

COS 4807 Assignment 4

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1 Question 1i

$$\exists x p(x) \rightarrow \exists y(p(y) \wedge \exists x q(x, y)) \quad (1)$$

$$\exists x p(x) \rightarrow \exists y(p(y) \wedge \exists z q(z, y)) \quad (2)$$

Using $A \rightarrow B \equiv \neg A \vee B$

$$\neg \exists x p(x) \rightarrow \exists y(p(y) \wedge \exists z q(z, y)) \quad (3)$$

$$\forall x \neg p(x) \rightarrow \exists y(p(y) \wedge \exists z q(z, y)) \quad (4)$$

$$\forall x (\neg p(x) \rightarrow \exists y(p(y) \wedge \exists z q(z, y))) \quad (5)$$

$$\forall x \exists y (\neg p(x) \rightarrow (p(y) \wedge \exists z q(z, y))) \quad (6)$$

$$\forall x \exists y \exists z (\neg p(x) \rightarrow (p(y) \wedge q(z, y))) \quad (7)$$

Using distributivity of \vee and \wedge

$$\forall x \exists y \exists z ((\neg p(x) \vee p(y)) \wedge (\neg p(x) \vee q(z, y))) \quad (8)$$

$$\forall x \exists y ((\neg p(x) \vee p(y)) \wedge (\neg p(x) \vee q(f(x), y))) \quad (9)$$

$$\forall x ((\neg p(x) \vee p(g(x))) \wedge (\neg p(x) \vee q(f(x), g(x)))) \quad (10)$$

$$\{\{\neg p(x), p(g(x))\}, \{\neg p(x), q(f(x), g(x))\}\} \quad (11)$$

2 Question 1ii

$$\forall x ((\forall y q(x, y) \vee \forall z q(z, x)) \rightarrow g(x, x)) \quad (12)$$

Using $A \rightarrow B \equiv \neg A \vee B$

$$\forall x (\neg (\forall y q(x, y) \vee \forall z q(z, x)) \vee g(x, x)) \quad (13)$$

$$\forall x ((\neg \forall y q(x, y) \wedge \neg \forall z q(z, x)) \vee g(x, x)) \quad (14)$$

$$\forall x ((\exists y \neg q(x, y) \wedge \exists z \neg q(z, x)) \vee g(x, x)) \quad (15)$$

$$\forall x \exists y ((\neg q(x, y) \wedge \exists z \neg q(z, x)) \vee g(x, x)) \quad (16)$$

$$\forall x \exists y \exists z ((\neg q(x, y) \wedge \neg q(z, x)) \vee g(x, x)) \quad (17)$$

Using distributivity of \vee and \wedge

$$\forall x \exists y \exists z ((q(x, x) \vee \neg q(x, y)) \wedge (q(x, x) \vee \neg q(z, x))) \quad (18)$$

$$\forall x \exists y ((q(x, x) \vee \neg q(x, y)) \wedge (q(x, x) \vee \neg q(f(x), x))) \quad (19)$$

$$\forall x ((q(x, x) \vee \neg q(x, g(x))) \wedge (q(x, x) \vee \neg q(f(x), x))) \quad (20)$$

$$\{\{q(x, x), \neg q(x, g(x))\}, \{q(x, x), q(f(x), x)\}\} \quad (21)$$

3 Question 1ii

$$\forall x \forall y (\exists z p(z) \leftrightarrow q(x, y)) \quad (22)$$

Using $A \leftrightarrow B \equiv (A \rightarrow B) \wedge (B \rightarrow A)$

$$\forall x \forall y ((\exists z p(z) \rightarrow q(x, y)) \wedge (q(x, y) \rightarrow \exists z p(z))) \quad (23)$$

$$\forall x \forall y ((\exists z p(z) \rightarrow q(x, y)) \wedge (q(x, y) \rightarrow \exists k p(k))) \quad (24)$$

Using $A \rightarrow B \equiv \neg A \vee B$

$$\forall x \forall y ((\neg \exists z p(z) \vee q(x, y)) \wedge (\neg q(x, y) \vee \exists k p(k))) \quad (25)$$

$$\forall x \forall y ((\forall z \neg p(z) \vee q(x, y)) \wedge (\neg q(x, y) \vee \exists k p(k))) \quad (26)$$

$$\forall x \forall y \forall z \exists k ((\neg p(z) \vee q(x, y)) \wedge (\neg q(x, y) \vee p(k))) \quad (27)$$

$$\forall x \forall y \forall z ((\neg p(z) \vee q(x, y)) \wedge (\neg q(x, y) \vee p(f(x, y, z)))) \quad (28)$$

$$\{\{\neg p(z), q(x, y)\}, \{\neg q(x, y), p(f(x, y, z))\}\} \quad (29)$$