# COS 4892 Assignment 2

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## 1 Question 1

We will define

$$Q = ((Q_1 \wedge B) \vee (Q_2 \wedge \neg B)) \tag{1}$$

Now we calculate from Equation 1

$$\begin{aligned} Q \wedge B &= ((Q_1 \wedge B) \vee (Q_2 \wedge B)) \wedge B \\ &= (Q_1 \wedge B \wedge B) \vee (Q_2 \wedge \neg B \wedge B) \\ &= Q_1 \wedge B \end{aligned} \tag{2}$$

and

$$\begin{aligned} Q \wedge B &= ((Q_1 \wedge B) \vee (Q_2 \wedge B)) \wedge B \\ &= (Q_1 \wedge B \wedge \neg B) \vee (Q_2 \wedge \neg B \wedge \neg B) \\ &= Q_2 \wedge \neg B \end{aligned} \tag{3}$$

## 2 Question 2.1

$$\{?\}x := 3.2z\{wy - 2w^2 < z\} \tag{4}$$

$$\{wy - 2w^2 < z\}x := 3.2z\{wy - 2w^2 < z\} \tag{5}$$

The post condition is not dependant on $x$. Thus there is notihing to replace by using the assignment Axiom

## 3 Question 2.2

$$\{?\}x := x - 1; y := y - 1\{z - 1 \leq y < x \leq w\} \tag{6}$$

$$\{z - 1 \leq y - 1 < x - 1 \leq w\}x := x - 1; y := y - 1\{z - 1 \leq y < x \leq w\} \tag{7}$$

$$\{z \leq y < x \leq w + 1\}x := x - 1; y := y - 1\{z - 1 \leq y < x \leq w\} \tag{8}$$

## 4 Question 2.3

$$\{?\}if even(x) \rightarrow x := x - 1 else odd(x) \rightarrow z := z + yx\{x \geq 0 \wedge z + yx = ab\} \tag{9}$$

Using the retrogressive theorem for the if statement:
Taking the case as of $S1 = even(x) \rightarrow x := x - 1$

$$\{?\}x : x - 1\{x \geq 0 \wedge z + yx = ab\} \tag{10}$$

$$\{x \geq 0 \wedge z + y(x - 1) = ab\}x : x - 1\{x \geq 0 \wedge z + yx = ab\} \tag{11}$$

$$\{x \geq 0 \wedge z + yx - y = ab\}x : x - 1\{x \geq 0 \wedge z + yx = ab\} \tag{12}$$

Taking the case as of $S2 = odd(x) \to z := z + yx$

$$\{?\}z : z + yx\{x \geq 0 \wedge z + yx = ab\} \tag{13}$$

$$\{x \geq 0 \wedge (z + yx) + yx = ab\}z : z + yx\{x \geq 0 \wedge z + yx = ab\} \tag{14}$$

$$\{x \geq 0 \wedge z + 2yx = ab\}z : z + yx\{x \geq 0 \wedge z + yx = ab\} \tag{15}$$

Now applying the retrogressive theorem for the if statement

$$\begin{aligned} \{(x \geq 0 \wedge z + yx - y = ab \wedge even(x)) \vee (x \geq 0 \wedge z + 2yx = ab \wedge odd(x))\} \\ if even(x) \to x := x - 1 else odd(x) \to z := z + yx \\ \{x \geq 0 \wedge z + yx = ab\} \end{aligned} \tag{16}$$