COS 4892 Assignment 1

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1 Question 1

For Formula

2 Question 2

Relations used:

$$p \Rightarrow q \equiv p \vee q \equiv p \tag{1}$$

$$p \equiv q \equiv (p \wedge q) \vee (\neg p \wedge \neg q) \tag{2}$$

$$p \wedge (p \vee q) \equiv p \tag{3}$$

$$p \vee (p \wedge q) \equiv p \tag{4}$$

3 Question 2.1

4 Question 2.2

$$(A \wedge \neg(B \vee \neg C)) \Rightarrow (\neg B \Rightarrow (A \wedge B)) \tag{5}$$

Only if Eqn 1

$$(A \wedge \neg(B \vee \neg C)) \Rightarrow (\neg B \vee (A \wedge B) \equiv (A \wedge B)) \tag{6}$$

Distributivity

$$(A \wedge \neg(B \vee \neg C)) \Rightarrow (((\neg B \vee A) \wedge (\neg B \wedge B)) \equiv (A \wedge B)) \tag{7}$$

Simplify

$$(A \wedge \neg(B \vee \neg C)) \Rightarrow (((\neg B \vee A) \wedge false)) \equiv (A \wedge B)) \tag{8}$$

Conjunction zero

$$(A \wedge \neg(B \vee \neg C)) \Rightarrow (false \equiv (A \wedge B)) \tag{9}$$

Disjunctive Normal Form

$$(A \wedge \neg(B \vee \neg C)) \Rightarrow ((false \wedge (A \wedge B)) \vee (\neg false \wedge \neg(A \wedge B))) \tag{10}$$

Conjunction zero

$$(A \wedge \neg(B \vee \neg C)) \Rightarrow (false \vee (\neg false \wedge \neg(A \wedge B))) \tag{11}$$

De Morgan and disjunction unit

$$(A \wedge \neg(B \vee \neg C)) \Rightarrow (true \wedge (\neg A \vee \neg B)) \tag{12}$$

Conjunction unit

$$(A \wedge \neg(B \vee \neg C)) \Rightarrow (\neg A \vee \neg B) \tag{13}$$

Only if Eqn 1

$$(A \wedge \neg(B \vee \neg C)) \vee (\neg A \vee \neg B) \equiv \neg A \vee \neg B \tag{14}$$

De Morgan

$$(A \wedge (\neg B \wedge C)) \vee (\neg A \vee \neg B) \equiv \neg A \vee \neg B \tag{15}$$

Distributivity

$$(A \vee (\neg A \vee \neg B)) \wedge ((\neg B \wedge C) \vee (\neg A \vee \neg B)) \equiv \neg A \vee \neg B \tag{16}$$

Disjunction Zero

$$true \wedge ((\neg B \wedge C) \vee (\neg A \vee \neg B)) \equiv \neg A \vee \neg B \tag{17}$$

Conjunction unit

$$(((\neg B \wedge C) \vee (\neg A \vee \neg B))) \equiv \neg A \vee \neg B \tag{18}$$

Distributivity

$$((\neg B \vee (\neg A \vee \neg B)) \wedge (C \vee (\neg A \vee \neg B))) \equiv \neg A \vee \neg B \tag{19}$$

Simplification

$$((\neg A \vee \neg B) \wedge (C \vee \neg A \vee \neg B)) \equiv \neg A \vee \neg B \tag{20}$$

Disjunctive normal Form Eqn 2

$$((\neg A \vee \neg B) \wedge (\neg A \vee \neg B \vee C) \wedge (\neg A \vee \neg B)) \vee (\neg((\neg A \vee \neg B) \wedge (\neg A \vee \neg B \vee C)) \wedge \neg(\neg A \vee \neg B)) \tag{21}$$

De Morgan

$$((\neg A \vee \neg B) \wedge (\neg A \vee \neg B \vee C) \wedge (\neg A \vee \neg B)) \vee ((\neg(\neg A \vee \neg B) \vee \neg(\neg A \vee \neg B \vee C)) \wedge (A \wedge B)) \tag{22}$$

Simplification of duplicate term

$$((\neg A \vee \neg B) \wedge (\neg A \vee \neg B \vee C)) \vee ((\neg(\neg A \wedge \neg B) \vee \neg(\neg A \vee \neg B \vee C)) \wedge (A \wedge B)) \tag{23}$$

De Morgan

$$((\neg A \vee \neg B) \wedge (\neg A \vee \neg B \vee C)) \vee (((A \wedge B) \vee (A \wedge B \wedge \neg C)) \wedge (A \wedge B)) \tag{24}$$

5 Question 2.3

$$(\neg A \vee \neg B) \Leftrightarrow (A \Rightarrow \neg B) \tag{25}$$

Using Only if Eqn 1

$$(\neg A \vee \neg B) \Leftrightarrow (A \vee \neg B \equiv \neg B) \tag{26}$$

using disjunctive normal form Eqn 2

$$(\neg A \vee \neg B) \Leftrightarrow ((A \vee \neg B) \wedge \neg B) \vee (\neg(A \vee \neg B) \wedge \neg\neg B) \tag{27}$$

using absorbtion Eqn 4

$$(\neg A \vee \neg B) \Leftrightarrow (\neg B) \vee (\neg(A \vee \neg B) \wedge \neg\neg B) \tag{28}$$

de morgans law

$$(\neg A \vee \neg B) \Leftrightarrow (\neg B) \vee (\neg A \wedge \neg B \wedge B) \tag{29}$$

Contradiction

$$(\neg A \vee \neg B) \Leftrightarrow \neg B \vee false \tag{30}$$

Unit disjunction

$$(\neg A \vee \neg B) \Leftrightarrow \neg B \tag{31}$$

Disjunctive normal form Eqn 2

$$((\neg A \vee \neg B)) \vee (\neg(\neg A \vee \neg B) \wedge \neg\neg B) \tag{32}$$

Absorbtion Eqn 3

$$\neg B \vee (\neg(\neg A \vee \neg B) \wedge \neg\neg B) \tag{33}$$

De Morgan

$$\neg B \vee (A \wedge B \wedge B) \tag{34}$$

Simplification

$$\neg B \vee (A \wedge B) \tag{35}$$

Distributivity

$$(\neg B \vee A) \wedge (\neg B \vee B) \tag{36}$$

simplification

$$(\neg B \vee A) \wedge true \tag{37}$$

Unit Conjunction

$$\neg B \vee A \tag{38}$$

6 Question 3

7 Quesstion 4