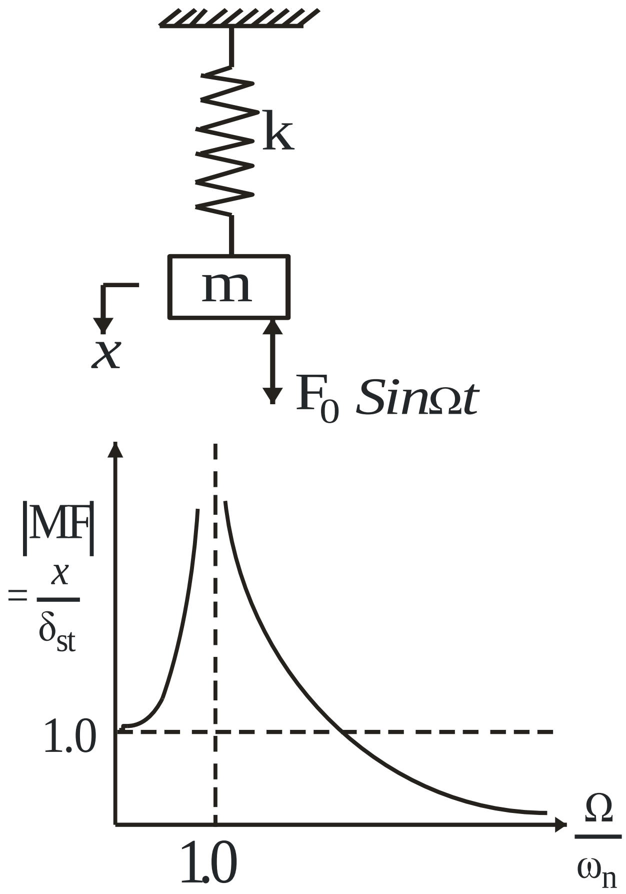
CONCEPTS IN DYNAMIC ANALYSIS

Simple spring mass system

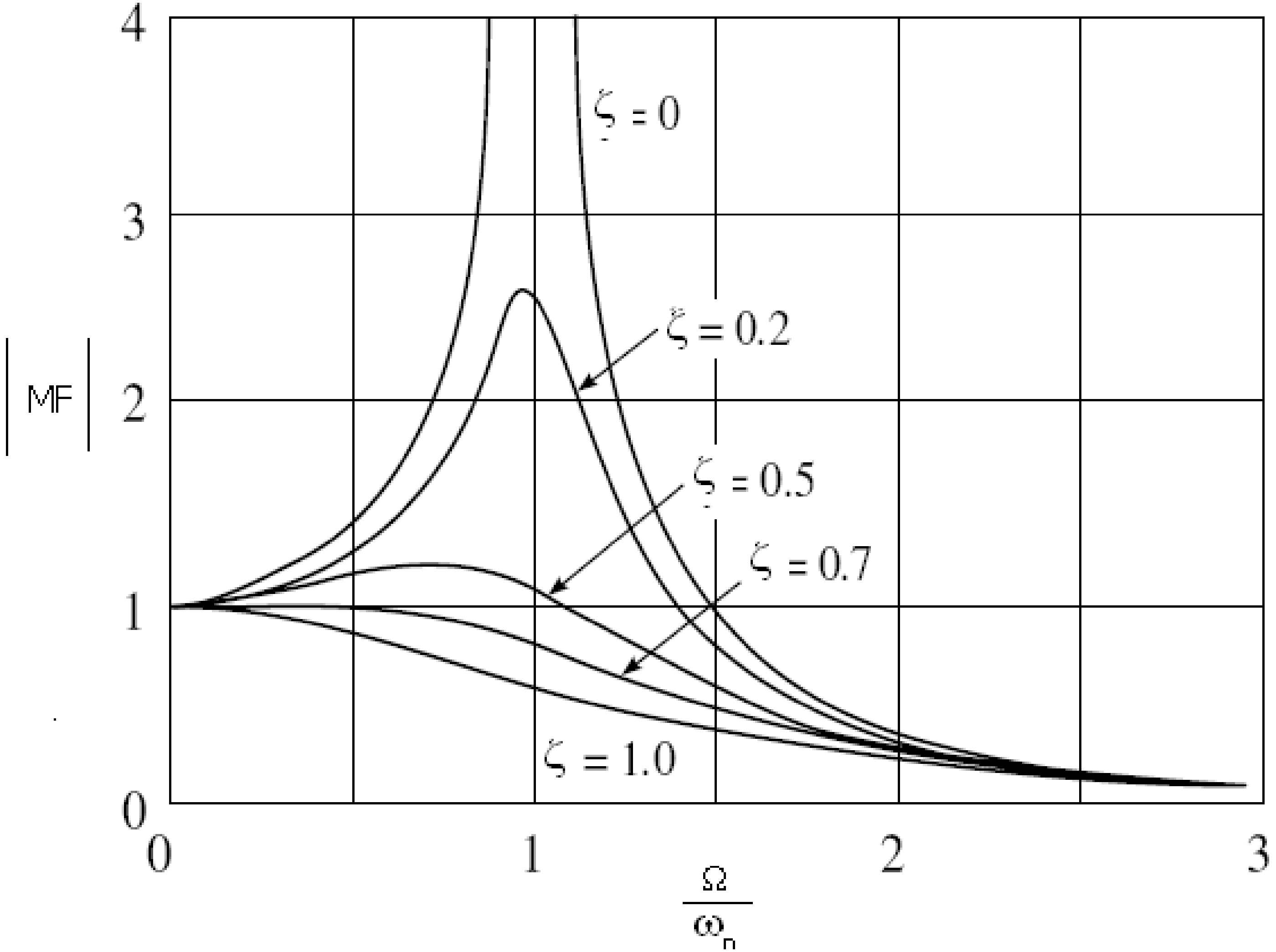
$$|MF| = \frac{1}{1-\left(\frac{\Omega}{\omega_n}\right)^2}$$

$$\Omega = \frac{1}{3}\omega_n, \quad |MF| = 1.125$$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$



Damped Forced Vibration

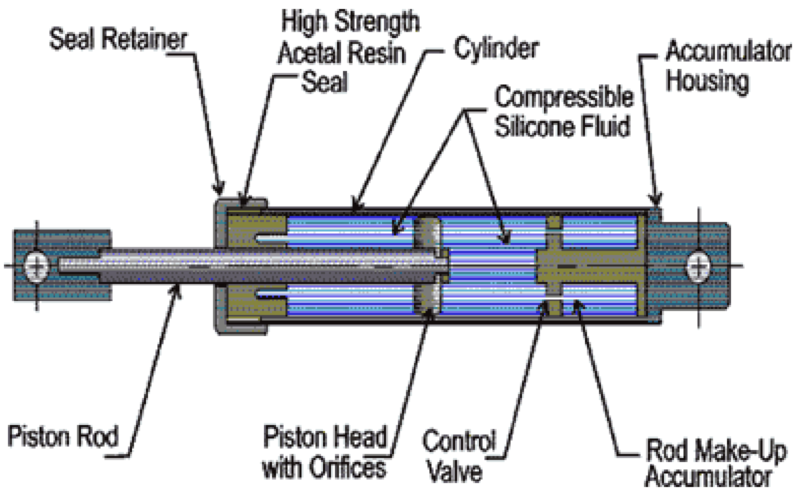


TRANSIENT VIBRATION ANALYSIS

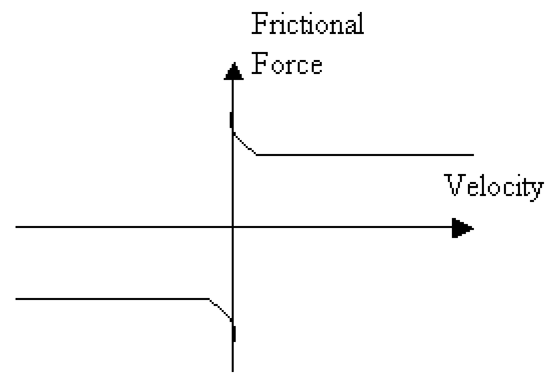
Modeling of damping is complex.

Sources of dissipation include:

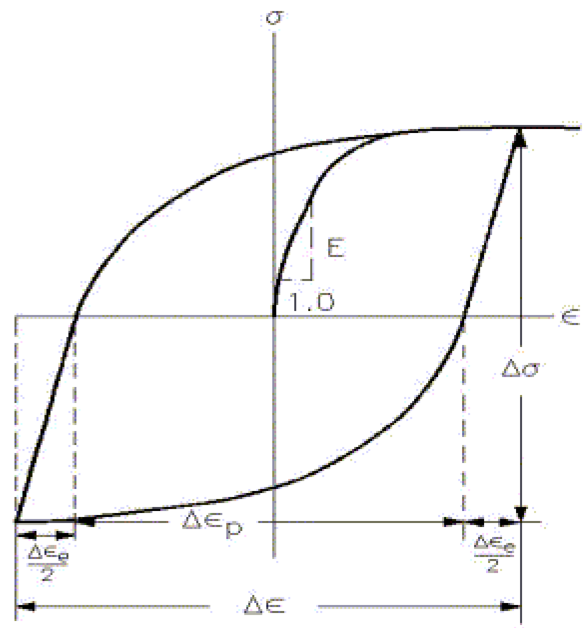
- Viscous friction
- Coulombic friction
- Hysteresis



Viscous Damper



Coulomb Friction



Material Hysteresis

TRANSIENT VIBRATION ANALYSIS

Generalised governing equations

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K]\{X\} = \{F(t)\}$$

Methods of solving :

- Direct Integration Methods
- Mode Superposition Method

Mode Superposition

Any deflected shape can be represented as linear combination of eigenvectors as

$$\{X\} = c_1\{U_1\} + c_2\{U_2\} + \cdots + c_n\{U_n\}$$

$$\begin{Bmatrix} X_1(t) \\ X_2(t) \\ \vdots \\ X_n(t) \end{Bmatrix} = \begin{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{Bmatrix}^1 & \begin{Bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{Bmatrix}^2 & \cdots & \begin{Bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{Bmatrix}^m \end{bmatrix}_{n\times m} \begin{Bmatrix} p_1(t) \\ p_2(t) \\ \vdots \\ p_m(t) \end{Bmatrix}_{m\times 1}$$

$$\{X(t)\} = [U]\,\{p(t)\}$$

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K]\{X\} = \{F(t)\}$$

$$\{X(t)\} = [U]\{p(t)\}$$

$$[M]\{U\}\{\ddot{p}\} + [C]\{U\}\{\dot{p}\} + [K]\{U\}\{p\} = \{F(t)\}$$

$$\{U\}^T[M]\{U\}\{\ddot{p}\} + \{U\}^T[C]\{U\}\{\dot{p}\} + \{U\}^T[K]\{U\}\{p\} = \{U\}^T\{F(t)\}$$

$$\{U\}^T[M]\{U\} = [I]$$

$$\{U\}^T[K]\{U\} = \begin{bmatrix} \ddots & & 0 \\ & \omega^2 & \\ 0 & & \ddots \end{bmatrix}$$

$$\{\ddot{p}\} + \{U\}^T[C]\ \{U\}\ \{\dot{p}\} + \begin{bmatrix} \ddots & & 0 \\ & \omega^2 & \\ 0 & & \ddots \end{bmatrix} \{p\} = \{U\}^T\{F(t)\}$$

$$m\ddot{x} + c\dot{x} + kx = f(t)$$

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = \frac{f(t)}{m}$$

$$critical\ \ damping, c_c = 2\sqrt{km}; \quad \xi = \frac{c}{c_c}; \quad \frac{c}{m} = 2\xi\omega$$

$$\ddot{x} + 2\xi\omega\dot{x} + \omega^2 x = \frac{f(t)}{m}$$

$$\{U\}^T\ [C]\ \{U\} = [c] = \begin{bmatrix} \ddots & & 0 \\ & 2\xi\omega & \\ 0 & & \ddots \end{bmatrix}$$

Mode superposition technique

$$\ddot{p}_1 + 2\xi_1\omega_1\dot{p}_1 + \omega_1^2 p_1 = \bar{f}_1(t)$$

$$\ddot{p}_2 + 2\xi_2\omega_2\dot{p}_2 + \omega_2^2 p_2 = \bar{f}_2(t)$$

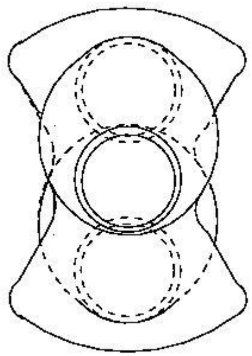
$$\vdots$$

$$\ddot{p}_m + 2\xi_m\omega_m\dot{p}_m + \omega_m^2 p_m = \bar{f}_m(t)$$

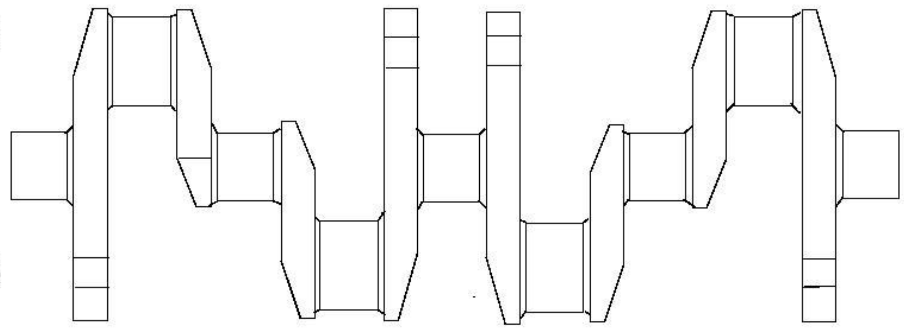
$$\ddot{p}_i + 2\,\xi_i\omega_i\dot{p}_i + \omega_i^2 p_i = \bar{f}_i\,(t)$$

$$^{-}p_i(t) = \frac{1}{\omega_i}\int_0^t \bar{f}_i(t)\sin\omega_i(t^{++}-\tau)\,d\tau + \alpha_i\sin\omega_i t + \beta_i\cos\omega_i t$$

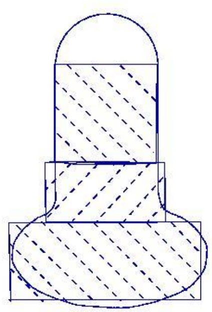
Typical crankshaft case study



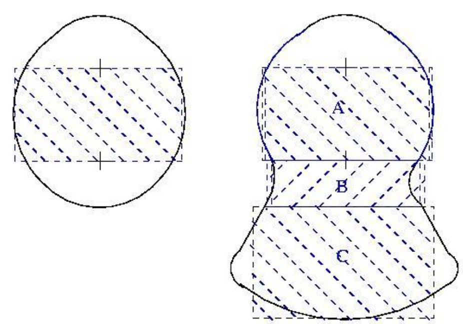
Side View



Front view

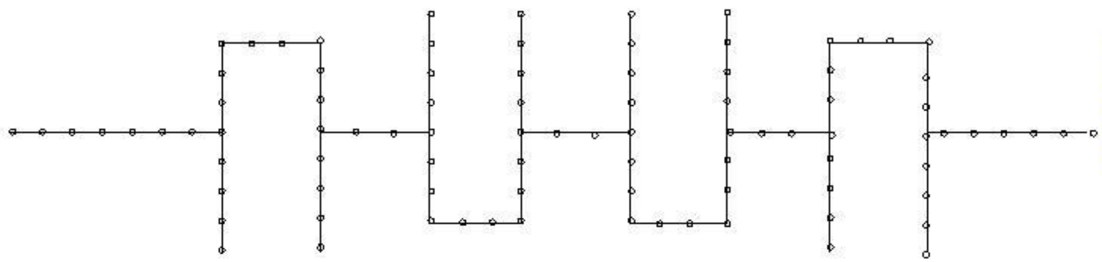


Okamura Model



Present Crankshaft Model

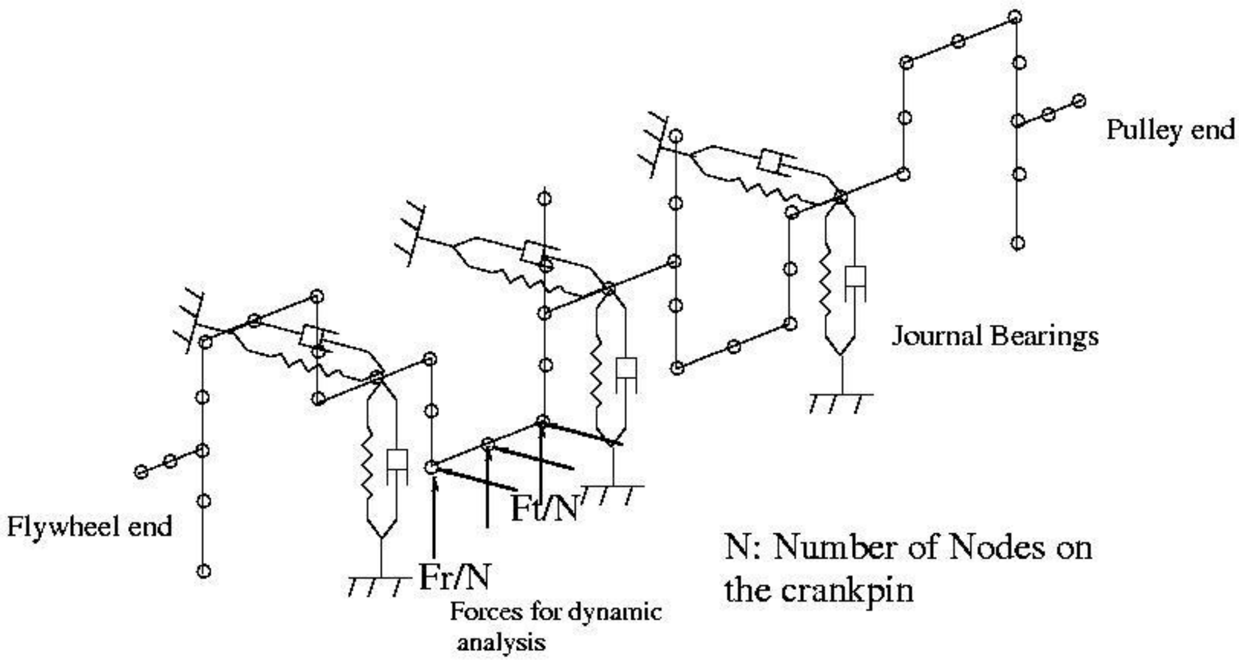
Beam Element Model



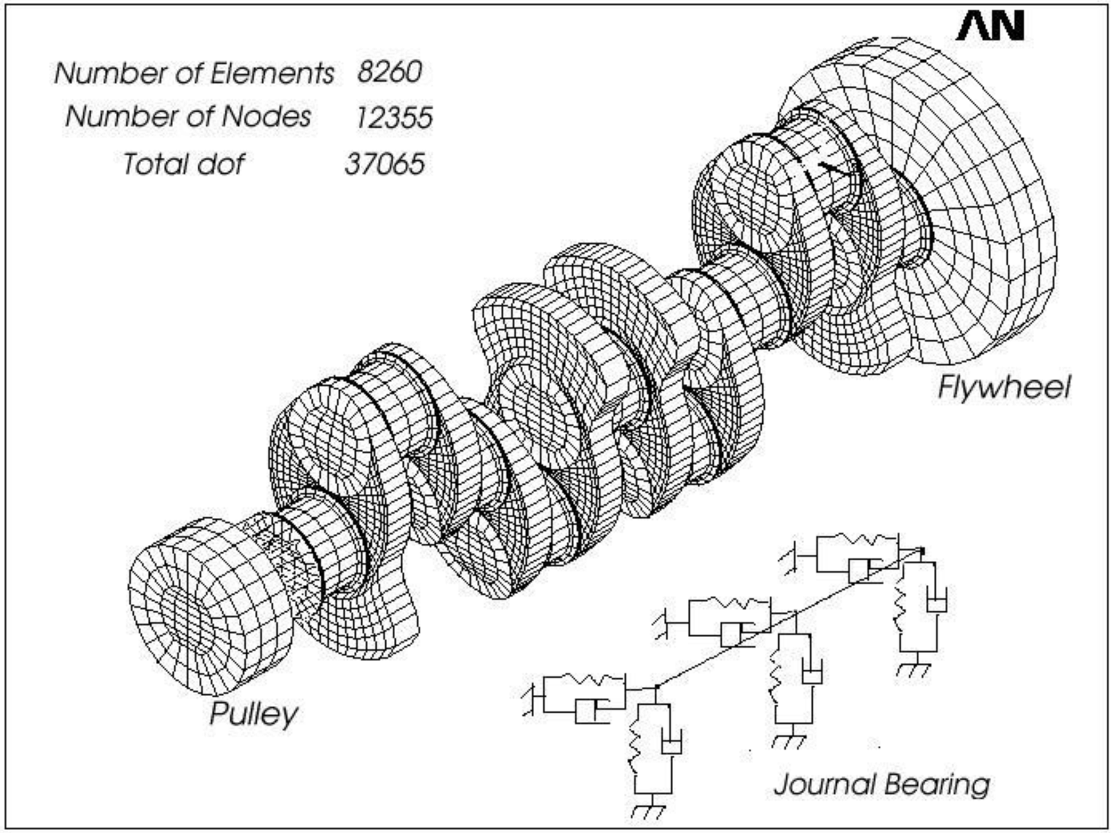
(In actual model)
No. of Elements:356
No. of Nodes :357
Total dof :2142

Beam element model for the crankshaft data given in Ref[10]

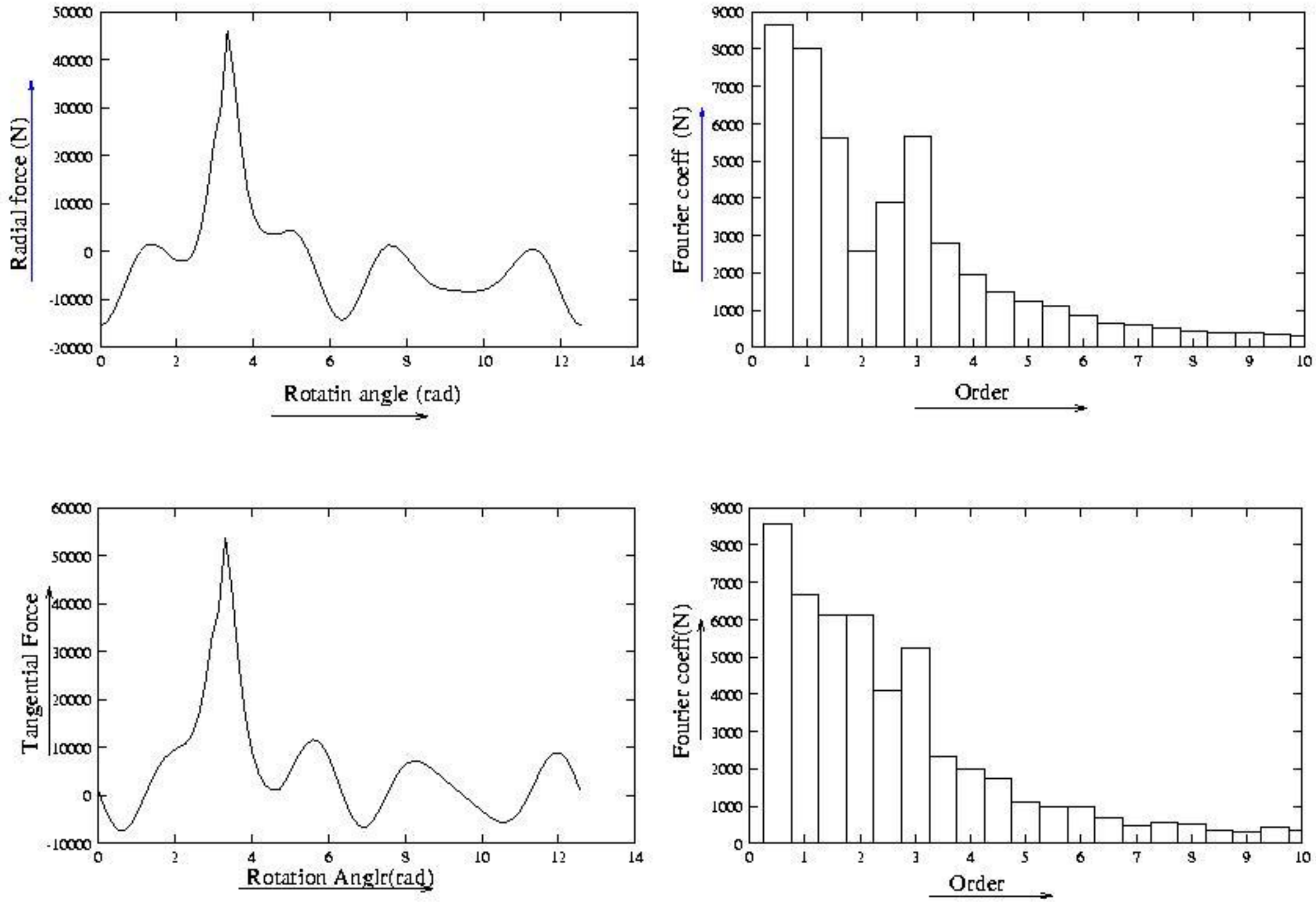
Number of Elements	:	375
Number of nodes	:	376
Total dof	:	2256



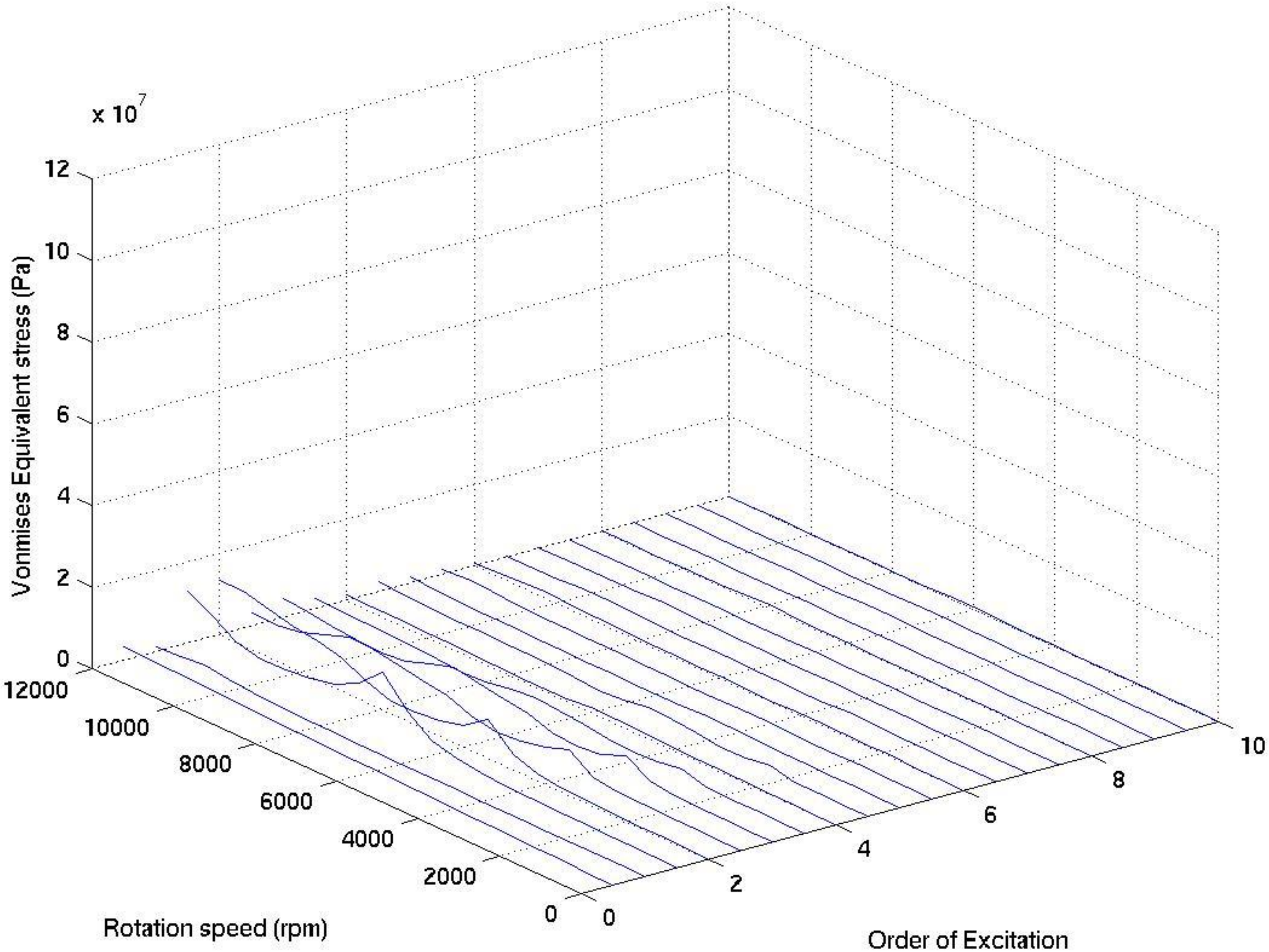
3-D Element Model



Dynamic Forces on the Crankshaft and their Fourier Decomposition



Dynamic Stresses on Crank-Pin (Beam Element Model)



Dynamic Stresses on Crank-Pin (3D Model)