Math 334 Homework #2 (Section 1.2)

There are five problems on this assignment.

1. (6 points) Let P(A^c ∩ B) = 0.4 and P(A^c ∩ B^c) = 0.5. Find P(A).

As $P(A \cap B)+P(A \cap Bc)+P(Ac \cap B)+P(Ac \cap Bc)=1$, $P(A \cap B)+P(A \cap Bc)=1-0.9 = 0.1$
$P(A) = P(A \cap B)+P(A \cap Bc) = 0.1$
$P(A) = 0.1$

2. Let Ω be a sample space, and let R, S, and T be three subsets of Ω, not necessarily disjoint. Each of these sets describes a specific "event." Give the expressions for these compound events listed below in terms of set notation. For example, "E and F both occur" would be represented by the expression E ∩ F.

(a) (2 points) All three events R, S, and T occur.

$R \cap S \cap T$

(b) (2 points) None of the events R, S, or T occur.

$Rc \cap Sc \cap Tc$

(c) (2 points) Only event S occurs, and not R or T.

$S \cap Rc \cap Tc$

(d) (2 points) Exactly one event occurs (but it can be any of R, S, or T).

$(R \cap Sc \cap Tc) \cup (S \cap Rc \cap Tc) \cup (T \cap Rc \cap Sc)$

(e) (2 points) At least two events out of the three occur.

$(R \cap S) \cup (R \cap T) \cup (S \cap T)$

3. (6 points) The formula for the probability of the union of two events can be extended to the union of three events as follows:

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

Use this formula to help solve the following problem:

A customer visiting a suit department of a certain store will purchase a suit with probability 0.32 a shirt with probability 0.30, a tie with probability 0.24. The customer will purchase both a suit and a shirt with probability 0.09, both a suit and a tie with probability 0.11, and both a shirt and a tie with probability 0.10. A customer will purchase all three items with probability 0.06. What is the probability a customer purchases none of these items?

$P(A \cup B \cup C) = 0.32+0.30+0.24–0.09–0.11–0.10+0.06 = 0.62$

The probability of not buying any is then 1 - 0.62 or 0.38

4.(6 points) Human visual inspection of solder joints on printed circuit boards can be very subjective. Part of the problem stems from the numerous types of solder defects (e.g., pad non-wetting, knee visibility, voids) and even the degree to which a joint possesses one or more of these defects. Consequently, even highly trained inspectors can disagree on the disposition of a particular joint.

In one batch of 10,000 joints:

- - Inspector A found 724 judged defective, (EVENT A)
- - Inspector B found 751 judged defective, (EVENT B)
- - 1,159 joints were judged defective by at least one of the inspectors. (EVENT C)

Suppose one of the 10,000 joints is randomly selected.

(a) What is the probability that the selected joint was judged to be defective by neither of the two inspectors?

If P(C) = 0.1159 | P(Neither) = 1 - P(C) = 0.8841

(b) What is the probability that the selected joint was judged to be defective by Inspector B but not by Inspector A?

$P(A \cap B) = 0.0724 + 0.0751 - 0.1159 = 0.0316.$
$P(B) - P(B \cap A).$
$0.0751 - 0.0316 = 0.0435$

5. (12 points) A computer consulting firm presently has bids out on three projects. Let Ai = {awarded project i} for i = 1, 2, 3, and suppose that: P(A1) = 0.22, P(A2) = 0.25, P(A3) = 0.28, P(A1 ∩ A2) = 0.11, P(A1 ∩ A3) = 0.05, P(A2 ∩ A3) = 0.07, P(A1 ∩ A2 ∩ A3) = 0.01.

For each of the following expressions, write a verbal description of the event AND find its probability:

(a) A1 ∪ A2

$P(A1) + P(A2) - P(A1 \cap A2) = 0.22 + 0.25 - 0.11 = 0.36$
At least one of projects 1 or 2 received an award

(b) $A1^c \cap A2^c$

$1 - P(A1 \cup A2) = 1 - 0.36 = 0.64$
Neither project 1 nor project 2 received an award

(c) $A1 \cup A2 \cup A3$

P(each individual) - P(any combination of 2) + P(Combination all 3)
$0.22 + 0.25 + 0.28 - 0.11 - 0.05 - 0.07 + 0.01 = 0.53$
At least one from project 1, 2, or 3, received an award

(d) $A1^c \cap A2^c \cap A3^c$

1 - P(event from part c)
$1 - 0.53 = 0.47$.
None of the projects received an award

(e) $A1^c \cap A2^c \cap A3$

$P(A3) - P(A1 \cap A3) - P(A2 \cap A3) + P(A1 \cap A2 \cap A3)$
$0.28 - 0.05 - 0.07 + 0.01 = 0.17$.
Only project 3 received an award

(f) $(A1^c \cap A2^c) \cup A3$

P(part b) + P(A3) - P(part e)
$0.64 + 0.28 - 0.17 = 0.75$.
Project 3 received an award, or neither of the first 2 projects received an award.