Chapter 8 :

8.1 : Pure Jump Processes :

$\{S(t)\} :=$ stochastic process where $S(t)$ is the state of the process at time $t>0$.

$T_1 < T_2 < \cdots :=$ point of time of the events

$$S(t) = S(0) + \sum_{j=1}^{\infty} I(T_j \le t) J_j , \; t>0 \quad (8.1)$$

where $I(\cdot) :=$ the indicator function

$J_j :=$ the change in the state $(\pm)$ of the process at time T_j

J_j : $+1$ for a repair event, -1 for a failure event

$S(t)$: piecewise constant, right-continuous in t with jumps at $T_1 < T_2 < \cdots$

$$S(t) = S(0) + \sum_{j=1}^{k} J_j = S(T_k) \quad \text{for } \forall t \in [T_k, T_{k+1}) \quad (8.2)$$

where: $T_k = \sum_{r=1}^{k} \Delta T_r , \; k=1,2,\ldots$

• Criterion for regularity: $\sum_{j=1}^{\infty} \Delta T_j$ where $T_0=0$; $T_\infty = \infty$ is divergent with probability $=1$

Proposition 8.1.1: $\{S(t)\}$ with $T_1 < T_2 < \cdots$, $T_0 = 0$ with non-negative random variables ΔT_j, $j=1,2,\ldots$, assume $\{\Delta T_j\}$ has infinite subsequences $\{\Delta T_{k_j}\}$ of independent r.v.'s : $E[\Delta T_{k_j}] = d > 0$, Then S is regular.

Proof.

Since $\sum_{j=1}^{\infty} \Delta T_{k_j} < \sum_{j=1}^{\infty} \Delta T_j$, Then $P\left(\lim_{m\to\infty} \frac{1}{m} \cdot \sum_{j=1}^{m} \Delta T_{k_j} = d\right) = 1$

k_j : infinite subsequences in $T_j - T_{j-1} = \Delta T_j$

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Proposition 8.1.2: $\{S(t)\}$ is pure jump process with $T_1 < T_2 < \cdots$, then $\lim_{t\to s^-} S(t)$ exists for $\forall s>0$ with probability $=1$.

Proposition 8.1.3: $\{S(t)\}$ is regular pure jump process with $T_1 < T_2 < \cdots$, $0 \le u < v < \infty$. Assume $\{T_j : u < T_j < v\} = \{T^{(1)}, \ldots, T^{(k)}\}$, where $T^{(1)} < \cdots < T^{(k)}$. $T^{(0)} = u$ and $T^{(k+1)} = v$, then:

$$\int_u^v S(t)\, dt = \sum_{j=0}^{k} S(T^{(j)}) \cdot \left(T^{(j+1)} - T^{(j)}\right)$$

Proposition 8.1.4: $\{S_1(t)\}, \ldots, \{S_n(t)\}$ is n collection of regular pure jump processes, $H(t) = H(S(t))$, where $S(t) = (S_1(t), \ldots, S_n(t))$, $T>0$. Then $\{H(t)\}$ is a regular pure jump process, $H(t) = H(S(t))$ is piecewise constant and right continuous continuous in t, and the # of jumps in any finite interval is finite with probability $=1$.

8.2 : Repairable Components :

$U_{ij} :=$ the j^{th} lifetime of the i^{th} component $\rightarrow$ absolutely continuous distribution F_i with positive mean $\mu_i < \infty$

$D_{ij} :=$ the j^{th} repair time of the i^{th} component $\rightarrow$ " — " — " — G_i with positive mean $\nu_i < \infty$

Availability of the i^{th} component at t $:= \boxed{A_i(t) = P(X_i(t) = 1) = E[X_i(t)]}$

The stationary availabilities : $\boxed{A_i = \lim_{t\to\infty} A_i(t) = \frac{\mu_i}{\mu_i + \nu_i} , \; i=1,\ldots,n}$

The system's availability at (t) : $\boxed{A_\phi(t) = P(\phi(\vec{X}(t)) = 1) = E[\phi(\vec{X}(t))] = h(\vec{A}(t))}$

The corresponding stationary availability : $\boxed{A_\phi = \lim A_\phi(t) = h(\vec{A})}$

$\underbrace{\psi_i(\vec{X}(t))}_{\text{critical state of component } i \text{ at } t} = \phi(1_i, \vec{X}(t)) - \phi(0_i, \vec{X}(t)) = 1$ and $I_B^{(i)}(t) = P(\psi_i(\vec{X}(t)) = 1) = E[\psi_i(\vec{X}(t))] = h(1_i, \vec{A}(t)) - h(0_i, \vec{A}(t))$

∴ The corresponding stationary measure : $I_B^{(i)} = \lim_{t\to\infty} I_B^{(i)}(t) = h(1_i, \vec{A}(t)) - h(0_i, \vec{A}(t))$