Modeling viscoelastic seismic wave propagation in Deccan flood basalt, western India

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Summary

Synthetic seismic response of a realistic model representing Deccan flood basalt was computed following elastic and viscoelastic formulation of wave propagation. The model was prepared by using measured data on core samples, obtained from KLR-1 borehole (18^0 03' 07'' N, 76^0 33' 20'' E), drilled at Killari in Deccan Volcanic Province of western India. Drilling results from KLR-1 borehole indicate that in Killari, a thin layer of sediments exists, which cannot be resolved seismically. Laboratory results indicate that there exists appreciable velocity contrast between different layers of basalts. We also observed differences in attenuation values in different layers of basalt. This makes, Deccan basalts, a motivating case for viscoelastic formulation of seismic wave propagation. In this paper we attempt to address the effect of velocity contrast and attenuation on seismic modeling by simulating the wave propagation in Deccan basalt, following viscoelastic formulations. We show that, the seismic waves are highly attenuated in the viscoelastic media, with slight velocity dispersion as compared to the elastic media. We can also infer that the viscoelastic formulations with three relaxation mechanisms (i.e. L=3) can better incorporate Q.

Introduction

The Deccan flood basalts of western India were formed due to episodic lava emplacement process and thus offer heterogeneity at various levels. The Deccan basalts are being studied for exploration of Mesozoic sediments, lying below the thick traps cover. In principle, seismic modelling can be used to model the heterogeneity of Deccan basalt on the seismic wavefield. Drilling results suggest that the Deccan basalts can be broadly divided into two categories namely massive and vesicular. The commonly used seismic modeling algorithms are mainly based on either the acoustic wave equation, which considers the medium to behave ideally or on elastic wave equations, which can better determine the amplitudes of P- and S- waves. However, elastic wave formulation does not account for amplitude decay due to inherent attenuation. Hence, in case of multilayered Deccan basalt, the wave propagation has to be anelastic and the viscoelastic wave formulation could be a better choice as it incorporates quality factor (Q) accurately. We know that Q depends upon the frequency, but it can be assumed to be a constant over the seismic frequency band (McDonal et. al., 1958). Many authors have proposed different approximations for the viscoelastic stress-strain relation, using a convolution integral. Carcione (1988) introduced the memory variable approach for viscoelastic wave propagation using pseudospectral method. An overview of all methods based on various rheological models, i.e., Generalized Maxwell Body and Generalized Zener Body (GZB), are given by Cao (2014). It should be noted that both GMB and GZB are equivalent (Moczo and Kristek, 2005).

Q can be approximated to a constant value using the Standard Linear Solid (SLS), also known as GZB, for a given set of parameters, which are stress relaxation time and strain relaxation time for each l^{th} mechanism of SLS body. However, determination of these parameters is an optimization problem. Blanch et. al., (1995) proposed 'tau-method' using which these parameters can be estimated optimally for SLS consisting of L elements over a given frequency range. In a recent study, Zhu et. al., (2013) showed that, the simulation results obtained for SLS, with L=1 and L=3 are equivalent for viscoelastic medium.

For Deccan basalts, where Q_p ranges from 33.07 – 1959.77 and 6.38 – 46.28 for massive and vesicular basalt respectively and Qs from 26.00 – 506.09 and 5.29 – 49.12, we can expect pronounced effect of dispersion and attenuation in seismic modeling. Thus, to see the importance of considering anelastic effects of Deccan basalts in seismic wave propagation, we generated synthetic seismograms following viscoelastic formulations of wave equation. The realistic Deccan basalt model parameters viz.: density, V_p, V_s, Q_p and Q_s, were obtained from the laboratory measurements carried out on 37 water saturated cores recovered from KLR-1 borehole. The KLR-1 borehole penetrated 338 m of Deccan flood basalts, followed by a further 270 m of 2.57 Ga old Archean crystalline basement.

Method

Elastic formulation for wave propagation

Seismic wave propagation in a given elastic medium can be defined by using different formulations viz., Displacement-Stress, Velocity-Stress, etc. For this study, we use velocity-stress (elastic) formulation, which can be written succinctly as:

$$\rho \frac{\partial v_i}{\partial t} = \partial_j \sigma_{ij} + \rho f_i \tag{1}$$

$$\frac{\partial \sigma_{ij}}{\partial t} = \lambda\, \dot{\varepsilon}_{k,k}\, \delta_{ij} + \mu \left(\dot{\varepsilon}_{i,j} + \dot{\varepsilon}_{j,i}\right) \tag{2}$$

where, $\dot{\varepsilon}_{i,j} = \frac{1}{2}\left(\frac{\partial v_j}{\partial x_i} + \frac{\partial v_i}{\partial x_j}\right)$

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($\dot{}$) represents time derivative of the function, ρ is density; v_i is velocity component; σ_{ij} represent stress components; f_i is external body force; λ, μ are lamé parameters; δ_{ij} is Kroneker delta function; and i, j, k ∈ {x,y,z}.

Eqs. (1) and (2) form a coupled differential equation and can be solved using Finite Difference method (FDM) (Graves, 1996). Figure (1) shows the distribution of field parameters ($\sigma_{xx}, \sigma_{zz}, \sigma_{xz}, v_x, v_z$) and material parameters (ρ, λ, μ) on a staggered grid that are used in elastic formulation. The numerical grid for the forward modeling was staggered in space and time, with fourth and second order accuracies, $O(\Delta x^4, \Delta t^2)$, respectively.

Viscoelastic formulation for wave propagation

For this study we have used the time domain vicoelastic formulation with memory variable approach (Carcione, 1988). The stress (σ_{ij}) strain (ϵ_{kl}) relationship for the viscoelastic case can be written in the form of convolution integral as,

$$\sigma_{ij} = C_{ijkl} * \dot{\epsilon_{kl}} = \dot{C}_{ijkl} * \epsilon_{kl} \tag{3}$$

where, C_{ijkl} is a fourth order tensor of elastic constants. For a Standard Linear Solid (SLS) comprised of L relaxation mechanisms, C_{ijkl} can be represented using a relaxation function, $\psi(t)$ of following form (with a correction factor $\frac{1}{L}$, introduced by Carcione (2007)):

$$\psi(t) = M_R\left[1 - \frac{1}{L}\sum_{l=1}^{L}\left(1 - \frac{\tau_\varepsilon^l}{\tau_\sigma^l}\right)e^{(-t/\tau_\sigma^l)}\right]H(t). \tag{4}$$

M_R is the relaxed modlus, τ_σ^l and τ_ε^l are stress and strain relaxation times, respectively.

Eq. (3) can be expressed in a more convenient form for 2D case, (Robertsson et. al., 1994)

$$\sigma_{ij} = \dot{\Lambda}(t) * \varepsilon_{kk}(t)\delta_{ij} + \dot{M} * \varepsilon_{ij} \tag{5}$$

Stress-strain and memory variable equations can be obtained by substituting eq. (4) into eq. (5),

$$\frac{\partial\sigma_{ij}}{\partial t} = \left[\Pi_R\left(1 - \sum_{l=1}^{L} T_l^S\right) - 2\,M_R\left(1 - \sum_{l=1}^{L} T_l^S\right)\right]\dot{\varepsilon}_{kk}\delta_{ij}. + 2M_R\left[1 - \sum_{l=1}^{L} T_l^S\right]\dot{\varepsilon}_{ij} - \sum_{l=1}^{L} r_{ijl}, \tag{6}$$

and,

$$\frac{\partial r_{ijl}}{\partial t} = \frac{-1}{\tau_{\tau l}} r_{ijl} + \frac{1}{\tau_{\sigma l}}\left(\Pi_R\,T_l^P - 2M_R T_l^S\right)\dot{\varepsilon}_{kk}\delta_{ij} + \frac{2}{\tau_{\sigma l}} M_R\,T_l^S\dot{\varepsilon}_{ij}, \tag{7}$$

where, $\Pi = \Lambda + 2M$; Π_R and M_R are relaxed modulus for respective fuctions and T_l^P, T_l^S are $\frac{1}{L}\left(1 - \frac{\tau_\varepsilon^{lp}}{\tau_\sigma^l}\right)$ and $\frac{1}{L}\left(1 - \frac{\tau_\varepsilon^{ls}}{\tau_\sigma}\right)$, respectively. Eqs. (6) and (7), together with Newton's stress balance equation, $\rho\frac{\partial v_i}{\partial t} = \frac{\partial\sigma_{ij}}{\partial x_i} + f_i$, forms a complete set of equations for viscoelastic modeling, which can be solved using FDM on a staggered grid (Bohlen, 2002). Figure (1) shows the distribution of field parameters ($\sigma_{xx}, \sigma_{zz}, \sigma_{xz}, v_x, v_z, r_{xx}, r_{zz}, r_{xz}$) and material parameters (ρ, Λ, M, Π) on a staggered grid that are used in viscoelastic formulation.

Determination of Q

$Q(\omega)$, for a given frequency range, can be defined as the ratio of real to the imaginary value of the complex modulus, M^C:

$$Q(\omega) = \frac{\Re e[M^C(\omega)]}{\Im m[M^C(\omega)]} \tag{8}$$

where, $M^C(\omega) = \mathscr{F}\{\,\partial_t[\psi(t)]\,\}$ (Carcione, 2007) and, $\mathscr{F}\{\}$ is the Fourier transform of the time derivative of relaxation function.

For a given relaxation function, Q can be written as,

$$Q(\omega) = \frac{\sum_{l=1}^{L}\left(\frac{\omega^2\tau_\sigma^{l\,2}\tau}{1+\omega^2\tau_\sigma^{l\,2}}\right) - L}{\sum_{l=1}^{L}\left(\frac{\omega\tau_\sigma^l\tau}{1+\omega^2\tau_\sigma^{l\,2}}\right)} \tag{9}$$

where, $\tau = \left(1 - \frac{\tau_\varepsilon^l}{\tau_\sigma^l}\right)$

Q can be optimized for the determination of τ_ε^l by minimizing the integral, ϕ, over a frequency range ω_1 and ω_2 (Blanch, 1995):

$$\phi = \int_{\omega_1}^{\omega_2}\left[Q^{-1}(\omega, \tau_\sigma^l, \tau_\varepsilon^l) - Q_0^{-1}(\omega)\right]^2 d\omega \tag{10}$$

and, ${\tau^l}_\sigma = \frac{1}{\omega_l}$ is the stress relaxation time.

However, Q is overestimated using tau-method and it may require appropriate scaling to match desired Q_0 as shown in Figure (2).

Boundary Conditions

We have applied absorbing boundary condition of damping nature to suppress edge reflections (Cerjen, 1985) to all sides of the model. Same boundary conditions are used for both elastic and viscoelastic cases. To ensure the stability of FD scheme the time step has to be bounded according to the relation, $dt < \frac{6h}{7\sqrt{3}v_{max}}$. The numerical dispersion can be constrained to be less than 5% if the grid size is less than $\frac{\lambda_{min}}{6}$ i.e., there has to be at least 6 grids per shortest wavelength (Bohlen, 2002).

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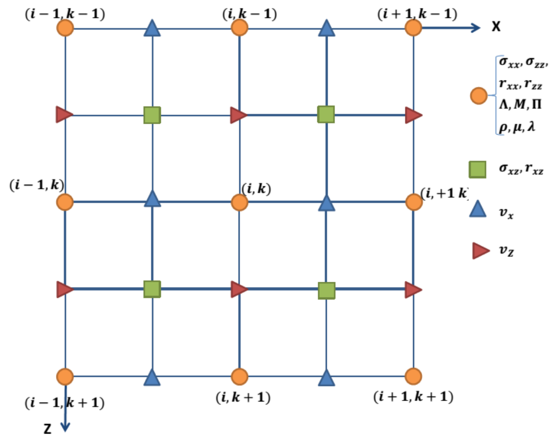

Figure (1): Composite arrangement for elastic and viscoelastic formulation on a staggered grid, depicting the position of field variables (stress, velocity, memory variables, etc.), material parameters (density, Lamé parameters, and relaxation functions). Field parameters common to elastic and viscoelastic, share same position.

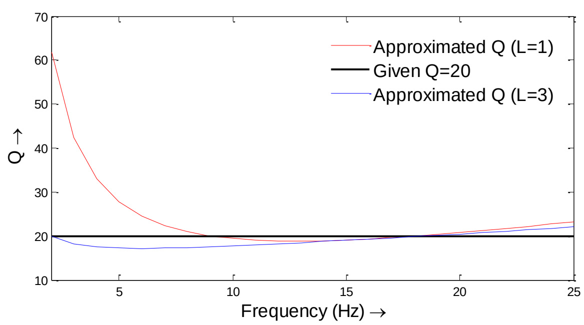

Figure (2): Comparison of approximated Q, for L=1 and L=3, with given Q, over a frequency range, 2 Hz – 25 Hz.

Laboratory Measurements

Total 37 water saturated basalt cores from Killari borehole were analyzed, following the standards laid down by the International Society of Rock Mechanics at rock mechanics lab, CSIR-NGRI. The velocity measurements were carried out using time-of-flight ultrasonic pulse transmission technique at room P-T conditions. Density of samples was measured by calculating it's mass to bulk volume ratio (Lakshmi et. al, 2013). Q^{-1} was estimated using the pulse broadening test as described in Rao et al., (2002), which provides a simple, accurate and repeatable results.

Deccan Basalt Model

Based on Killari borehole cores, a stratified, laterally homogeneous model was prepared for the Deccan traps as shown in Figure (3). Dimension of the Deccan basalt model is 1000m x 600m. It was discretized with a uniform sampling interval of 1m along X- and Z- directions ($\Delta x = \Delta z = 1$m) and 25 grid points were dedicated for absorbing layers. The first layer represents younger Ambenali formation comprising of high density massive basalts. Second layer represents low velocity vesicular basalts of Poladpur formation. Third layer represents massive basalts from older Poladpur formation and the fourth layer is the Archean basement. We can see that in Deccan traps, different basalt layers exhibiting different physical properties; hence it is required to consider the viscoelastic formulations.

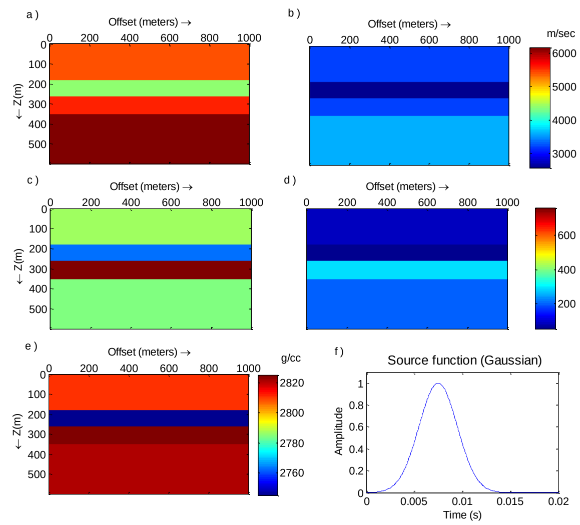

Figure (3): A stratified velocity model of Deccan basalt. a) Vp; b) Vs; c) Qp; d) Qs; e) Density, $\boldsymbol{\rho}$; f) Source function (Gaussian; central frequency: 20Hz)

Results and Discussion

Following the above mentioned methodology for elastic and viscoelastic media, we generated synthetic seismograms for the Deccan basalts model (Figure 3). We observed that, for the numerical stability, better results were obtained using Courant number $\left(\frac{v_{max}\Delta t}{\Delta x}\right) < 0.3$. By comparing the seismograms computed using elastic and viscoelastic (for L=1 and L=3) formulations, we can see that arrival times for different phases are similar (Figure 4); however a slight difference can be attributed to the velocity dispersion phenomenon in viscoelastic media (Figure 5). As mentioned in the methodology, velocity for viscoelastic media depends on the complex modulus, $\boldsymbol{M^c(\omega)}$, which is a function of frequency, hence phase velocities depend on frequencies present in the seismic signal. We can also see the effect of attenuation in viscoelastic formulations (Figure 4). As compared to the elastic formulation, viscoelastic (both L=1 and L=3) clearly shows attenuation for all phases. The deeper events are barely visible in

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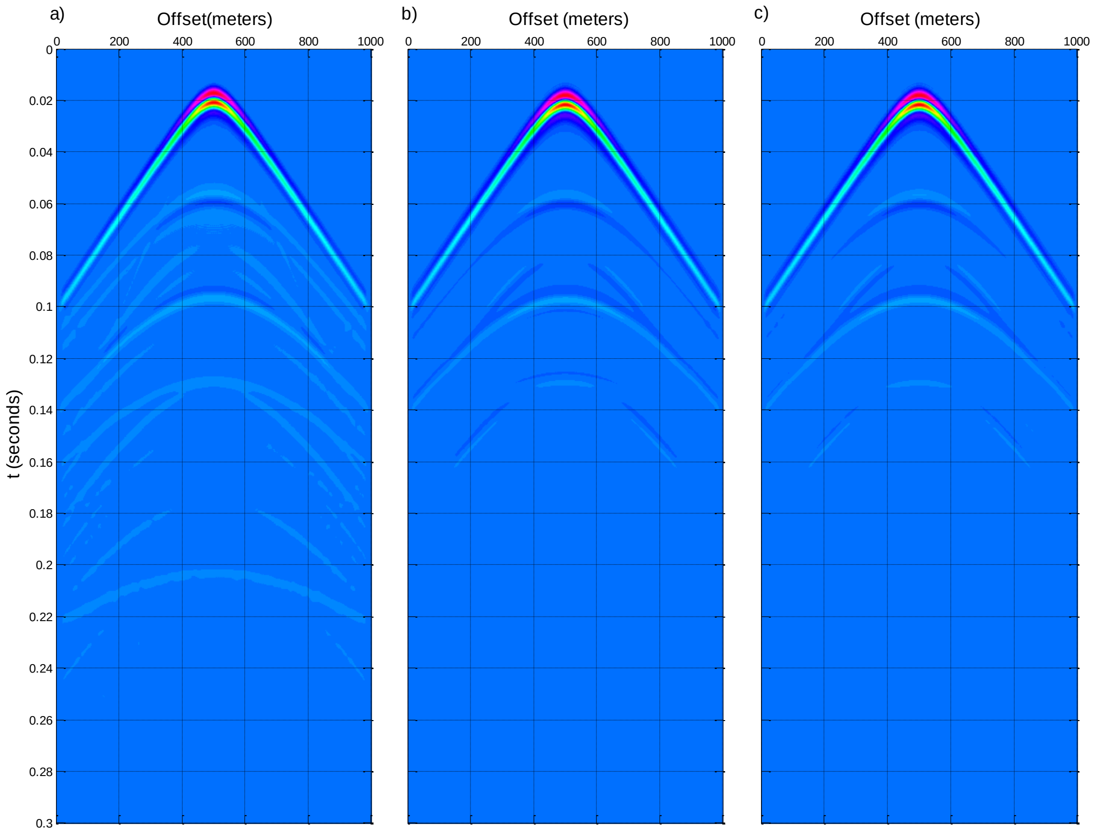

Figure (4): Comparison of synthetic seismograms generated using the (a) elastic and viscoelastic for (b) L=1 and (c) L=3, relaxation mechanisms.

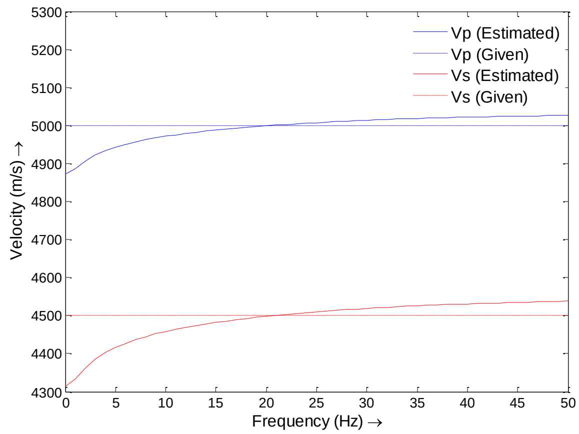

Figure (5): Velocity dispersion curve for viscoelastic formulation (L=3).

viscoelastic section. It is worth to mention that no gain function is applied to these seismograms. On the basis of attenuation curve (Figure 2), we can infer that viscoelastic formulation with three relaxation mechanisms (i.e., L=3) produce better results and, dispersion in Q is less at lower frequencies. Thus, from the preliminary results, we suggest that the relaxation mechanism for viscoelastic media may not be generalized and it is a subject of further investigation.

Conclusions

We have investigated the applicability of viscoelastic formulation for Deccan basalt model and found that most of the events are highly attenuated in different layers of basalts. Higher attenuation would cause the excessive amplitude reduction of sub-basalt events. However, the velocity dispersion for this model is not significant. Thus, seismic modeling can be improved by considering attenuated model in such situations.

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EDITED REFERENCES
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