

Homework 02

Please note that you should type your assignment using either L^ATEX or Word. Both templates will be provided. **Hand-written assignments will not be graded.** You need to submit a **pdf** version on Gradescope by the due date given on Canvas.

1. (20 points) What conclusion can you draw from the given statement?
- (a) (5 points) If you press the button, nothing will happen. You press the button.

Solution:

Nothing will happen

- (b) (5 points) If Jack is taking Algorithms, then he has passed Discrete Math. Jack has not passed Discrete Math.

Solution:

Jack is not taking algorithms

- (c) (5 points) All irrational numbers are real numbers. x is an irrational number.

Solution: x is a real number

- (d) (5 points) Jenna will give a speech or she will give a speech and write a paper.

Solution:

Jenna will definitely give a speech

2. (10 points) Find the negation of the following statements. Write your final negation as an English sentence.

- (a) (5 points) If I take Database Management and Linear Algebra this semester, I will not take Algorithms.

Solution:

I will take Linear Algebra, Database Management, and Algorithms this summer

- (b) (5 points) It is neither a weekend nor a holiday.

Solution:

It is either a weekday or a holiday

3. (10 points) Justify each step in the proof sequence below by writing the line numbers each step follows from. You do not need to write the names of any laws.

$$(A \rightarrow (B \vee C)) \wedge \neg B \wedge \neg C \rightarrow \neg A$$

Solution:

This argument is valid

Proof.

1. $(A \rightarrow (B \vee C))$ (Assume, 1)
2. $\neg B$ (Assume, 1)
3. $\neg C$ (Assume, 1)
4. $\neg B \wedge \neg C$ (2 and 3)
5. $\neg(B \vee C)$ (4)
6. $\neg A$ (1 and 5)

4. (20 points) Translate and write out the following argument into a single propositional logic statement (using variables and logical connectives), then provide a proof sequence to establish the validity of this argument.

If the suspect is guilty, then the knife was in the kitchen. Either the knife was not in the kitchen or Richard saw the knife. If the knife was not in the garage, it follows that Richard didn't see the knife. Furthermore, if the knife was in the garage, then the hammer was in the garage. But we all know that the hammer was not in the garage. Therefore, the suspect is innocent.

Let I be the suspect is innocent, Let K be the knife was in the kitchen, Let R be Richard saw the knife, Let G be the knife was in the garage, and Let H be the hammer was in the garage.

Solution:

$$(\neg I \rightarrow K) \wedge (\neg K \vee R) \wedge (\neg G \rightarrow \neg R) \wedge (G \rightarrow H) \wedge \neg H \rightarrow I$$

1. $\neg I \rightarrow K$ (Premise)
2. $\neg K \vee R$ (Premise)
3. $\neg G \rightarrow \neg R$ (Premise)
4. $G \rightarrow H$ (Premise)
5. $\neg H$ (Premise)
6. $\neg G$ (4, 5)
7. $\neg R$ (3, 6)
8. $\neg K$ (2, 7)
9. I (1, 8)

5. (40 points) Use propositional logic to prove that the following arguments are valid:

(a) (10 points) $(A \rightarrow B) \wedge (A \rightarrow (B \rightarrow C)) \rightarrow (A \rightarrow C)$

Solution: The argument is valid

Proof.

1. $(A \rightarrow B) \wedge (A \rightarrow (B \rightarrow C))$ (Premise)
2. $A \rightarrow B$ (1)
3. $A \rightarrow (B \rightarrow C)$ (1)
4. A (Assumption)
5. B (2, 4)
6. $B \rightarrow C$ (3, 4)
7. C (5, 6)
8. $A \rightarrow C$ (4-7)

(b) (10 points) $\neg(\neg A \vee B) \wedge (\neg C \rightarrow B) \wedge (D \vee B) \rightarrow (A \wedge C \wedge D)$

Solution: Argument is valid

Proof.

1. $\neg(\neg A \vee B)$ (Premise)
2. $\neg C \rightarrow B$ (Premise)
3. $D \vee B$ (Premise)
4. $\neg(\neg A \vee B) \rightarrow (\neg\neg A \wedge \neg B)$
5. $\neg\neg A \wedge \neg B$ (1, 4)
6. A (5)
7. $\neg B$ (5)
8. $\neg C \rightarrow B$ (2)
9. $\neg C$ (7, 8)
10. C (9)
11. $D \vee B$ (3)
12. D (7, 11)
13. $A \wedge C \wedge D$ (6, 10, 12)
14. $\neg(\neg A \vee B) \wedge (\neg C \rightarrow B) \wedge (D \vee B) \rightarrow (A \wedge C \wedge D)$ (1-13)

(c) (10 points) $(A \rightarrow (B \rightarrow C)) \rightarrow (B \rightarrow (A \rightarrow C))$

Solution:

Proof.

1. $A \rightarrow (B \rightarrow C)$ (Premise)
2. B (Assumption)
3. $B \rightarrow C$ (1, 2)
4. A (Assumption)
5. C (3, 4)
6. $A \rightarrow C$ (4-5)
7. $B \rightarrow (A \rightarrow C)$ (2-6)
8. $(A \rightarrow (B \rightarrow C)) \rightarrow (B \rightarrow (A \rightarrow C))$ (1-7)

(d) (10 points) $A \wedge (B \vee \neg C) \wedge \neg(A \wedge B) \rightarrow (\neg B \wedge \neg C)$

Solution:*Proof.*

1. $A \wedge (B \vee \neg C) \wedge \neg(A \wedge B)$ (Premise)
2. A (1)
3. $B \vee \neg C$ (1)
4. $\neg(A \wedge B)$ (1)
5. $\neg B \vee \neg A$ (4)
6. $\neg A \vee \neg B$ (5)
7. $\neg A$ (2, 6)
8. $\neg C$ (3, 7)
9. $\neg B \wedge \neg C$ (8, 7)
10. $A \wedge (B \vee \neg C) \wedge \neg(A \wedge B) \rightarrow (\neg B \wedge \neg C)$ (1-9)