Please note that you should type your assignment using either $\LaTeX$ or Word. Both templates will be provided. **Hand-written assignments will not be graded.** You need to submit a **pdf** version on Gradescope by the due date given.

1. (10 points) Check whether the following sentences are propositions or not:
   (a) (2 points) Taking notes can be helpful.
   Yes
   (b) (2 points) Golden is the sun.
   No
   (c) (2 points) How did this happen?
   No
   (d) (2 points) A monkey is not a type of animal.
   Yes
   (e) (2 points) Stand up or sit down.
   No

2. (16 points) Rewrite each of the following statements in the form "If A, then B."
   (a) (4 points) A door can be opened without a key; therefore it is not locked.

   Solution:

   If a door can be opened without a key, then it is not locked

   (b) (4 points) Playing an instrument well follows from constant practice and dedication.

   Solution:

   If you constantly practice and are dedicated, then you can play an instrument well

   (c) (4 points) You can move a king to a square in chess only if that will not cause it to be captured.

   Solution: If it will not result in the king being captured, then you can move a king to that square

   (d) (4 points) Being able to drive well is a necessary condition for getting a driver's license.

   Solution: If you have a drivers license, then you can drive well.

3. (4 points) Several forms of negation are given for each of the following statements. Which one(s) is/are correct?

   (a) (2 points) It is sunny and it is not raining.

      A. If it is sunny, then it is raining.
      B. It is not sunny or it is raining.
      C. It is not sunny and it's raining.

Solution: B

   (b) (2 points) If I work, I will be tired.

      A. If I work, then I won't be tired.
      B. I work, but I am not tired.
      C. I don't work, but I am tired.

Solution: B

4. (20 points) Write truth tables for the following propositions.

   (a) (10 points) $(\neg B \rightarrow \neg A) \wedge (A \vee B)$

Solution:

A	B	¬A	¬B	¬B -> ¬A	A \/ B	(¬B -> ¬A)/\(A \/ B)
T	T	F	F	T	T	T
T	F	F	T	F	T	F
F	T	T	F	T	T	T
F	F	T	T	T	F	F

(10 points) $(A \rightarrow (\neg A \vee B)) \leftrightarrow (B \wedge A)$

Solution:

A	B	¬A	¬A \/ B	A->(¬A \/ B)	(B/\A)	(A->(¬A \/ B) <->(B/\A)
T	T	F	T	T	T	T
T	F	F	F	F	F	T
F	T	T	T	T	F	F
F	F	T	F	T	F	F

5. (20 points) Determine whether each of the following propositions is a tautology, satisfiable but not a tautology, or a contradiction. If it is a tautology or a contradiction, please give the proof (truth table or simplification using equivalence laws). If it is satisfiable, please give a truth assignment and a false assignment.

(a) (5 points) $(A \vee B \vee \neg A) \rightarrow (\neg(C \vee \neg B) \wedge C)$

Solution: F (Contradiction)

(A \/ ¬A \/ B) -> (¬(C \/ ¬B) /\ C)

(T \/ B) -> ((¬C /\ B) /\ C)

T -> (C /\ ¬C /\ B)

T -> (F /\ B)

T -> F

F (Contradiction)

(b) (5 points) $B \rightarrow \neg(A \vee \neg B)$

Solution: Truth argument A is False B is False

False argument A is True B is True

A	B	¬B	¬(A \/ ¬B)	B -> (¬(A \/ ¬B))
T	T	F	F	F
T	F	T	F	T
F	T	F	T	T
F	F	T	F	T

(c) (5 points) $\neg(A \wedge B) \vee (B \leftrightarrow A)$

Solution: Tautology

A	B	A <-> B	¬(A /\ B)	¬(A /\ B) \/ (A <-> B)
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	T	T	T

(d) (5 points) $(A \leftrightarrow (B \vee C)) \wedge ((\neg(C \wedge \neg B) \rightarrow A) \rightarrow \neg A)$

Solution: satisfiable

Truth argument (A B and C are False) False argument (A B and C are True)

A	B	C	¬A	B\/C *1	A <-> (rBe)	¬(C\/¬B)	rBe->A	rBe ->¬A *2	*1 /\ *2
T	T	T	F	T	rBe = row before T	F	T	F	F
T	T	F	F	T	T	T	T	F	F
T	F	T	F	T	T	F	T	F	F
T	F	F	F	F	F	F	T	F	F
F	T	T	T	T	F	F	T	T	F
F	T	F	T	T	F	T	F	T	F
F	F	T	T	T	F	F	T	T	F
F	F	F	T	F	T	F	T	T	T

6. (20 points) Prove the following using a series of equivalence laws (NOT TRUTH TABLES):
   (a) (10 points) $((\neg B \wedge C) \vee (C \wedge B)) \wedge (\neg(A \rightarrow \neg B) \vee \neg C) \equiv C \wedge A \wedge B$

Solution: $C \wedge A \wedge B$

Proof.

$(\neg B \wedge C) \vee (C \wedge B) \equiv C \wedge (\neg B \vee B)$

$(C) \wedge (T) \equiv C \quad \wedge \quad \neg(A \rightarrow \neg B) \equiv (A \wedge B) \quad \vee \quad \neg C)$

$C \wedge (A \wedge B)) \vee (C \wedge \neg C)$

$C \wedge (A \wedge B) \vee (F)$

$C \wedge A \wedge B$

   (b) (10 points) $((A \rightarrow B) \wedge (\neg B \rightarrow A)) \vee \neg B$ is a tautology.

Solution: *Tautology*

Proof. $((\neg A \vee B) \wedge (B \vee A)) \vee \neg B$

$(\neg A \vee B) \wedge B = B \ \wedge \ (\neg A \wedge A) \vee (B \wedge A) \equiv B \wedge A$

$B \vee (B \wedge A) \equiv B$

$B \vee \neg B$

T

Tautology

7. (10 points) Simplify the following formula as much as possible (resulting in an **equivalent** formula with as few letters/symbols as possible):
   $(\neg(A \leftrightarrow B) \vee (\neg A \wedge \neg B)) \vee (A \rightarrow C)$

Solution: $\neg A \lor (\neg B \land A) \lor C$

$A \leftrightarrow B \equiv (A \land B) \lor (\neg A \land \neg B)$

$\neg(A \leftrightarrow B) \equiv \neg(A \land B) \land \neg(\neg A \land \neg B)$

$(\neg A \lor \neg B) \land (A \lor B)$ $\qquad$ **$A \rightarrow C \equiv \neg A \lor C$**

Full so far: $(((\neg A \lor \neg B) \land (A \lor B)) \quad \lor (\neg A \land \neg B)) \quad \lor (\neg A \lor C)$

$(\neg A \land A) \lor (\neg A \land B) \lor (\neg B \land A) \lor (\neg B \land B) \quad \lor (\neg A \land \neg B)) \quad \lor (\neg A \lor C)$

$F \quad \lor (\neg A \land B) \lor (\neg B \land A) \lor (F) \quad \lor (\neg A \land \neg B)) \quad \lor (\neg A \lor C)$

$(\neg A \land B) \lor (\neg B \land A) \lor (\neg A \land \neg B)) \quad \lor (\neg A \lor C)$

$\neg A \lor (\neg B \land A) \quad \lor (\neg A \lor C)$

$\neg A \lor (\neg B \land A) \lor C$