COS 4807 Assignment 1

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1 Question 1.1

For Formula

$$(((p \vee q) \vee r) \wedge (\neg p \wedge \neg q)) \rightarrow r \tag{1}$$

To prove the formula is valid we need to show it is true for all possible interpretations. We will use 0 to represent false and 1 to represent true

p	q	r	$\neg p$	$\neg q$	$p \vee q$	$(p \vee q) \vee r$	$\neg p \wedge \neg q$	$(((p \vee q) \vee r) \wedge (\neg p \wedge \neg q))$	$(((p \vee q) \vee r) \wedge (\neg p \wedge \neg q)) \rightarrow r$
0	0	0	1	1	0	0	1	0	1
0	0	1	1	1	0	1	1	1	1
0	1	0	1	0	1	1	0	0	1
0	1	1	1	0	1	1	0	0	1
1	0	0	0	1	1	1	0	0	1
1	0	1	0	1	1	1	0	0	1
1	1	0	0	0	1	1	0	0	1
1	1	1	0	0	1	1	0	0	1

Since this formula is true in all cases (last column), it is valid (tautology).

2 Question1.2

For Formula

$$(p \rightarrow (q \rightarrow r)) \leftrightarrow ((p \wedge q) \wedge \neg r) \tag{2}$$

to show that it is unsatisfiable we need to show that it is false for all interpretations

p	q	r	$q \rightarrow r$	$p \rightarrow (q \rightarrow r)$	$p \wedge q$	$\neg r$	$(p \wedge q) \wedge \neg r$	$(p \rightarrow (q \rightarrow r)) \leftrightarrow ((p \wedge q) \wedge \neg r)$
0	0	0	1	1	0	1	0	0
0	0	1	1	1	0	0	0	0
0	1	0	0	1	0	1	0	0
0	1	1	1	1	0	0	0	0
1	0	0	1	1	0	1	0	0
1	0	1	1	1	0	0	0	0
1	1	0	0	0	1	1	1	0
1	1	1	0	1	1	0	0	0

From this we can see that the Formula is unsatisfiable for all interpretations of p q and r

3 Question1.3

For Formula

$$(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow r) \tag{3}$$

we need to show that there exists at least 1 interpretation for which this formula is false and at least one interpretation for which this formula is true

p	q	r	$q \to r$	$p \to q$	$p \to (q \to r)$	$(p \to q) \to r$	$(p \to (q \to r)) \to ((p \to q) \to r)$
0	0	0	1	1	1	0	0
0	0	1	1	1	1	1	1
0	1	0	0	1	1	0	0
0	1	1	1	1	1	1	1
1	0	0	1	0	1	1	1
1	0	1	1	0	1	1	1
1	1	0	0	1	0	0	1
1	1	1	1	1	1	1	1

The above trusth table shows that the formula is true for some interpretations and thus the Formula is satisfiable. Also the truth table shows that the formula is false for some interpretations and thus the formula is falsifiable

4 Question2.1

Given Formula

$$F_1 = (((p \vee q) \vee r) \wedge (\neg p \wedge \neg q)) \to r \tag{4}$$

We need to show that

$$\nu_I(F_1) = T \tag{5}$$

for any interpretation of p, q and r.
Using proof by contradiction we assume

$$\nu_I(F_1) = F \tag{6}$$

then

$$\nu_I(r) = F \tag{7}$$

and

$$\nu_I(((p \vee q) \vee r) \wedge (\neg p \wedge \neg q)) = T \tag{8}$$

The subformula

$$((p \vee q) \vee r) \wedge (\neg p \wedge \neg q) \tag{9}$$

is made up of the conjuction between sub formulas

$$(p \vee q) \vee r \tag{10}$$

and

$$\neg p \wedge \neg q \tag{11}$$

Thus the statement 8 can only be valid if both the statements

$$\nu_I((p \vee q) \vee r) = T \tag{12}$$

and

$$\nu_I(\neg p \wedge \neg q) = T \tag{13}$$

are true.

But formula 11 is the negation of formula 10 by de Morgans law. Taking into account that $\nu_I(r) = F$. Thereore we have a contradiction. Statement 8 cannot be true because the conjunction of a formula and its negation cannot be True for any interpretation. That means our assumption in statement 6 is incorect and therefore statement 5 is true. And since F_1 is true for all interpretations, it is a valid formula.

5 Question 4.1

For formula

$$(((p \vee q) \vee r) \wedge (\neg p \wedge \neg q)) \to r \tag{14}$$

Applying the β formula for implication gives

$$\neg(((p \vee q) \vee r) \wedge (\neg p \wedge \neg q)) \tag{15}$$

and

$$r \tag{16}$$

Where Formula 16 is satisfiable.
Continueing for Formula 15 by applying the β formula for $\neg(B_1 \wedge B_2)$ we get

$$\neg((p \vee q) \vee r) \tag{17}$$

and

$$\neg(\neg p \wedge \neg q) \tag{18}$$

Continueing for Formula 17 and applying α formula $\neg(A_1 \vee A_2)$ we get

$$\neg(p \vee q), \neg r \tag{19}$$

Then applying formula $\neg(A_1 \vee A_2)$ again we get

$$\neg p, \neg q, \neg r \tag{20}$$

Which is satisfiable.
Going back to Formula 18 which can by simplified to

$$p \vee q \tag{21}$$

Then applying β formula $B_1 \vee B_2$ we get formulas

$$p \tag{22}$$

and

$$q \tag{23}$$

Both of which are satifyable. Now looking at all the leaf nodes namely Formalas 16, 20, 22 and 23 we can see that they are all satisfiable. Thus Formula 14 is valid.

6 Question 4.2

$$(p \to (q \to r)) \leftrightarrow ((p \wedge q) \wedge \neg r) \tag{24}$$

By substituting for double implication operator:

$$(p \to (q \to r)) \to ((p \wedge q) \wedge \neg r), ((p \wedge q) \wedge \neg r) \to (p \to (q \to r)) \tag{25}$$

Substituting for the implication operator in the first term gives:

$$\neg(p \to (q \to r)), ((p \wedge q) \wedge \neg r) \to (p \to (q \to r)) \tag{26}$$

and

$$((p \wedge q) \wedge \neg r), ((p \wedge q) \wedge \neg r) \to (p \to (q \to r)) \tag{27}$$

Equation 26 becomes

$$p, \neg(q \to r)), ((p \wedge q) \wedge \neg r) \to (p \to (q \to r)) \tag{28}$$

then

$$p, q, \neg r, ((p \wedge q) \wedge \neg r) \rightarrow (p \rightarrow (q \rightarrow r)) \tag{29}$$

Substituting for the implication in the above formula gives:

$$p, q, \neg r, \neg((p \wedge q) \wedge \neg r) \tag{30}$$

and

$$p, q, \neg r, (p \rightarrow (q \rightarrow r)) \tag{31}$$

Formula 30 becomes

$$p, q, \neg r, \neg(p \wedge q) \tag{32}$$

and

$$p, q, \neg r, r \tag{33}$$

We can see that Formula 33 is unsatifyable. Then Formula 32 becomes

$$p, q, \neg r, \neg p \tag{34}$$

and

$$p, q, \neg r, \neg q \tag{35}$$

Both of which are unsatifyable. Now having done all the leaves under Formula 30, we continue with the leaves under Equation 31.

Equation 31 becomes

$$p, q, \neg r, \neg p \tag{36}$$

which is unsatifyable and

$$p, q, \neg r, q \rightarrow r \tag{37}$$

which becomes

$$p, q, \neg r, \neg q, r \tag{38}$$

Having finished all the leaves under Formula 26 we continue with Formula 27 which becomes

$$p, q, \neg r, ((p \wedge q) \wedge \neg r) \rightarrow (p \rightarrow (q \rightarrow r)) \tag{39}$$

But Formula 39 is the same as Formula 29. Formula 29 will have the same leaf nodes as Formula 39.

Thus we have all the leaf nodes namely Formulas 33, 34, 35, 36 and 38. All of these Formulas are unsatisfiable, therefore the original Formula 24 is unsatisfiable.