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a) $\int_1^3 \frac{x(-x+4)}{9} dx$ $\int_1^3 \frac{4x}{9} - \frac{x^2}{9}$ $\frac{2}{9}x^2 - \frac{x^3}{27}\Big|_1^3$ $2-1-\left(\frac{2}{9} - \frac{1}{27}\right)$

$$\boxed{\frac{22}{27}}$$

b) $\int_0^x \frac{\alpha \theta^\alpha}{(y+\theta)^{\alpha+1}} dy$ ~~$\int_0^x f(y) dy$~~ $\int_0^x f(y) dy$ $\boxed{0}$

c) $\int_0^1 t^2 e^{at} dt$

$\begin{array}{ll} t^2 & e^{at} \\ 2t & \frac{e^{at}}{a} \\ 2 & \frac{e^{at}}{a^2} \\ 0 & \frac{1}{a^3}e^{at} \end{array}$

$t^2 \frac{e^{at}}{a} - 2t\frac{e^{at}}{a^2} + \frac{2e^{at}}{a^3}\Big|_0^1$ $\boxed{\frac{e^a}{a} - \frac{2e^a}{a^2} + \frac{2e^a}{a^3} - 2}$

d) $\int_1^\infty \frac{x}{(1+x^2)^2} dx = \lim_{a\to\infty} \int_1^a \frac{x}{(1+x^2)^2} = \lim_{a\to\infty}\left(-\frac{1}{1+2a^2} + \frac{1}{4}\right) = \boxed{\frac{1}{4}}$