Generalized Momentum and Flexible Asset Allocation (FAA)
An Heuristic Approach

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December 24, 2012, draft v0.98

Abstract

In this paper we extend the timeseries momentum (or trendfollowing) model towards a generalized momentum model, called Flexible Asset Allocation (FAA). This is done by adding new momentum factors to the traditional momentum factor R based on the relative returns among assets. These new factors are called Absolute momentum (A), Volatility momentum (V) and Correlation momentum (C). Each asset is ranked on each of the four factors R, A, V and C. By using a linearised representation of a loss function representing risk/return, we are able to arrive at simple closed form solutions for our flexible asset allocation strategy based on these four factors. We demonstrate the generalized momentum model by using a 7 asset portfolio model, which we backtest from 1998-2012, both in- and out-of-sample.

Keywords: Tactical Asset Allocation, Momentum, Trendfollowing

JEL Classification: C00, C10, G00, G11

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1. Introduction: momentum

Since Faber (2007 and 2010) there is a renewed interest in Tactical Asset Allocation (TAA). Faber based his TAA model on two anomalies: momentum (Faber 2010) and moving averages (MA) strategies (Faber 2007). The momentum anomaly is well known for centuries, see Antonacci (2012) for a nice review. The core of the momentum anomaly is that assets often continue their price momentum, defined as the change in price over a given look-back period (eg. 12 or 6 months). Therefore one should buy assets with the highest momentum and sell assets with the lowest momentum. We will call this the relative or return momentum model. In the subsequent we will not consider short selling, so only the first rule applies: higher relative momentum makes an asset more attractive to go long. As Faber does, we will therefore pick each month the best subset based on the momentum sorting of our universe of assets.

The MA anomaly is also known for centuries, see also Thomas (2012), Baltas (2012) and

[1] We want to thank Bas Nagtzaam, Steve LeCompte and Olga van Putten for comments on earlier drafts of this paper and Menno Dreischor for some earlier discussions.

Glabadanidis (2012) for an overview. The core of the MA anomaly is that assets are more (less) attractive when their prices are above (below) their moving average over a given look-back period (eg. 10 months or 200 days). The TAA approach of Faber relies on both methods: the relative momentum and MA. The advantages of his TAA model is its simplicity: anyone with access to a spreadsheet program and some historical data (eg. monthly asset prices) can perform the relevant backtests and determine the best asset allocation for the future. No daily variances or covariances enter into the equation, just monthly prices will do.

In this paper we will extend the relative momentum model towards absolute momentum and two second- order characteristics: volatility and correlation. We will use very simple linear functions to combine relative and absolute momentum with volatility- and correlation-momentum. We will call this the generalized momentum model. We show backtests for this model based on seven global index funds from 1998 on, including both in-sample and out-of-sample tests.

2. More sophisticated models

Besides simple approaches based on momentum and MA, some more sophisticated methods have recently been considered for TAA. Most of these methods are based on so-called minimum- or mean-variance models or models that optimize a risk contribution or diversification measure, by choosing optimal weights wi for each asset class i under restrictions. An overview and demonstration of these methods can be found in Ang (2012) and Butler (2012). All these methods have in common that the optimal asset allocation does not exist in a closed form or as a simple rule, as with momentum and MA.

All these sophisticated models require a quadratic programming or even more complex optimization, in particular if one wants to take into account practical restrictions such as long-only, fully-invested and no leverage (0<=wi<=1, sum wi =1). Most of these sophisticated methods also use look-back periods of several months (like TAA and MA do), mostly for estimating covariance matrices. These estimated covariance matrices can be the source of non-robust estimators for the asset weights wi. As a result, sometimes undesirable singular solutions (eg. corner solutions with many wi=0) show up (see eg. Dori 2012). Most of these sophisticated models also only use second-order information like volatilities and correlations (or covariance matrices), but seldom first-order returns as in momentum.

In this paper we try to use similar heuristic (simple) methods as Faber while at the same time taking into account more characteristics then price alone, as is the case of the more sophisticated models, like minimum variance. We will show that as an additional advantage, our solutions are not only very simple to calculate but also very regular (non-singular). Our objective herewith is (much like Faber) to select each month the best subset of equal-weighted funds out of a larger universe of funds by applying a generalized momentum model.

3. Absolute and relative momentum

Before going into more sophisticated TAA strategies (which we will call Flexible Asset Allocation or FAA strategies), we alter the Faber model slightly. Faber (and many others) replaces an asset (or fund) by cash (or some risk-free asset like T-Bills) when its price goes below its MA, given some MA lookback period. Successful MA lookback periods are eg. 10 months or 200 trading days. This is done eg. monthly, sometimes together with the relative momentum sorting on returns.

Instead of such MA rule we will use an absolute momentum measure, which fits the corresponding momentum model better in our opinion since it is directly related to the price momentum. The cash rule becomes now: go to cash if the price momentum ri is below a threshold rmin. A common threshold is rmin=0 or rmin=risk-free rate. We call this the *absolute momentum* rule, similar to Antonacci (2012) and others. If we would have allowed short selling, an alternative would be to sell the asset short when r<rmin, as is common for trend-following strategies in the managed futures industry (see eg Hurst, 2012). In this paper we will assume rmin=0.

We will assume that the lookback period for the relative (return) momentum equals that of the absolute momentum, eg. 6 months. Now the momentum rule becomes: select each month an equal-weighted portfolio of the best N funds out of the U funds in the universe, sorting funds by relative momentum (higher is better), while replacing funds by cash when their absolute momentum is negative. As we will see below, this simple sorting procedure will become the cornerstone of our generalized momentum method.

4. Data, universe and selected portfolio

Assume we use only the relative and absolute momentum rules to select funds out of a universe of assets or funds. There are U funds in our universe. As Faber we will consider worldwide ETFs for broad indices based on stocks, bonds, REITS and commodities as assets. Since ETFs have only been available for a limited number of years, we will use index funds as a proxy in our backtests here to cover also the previous time period.

To be more specific our example universe consists of 7 index funds (so U=7), i.e. 3 for global stocks (VTSMX, FDIVX, VEIEX) covering US, EAFE and EM regions, 2 for US bonds (VFISX, VBMFX) and a commodity and REIT index fund (QRAAX, VGSIX). Users only interested in recent years can use the corresponding ETFs (eg. VTI, VEA, VWO, SHY, BND, GSG, and VNQ) which follow the same indices as our index funds.

Our dataset is based on the daily closing prices from mid 1997 till end 2012. All our prices are in USD. As proxy for cash returns we will use VFISX (2-3 year US government bond), and therefore any fund with ri<0 (negative return momentum) will be replaced by VFISX in the monthly selection. Below we will build up our method step-by-step, starting with traditional relative or Return momentum and adding Absolute momentum, Volatility momentum and Correlation momentum. We will call these the four momentum factors (indicated by resp. R, A, V, C).

Each step is illustrated with a numerical backtest example, using the data described above. Since we want to check our results out-of-sample, we do all the backtesting first with an in-sample (learning) period of nearly 8 years from January 2005 till December 2012, covering most of the recent business cycle and including the 2008 financial crisis. After all the modeling is done, we will test our final model out-of-sample for a period of 7 years from 1998-2005. At the end we will consider extended models including unequal weighting, leverage and transaction costs and discuss the risks of datasnooping.

5. Example 1: Relative momentum (factor R)

Our first heuristic is very similar to Faber's: we will select an equal-weighted portfolio of the best N=3 funds (ETFs, assets) monthly out of the universe of the U=7 funds, using only relative momentum for the sorting. Sometimes our relative momentum is called relative strength (RS, see Faber 2010) or timeseries momentum (see Thomas 2012). We will also use the term return momentum, to contrast better with volatility and correlation momentum.

At the end of each month (at the close of the last day) we will determine the momentum model and trade the funds (or ETFs) selected at the (open of the) first day of the new month. For the moment we will disregard transaction costs, but consider them in paragraph 12. As non-US citizens, we are not interested in US taxes.

In the subsequent examples, we will fix both N=3 (approximately 40% of U) and a lookback period of 4 months for measuring momentum. We have done many backtests on many different universes, from mutual (index) funds to ETFs and from this simple 7 fund universe to much larger universes up to 200 funds and found these two choices (40% and 4 months) to be optimal most of the times. This includes data ranging from 1986 to present, including the financial crisis (2008), the internet bubble and Black Monday (Oct. 1987).

The chosen length of the lookback period of 4 months is also close to the average length of the combined momentum over 1, 3 and 12 months, which is often used in managed futures (see eg. Hurst 2012). We might also search for each universe for the best N and length of the lookback period in our backtests, but then we are capitalizing on the backtest data and are becoming datasnooping prone. See also paragraph 13.

In this example we will therefore start with the simple equal weighted *relative momentum* solution (with N=3, lookback=4m, 3Jan2005-11Dec2012) and compare that to our *benchmark*, the equal weighted (EW) Buy & Hold portfolio with all 7 funds, rebalanced monthly. The results of our backtests in-sample are

> **Rel. (R) momentum**: 03Jan2005-11Dec2012, 3/7, 4m/4m/4m, 100/0/0%:
> R=9.1%,V=14.5%,D=-29.2%,S0=0.63,S2.5=0.45,S5=0.28,W=63.5%,T=2.9,O=1.63,Q0=0.31,Q5=0.14
>
> **Bench**: 03Jan2005-11Dec2012, 7/7:
> R=5.6%,V=16.6%,D=-46.3%,S0=0.34,S2.5=0.19,S5=0.04,W=63.5%,T=0.1,O=1.42,Q0=0.12,Q5=0.01

where the following statistics are shown: R= annual Return, V= annual Volatility, D= monthly maxDrawdown, Sx=Sharpe ratio (hurdle x%), W= Winning months, T= yearly Turnover, O=Omega, Qx= Calmar ratio (hurdle x%). The Omega and Calmar ratio are similar to the Sharpe ratio, but with respectively the average negative return (R<0) and the maxdrawdown D instead of the volatility in the denominator. They are therefore more sensitive for downward volatility. All our statistics are annual (eg. Return is 9.1% per annum) based on monthly measurement, with the exception of annual Volatility, which is based on daily measurements. The parameters 4m/4m/4m, 100/0/0% refer to the length of the lookback period (4m) and weight (100%, 0%, 0%) of the momentum factors R, V and C respectively, to be explained below. The lookback period for absolute momentum (A) is always the same as for return momentum (R).

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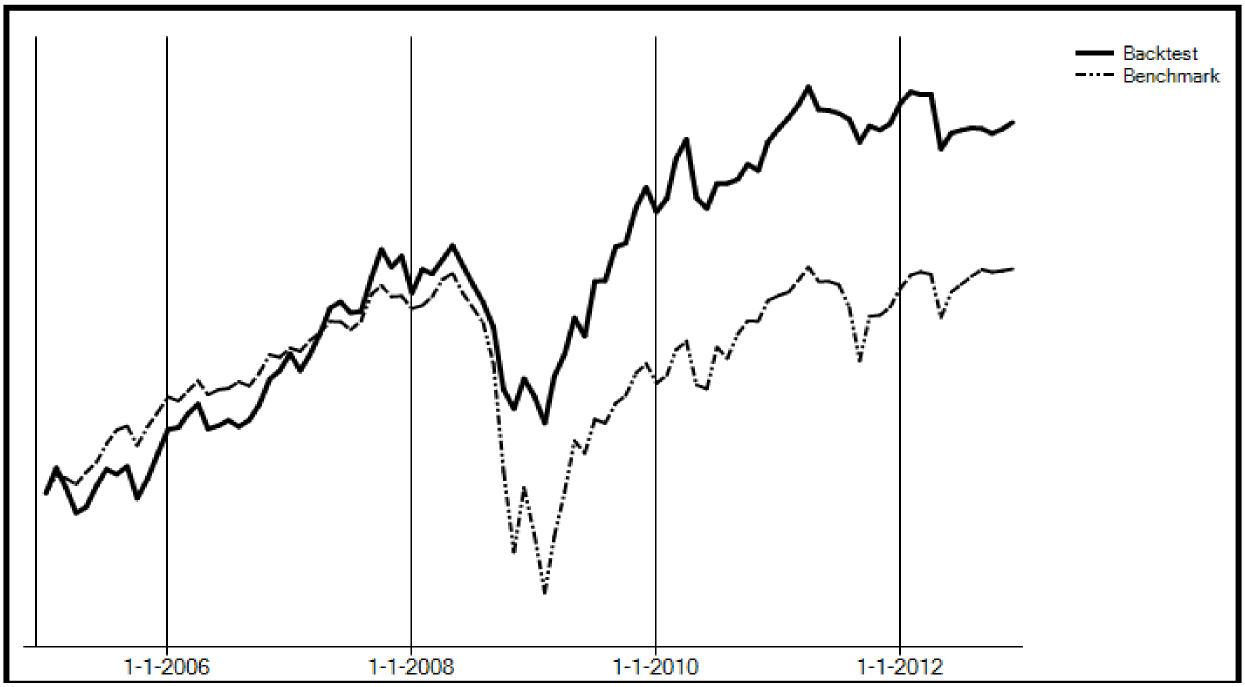

Fig. 1 Relative (R) momentum and the B&H benchmark (in-sample)

In the graph above (all graphs have log scales) the dashed line is the Equally Weighted (EW) “Buy & Hold” benchmark (B&H, 7/7 EW, no momentum, monthly rebalanced) while the bold line is the result of the relative momentum model (N=3, 4m). As the statistics (and figure) illustrate, introducing relative momentum shows a substantial improvement: R= 9.1% (bench 5.6%), V= 14.5% (16.6%), D= 29.2% (46.3%), S2.5= 0.45 (0.19), O= 1.63 (1.42), and Q5= 0.14 (0.01). We will identify relative (or Return) momentum as “the factor R”.

6. Example 2: Absolute momentum (factor A)

Notice that both our momentum results (3 out of 7) as the B&H benchmarks above both suffer from a heavy drawdown (of 29% and 46% resp.) in 2008. Now we will repeat the analysis with absolute momentum (factor A) added to our relative momentum (factor R), with a threshold of rmin=0 (and again N=3, 4m) and VFISX as cash proxy fund (cpf). So every fund i with a return momentum ri<=0 over the lookback period will be replaced by cash. This procedure is similar to Faber (2007) but with his MA rule replaced by our absolute momentum rule. Notice that we will always use the same lookback period (e.g. 4 months) for absolute momentum (factor A) as for relative momentum (factor R), in contrast to Faber’s MA strategy.

The resulting statistics of the in-sample backtest are:

R & A momentum: 03Jan2005-11Dec2012,cpf VFISX, 3/7, 4m/4m/4m, 100/0/0%:
R=11.7%,V=12.8%,D=-12.6%,S0=0.92,S2.5=0.72,S5=0.53,W=67.7%,T=2.9,O=2.04,Q0=0.93,Q5=0.53

Compare this R &A model with the R model above with only relative momentum for N=3. The return R is improved (R=11.7% for the R&A model, was 9.1% for the R-only model), as is the volatility V=12.8% (was 14.5%) and in particular the maxdrawdown D=12.6% (was 29.2%). The relative/absolute momentum graph (using both factors R and A) and the benchmark (dashed) becomes:

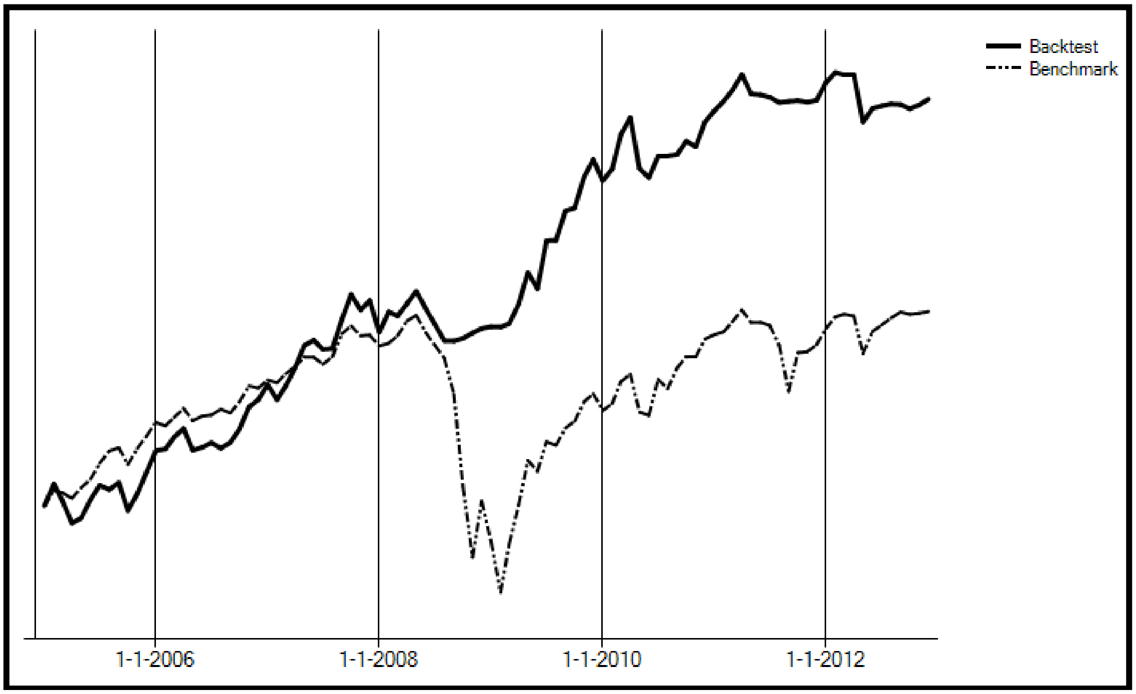

Fig. 2 Relative and absolute (RA) momentum against the benchmark (in-sample)

This shows the effectiveness of the absolute momentum rule and the factor A. In particular it improves (lowers) the maxdrawdown D, therefore we sometimes call absolute momentum "cash" or "crash" protection (CP) since it shifts the portfolio towards cash in bear (crash) markets.

The next list shows the selections for months where the absolute momentum rule applies, and the percentage of Cash (VFISX) assigned over the 8 years. The credit crisis (bear-market) of 2008 is easily recognized in this list. Eg. in October and November 2008 the model was 100% in cash while in September, December 2008 and January 2009 for 67% (Note: $CASH$=VFISX).

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02-Sep-2008 - 01-Oct-2008: $CASH$ (66.6%), VBMFX (33.3%)
01-Oct-2008 - 03-Nov-2008: $CASH$ (100%)
03-Nov-2008 - 01-Dec-2008: $CASH$ (100%)
01-Dec-2008 - 02-Jan-2009: $CASH$ (66.6%), VBMFX (33.3%)
02-Jan-2009 - 02-Feb-2009: $CASH$ (66.6%), VBMFX (33.3%)
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7. Volatility and Risk Parity (RP)

The covariance matrix of returns has not entered the equation yet. This matrix has on its diagonal the squared volatilities vi for asset i. When we assume there is no correlation between funds, all sophisticated models come down to a simple rule: you should weigh your fund i with weights wi proportional to the inverse of each volatility vi (with 0<=wi<=1, sum wi =1). This rule is also known as the Risk Parity (RP) rule by some. When we don't assume a diagonal covariance matrix, more sophisticated strategies are used, eg. Equal Risk Contribution or ERC, see Maillard (2009) and Bruder (2012).

Of course we should base the volatilities on an estimate, eg. the observed volatility of daily (close) prices of the fund over a given lookback period of 4 months. So at this point we need daily data. The RP solution for each month for our 7 fund universe is then simple a set of 7 different weights, each inverse proportional to their volatilities.

Since we don't take into account returns and only use volatilities, the portfolio in case of RP often becomes heavily populated with bonds in recent years. Alternatively, in order to use equal weighting one might select the N (eg. 3) funds with the largest weights and rescale them to be equal and sum to 1. Probably, you will often end up with mostly bonds in this portfolio. The generalized momentum model might be a fix for this problem.

8. The loss function and generalized momentum

To bring volatility and other factors into the equation, we introduce the "loss function" L for our portfolio, with higher "loss" L meaning worse risk/return.

L=L(r, v),

with r, v the vectors of returns ri resp. volatilities vi of our U assets, and with the following derivatives:

dL/dri<0 (higher return ri is better), dL/dvi>0 (lower volatility vi is better)

We can use the loss function L on our universe (U funds), on the selected portfolio (N funds) or even on an individual fund i. It simply allows us to sort different portfolio's or funds, where lower loss L is always better according to some notion of risk/return.

An example of such a loss function is the inverse of the well known Sharpe function S(r,v) of the portfolio, with

S(r,v) = (w'r - rmin)/(w'Hw)

with w=w1..wn, r=r1..rn, rmin=risk-free rate, H=covar matrix of returns (equal to a diagonal matrix when all correlations are zero), but other loss functions are also possible of course. In fact, we are indifferent to the actual shape of the loss function as long as it obeys the simple rules dL/dri<0 and dL/dvi>0.

To arrive at our heuristic solution, we consider a simple Taylor expansion Li of the loss function L(r, v) wrt. (monotonic functions of) the returns ri and volatilities vi of fund i around a point in the ri/vi space. To stay close to the ordinary momentum, we will use a generalized ranking function for ri and vi (inspired by etfreplay.com):

$$Li = wR * rank(ri) + wV * rank(vi), \qquad (1)$$

where e.g. rank(ri) = 1 (best) when the return ri of fund i is the *maximum* return over all our 7 funds in a month and rank(ri) =7 (worst) when fund i has the minimum return over all our 7 funds. Similarly, rank(vi)=1 (best) when the volatility of fund i is the *minimum* volatility over all 7 funds in the universe and rank=7 (worst) when maximum. One can consider Li as a generalized ranking function, since it is simply a linear function of the ranks of the factors R and V, where rank=1 is always best.

The weights wR and wV determine the importance of (higher) return rank versus (lower) volatility rank in our generalized momentum function. Both weights should be non-negative.

Now, given wR and wV, we can sort all funds on Li and select the best N (say 3 out 7) funds. In fact, we replaced ranking on ri (as we did with relative return momentum) by ranking on Li, the generalized ranking. Since the sorting does not change when Li is multiplied by a constant, we can normalise the weights wR and wV, eg. by wR=1 or wR+wV=1.

Because we don't know the true return and risk, we will redefine the return ri and volatility vi as the corresponding estimate each month over the lookback period (here 4 months). So ri and vi are proxies of the true measures, and calculated each month using historical observations over the previous months (the lookback period). Together with relative (R) and absolute (A) momentum, we add volatility (V) momentum as third factor, in order to arrive at factors R, A and V in our generalized momentum model.

9. Example 3. Generalized momentum with factors R, A and V

Our model now contains various parameters: N (number of selected funds), wR, wV (weights for return and volatility) and a lookback period for each factor R, A and V, which we can all optimize in order to arrive at an optimal FAA strategy. However, optimizing over too many parameters will lead to serious datasnooping problems (see par. 13), so we will try to keep the degrees of freedom as low as possible.

Therefore, we will fix N=3 in our example and use a fixed lookback period of 4m for all factors, as we did before. Given the normalization of wR and wV parameters, we fix wR=1 and are left with only one degree of freedom: wV, which we will also fix at 50% for the moment, giving more weight to the (rank of the) Return factor than to the (rank of the) Volatility factor. This way we will end up with higher returns for the same risk/return figure, thereby lowering the leverage needed to arrive at a certain volatility level (see par. 13).

We will then use the generalized momentum function Li (eq. 1) to sort all funds on Li. To protect for crashes, we replace all funds i with ri<0 by Cash (VFISX in our example) after sorting on Li and selecting the best N funds. To estimate ri and vi, we will use a fixed lookback period of 4 months as before.

For the fixed parameters N=3, wR=1, wV=50% and a lookback period for R and V of 4 months, the backtest (in-sample) results in

R,A,V momentum: 03Jan2005-11Dec2012,cpf VFISX, 3/7, 4m/4m/4m, 100/50/0%:
R=12.5%,V=11.7%,D=-11.4%,S0=1.07,S2.5=0.86,S5=0.64,W=70.8%,T=3.1,O=2.25,Q0=1.10,Q5=0.66

Compared to the model with relative and absolute (R&A) momentum and no volatility(V), the return is improved (R=12.5%, was 11.7%), as the volatility V=11.7% (was 12.8%) and the maxDrawdown D=11.4% (was 12.6%). The corresponding graph is

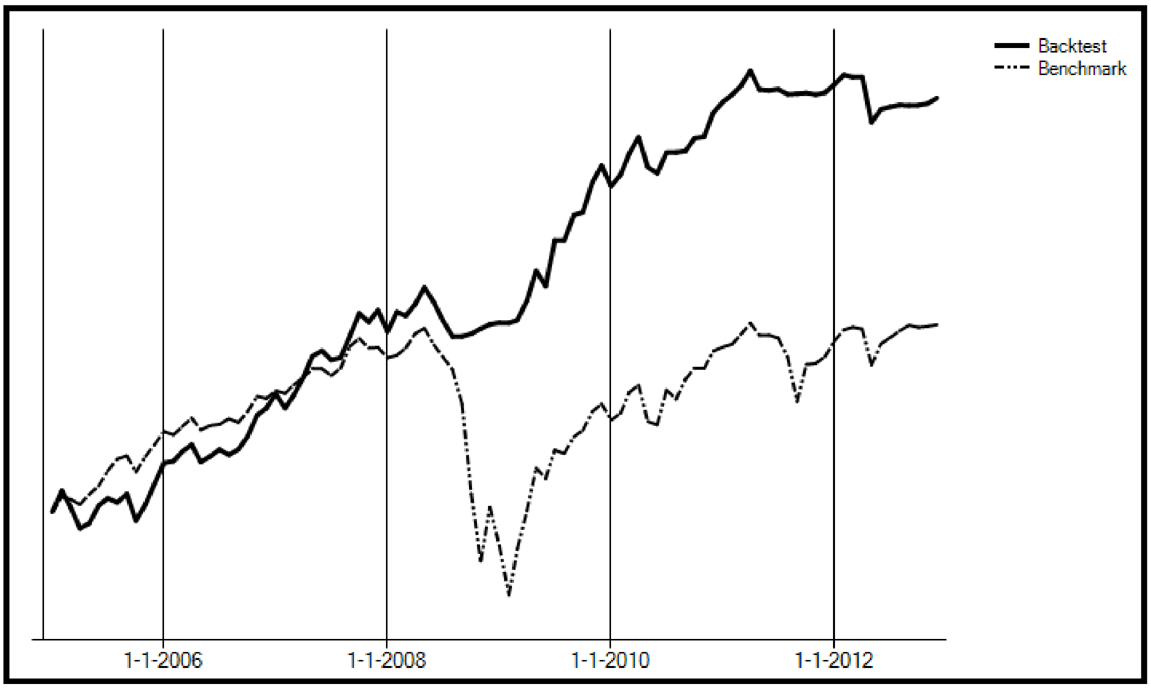

Fig. 3 Relative, absolute, and volatility (RAV) momentum (in-sample)

10. Correlations

The final factor is correlation. As shown in Varadi (2012), diversification of a portfolio is improved by lower average asset correlation, ie. by lower non-diagonal elements of the correlation matrix. With U assets or funds, there are (U-1) non-diagonal correlations per fund i. So we are able to rank the U funds in the universe according to their average correlation with respect to the other U-1 funds. The generalized ranking function (normalized by ranks) is now easily extended with correlation ci:

$$Li = wR * rank(ri) + wV * rank(vi) + wC * rank(ci) \quad (1)$$

where ci is the average (non-diagonal) correlation of fund i with all U-1 other funds and rank(ci)=1 and rank(ci)=U when ci is minimum respectively maximum over all U funds. In this way, funds with lower average correlations within the universe are more preferred since they will diversify better, and reduce volatility by their low or negative correlations.

Again the weight wC should be non-negative, and some normalization again applies on the three weights wR, wV and wC. Correlation will be our factor C in our generalized momentum model, consisting now of four factors: R, A, V, and C.

11. Example 4. Correlations

Using the generalized momentum function with wC>0

$$Li=wR * rank(ri) + wV * rank(vi) + wC * rank(ci), \quad (2)$$

we can sort each month the funds and select the best N funds in our equal weighted portfolio, and replace all funds with negative return momentum by cash (VFISX). Assuming a fixed lookback period of 4 months (for return, volatility and correlation) and a selection of the best N=3 funds out of 7, we run our backtest once weights wR, wV and wC are given. We will (somewhat arbitrarily) use the same weight for Correlation as for Volatility: wV=0.5 and wC=0.5 (while wR=1 by normalization). Then we find (see also fig. 4):

R,A,V,C momentum: 03Jan2005-11Dec2012,cpf VFISX, 3/7, 4m/4m/4m, 100/50/50%:
R=14.7%,V=9.2%,D=-7.4%,S0=1.60,S2.5=1.33,S5=1.06,W=74.0%,T=3.0,O=3.40,Q0=1.98,Q5=1.31

Now we see that diversification works as well: by introducing correlation momentum (factor C) in addition to the factors R, A and V, nearly all statistics improve: R= 14.7% (was 12.1% in the RAV model)), Volatility V=9.2% (was 11.7%), and maxDrawdown D=7.4% (was 10.4%). Because of the better return and lower risk, the return/risk statistics of the RAVC model are better: Sharpe S25=1.33 (was 0.86 in the RAV model), Omega O=3.40 (was 2.25) and Calmar Q5=1.31 (was 0.66) all improve substantially, showing that diversification works.

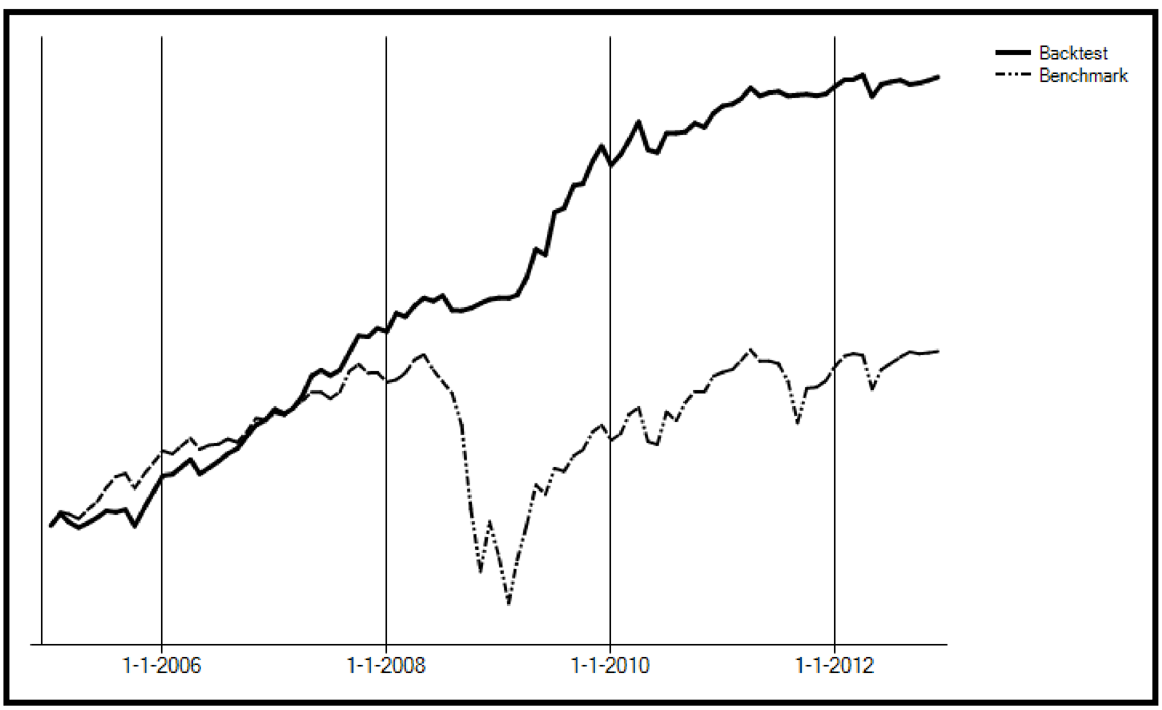

Fig. 4 Relative, absolute, volatility and correlation (RAVC) momentum (in-sample)

12. Extensions

Up till now, we used the loss function to rank funds and take the best N out of U funds for an *equal-weighted* (EW) portfolio of N funds. We can also use the loss function for unequal weights of funds. A possible unequal weight function could be based on the ranks of Li, with the best fund i with the lowest Li having rank 1 and the largest weight, etc. So the weights are then eg. (N+1-ranki)/sum(ranki). So when N=3 the weights become 3/6, 2/6, 1/6, with the highest weight for the best fund on Li. We can also mix the equal weight with the ranked weight with the formula wi= (1-a)*1/N + a * ranki/sum(ranki). For a=0.5 and N=3 this result in the weights 2.5/6, 2/6 and 1.5/6, while with a=0 we arrive at equal weights .

Before, we focused on the generalized rank version (1) of the loss function (see par. 8 and 11). Extensions with other shapes of the partial loss function can easily be developed. We give one example, where we focus on other type of monotonic functions than ranking, like logarithmics. For example we can maximize –Li where

$$-Li = wR * \ln (ri/rmax) + wV * \ln (vmin/vi) + wC * \ln ((cmin+1)/(ci+1)) \quad (3)$$

where ln(x) is the natural log of x and where rmax is the best (max) return over all funds, vmin is the best (min) volatility over all funds and cmin is the best (lowest) correlation ci over all funds. This way we apply weights wR, wV and wC to arguments ranging from minus infinity (worst) to 0 (ln(1)=best) and rank funds on –Li, where higher is better. Remember that funds with ri<0 are replaced by cash, and that vi>0 and -1<=cmin<=ci<=1.

By using the logarithmic function ln(x) we easily see that when wR=wV and wC=0, ranking of Li will be identical to the ranking of the return/risk ratio, ri/vi. When ri is measured in excess to the risk-free rate, this ratio becomes the well-known Sharpe ratio, which is maximized through -Li. You might therefore consider equation (3) as a (local) extension of the Sharpe measure, now including correlations as well.

We can also introduce models with different parameters (eg different lookback periods for the factors R, A, V and C), an additional momentum factor (eg. beta or value), or introduce an excess rate over a risk-free rate rmin for returns (ei=ri-rmin) and similar displacements for volatility and correlations, to take into account certain boundary values. Attention should also be paid to the absolute momentum rule in relation to the generalized momentum function, since some Li variants demand that ri>0 (eg. because ln(ri) is undefined when ri<=0).

Finally, one can "*optimize*" any generalized momentum model above by looking for the "best" parameters with respect to certain statistics. For example, we can easily improve on our (A,R, V,C) model in par. 11 by searching for better N, lookback months mR, mV, mC, and weights wR, wV and wC (given some normalizations).

As an example, we did some trial-and-error with just wV and wC as degrees of freedom. Then, assuming the same 4m lookback period for all factors (mR=mV=mC=4m) and wR=1 by normalization, we find the following optimal (or "best") solution with respect to the Calmar ratio Q5 with wV=80% and wC=60%:

> **R,A,V,C momentum**: 03Jan2005-11Dec2012,cpf VFISX, 3/7, 4m/4m/4m, **100/80/60%**:
> R=13.0%,V=7.4%,D=-5.2%,S0=1.76,S2.5=1.42,S5=1.08,W=71.9%,T=2.6,O=3.75,Q0=2.49,**Q5=1.53**

This "optimization" clearly improves Q5=1.53 (was 1.31), V=7.4% (was 9.2%) and D=5.2% (was 7.4%). Return R=13.0 (was 14.7%) is slightly worse.

We might also introduce *transaction costs* into this optimized model. It turns out that as long as transactions (switching) costs stay below 1.20% the return R wil be better than in the benchmark (R=5.7%, see par. 2 above). So even with heavy switching costs the model improves over the Buy & Hold benchmark..

To compare our solution to a Risk Parity strategy with a normalized SPY-like volatility of V=15% annual, we can use some leverage (i.e. 2x) to arrive at this volatility in our backtests. We also add transaction costs (0.1% per transaction) and *interest costs* of leverage (3% annual) to the model. Then we arrive at (see fig. 5):

R,A,V,C momentum: 03Jan2005-11Dec2012,cpf VFISX, lev=2,cost%j=3,tc=0.1%,3/7, 4m/4m/4m, 100/80/60%:
R=22.6%,V=14.7%,D=-10.7%,S0=1.54,S2.5=1.37,S5=1.20,W=70.8%,T=2.6,O=3.21,Q0=2.12,Q10=1.18

Now we have a Return of R=22.6% with a Volatility of nearly 15% and a maxDrawdown D=10.7%. Remember that we are now (with optimized wV and wC) possibly relying on datasnooping. Therefore we will test both the default solution (with N=3, m=4m, wR/wV/wC=100/50/50%) as well as the "best" solution (with 4m, 100/80/60% and 2x leverage with costs) in the next paragraph *out-of-sample*.

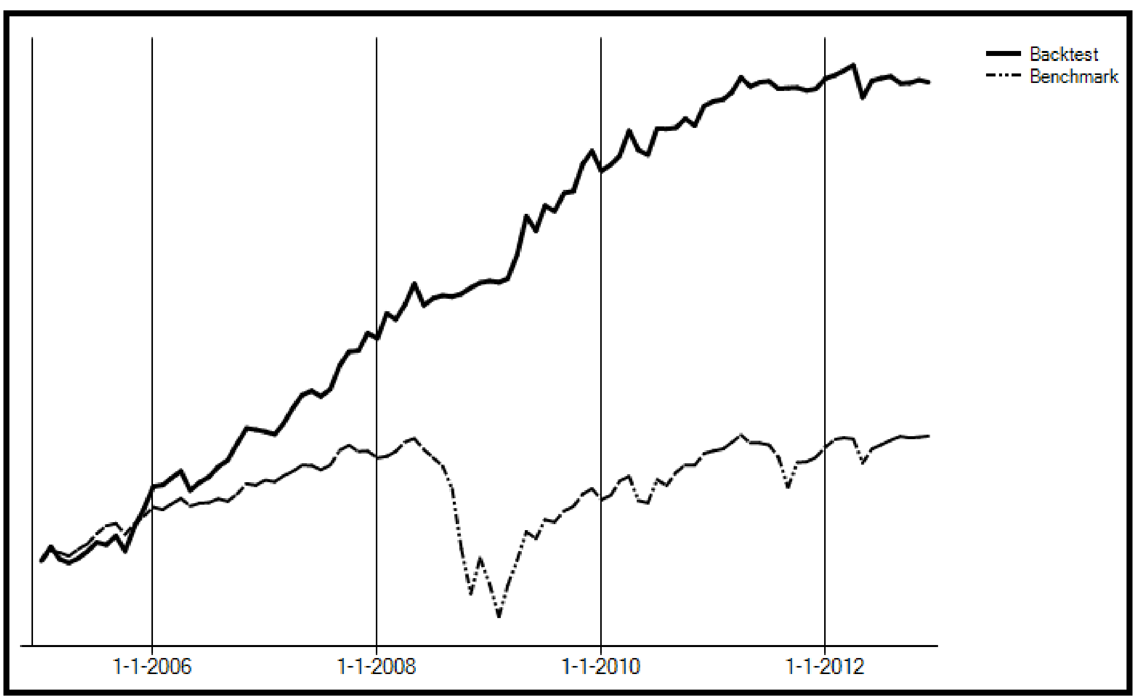

Fig. 5 Optimized model with RAVC momentum, including leverage costs and transaction costs (in-sample)

13. Datasnooping: out-of-sample testing

In the last paragraph, we presented the “optimal”solution in terms of Q5 by searching for the best parameters like weights wV and wC, for the in-sample period 2005-2012. We also presented the solution in terms of the default parameters. In both cases there is probably a risk of “datasnooping”, although the risks are of course greater when we optimize the parameters instead of using default parameters. The risk of finding such “best-fit” solution is that we twist the model (by changing parameters) to follow the data so that any dataset will be forced to come up with some attractive return/risk figures. This is called datasnooping or datamining.

To test for datasnooping, we use the in-sample data for learning and the out-of-sample data for testing. Since we have data from Jan. 1998 we will use the period 1998-2004 as out-of-sample period.

First we will test the set of default parameters and check the B&H benchmark in this “out-of-sample” period:

R,A,V,C momentum: 02Jan1998-03Jan2005,cpf VFISX, 3/7, 4m/4m/4m, 100/50/50%
R=13.4%,V=7.7%,D=-5.9%,S0=1.73,S2.5=1.41,S5=1.08,W=75.9%,T=2.9,O=3.93,Q0=2.28,Q5=1.43

Bench: 02Jan1998-03Jan2005,7/7, 4m/4m/4m, 100/50/50%:
R=8.3%,V=9.8%,D=-15.2%,S0=0.85,S2.5=0.60,S5=0.34,W=61.4%,T=0.1,O=1.89,Q0=0.55,Q5=0.22

The graph for the entire period 1998-2012 (in- and out-of-sample) is

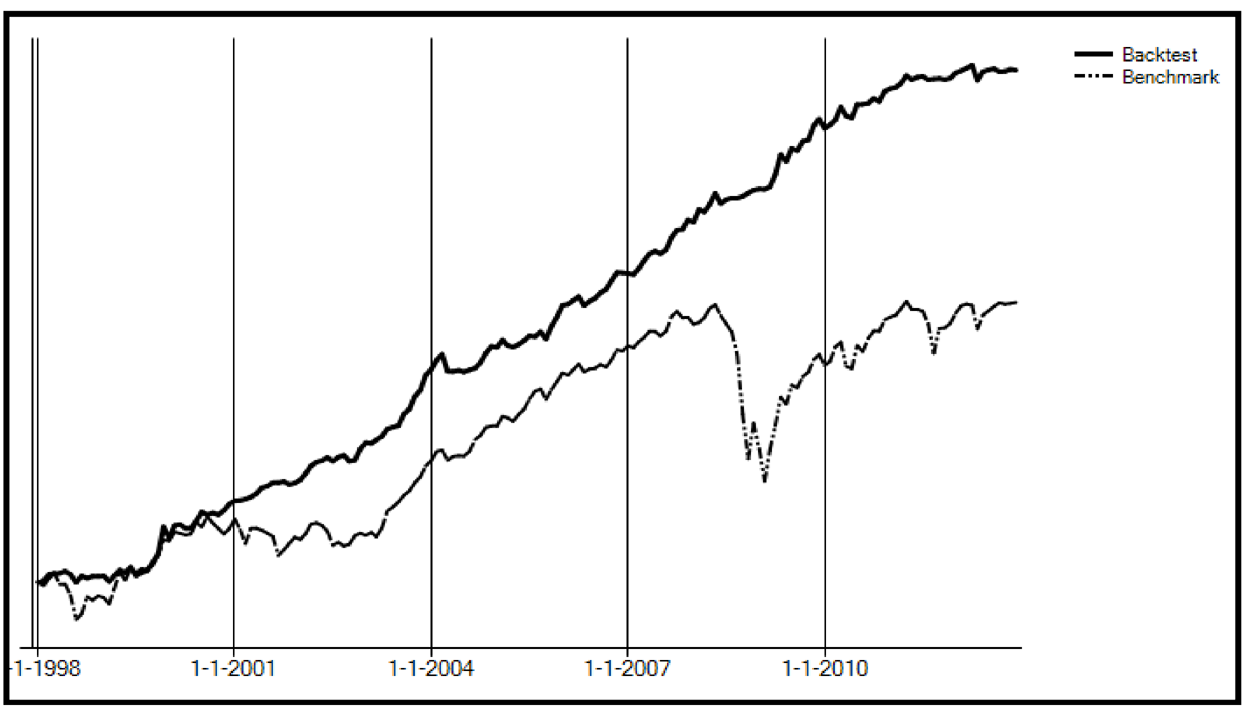

Fig. 6. Default model with A, R, V and C momentum (in- and out-of-sample)

It is clear that the out-of-sample test is rather successful: most statistics (R, D, S, W, and O) are good and better than the statistics of the same model in the in-sample period. Also they are all also much better than the B&H benchmark in the same period.

Now we will test the in-sample optimized parameters out-of-sample (with leverage and costs). Since we use a leverage of 2x, we will also use a Calmar ratio (Q10) based on a risk-free hurdle of 10% instead of 5%.

R,A,V,C momentum: 02Jan1998-03Jan2005,cpf VFISX, lev=2,cost%j=3,tc=0.1%,3/7, 4m/4m/4m, 100/80/60%
R=21.6%,V=14.5%,D=-12.9%,S0=1.49,S2.5=1.31,S5=1.14,W=74.7%,T=2.1,O=3.29,Q0=1.67,Q10=0.90

Bench: 02Jan1998-03Jan2005,7/7:
R=8.3%,V=9.8%,D=-15.2%,S0=0.85,S2.5=0.60,S5=0.34,W=61.4%,T=0.1,O=1.89,Q0=0.55,Q10=-0.11

As we see, the Volatility in the out-of-sample period is again close to 15%. Due to the leverage (2x) the maxdrawdown D=12.9% is substantial in the out-of-sample period for the optimized model but still not larger than the D=15.2% in the benchmark (without leverage or costs). Additionally, the R is much better (R=21.6% compared to R=8.3% in the unlevered B&H bench), driving also all return/risk statistics (S, W, O, Q) much higher. The graph for the whole period (in- and out-of-sample) of the optimized and leveraged RAVC model is:

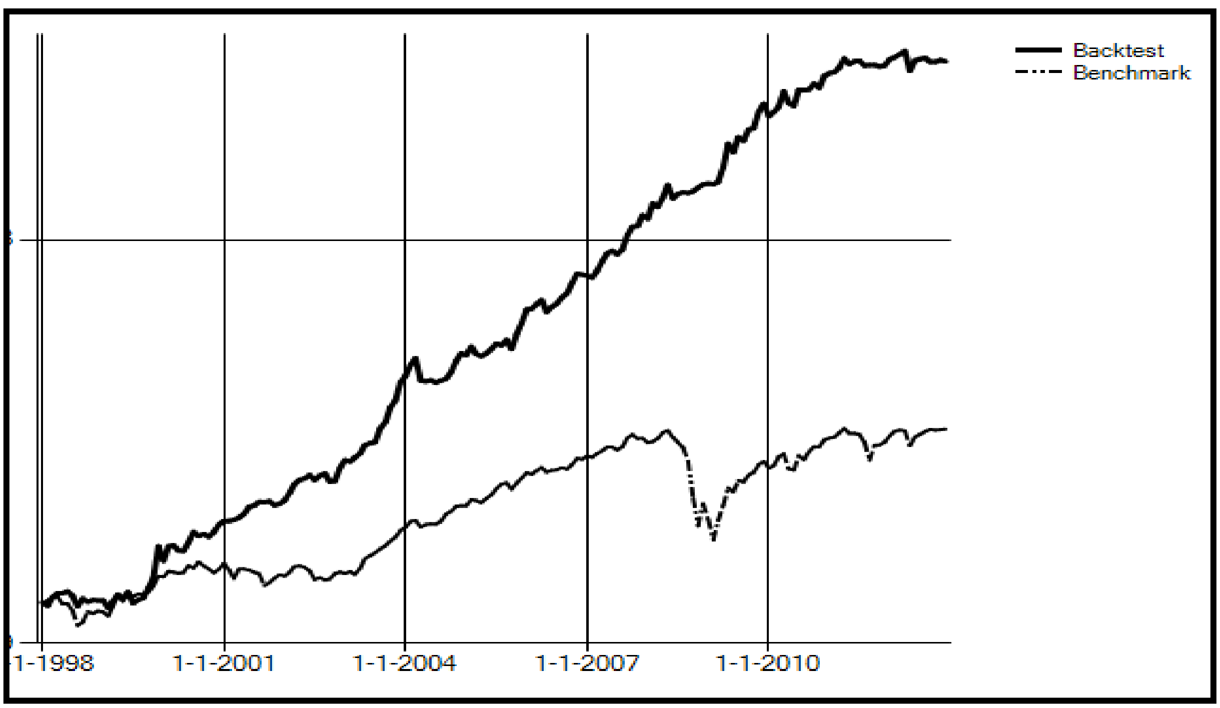

Fig. 7. Optimized model with RAVC momentum, leverage and costs (in- and out-of-sample). NB. The benchmark is without leverage and costs.

14. Robustness

Besides in- and out-of-sample testing to test for datasnooping, we might also employ *robustness tests* as an indication of (the lack of) datasnooping. We will consider two kinds of robustness here: first, what happens if we change our *model* (parameters) slightly, and second, what happens if we change our *data* slightly.

First we will check for the *model robustness* of the default backtest with lookback period of four months and the default weights of 100/50/50% for factors R,V and C for the whole period from 1998 on. This default backtest (with RAVC momentum) and the B&H benchmark over 1998-2012 yields the following results:

R,A,V,C momentum: 02Jan1998-14Dec2012,cpf VFISX, 3/7, **4m/4m/4m, 100/50/50%:**
R=14.2%,V=8.5%,D=-7.4%,S0=1.67,S2.5=1.38,S5=1.08,W=75.0%,T=2.9,O=3.65,Q0=1.92,Q5=1.25

Bench: 02Jan1998-14Dec2012, 7/7:
R=6.8%,V=13.8%,D=-46.3%,S0=0.50,S2.5=0.31,S5=0.13,W=62.8%,T=0.0,O=1.58,Q0=0.15,Q5=0.04

Here we use a lookback period of length 4 months for all factors (R/A, V,C) for the default backtest. In the figure below we present the annual Return R, the Sharpe S5 and the Calmar ratio Q5 (both using a hurdle of 5%) for different length of the lookback period (1,2,3,4,5,6, 9 and 12 months), again the same for all factors R,A,V,C.

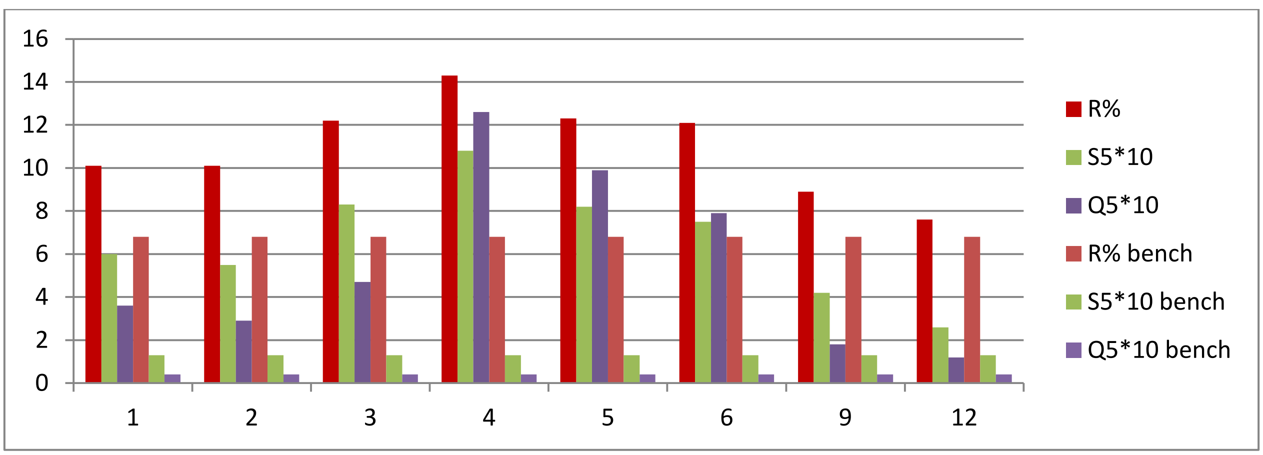

Figure 8. Annual return R, Sharpe S5 and Calmar Q5 by lookback period (months), 1998-2012

As shown, it is clear that a lookback period of length 4 months is optimal in terms of R, S5 and Q5. When we compare these results to the benchmark (with R=6.8%, S5=0.13, Q5=0.04), it is also clear that they all are better than the benchmark figures.

We can also check the levered model of par. 12 for robustness, with the optimal (best) in-sample parameterization applied to the whole period from 1998-2012 (see also fig. 7):

R,A,V,C momentum: 02Jan1998-14Dec2012,cpf VFISX,lev=2,cost%j=3,tc=0.1%,3/7,**4m/4m/4m, 100/80/50%:**
R=23.2%,V=15.2%,D=-16.5%,S0=1.53,S2.5=1.36,S5=1.20,W=70.6%,T=2.5,O=3.30,Q0=1.41,Q10=0.80

Bench: 02Jan1998-14Dec2012, 7/7:
R=6.8%,V=13.8%,D=-46.3%,S0=0.50,S2.5=0.31,S5=0.13,W=62.8%,T=0.0,O=1.58,Q0=0.15,Q10=-0.07

To test for model robustness of all parameters involved, we backtest all simple deviations (plus and minus a step) of the 6 parameters from this best solution: the number N (+/- one), the lookback lengths mR, mV, mC (+/1 one month) for the factors RAVC and the weights wV and wC (+/- 10%, with wR=100% as normalization). The results are (first line is best):

02Jan1998-14Dec2012,cpf VFISX,lev=2,cost%j=3, tc=0.1%
3/7,4m/4m/4m, 100/80/50%: R=23.2%,V=15.2%,D=-16.5%,S0=1.53,S2.5=1.36,S5=1.20,W=70.6%,Q0=1.41,T=2.5,O=3.30,Q10=0.80

4/7, 4m/4m/4m,100/80/50%:R=19.0%,V=14.4%,D=-17.4%,S0=1.32,S2.5=1.14,S5=0.97,W=68.9%,Q0=1.09,T=2.1,O=2.70,Q10=0.51
2/7,4m/4m/4m, 100/80/50%: R=15.4%,V=16.5%,D=-19.6%,S0=0.93,S2.5=0.78,S5=0.63,W=68.9%,Q0=0.78,T=3.0,O=2.37,Q10=0.27
3/7,**5m**/4m/4m, 100/80/50%: R=19.2%,V=16.7%,D=-18.9%,S0=1.15,S2.5=1.00,S5=0.85,W=65.0%,Q0=1.02,T=2.3,O=2.56,Q10=0.49
3/7,**3m**/4m/4m, 100/80/50%: R=18.8%,V=15.7%,D=-23.5%,S0=1.20,S2.5=1.04,S5=0.88,W=68.3%,Q0=0.80,T=3.0,O=2.78,Q10=0.37
3/7,4m/**5m**/4m, 100/80/50%: R=23.2%,V=14.9%,D=-12.9%,S0=1.55,S2.5=1.38,S5=1.22,W=70.6%,Q0=1.79,T=2.5,O=3.38,Q10=1.02
3/7,4m/**3m**/4m, 100/80/50%: R=21.1%,V=14.8%,D=-15.3%,S0=1.42,S2.5=1.25,S5=1.08,W=68.3%,Q0=1.38,T=2.6,O=3.05,Q10=0.72
3/7,4m/4m/**5m**, 100/80/50%: R=23.0%,V=15.0%,D=-12.9%,S0=1.53,S2.5=1.36,S5=1.20,W=71.1%,Q0=1.77,T=2.4,O=3.33,Q10=1.00
3/7,4m/4m/**3m**, 100/80/50%: R=23.7%,V=14.8%,D=-12.9%,S0=1.60,S2.5=1.43,S5=1.26,W=71.7%,Q0=1.83,T=2.6,O=3.52,Q10=1.05
3/7,4m/4m/4m, **110**/80/50%: R=23.6%,V=16.1%,D=-16.5%,S0=1.47,S2.5=1.31,S5=1.15,W=71.1%,Q0=1.43,T=2.5,O=3.14,Q10=0.82
3/7,4m/4m/4m, **90**/80/50%: R=21.4%,V=14.4%,D=-12.9%,S0=1.48,S2.5=1.31,S5=1.14,W=70.6%,Q0=1.65,T=2.4,O=3.27,Q10=0.88
3/7,4m/4m/4m, 100/**90**/50%: R=21.3%,V=15.1%,D=-12.9%,S0=1.41,S2.5=1.24,S5=1.08,W=69.4%,Q0=1.64,T=2.4,O=3.13,Q10=0.87
3/7,4m/4m/4m, 100/**70**/50%: R=23.7%,V=16.1%,D=-16.5%,S0=1.48,S2.5=1.32,S5=1.16,W=71.7%,Q0=1.44,T=2.5,O=3.15,Q10=0.83
3/7,4m/4m/4m, 100/80/**60**%: R=22.3%,V=14.6%,D=-12.9%,S0=1.53,S2.5=1.36,S5=1.19,W=72.8%,Q0=1.73,T=2.4,O=3.28,Q10=0.95
3/7,4m/4m/4m, 100/80/**40**%: R=22.7%,V=15.9%,D=-17.4%,S0=1.42,S2.5=1.27,S5=1.11,W=68.9%,Q0=1.31,T=2.5,O=3.03,Q10=0.73

At each line, the deviations (with bold parameters) of the best (optimal) solution (first line) are presented. This results in 1+2x6=13 backtests. Over all these 13 observations, we take the minimum and maximum value for each parameter and arrive at the following *model robustness ranges* (min-max):

R=15-24%, V=14-17%, -D=13-24%, S0=0.9-1.6, S25=0.8-1.4, S5=0.6-1.3, W=65-73%, O=2.4-3.5, Q0=0.8-1.8, Q10=0.3-1.1

Alternatively, we can change the *data* to test for robustness. In this case we simply drop each of our 7 funds and run the backtest again with the given parameters (moving to cash with r=0 when dropping cpf=VFISX). This gives us 7 deviations plus one best solution (at the first line) is 8 observations:

02Jan1998-14Dec2012,cpf VFISX, lev=2,cost%j=3,tc=0.1%:
3/7,4m/4m/4m, 100/80/50%: R=23.2%,V=15.2%,D=-16.5%,S0=1.53,S2.5=1.36,S5=1.20,W=70.6%,Q0=1.41,T=2.5,O=3.30,Q10=0.80, **best**

3/6, 4m/4m/4m, 100/80/50%: R=22.6%,V=15.5%,D=-19.8%,S0=1.46,S2.5=1.30,S5=1.14,W=68.9%,Q0=1.14,T=2.2,O=3.14,Q10=0.64, drop VTSMX
3/6,4m/4m/4m, 100/80/50%: R=20.7%,V=14.5%,D=-17.4%,S0=1.43,S2.5=1.26,S5=1.08,W=68.9%,Q0=1.19,T=2.3,O=2.95,Q10=0.62, drop FDIVX
3/6,4m/4m/4m, 100/80/50%: R=18.4%,V=12.9%,D=-15.9%,S0=1.43,S2.5=1.23,S5=1.04,W=70.0%,Q0=1.16,T=2.2,O=3.02,Q10=0.53, drop VEIEX
3/6,4m/4m/4m, 100/80/50%: R=22.4%,V=18.4%,D=-21.5%,S0=1.22,S2.5=1.08,S5=0.95,W=66.1%,Q0=1.04,T=2.3,O=2.55,Q10=0.58, drop VBMFX
3/6,4m/4m/4m, 100/80/50%: R=21.3%,V=18.6%,D=-20.7%,S0=1.15,S2.5=1.01,S5=0.88,W=65.0%,Q0=1.03,T=2.8,O=2.49,Q10=0.55, drop VFISX
3/6,4m/4m/4m, 100/80/50%: R=18.1%,V=15.6%,D=-20.2%,S0=1.16,S2.5=1.00,S5=0.84,W=66.7%,Q0=0.90,T=2.4,O=2.63,Q10=0.40, drop VGSIX
3/6,4m/4m/4m, 100/80/50%: R=16.2%,V=14.8%,D=-19.0%,S0=1.09,S2.5=0.92,S5=0.76,W=69.4%,Q0=0.85,T=2.5,O=2.53,Q10=0.33, drop QRAAX

Over all these 1+7=8 observations, we also take the minimum and maximum value for each parameter and arrive at the following *data robustness ranges* (min-max):

R=16-23%, V=13-19%, -D=16-21%, S0=1.1-1.5, S2.5=0.9-1.3, S5=0.7-1.1, W=65-70%, O=2.5-3.1, Q0=0.8-1.2, Q10=0.3-0.6

which is in the same order of magnitude as the *model robustness* ranges. Combining both min-max ranges we arrive at the combined (and widest) robustness ranges:

R=15-24%, V=13-19%, -D=13-24%, S0=0.9-1.6, S25=0.8-1.4, S5=0.6-1.3, W=65-73%, O=2.4-4.2, Q0=0.8-1.8, Q10=0.3-1.1

If we compare the “worst” robustness scores (highest V=19%, D=24%, lowest R=15%, S5=0.6, Q0=0.8, etc) of this list with the benchmark parameters, we arrive at the following figure.

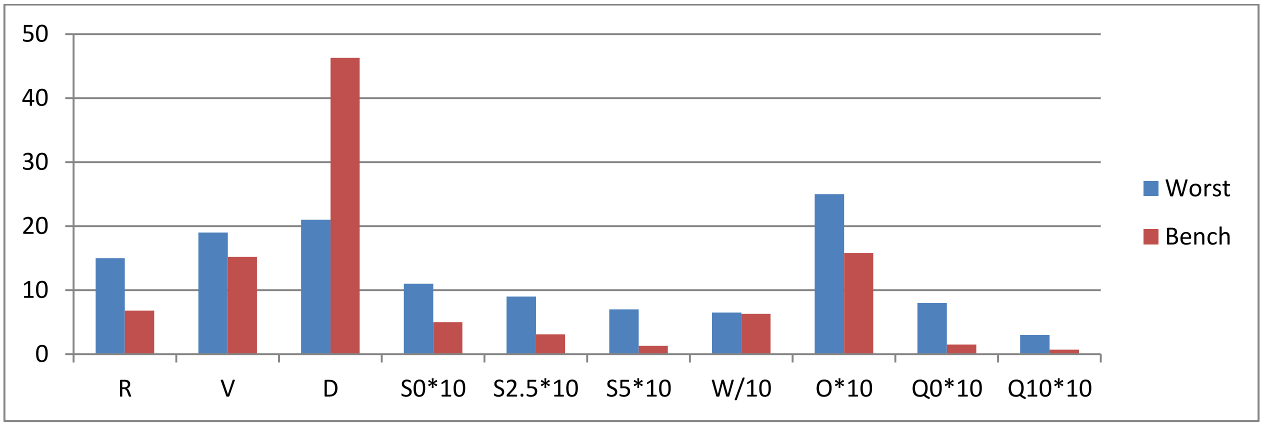

Fig. 9. Comparing the “worst” robustness scores with the benchmark scores, 1998-2012

From the figure we see that all these “worst” robustness scores are (much) better than the corresponding benchmark scores, except for volatility V (where the bench is slightly better, due to our normalization towards V=15%). Return/risk is also much better as indicated by the Sharpe S, and Calmar Q ratio’s, which are all at least twice the corresponding benchmark figures.

Of course, this result depends on the size of the (model and data) deviations, but at least it shows that our best (optimized in-sample) results are robust to a certain extend. Together with the out-of-sample test, this gives some indication that our results are not purely the result of datasnooping. Besides testing the robustness of a backtest against the benchmark, we can also use this kind of robustness checks to compare different backtest models.

Alternatively, we might have drawn *random* samples in both the 6 model and the 7 data dimensions and compute averages and ranges (eg. twice the standard deviations over the samples), but as long as the samples are inside the above deltas the ranges will also be within our min-max ranges computed above. The position and width of these ranges are again a possible indication of the (lack of) robustness of our solution.

15. Conclusions

We have extended the traditional momentum model with 3 factors towards a more general momentum model: Absolute momentum (A), Volatility momentum (V), and Correlation momentum (C). We have shown that by using a linear ranking (or other monotonic) function for these four factors we are able to come up with simple but powerful asset allocation strategies, which we called Flexible Asset Allocation (FAA).

Specifically, at the end of each month, we allocate our money to an equally weighted portfolio of N out of U assets with the highest generalized momentum over the past m months. The model parameters are the weights and lookback period for the return, volatility and correlation momentum and the number N, given some normalizations and a universe of U assets or funds.

We can test each set of parameters by running backtests, using dividend-adjusted closing prices for all assets.

In applying our generalized momentum model, there is no need to run complex computational procedures like quadratic optimization under restrictions, while we can incorporate all first and second-order characteristics (like returns and covariances) of the total return series of our assets. All computations can be done using e.g. spreadsheets. There is also less chance to arrive at singular corner solutions as in the case of quadratic optimizations under restrictions.

We have illustrated our generalized momentum model by demonstrating the monthly strategy step-by-step with several backtests with a simple 7-asset portfolio (including global index funds for stocks, government bonds, commodities and REITs) from January 1998 to December 2012, both in-sample and out-of-sample using default values for most of the parameters of our model. We find using our generalized momentum model much better solutions than with the B&H benchmark, both in terms of return as risk.

Finally, we tested the degree of datasnooping in our generalized momentum model by computing an out-of-sample test based on simple in-sample model optimization of our 7-asset example. Additionally we performed some robustness tests in both the model and data dimensions to arrive at minimum and maximum ranges for results like return, volatility and drawdown in our example. We demonstrated that our solutions show strength even out-of-sample and are relatively robust for moderate changes in model parameters and data.

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