1. (a) Since μ is the counting measure, $\mu(A) = 0 \Rightarrow A = \emptyset \Rightarrow \nu(A) = 0$, since ν is a measure. Thus, ν is absolutely continious with respect to μ. Since ν is the counting measure, f is the probability mass function $f(n) = \frac{1}{2^n}$.

   (b) $\mu\left(\mathbb{R}\backslash(0,1)\right) = 0$ but $\nu\left(\mathbb{R}\backslash(0,1)\right) > 0$, so $\mu(A) = 0 \not\Rightarrow \nu(A) = 0$ and ν is not absolutely continious with respect to μ.

2. Let S_μ and S_ν be the supports of μ and ν. If the two measures are mutually singular, $S_\mu \cap S_\nu = 0$ and $S_\mu, S_\nu \subseteq \Omega$, so $S_\mu \subseteq \Omega \backslash S_\nu$. Thus, $0 \leq \nu(S_\mu) \leq \nu(\Omega \backslash S_\nu) = 0$, so μ is not absolutely continious with respect to μ. By an identical argument, ν is not absolutley continious with respect to ν.