CSCI-358 Spring 2025 **Collaborators (CWIDs only):___________**

Homework 05

Please note that you should type your assignment using LATEX or Word. Both templates will be provided. **Hand-written assignments will not be graded.** You need to submit a **pdf** version on Gradescope by the due date given on Canvas.

1. (10 points) Which of the following statement(s) is/are true? Click the box next to any true statement to mark it. If the checkboxes don't work with your version of Word, you can highlight or otherwise indicate the correct answers instead.

   (a) (5 points) Let $S = \{x \mid x = 2n + 1, n \in Z^+\}$ and $E = \{x \mid x = n + 2, n \in Z^+\}$

   ☐$1 \in S$
   ☐$S - E = \emptyset$
   ☐$S \cap E = S$
   ☐$Z \subseteq E$
   ☐$9 \in (S \cap E)$

   (b) (5 points) Let $S = \{\emptyset, \{1, 3, 5\}, \{6, 8\}, \{1, 3\}, \{4, 5\}\}$

   ☐$\{\emptyset\} \subseteq S$
   ☐$\{\{4, 5\}\} \in 2^S$
   ☐$\{1, 3\} \subseteq S$
   ☐$6 \in S$
   ☐$\{8\} \in S$

2. (12 points) Are the following statements true or false for all arbitrary sets A, B, and C?

   (a) (3 points) $|A| + |B| = |A \cup B|$
   True False

   (b) (3 points) $(A - B)' = A' - B'$
   True False

   (c) (3 points) $(A - C) \cup (B - C) = (A \cup B) - C$
   True False

   (d) (3 points) $A - B = (A - B) \cap A$
   True False

3. (8 points) Describe each of the following sets by listing its elements. **(Assume ℕ denotes the set of all non-negative integers for all questions on this homework.)**

   a. $\{x \mid x \in R \text{ and } x^2 = 11\}$

   b. $\{x \mid x \in Z \text{ and } x^2 + 4x - 5 = 0\}$

c. $\{x \mid x \in N \text{ and } \exists y(y \in \{1, 2, 3\} \text{ and } x < y\}$

d. $\{x \mid x \in N \text{ and } x \leq 10 \text{ and } \sqrt{x} \notin Z\}$

Solution:

a. {-√11,√11}
b. {-5,1}
c. {0,1,2}
d. {2,3,5,6,7,8,10}

4. (5 points) How many *unique* sets are described in the following? What are they (in their simplest form)?

- $\{6,\ 8,\ 10\}$
- $\{x \mid x\ is\ the\ first\ letter\ of\ cats,\ dogs,\ or\ bunnies.\}$
- $\{6,\ c,\ 8,\ b,\ 10,\ d\}$
- $\{x \mid x \in N\ and\ 6 \leq x \leq 10\ and\ x\ is\ even.\}$
- $\{x \mid x \in \{6,\ 8,\ 10\}\ and\ \sqrt{x} \in Z^{+}\}$
- $\{x \mid 2x\ =\ k\ for\ k \in \{16,\ 12,\ 20\}\}$
- $\{d,\ b,\ c\}$
- $\{x \mid x \in \{6,\ 8,\ 10\}\ and\ \sqrt{x} \in\ R\}$

Solution: There are 4 unique sets

1. **{6,8,10}**
2. **{b,c,d}**
3. **{6,c,8,b,10,d}**
4. **{∅}**

5. (15 points) Let S be the set of all Mines students. Consider the following subsets of S:

A = set of all computer science majors
B = set of all computer science majors on the data science track
C = set of all students who are double-majoring
D = set of all female-identifying students
E = set of all first-generation students

Using set operations, describe each of the following sets in terms of $A,\ B,\ C,\ D$, and E.

(a) (5 points) The set of all students who are double majoring either not in computer science at all or in computer science on the data science track.

Solution: C ∩ (A’ U B)

(b) (5 points) The set of all non-computer science majors who are neither first-generation nor identify as female.

Solution: A’ ∩ D’ ∩ E’

(c) (5 points) The set of all female-identifying or first-generation students who are majoring in computer science.

Solution: A ∩ (D U E)

6. (20 points) Let

$A = \{p, q, r, s\}$
$B = \{r, t, v\}$
$C = \{p, s, t, u\}$

be subsets of $S = \{p, q, r, s, t, u, v, w\}$. Find:

(a) (5 points) $A - (B \cup C)$

Solution: {q}

(b) (5 points) C'

Solution: {q,r,v,w}

(c) (5 points) $A \times B$

Solution:

{(p,r),(p,t),(p,v),

(q,r),(q,t),(q,v),

(r,r),(r,t),(r,v),

(s,r),(s,t),(s,v)}

(d) (5 points) $(A \cup B) \cap C'$

Solution: {q,r,v}

7. (30 points) Let A, B, and C be arbitrary sets.

(a) (10 points) Prove that $\left(A' \cup (A \cap C) \cup \left(A \cap C'\right)\right) \cap \left(B' \cap \left(A' \cup B\right) \cap (A \cup B)\right) = \emptyset$ using a sequence of set identities.

Solution:∅

Proof. (A′∪(A∩C)∪(A∩C′))∩(B′∩(A′∪B)∩(A∪B))

A∩C∪A∩C′=A because C∪C′=U and A′∪A =U so first part simplifies to U

B′∩B=∅, then (B′∩A′)∩(A∪B) = ∅

U∩∅=∅

(b) (10 points) Prove that if $A - B = A$ and $B \subseteq A$, then $B = \emptyset$.

Solution:

Proof. A−B={x ∈ A | x /∈ B}

The condition, A−B=A means that every element of A must not be in B, which means B must be the empty set.

Therefor B=∅

(c) (10 points) Prove that if $2^A \subseteq 2^B$, then $A \subseteq B$

Solution:

Proof. Every subset of A is also a subset of B

Assume x∈A This is a subset of A, and given the statement in the question, {x} ⊆ B

As x is arbitrary, every element of A is also in B

A ⊆ B