q}$ 1 and 5
7. $\bar{p}r$ 3 and 6
8. $\bar{p}$ 2 and 7
9. $\bar{q}r$ 1 and 4
10 $\bar{q}$ 2 and 9

No boxes have been found so this formula must be satisfiable

3 Question 3

Forumla A:

$$p \to q \tag{16}$$

$$\neg p \vee q \tag{17}$$

$\{\bar{p}q\}$

Formula B:

$$q \to r \tag{18}$$

$$\neg q \vee r \tag{19}$$

$\{\bar{q}r\}$

Negation of the single formula

$$\neg(p \to (q \wedge r)) \tag{20}$$

$$\neg(\neg p \vee (q \wedge r)) \tag{21}$$

$$p \wedge (\neg q \vee r) \tag{22}$$

$$(p \wedge \neg q) \vee (p \wedge \neg r) \tag{23}$$

$$((p \wedge \neg q) \vee p) \wedge ((p \wedge \neg q) \vee \neg r) \tag{24}$$

$$(p \vee p) \wedge (p \vee \neg q) \wedge (\neg r \vee p) \wedge (\neg r \vee \neg q) \tag{25}$$

Which is $\{p, p\bar{q}, p\bar{r}, \bar{q}\bar{r}\}$

Now for the resolution refutation

1. p
2. $p\bar{q}$
3. $p\bar{r}$
4. $\bar{q}\bar{r}$
5. $\bar{p}q$
6. $\bar{q}r$
7. q 1 and 5
8. $q\bar{r}$ 3 and 5
9. $\bar{r}$ 4 and 7
10 $\bar{r}$ 4 and 8
11 r 6 and 7
12 $\square$ 10 and 11

Because resolution refutation shows that the set of clauses is unsatidfiable we can conslude that $\{p \to q, q \to r\} \models p \to (q \wedge r)$