Inversion of well log data using improved shale model for determination of petrophysical parameters

Ajay Malkoti¹, Nimisha Vedanti¹ and A.K. Bhattacharya²

¹Academy of Scientific and Innovative Research, CSIR-NGRI, India-500007

² Indian Institute of Technology, Kharagpur, West Bengal, India-721302

*Corresponding Author: ajmalkoti@gmail.com

ABSTRACT

Estimation of petrophysical parameters from well logs is an important procedure in reservoir characterization. Here we present a method, based upon improved estimates of average mineralogical composition of shale, to estimate the petrophysical parameters from the well log data of a sandstone reservoir. The shale response is dependent of its distributional characteristics which is difficult to model. Shale response has a large influence on the inversion algorithm, which affects the parameter estimation. Thus to mitigate its effect in forward modelling we used the improved estimates of an average shale-mineralogical-composition model (SMCM). A genetic algorithm (GA) based inversion was carried out for correct estimation of petrophysical parameters. This improved algorithm was tested for the applicability on the synthetic data and then applied on - well log data of Ankleshwar field, cambay Basin, India. The result for synthetics exibited good match with the assumed model and worked well even in presence of large noise. In Ankleshwar reservoir, parameters estimated using this method were compared with the industry provided values and with earlier studies. It was found that our results are in good agreement with them. The average error between the ankleshwar well data and its synthetics, generated for inverted parameters, was found to be about 6.27%. Major advantages of this approach are mitigation of cumulative error, enhanced resolution and capability to generate missing logs. This algorithm has also demonstrated its capability in delineating the finer details of the formation.

Key words: Inversion, Genetic Algorithm(GA), shale-mineralogical-composition model (SMCM), Well-log data, Ankleshwar field.

INTRODUCTION

In exploration geophysics, well-log data is widely used to estimate the petrophysical parameters of the formation in the vicinity of the borehole. Some useful parameters are porosity, permeability, fluid saturation and mineral composition of the formation. Inversion methods are used to obtain estimates of these vital petrophysical parameters necessary to characterize the formation. Efficacy of an inversion scheme depends upon the choice of model used for obtaining log. Along with that, the inversion schemes (linear or some local optimization method) suffer from several drawbacks (Menke, 2012; Sen and Stoffa, 2013). Thus, accurate estimation of petrophysical parameters from the well logs requires a realistic model which can describe the log behaviour/response accurately and a robust inversion scheme.

The log response is generally computed using the weighted sum of the log responses for each mineral/fluid present in rock, called linear model. Here weights/parameters are assigned on the basis of fractional percentage of respective minerals/fluids. In practice, we are interested in determining depth distribution of these parameters (e.g. porosity, water and hydrocarbon saturation, shale and sand volume, etc.). It is an inverse problem which can be treated using linear inversion techniques (Menke, 2012).

Many authors such as Mayer & Sibbit (1980), Alberty and Hashmy (1984), and Mezzatesta, et al., (1988) utilised the various logs such as Gamma ray log (GR), Self-potential log (SP), Neutron log (NPHI), Density log (DEN), Sonic log (DT) jointly to estimate petrophysical parameters. It is important to note that some parameters (viz. shale) may represent the volume of a mineral assemblage which may vary. For linear problem we assume that constitution of this mineral assemblage is known. Usually, the ratio of number of logs (or data) to parameters in such linear problems varies between 1 and 2 which is quite low for parameter estimation. To increase data to parameter ratio we can include more data by using other logs which are governed by non-linear relationships.

Non-linear models have a non-proportional relation between the input and output of the model. The forward model for well logs depends upon the fractional volume content of the mineral/fluid present in matrix/pore as well as on the structural or distributional (e.g. dispersed, laminar, etc.) character of the rock. For example, various models proposed in literature for resistivity logs are non-linear in nature. The resistivity log can be explained using a non-linear empirical relation given by Archie (1942). It is important to note that this model does not work for every geological scenario. For example in case of shaly-sands, resistivity log may show anomalous high values of

conductivity due to high cation exchange capacity (CEC) of clay minerals (Serra, 1984). This effect is observed when the aluminium ions present over the surface of clay gets replaced by other ions such as Fe++ or Mg++. It makes aluminosilicate sheets of clay negatively charged, causing accumulation of loosely held positive charge ions near its surface. These accumulated positive ions can be easily mobilized, resulting in high conductivity. CEC depends upon the surface area or distributional characteristics of shale which makes its estimation hard for shaly formation, as shale occurs in various forms (laminar, dispersed and structural) with different surface areas. Clay minerals (Kaolinite, Illite, Montmorillonite, Vermiculite, etc.) present in shale have different CEC due to different physical and molecular structures. Various authors made attempts to address this issue and proposed different models for shaly formation (Worthington, 1985). Simandoux (1963) approximated this effect by considering total resistance of pore fluid and shale as two resistors in parallel. Different models have been proposed for laminated clay (Poupon et al., 1954) and dispersed clay (Schlumberger, 1975) to take care of clay distribution. However some resistivity models e.g. Waxman and Smith model (WSM) (1968) and Dual water model (DWM) (Clavier et al., 1977) are distribution independent. WSM assumes that the clay particles increase the conductivity of the formation while DWM assumes that only cations increase the conductivity of clay bound water. However these two models require some additional parameters e.g. WSM requires estimates of specific cation conductance and CEC whereas DWM requires resistivity and saturation values for bound and free water in the formation. Hence use of these models is not advisable as they involve extra parameters to calculate the log response. In this study we have used the Indonesian formulae (Poupon and Leveaux, 1971) to calculate the resistivity log response due to two reasons; first, it's practically proven and second, the parameters used to define the response do not add any new parameter to our set of parameters. In the forward modelling the SMCM plays an important role. Most of the available literature on estimating SMCM is based upon argillaceous part of clay (Perrin, 1971; Ridgway 1982; Shaw, 1981; Sellwood & Sladen 1981). Hiller (2006) provided a new average SMCM for which he analysed 105 samples and estimated both, argillaceous as well as non-argillaceous component of shale. This kind of data was not available earlier. Thus using this model, the log response for the shale can be determined more precisely using appropriate inversion technique.

From the literature it is evident that several inversion techniques exist. However, a close scanning of these details from the literature, it is noticed from the linear and nonlinear inversion techniques GA based Stochastic inversion is better. Our focus is on reasonable understanding of Ankleswar reservoir composition by proper use of

available borehole data, as deligently as possible with faster convergence, limiting computational costs. As such we do not go into details of relative merits and limitations of various inversion methods. Bearing this in mind, we will be using the forward modelling set of equation as described in (Dobroka & Sazabo, 2011, Szucs & Civan, 1996) with improved estimates of SMCM. Non-linear nature of modelling equations limits the use of linear inversion methods and/or local optimization schemes (Gill et al., 1981; Dimri, 1992; Vedanti et al., 2005). Thus, inversion for the petrophysical parameters will be carried out using the global optimization approach. In the current study we employ the Genetic Algorithm (GA) (Holland, 1975) to estimate the petrophysical parameters. The applicability of the method is demonstrated on synthetic log data and then it is applied on well log data of Ankleshwar oilfield, situated in Cambay basin, India. The estimated petrophysical parameters form Ankleshwar logs are compared with that of provided by the Oil and Natural Gas Corporation Ltd. (ONGC) and Vadapalli et al., (2014) and the results are in good agreement.

Theory and Algorithm

In a borehole, the geologic formation consists of two main components - 1) lithological matrix, and 2) fluid(s) entrapped in this matrix. The fluid filled pores may contain water or hydrocarbon and the lithology may consists of different rocks e.g. shale, sandstone, etc. The rocks again can be treated as an assemblage of different minerals. The log response equation assumes that the logging tools respond mainly to the compositional characteristics of rock, not to their structure (Serra, 1984; Schlumberger, 1989). It means that the components present at a given depth only affect the behaviour of log at that depth. Assuming this, we can calculate the synthetic log at any depth by summing weighted individual response of tool, corresponding to each component, as:

$$L^{LOG} = \phi_e S_{xo} L_w^{LOG} + \phi_e (1 - S_{xo}) L_{hc}^{LOG} + V_{sh} L_{sh}^{LOG} + (1 - V_{sh} - \phi_e) \sum_{i=1}^{n} V_i L_i^{LOG}$$
 (1)

Here L^{LOG} represent the log response (L) for a given log tool for a mineral/fluid at 100% saturation of the material indicated in subscript. The subscripts 'w', 'h', 'sh', 'i' represents the log reading in water/mud filtrate, hydrocarbon, shale and ith mineral/fluid in matrix respectively. Parameters act as weight here represented by f_{e} , effective porosity; S_W and S_{XO} , saturation in un-invaded zone and flushed zone respectively and V_i the volume of ith mineral/fluid. Eq (1) can be used to determine the response of many logs e.g. GR, SP, Neutron, Density, etc. GR log response can be written as:

$$L^{GR} = \phi [L_w^{GR} S_{xo} + L_{hc}^{GR} (1 - S_{xo})] + V_{sh} L_{sh}^{GR} + \sum_{i=1}^{n} V_i L_i^{GR}$$
 (2)

Average Shale content	Hiller (2006)
Quartz	23.9
Feldspar	3.7 (K-spar) 2.4 (Plag.)
Carbonate	7.5 (Calcite) 1.3(Dolomite) 0.5 (Siderite)
Fe-Oxide	0.8
Clay minerals	47.7 (Di-clay) 7.5 (Tri-clay)
Other minerals	0.5 (Pyrite)
Organic matter	

Table 1. The table above shows the average composition of average shale.

Log response equation for resistivity of flushed zone (R_{XO}) and uninvaded zone (R_W) can be written using Indonasian Equation as:

$$\frac{1}{\sqrt{R_{xo}}} = \left[\frac{v_{sh}^{1 - \frac{V_{sh}}{2}}}{R_t} + \sqrt{\frac{\phi^m}{aR_{xo}}} \right] \sqrt{S_{xo}^n}$$

$$1 \qquad \left[v_{sh}^{1 - \frac{V_{sh}}{2}} + \sqrt{\frac{\phi^m}{aR_{xo}}} \right] \sqrt{S_{xo}^n}$$
(3)

 $\frac{1}{\sqrt{R_{w}}} = \left[\frac{v_{sh}^{1 - \frac{V_{sh}}{2}}}{R_{t}} + \sqrt{\frac{\phi^{m}}{aR_{w}}} \right] \sqrt{S_{w}^{n}}$ (4)

Here R_t , R_W , R_{XO} , represent the true resistivity of formation, resistivity of formation water, and resistivity of flushed zone water respectively. Cementation constant, saturation exponent, and tortuosity factor are represented by 'm', 'n' and 'a' respectively.

It can be seen that logs governed by eq (1) are very much dependent on L_w , L_{hc} , L_{sh} , and L_i . The response with respect to given SMCM is very important since its contents greatly affect the linear response equations (e.g. GR and SP) while its fractional volume affects the resistivity log. In this study we have used the average SMCM model given by Hiller (2006) (Table 1). The fractional volume of all the components was estimated utilizing the XRF data, which is more accurate than earlier methods/data.

Using the improved shale model we obtain L_{sh}^{log} and thus provided improved forward log response equations (eq. 1, 3, 4). Using these equations we can obtain petrophysical parameters using GA inversion.

The non-linear nature of the log response equations insinuates us to use the global optimization technique. So in this study we have used the evolution based optimization method called Genetic Algorithm (GA) given by John Holland (1975). In this technique each solution is treated as an individual or a chromosome. We used "Binary ladder" computational notation as it simplifies the application of GA operators (viz. crossover, mutation, selection, etc.) (Goldberg, 1989). It starts by initializing a pool of

individuals, called population. From the population, two individuals are selected randomly and crossover operator is applied to them. This operator makes them exchange part of their chromosome after the crossover point. Mutation operator introduces variation in these two chromosomes by changing or flipping the random bit of their chromosome. Thus obtained individuals (chromosome) are evaluated on the basis of a fitness function, which determines its chances of survival/selection for the next population set. Finally a selection process is used to determine which individual will go to next generation. Its analogue can be treated as roulette wheel where each individual of population have area proportional to its fitness and the individual having more area will have more chance of selection. However this does not guarantee that an individual with high fitness must get selected so an individual with low fitness may also remain in next population. The above process is performed over a given number of generations or till the termination criteria is met. In short we apply all above genetic operator in given sequence to create a next generation from the present generation.

The results obtained after optimization can be accessed for its quality by degree of fitness of data. For this particular problem of petrophysical parameter estimation we define the fitness function as difference between observed real well log and synthetics in least square sense.

$$f = \sqrt{\frac{1}{N} \sum_{i}^{N} \left[\frac{d_{i}^{obs} - d_{i}^{syn}}{d_{i}^{obs}} \right]^{2}}$$
(5)

While minimizing the above fitness function different constraints can be imposed on parameters. First constraint imposed on any component is because of the maximum fractional volume (100%) it can occupy in a lithology (i.e. $0 < V_{sh}$, V_{ma} , V_i , $\phi < 1$). For our reservoir, we have constrained the porosity in range of 5-35% (i.e. $0.05 < \phi < 0.35$) and water saturation for flushed zone and uninvaded zone as $0.6 < S_{xo} < 1$ and $0.2 < S_w < 1$ - respectively. An additional

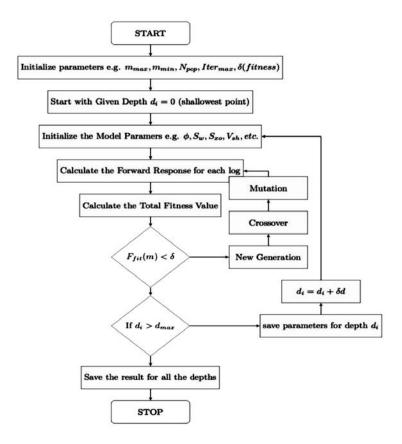


Figure 1. Algorithm for optimization of well log parameters at each depth point.

constraint equation, ϕ – V_{sh} + V_{mc} + V_i =1, can be imposed on parameters as the total volume occupied by all minerals along with void space (or porosity) in a rock is the volume of unit cube.

Numerical implementation of GA optimization is shown in Figure 1. In first step the parameters required by GA optimization algorithm, such as m_{min} and m_{max} , (vectors containing the minimum and maximum value for each parameters respectively), Itermax (maximum no of generation), δ (minimum error to terminate the loop), etc are initialized. GA optimization of fitness function is done at each depth point (d_i) starting from shallowest to deepest depth point (d_{max}) by stepping through δd . To retrieve the petrophysical parameters viz. φ, S_w, S_{xo}, V_{sh}, etc., Optimization at a depth is achieved by application of genetic operators on a set of population repeatedly until the criteria of maximum generation or δ has met. As soon as either of these conditions meet, the results (optimized parameters) are saved and it proceeds for the next depth point.

In this algorithm we have assumed that the response recorded by a logging tool is affected by the minerals/fluids present in horizontal section of the formation at that depth and thus the parameters obtained after inversion belong to that depth point only. As this inversion algorithm is associated with a particular depth point we can call it a point inversion algorithm. However in reality, the log response is also affected by the nearby layers thus to account for this effect, a weighted average response of adjacent layers may be assigned to that depth point. It should be noted that in order to use this method to invert real field data, it is desired to have some priori geological knowledge to limit number of components. Generalizing a model by considering a large number of components reduce the data to parameter ratio and makes the problem less over-determined or even-determined.

APPLICATIONS

Synthetic data application:

The inversion algorithm described above has been tested on synthetic log data. Synthetic logs were generated using an assumed petrophysical model shown in Figure 2. In this model, first and fourth layer are assigned general values for overburden and underburden, while second and third layers in this model correspond to the cap and reservoir rocks respectively. The second layer 6m-10m has very low porosity and high amount of shale (V_{sh} = 50%). The third layer (10-13 m) is mainly composed of sand with negligible

fraction of shale possessing a high porosity $\phi = 30\%$. This layer has some residual oil as the difference in saturation of flushed zone and of un-invaded zone is non zero i.e. $S_{xo} - S_w = 0$. Using this model, ideal response of logs (Figure 3) was generated using forward modelling eq (1), (3), and (4). Synthetic log was generated by addition of Gaussian noise to ideal responses. Experiments were carried out with various levels of noise, however here we have shown only large noise (10%) case is shown in Figure 3. Petrophysical parameters obtained after inversion are shown in Figure 3 and we found that this algorithm worked well on synthetic data logs.

Real data application:

We applied the algorithm on well logs of Ankleshwar field of Cambay basin, India (Figure 4) to estimate the petrophysical parameter. Ankleshwar oil field has four major formations viz. Telwa, Ardol, Kanwa, and Hazad. Telwa and Kanwa are primarily shale while the Ardol and Hazad are alternation of sandstone and shale. Hazad formation possesses the reservoir characteristics and out of its different sand layers (S1, S2, S3 and S4) only two (S3 and S4) are major producing layer. These two layers are being studied for CO₂ enhance oil recovery (EOR). We used one of the Ankleshwar well (ANK-W1) data which has been well-studied for reservoir characterization and thus it was considered as a standard for comparing our results. A thin coal layer is also present in payzone being studied but we have not considered coal in our parameter set. The reason is- first, it would be having a negligible effect on the log response; second, for inversion it would increase the number of parameters and thus might lead to erroneous results. With this a priori information about lithology, we have carried out the inversion only for the S4 sand layer of Hazad formation. The results for petrophysical parameters obtained after GA based inversion are presented in Figure 5 and compared with the results provided by the industry (ONGC) as well as with the earlier studies carried out by Vadapalli et al., (2014).

RESULTS AND DISCUSSION

For inversion of the noisy-synthetic-well-logs (Figure 3) using the GA based inversion. We have used 6 logs and 7 constraining equations to retrieve 5 parameters. Number of parameter to be estimated is reduced to 4 due to constraining equation on total volume and thus give us the data to parameter ratio is 3.25. Synthetic study result shows that even for very high value of noise in synthetics, this scheme could recover the petrophysical parameters (Figure 2) satisfactorily except for S_{xo} , which was not retrieved accurately and its value is restricted nearby 0.8. The reason for such behaviour for S_{xo} could be lesser number

of constraints for its estimation. S_{xo} represent water filled pores which have zero effect on GR and SP logs and thus resulting in lesser number of constraining equations for S_{xo} . However the valued for S_w were much more consistent unlike S_{xo} . Reason behind this is S_w influences the deep resistivity log, while having a negligible effect on other logs. Hence for S_w the inversion process is reduced to a one to one mapping from data to parameter space, i.e. from resistivity log directly.

Inversion results for Ankleshwar field well data show that the real logs have a good match with the synthetics logs, generated using the inverted parameter (Figure 5) with an average error of 6.27%. The small mismatch between the results can be attributed to the limitations of our model which comes primarily from two sources; first, due the relationships which govern the forward model and second, the number of constraining equations to maintain the threshold ratio. The forward modelling should be accurate so that it can generate the log response precisely. We have discussed several models in introduction but still there is a requirement of one single model to describe the effects caused due to contents as well as due to structure. Presence of several parameters brings non uniqueness of the solution into the picture, which can be mitigated only by constraining the parameter's bound. That is why we assert that our inversion requires the ratio of modelling equations (including constraints) and model parameters to be as large as possible. To explain this, let us consider two parameters constrained by three or more equations. In a two dimensional plot these constraining lines would cover some closed area, known as feasible region. If we add more and more such constraints, the feasible region becomes smaller and when this area becomes small enough we may get best possible solutions. If we have less data or constraints for a given depth point then we will have lesser options for selection of mineralogical components. Here we can use available geological information to ignore some components and deal with lesser number of parameters. Thus in this study we limited our model to the reservoir formation whose lithology is mostly known.

We found that the GA estimated parameter V_{sh} shows a strong correlation with given GR and SP logs. The S4 layer of Hazad formation can be further divided into different sand layers viz. S4.1 (1112.5-1116.0m), S4.2 (1119.0-1122.0m), S4.3 (1124.0-1130.5m), and S4.4 (1134.0-1138.0m). These sand layers presence can be clearly seen as the zones marked by high porosity, low GR and low shale. A comparision of parameters estimated using GA algorithm with that provided by the industry (ONGC) and Vadapalli et al., (2014) is presented in Figure 5. Industry has provided ϕ and S_w values, whereas the Vadapalli et al., (2014) provided ϕ , Vsh, and Vsd values. It can be observed that the ϕ estimates for all three are in good agreement. However, our method predicts significantly higher value

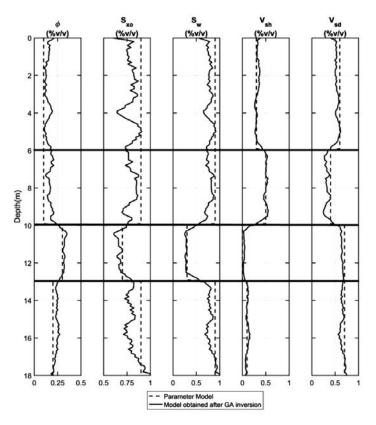


Figure 2. Parameter model (Porosity, ϕ ; Water saturation, S_w or S_{xo} ; Shale Volume, V_{sh} ; and Sand Volume, V_{sd}) used to create the synthetic logs and the inverted parameters obtained, using the noisy synthetic logs.

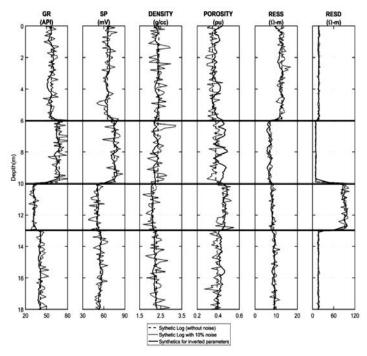


Figure 3. The response for the assumed parameter model (shown in Figure 2) without noise (ideal case) and with 10% noise (realistic case) are shown above. The logs generated for parameters obtained after inversion of noisy data are also shown. Various logs shown above are- Gamma Ray- GR; Self Potential-SP; Density-DENS; Neutron Porosity-NPOR; Shallow Resistivity- RESS; and Deep Resistivity- RESD.

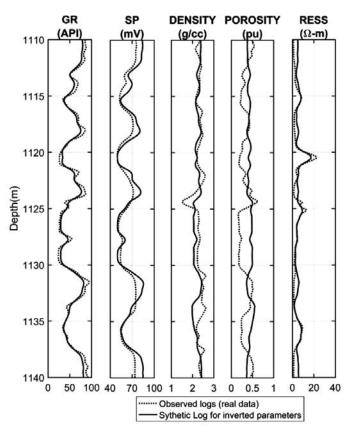


Figure 4. Comparison of real/observed borehole log with the synthetic log (generated using inverted parameters) for Ankleshwar borehole data.

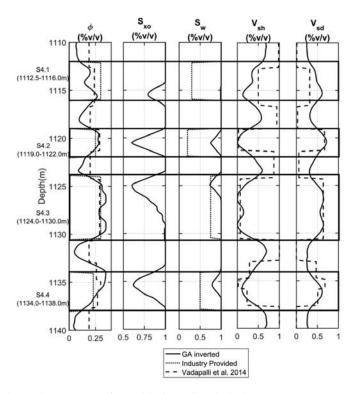


Figure 5. Comparison of petrophysical parameters for Ankleshwar borehole data. GA inversion results, industry provided results and results from Vadapalli et al., (2014) are shown.

of S_w than the industry provided values. This difference between GA inverted results and the values provided by the industry could be a result of assumptions, or some error made in conventional procedure used by the industry. The error originated at some step may grow while going through subsequent steps in processing and finally produce cumulatively large error. In our method we mitigated this cumulative error problem as we are utilizing all the logs simultaneously for parameter estimation. Our results are also in good agreement to all three parameters (ϕ , V_{sh} and V_{sd}) estimated by Vadapalli et al., (2014). In addition to this our method brings out the finer features of well log by detecting the sub layers of shale and sand present within the formation which is another advantage of our approach. The estimated petrophysical properties using this method can be used directly for interpretation as well for generation of missing log(s). Often, in industry provided data, some logs are missing which might be required by a researcher for further processing/interpretation. In this scenario we can generate these logs using log response equations for which we can use our inverted petrophysical parameters model as an input. However its practical application is yet to be tested and verified.

CONCLUSIONS

We have carried out inversion of well log data for estimation of the petrophysical parameters using a non-linear model with precise estimates of shale using the GA optimization technique. This technique was successfully tested on synthetic data and then applied on Ankleshwar field well data. The field data inversion was carried out for the S4 sand layer of Hazad formation which is being considered for CO2 -EOR and we found that our results are in good agreement with the information provided by the industry. This technique has limitation if there are a large number of parameters with few constraints. In this scenario parameters must be limited by some a priori knowledge of lithology of the formation. In this scenario parameter must be limited by some means. However the advantages of this method are-applicability to any formation, robust inversion method as it works even in presence of high noise, mitigating the error propagation problem to some extent by simultaneously inverting data for different parameters, finer resolution as it brings out the finer details of lithology, and capability of generating missing logs.

ACKNOWLEDGEMENTS

AM would like to express sincere thanks to University Grant Commission for JRF scholarship. Authors are thankful to ONGC for providing the data. We thank Director NGRI for giving permission to publish this paper. We also thank Dr. Dhananjay Kumar and Chief Editor of JIGU for their valuable suggestions.

Compliance with Ethical Standards

The authors declare that they have no conflict of interest and adhere to copyright norms.

REFERENCES

- Alberty, M., and Hashmy, K., 1984. Application of ULTRA to log Analysis, SPWLA 25th Annual Logging Symposium, pp: 1-17.
- Archie, G. E., 1942. The electrical resistivity log as an aid in determining some reservoir characteristics, Petroleum Transactions of AIME, v.146, pp. 54–62.
- Clavier, C., Coates, G., and Dumanoir, J., 1977. The theoretical and experimental bases for the "dual water" model for the interpretation of shaly sands, Society of Petroleum Engineers Journal, v.24, no.2, pp. 153-168.
- Dimri, V. P., 1992. Deconvolution and inverse theory application to geophysical problems, Elsevier Science Publishers, Amsterdam.
- Dobroka, M., and Sazabo, N. P., 2011. Interval inversion of well-logging data for objective determination of textural parameters. Acta Geophysica, v.59, no.5, pp: 907-934.
- Gill, P. E., Murray, W., and Wright, M. H., 1981. Practical Optimization, Academic Press.
- Goldberg, D. E., 1989. Genetic Algorithms in Search, Optimization, and Machine Learning. Addison-Wesley Longman Publishing Co., Inc., Boston, MA, USA.
- Hiller, S., 2006. Appendix A. Mineralogical and chemical data, in GM Reeves, I Sims, and JC Cripps eds., Clay materials used in construction: London. Geological Society, Engineering Geology Special Publications, v.21, pp. 449-459.
- Holland, J. H., 1975. Adaptation in Natural and Artificial Systems: An Introductory Analysis with Applications to Biology, Control, and Artificial Intelligence. The University of Michigan Press, Ann Arbor.
- Mayer, C., and Sibbit, A., 1980. GLOBAL, a new approach to computer-processed log interpretation. Presented at the 55th Ann. Fall Tech. Conf. at Dallas, Tex, Soc. Petr. Eng. AIME.
- Menke, W., 2012. Geophysical Data Analysis: Discrete Inverse Theory, Third Edition, Academic Press (Elsevier).
- Mezzatesta, A., Rodriguez, E., and Frost, E., 1988. Optima: a statistical approach to well log analysis. Geobyte, v.3, pp: 56-69.
- Perrin, R. M. S., 1971. The Clay Mineralogy of British Sediments. Mineralogical Society, London, pp: 247.
- Poupon, A., Loy, M. E., and Tixier, M. P., 1954. A Contribution to Electric Interpretation in Shaly Sands, Journal of Petroleum Technology, pp. 29–34.
- Poupon, A., and Leveaux, J., 1971. Evaluation of Water Saturations in Shaly Formations. Society of Petroleum and Well Log

- Analyst Transcation, 12th Annual Logging Symposium, pp: 1-2.
- Ridgway, J. M., 1982. Common Clay and Shale:Great Britain Institute of Geological Science, Mineral Resources Consultative Committee. Mineral Dossier 22.
- Schlumberger, 1975. A Guide to Wellsite Interpretation for the Gulf Coast. Schlumberger Offshore Services.
- Schlumberger, 1989. Log Interpretation Principle/Applications, Schlumberger Educational service.
- Sellwood, B. W., and Sladen, C. P., 1981. Mesozoic and Tertiary argillaceous units, distribution and composition. Quarterly Journal of Engineering Geology, v.14, pp. 263-276.
- Sen, M., K., and Stoffa, P. L., 2013. Global Optimization Methods in Geophysical Inversion. Cambridge University Press.
- Serra, O., 1984. Fundamentals of well-log interpretation volume 1-The acquisition of logging data. Development in Petroleum Science No 15A, Elsevier, Amsterdam..
- Shaw, H. F., 1981. Mineralogy and petrology of the argillaceous sedimentary rocks of the UK, Quarterly Journal of Engineering Geology, v.14, pp. 277-290.

- Simandoux, P., 1963. Dielectric measurements in porous media and application to shaly formation: Revue de L'Institut Français du Pétrole, v.18, pp. 193–215.
- Szucs, P., and Civan, F., 1996, Multi-layer well log interpretation using the simulated annealing method, Journal of Petroleum Science and Engineering, v.14, no.3, pp: 209-220
- Vadapalli, U., Srivastava, R. P., Vedanti, N., and Dimri V. P., 2014. Estimation of Permeability of a sandstone reservoir by fractal and Monte Carlo simulation approach: a case study, Nonlinear Processes in Geophysics, v.21, pp: 9-18.
- Vedanti, N., Srivastava, R. P., Sagode, J., and Dimri, V. P., 2005. An efficient non-linear Occam's inversion algorithm with analytically computed first and second derivatives for DC resistivity sounding, Computers & Geosciences, v.31, pp: 319-328.
- Waxman, M. H., and Smits, L. J. M., 1968. Electrical conductivities in oil-bearing shaly sand, Society of Petroleum Engineers Journal, v.8, pp: 107–122.
- Worthington, P.F., 1985. The evolution of shaly-sand concepts in reservoir evaluation, The Log Analyst, pp: 23–40.