

Strategic Demand Information Sharing Between Competitors

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We study the incentives of two competing firms — the building block of any supply chain with downstream competition — to exchange private demand signals when demand is uncertain. The novelty and fundamental contribution of our paper lies in its consideration of ex–post or strategic information sharing, where the decision to share or to conceal a private demand signal is made after observing the demand signal. Using a modeling framework with binary demand and binary demand signals, we demonstrate, for the first time in the literature, the existence of a pooling information sharing equilibrium: under certain conditions, competing firms want to share both low– and high–demand signals after receiving them. In addition, we also study the ex–ante version of the model mainly to facilitate comparison with its ex–post counterpart and to benchmark against existing results in the literature. Our analysis of the ex–ante case provides a theoretical proof for as well as an extension of related findings in the existing literature. We also show that not all ex–ante agreements to share information can survive ex–post, and that competitors have an incentive to make sure that any information exchanged is verifiable. As such, our results shed new light on a fundamental piece of the study of information flows in supply chains.

Key words: Information sharing, Cournot competition, Bayes–Nash equilibrium

1. Introduction

Firms collaborate in a wide variety of areas — product development, marketing, technology and information sharing, to name a few. Information sharing between firms can be broadly classified into two types: vertical and horizontal. The benefits of vertical information sharing in supply chains have been widely studied (see Chen 2003). In contrast, horizontal information exchange has seen less recent research; the research that exists is largely limited to the Economics literature of the 1980’s and early 1990’s, which investigates the incentives for firms to sign information sharing agreements prior to observing the demand signal, i.e., ex–ante information sharing. Nevertheless, there has been recent interest in competitors acquiring from and leaking information to each other through supply chain relationships (e.g., Anand and Goyal 2009). As suggested by Chen (2003), the

basic incentives and disincentives for competitors to exchange information are critical foundations for these studies. We believe that these basic incentives and their implications for supply chain relationships have yet to be well understood, especially in situations where the information sharing decision can be made after observing the demand signal, i.e., strategic or ex-post information sharing. Our goal is to partially fill this gap and to demonstrate the sensitivity of the traditional results to various assumptions.

Under some specific assumptions about demand information (e.g., normally distributed uncertainty and signals) and decision timing (e.g., ex-ante commitments to share information before signals are observed), the set of classic results in the Economics literature suggests that, in equilibrium, exchange of demand signals among competitors should not occur (e.g., Clarke 1983, Gal-Or 1985, and Li 1985). However, Kirby (1988), Raith (1996), and Malueg and Tsutsui (1998) demonstrate that an equilibrium analysis of horizontal information sharing can be sensitive to modeling assumptions regarding cost and demand. Evidence from business practice also suggests that information sharing between firms does happen more frequently than the classical results suggest. According to an extensive study undertaken by the Organisation for Economic Co-operation and Development (OECD), competitors exchange information in two ways: directly and through industry/trade associations (OECD 2010). More importantly, the OECD study argues that competing firms, in many cases, stand to benefit from the exchange of demand information. Prof. Kai-Uwe Kuhn remarks that “[d]emand data is of great importance for long run decisions on investment, but also for short run adjustments of firm strategy to market changes or internal incentive schemes that benchmark on market performance. There are a great many potential efficiencies from exchanges of such data and they are routinely included in the statistical programs of trade associations.” (OECD 2010).

In our work, we explore the sensitivity of the information sharing decision along another dimension that we believe is particularly important to Operations Management (OM) researchers: the timing of decision-making. To this end, we model two competing firms — the building block of any supply chain with downstream competition — who receive private signals about market demand, and we analyze their incentives to share demand signals with each other. Unlike most of the Economics literature, which uses a framework of normally distributed demand and signals, we model both the demand and the related signals as binary random variables as in Malueg and Tsutsui (1998). This is a standard modeling framework in the OM literature. In addition, we allow firms to agree to share information before (ex-ante) or after (ex-post or “strategically”) they observe their demand signals.

Our results in the ex-ante case analytically prove and extend findings by Malueg and Tsutsui (1998), who demonstrate, without proof, the existence of an information sharing equilibrium under

binary information structures. However, our interest in the ex-ante case is mainly to facilitate comparison with ex-post information sharing as well as to benchmark against the existing literature. The main contribution and novelty of this paper lies in its consideration of the ex-post case (also known as “strategic information sharing”), in which firms retain the flexibility to strategically share or conceal their private demand information after they have observed it. This is arguably the more realistic scenario in practice for two reasons: 1) Firms might be reluctant to sign blanket information sharing agreements due to antitrust concerns; 2) Agreements between competitors to exchange information are by their very nature non-binding; as the OECD study argues, participation in the statistical programs that facilitate exchange of information among trade association members should be voluntary (OECD 2010). Therefore, an analysis of the incentives to exchange demand information after obtaining it is necessary to study the robustness of any ex-ante agreements to do so. We investigate the incentives for horizontal information sharing first under the assumptions of truthful and mutual information sharing. The mutual or quid pro quo model of information sharing is used in Kirby (1988) and is especially relevant in the context of information sharing mechanisms employed by trade associations. In many cases, trade associations restrict access to information only to firms that participate in the study/survey.

By analyzing ex-post information sharing (i.e., deciding whether or not to share demand signals after these signals are observed), we show the existence of a pooling information sharing equilibrium. Interestingly, though, not all ex-ante information sharing agreements can survive ex-post. Furthermore, we demonstrate that, in some cases, firms can avoid underproduction and thus excessively high prices by exchanging private demand information. This finding is critical and supports Kuhn’s statement that information exchange between competitors can increase efficiency and does not necessarily harm competition. We believe that our results have important implications for supply chain research. For example, given our finding that demand information sharing can sometimes be beneficial, a retailer might sometimes favor leaking information to its competitor. Given that incentives for ex-post and ex-ante information sharing can differ, one might expect ad hoc supply chain arrangements to behave differently from contracted ones.

We also extend our analysis to consider the possibility that information sharing between firms can be non-mutual and non-truthful. The issue of non-truthful information sharing may not be a real issue in practice. As the OECD study points out, “announced plans can often be verified later or revoked, and wrong announcements can be punished, which in long-term commercial relationships may be enough to create credibility.” (OECD 2010). In other words, exchanging non-verifiable information defeats the very purpose of exchanging information. Nevertheless, we show that firms never exchange information in a non-mutual fashion and we also find that if competitors exchange non-verifiable information, they end up lying about their true demand signals, which results in

lower profits compared to the case whether they exchange verifiable information. Therefore, our work demonstrates that, in many circumstances, it is in the best interest of the competing firms to make sure there are mechanisms in place, such as independent trade associations, which can guarantee that all information being exchanged is verifiable and thus truthful.

1.1. Literature Review

Vertical information sharing has received much attention in the OM literature, e.g., Chen (1998), Aviv and Federgruen (1998), Gavirneni et al. (1999), Lee et al. (2000), Gal-Or (2008) and Ha et al. (2011). Vertical information exchange is usually a win-win situation for all supply chain partners. Having access to downstream information benefits the entire supply chain through increased product availability and lower costs. On the other hand, it is difficult to predict the impact of horizontal information sharing on the profitability of firms competing within the same supply chain or across different supply chains. Although having access to the competitor's information helps a firm make more informed decisions, it typically comes at a cost — a firm has to give up its private information in return for the competitor's information. Hence, a firm's information advantage, if any, is lost.

Horizontal information sharing between competitors has also been widely studied. Some of the classic works include Clarke (1983), Gal-Or (1985) and Li (1985), which study incentives for Cournot oligopolists to sign demand information sharing agreements ex-ante. Under the assumptions of normally distributed demands and signals (except for Li 1985, who assumes signals that satisfy the linear conditional expectation property), these papers show that “no information sharing” is the unique equilibrium. However, subsequent works have shown that the “no information sharing” result is sensitive to modeling assumptions. Kirby (1988) demonstrates that the “no information sharing” result can be reversed if the production cost function is sufficiently steep; Raith (1996) shows that the information sharing strategy depends on a variety of factors such as the type of competition, the product under consideration, as well as the kind of uncertainty, i.e., cost or demand. For a special case of our model with binary demand and signals, Novshek and Thoman (2006) demonstrate that the introduction of a capacity choice stage before the information sharing decision can lead to an information sharing equilibrium. Wu et al. (2008) show a similar result. Using a more general model than Novshek and Thoman (2006), we show that commitment to information sharing can be an equilibrium, even in the absence of a capacity decision. Employing the same model as ours, Malueg and Tsutsui (1998) demonstrate, without proof, that information sharing can be an equilibrium in the ex-ante case. We provide an analytical proof and a more complete characterization of the equilibrium properties than Malueg and Tsutsui (1998).

Our main contribution is to consider ex-post information sharing, i.e., strategic information sharing, a problem that is not well-studied in the existing literature. Chu and Lee (2006) study

strategic vertical demand information sharing between a more-informed retailer and a manufacturer. We are aware of only one paper that deals with ex-post information sharing in an oligopolistic setting; Jansen (2008) uses a binary demand model similar to ours but restricts his attention to the special case of equally likely demand states and demand signals that either completely reveal the true demand state or provide no information. An implication of Jansen's restrictive assumptions is that a separating equilibrium, whereby firms reveal low-demand signals and conceal high-demand signals, is the unique equilibrium. By employing a more general model, we show the existence of a pooling information sharing equilibrium, wherein firms share both high- and low-demand signals. Horizontal information sharing between competitors has also been studied in other contexts, not closely related to ours. For example, Bernstein and DeCroix (2006) and Zhang (2006) examine the value of horizontal inventory information sharing between two suppliers supplying to a single manufacturer.

Finally, an interesting intersection of the literatures on vertical and horizontal information sharing are papers that study vertical information sharing resulting in horizontal information leakage. In these papers, the information sharing decisions of downstream retailers mainly depend on whether the upstream manufacturer will leak the information shared to their competitors. In Li (2002), retailers can share their demand signals with a common upstream manufacturer who sets a wholesale price based on the shared signals. This results in unintentional information leakage because the wholesale price is observed even by retailers that did not share their demand signals. The author concludes that no information sharing is the unique equilibrium. A closely related work is Jain et al. (2011). In a setting similar to Li (2002)'s, Anand and Goyal (2009) study information leakage from an incumbent firm to an entrant firm through a common supplier. Unlike most information sharing models, they endogenize the information acquisition decision and find that the incumbent either does not acquire information or acquires information and always passes it on to the manufacturer, who in turn leaks it to the entrant firm. In our model, there is no unintentional information leakage — firms share information only if they want to do so.

1.2. Paper Outline

The rest of the paper is organized as follows. In §2, we detail our strategic (i.e., ex-post) information sharing model and formulate conditions for the existence of an information sharing equilibrium. We develop some intuition for the incentives of competitors to share/conceal information in §3. In §4, we demonstrate both analytically and numerically how firms behave in equilibrium under ex-post information sharing. In §5, we extend our analysis to include exchange of non-verifiable information as well as non-mutual information sharing. In §6, we present the equilibrium analysis under ex-ante information sharing and also compare the results with ex-post information sharing. Finally, in §7, we summarize our findings and state our conclusions.

2. Model and Preliminary Results

We model the horizontal information sharing problem as a two-stage game between two competing firms, indexed by $\ell \in \{1, 2\}$. We use ℓ' to denote the competitor to firm ℓ . Demand uncertainty is binary; α is a random variable representing the true demand state and α_H and α_L are its high and low realizations, respectively. In the first stage, each firm observes a private signal about the true demand state, which could be either *high* (H) or *low* (L). An example of a signal could be a demand forecast obtained through customer surveys or focus group interviews. The prior probabilities associated with high and low demand are r_H and $r_L = 1 - r_H$ respectively, i.e., $P(\alpha = \alpha_H) \equiv r_H$ and $P(\alpha = \alpha_L) \equiv r_L$. Firm ℓ does not know the true demand state but receives a private signal, denoted by S_ℓ , which could also be either H or L . We model signal accuracy in an explicit way using conditional probabilities, i.e.,

$$P(S_\ell = j | \alpha = \alpha_i) \equiv p_{ij}, \quad \ell \in \{1, 2\}, \quad i, j \in \{H, L\}.$$

The signal accuracy is the same for both firms, i.e., firms are symmetric in their ability to estimate demand. To simplify the notation, we further define:

$$p \equiv p_{ii} \equiv 1 - p_{ij} \quad \text{where } i, j \in \{H, L\}, \quad j \neq i.$$

Thus, larger values of p indicate more accurate demand signals. We assume that $P(S_\ell = j | \alpha = \alpha_i, S_{\ell'} = j') = P(S_\ell = j | \alpha = \alpha_i)$, $\ell, \ell' \in \{1, 2\}$, $j, j' \in \{H, L\}$, i.e., the firms' signals are independent, conditional on the true demand state. In the second stage, firms choose production quantities in a Cournot duopoly with an inverse demand function given by

$$p(Q) = \alpha - Q,$$

where Q represents the total production quantity of the two firms. Firms determine production quantities based on information available at the beginning of the second stage and without observing the true value of α . This information depends on whether or not firms decide to share their private information. We will elaborate on this decision in the remainder of this section. Because our focus is on sharing demand information, we assume away production costs, that is we assume that they are common knowledge, the same for both firms, and normalized to zero. As we mentioned before, we also assume for much of the paper that information, if exchanged, is truthfully exchanged. This is typical in the literature on horizontal information sharing (e.g., Clarke 1983, Gal-Or 1985 and Li 1985), as an ex-ante commitment to share information has little meaning unless information is shared truthfully. We will examine in Section 5 the implications of non-truthful information sharing in the ex-post setting.

In the ex-post version of our model, each firm receives a private signal about the true demand state, thereby resulting in a game with incomplete information. Therefore, we will use the concept of Bayes Nash Equilibrium (BNE) to determine the equilibrium information sharing strategy. Each firm's type is the private demand signal it receives at the beginning of the first stage. The action space of each firm is {share, not share} in the first-stage game and \mathbb{R}_+ (i.e., non-negative production quantities) in the second-stage game. In the ex-ante version, firms are not distinguished by their types and hence a standard Nash Equilibrium concept applies to that setting.

2.1. Ex-Ante vs. Ex-Post Information Sharing

Before we proceed to the analysis, we wish to point out, at a high level, the difference between ex-ante and ex-post information sharing. As we mentioned in Section 1, our interest in the ex-ante model is mainly to facilitate comparison with ex-post information sharing, which is the main focus of this paper. Figure 1 provides the sequence of events in the two settings.

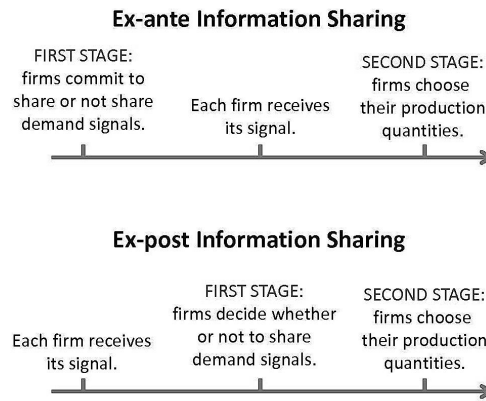


Figure 1 Timeline for ex-ante and ex-post information sharing.

Notice that under ex-ante information sharing, firms commit to share or conceal their private signals *before* actually observing them. In contrast, in the ex-post information sharing scenario, firms make the decision regarding sharing their demand signals *after* observing them. Hence, firms can make use of the extra information (demand signal) available to them to update their beliefs about the true demand before they make the sharing decision. Additionally, under ex-post information sharing, a firm's action (share/not share) might provide its competitor some information regarding the firm's signal. This is not the case under ex-ante information sharing, where actions are not contingent on the signals received.

In what follows, through Section 5, our discussion will focus on ex-post information sharing. In the next subsection, we show how firms can use the information available to them at the beginning of the first and second stages to update their beliefs about the true demand state as well as

the other firm's demand signal. These beliefs are then used to develop profit expressions for the firm under different information sharing strategies (share/not share). In subsequent sections, we use these profit expressions to characterize the equilibrium production quantities and information sharing strategies.

2.2. Belief Updates in the Ex-Post Setting

At the beginning of the first stage, both firms observe their demand signals, which they use to update their beliefs about the true demand state. Since firms are symmetric, we drop the subscript referring to firms whenever possible. Let $\beta_j(i) \equiv P(\alpha = \alpha_i | S_\ell = j)$, $i, j \in \{H, L\}, \ell \in \{1, 2\}$. A simple application of Bayes rule yields

$$\begin{aligned} \beta_j(i) &= P(\alpha = \alpha_i | S_\ell = j) \\ &= \frac{P(S_\ell = j | \alpha = \alpha_i) P(\alpha = \alpha_i)}{P(S_\ell = j)} \\ &= \frac{p_{ij} r_i}{p_{Hj} r_H + p_{Lj} r_L}. \end{aligned} \quad (1)$$

In addition to updating its belief about the true demand state, a firm constructs beliefs about its competitor's signal, given its own signal. Let $\delta_{j'}(j) \equiv P(S_\ell = j | S_{\ell'} = j')$, $\ell, \ell' \in \{1, 2\}, j, j' \in \{H, L\}$. By conditioning firm ℓ 's signal on the true demand state, firm ℓ' computes belief $\delta_{j'}(j)$ as follows.

$$\begin{aligned} \delta_{j'}(j) &= P(S_\ell = j | \alpha = \alpha_H, S_{\ell'} = j') P(\alpha = \alpha_H | S_{\ell'} = j') \\ &\quad + P(S_\ell = j | \alpha = \alpha_L, S_{\ell'} = j') P(\alpha = \alpha_L | S_{\ell'} = j') \\ &= P(S_\ell = j | \alpha = \alpha_H) P(\alpha = \alpha_H | S_{\ell'} = j') \\ &\quad + P(S_\ell = j | \alpha = \alpha_L) P(\alpha = \alpha_L | S_{\ell'} = j') \\ &= p_{Hj} \beta_{j'}(H) + p_{Lj} \beta_{j'}(L). \end{aligned} \quad (2)$$

Now, consider the information that a firm possesses at the beginning of the second stage. Having observed their demand signals, we assume that firms simultaneously communicate their decisions to share or conceal their signals. An alternate assumption could be sequential communication of information sharing decisions but it brings additional complications like first-mover advantage/disadvantage and takes the focus away from the central issue of information sharing. An important implication of the simultaneous communication assumption is that firms get to know the other firm's share/conceal decision regardless of their own decision. In Section 5, we relax the simultaneous communication assumption and allow for non-mutual or unilateral communication. When firms decide to share or conceal information ex-post, a firm observes its competitor's signal only if both are willing to share their signals. Otherwise, a firm knows its own signal and the competitor's intention to share/conceal its signal. Before discussing how firms update their beliefs

further, we introduce some additional notation. Let $X_\ell \in \{S, NS\}$ denote the action of firm ℓ in the first stage *after* observing its private signal. Here S and NS stand for “Share” and “Not Share” respectively. After observing its private signal, each firm decides whether or not to share this information with its competitor in exchange for the other firm’s signal. Let $\sigma_{\ell j}(x) \equiv P(X_\ell = x | S_\ell = j)$ represent the probability that firm ℓ chooses action x having seen signal j , $j \in \{H, L\}$, $x \in \{S, NS\}$. In our work, we consider only *pure* strategies and hence these probabilities are either 0 or 1.

We first focus on how firms update their beliefs about true demand once they have decided whether or not to share their private signals and observed their competitor’s action (and possibly their competitor’s signal, too). First, suppose that firms observe their signals and agree to share them. In this case, each firm knows its own signal as well as its competitor’s. On the other hand, if one firm decides not to share its signal, then neither firm has access to its competitor’s signal. Instead, each firm will know its own signal and its competitor’s action. As a result, the following four cases are possible at the beginning of the second stage, i.e., right before determining production quantities:

1. A firm knows its own signal and that its competitor’s signal is H .
2. A firm knows its own signal and that its competitor’s signal is L .
3. A firm knows its own signal and that its competitor is not willing to share its signal.
4. A firm knows its own signal, does not know its competitor’s signal but knows that its competitor is willing to share its signal. This is possible if the firm under consideration decided not to share its signal.

The first two cases arise when both firms are willing to share information. The last two cases arise when at least one of the firms is not willing to share its private information. We denote a firm’s information about its competitor by ϕ , where $\phi \in \Phi = \{SH, SL, NS, S\emptyset\}$. The four possibilities listed above are represented by $\phi = SH$, $\phi = SL$, $\phi = NS$, and $\phi = S\emptyset$ respectively. Using its own signal and information about its competitor, at the beginning of the second stage each firm can calculate $\mu_{\ell j}(i|\phi) \equiv P(\alpha = \alpha_i | S_\ell = j, \phi)$.

When $\phi = SH$ or $\phi = SL$, a firm knows its competitor’s signal. In this case, the knowledge that the competing firm is willing to share its signal has no additional value. In this case, $\mu_{\ell j}(i|Sj') \equiv P(\alpha = \alpha_i | S_\ell = j, S_{\ell'} = j')$. Here, $i, j, j' \in \{H, L\}$ and $\ell, \ell' \in \{1, 2\}$. Applying Bayes rule,

$$\begin{aligned}
 \mu_{\ell j}(i|Sj') &= P(\alpha = \alpha_i | S_\ell = j, S_{\ell'} = j') \\
 &= \frac{P(S_\ell = j, S_{\ell'} = j' | \alpha = \alpha_i) P(\alpha = \alpha_i)}{P(S_\ell = j, S_{\ell'} = j')} \\
 &= \frac{P(S_\ell = j | \alpha = \alpha_i) P(S_{\ell'} = j' | \alpha = \alpha_i) P(\alpha = \alpha_i)}{P(S_\ell = j | S_{\ell'} = j') P(S_{\ell'} = j')} \\
 &= \frac{p_{ij} p_{ij'} r_i}{(p_{Hj} \beta_{j'}(H) + p_{Lj} \beta_{j'}(L)) (p_{Hj'} r_H + p_{Lj'} r_L)}, \tag{3}
 \end{aligned}$$

where the last equality makes use of equation (2).

When $\phi = S\emptyset$ or $\phi = NS$, each firm only observes the action of the other firm. So, $\mu_{\ell j}(i|S\emptyset) = P(\alpha = \alpha_i|S_\ell = j, X_{\ell'} = S)$ and $\mu_{\ell j}(i|NS) = P(\alpha = \alpha_i|S_\ell = j, X_{\ell'} = NS)$. We can calculate these probabilities as follows.

$$P(\alpha = \alpha_i|S_\ell = j, X_{\ell'} = x') = \frac{P(S_\ell = j|\alpha = \alpha_i) P(X_{\ell'} = x|\alpha = \alpha_i) P(\alpha = \alpha_i)}{P(X_{\ell'} = x'|S_\ell = j) P(S_\ell = j)},$$

where

$$\begin{aligned} & P(X_{\ell'} = x'|S_\ell = j) \\ &= [P(X_{\ell'} = x'|S_{\ell'} = H) P(S_{\ell'} = H|\alpha = \alpha_H) + P(X_{\ell'} = x'|S_{\ell'} = L) P(S_{\ell'} = L|\alpha = \alpha_H)] \\ & \quad P(\alpha = \alpha_H|S_\ell = j) \\ &+ [P(X_{\ell'} = x'|S_{\ell'} = H) P(S_{\ell'} = H|\alpha = \alpha_L) + P(X_{\ell'} = x'|S_{\ell'} = L) P(S_{\ell'} = L|\alpha = \alpha_L)] \\ & \quad P(\alpha = \alpha_L|S_\ell = j) \\ &= [\sigma_{\ell'H}(x')p_{HH} + \sigma_{\ell'L}(x')p_{HL}] \beta_j(H) + [\sigma_{\ell'H}(x')p_{LH} + \sigma_{\ell'L}(x')p_{LL}] \beta_j(L). \end{aligned}$$

We still need to derive an expression for $P(X_{\ell'} = x'|\alpha = \alpha_i)$. Using Bayes rule, we get

$$\begin{aligned} P(X_{\ell'} = x'|\alpha = \alpha_i) &= P(X_{\ell'} = x'|S_{\ell'} = H, \alpha = \alpha_i) P(S_{\ell'} = H|\alpha = \alpha_i) \\ & \quad + P(X_{\ell'} = x'|S_{\ell'} = L, \alpha = \alpha_i) P(S_{\ell'} = L|\alpha = \alpha_i) \\ &= P(X_{\ell'} = x'|S_{\ell'} = H) P(S_{\ell'} = H|\alpha = \alpha_i) \\ & \quad + P(X_{\ell'} = x'|S_{\ell'} = L) P(S_{\ell'} = L|\alpha = \alpha_i) \\ &= \sigma_{\ell'H}(x')p_{iH} + \sigma_{\ell'L}(x')p_{iL}. \end{aligned}$$

In the above derivation, the second equality holds because strategies (actions of firms in the first stage) depend only on observed signals. Finally, combining the above derived expressions yields

$$\begin{aligned} & P(\alpha = \alpha_i|S_\ell = j, X_{\ell'} = x') \\ &= \frac{p_{ij}(\sigma_{\ell'H}(x')p_{iH} + \sigma_{\ell'L}(x')p_{iL})r_i}{[(\sigma_{\ell'H}(x')p_{HH} + \sigma_{\ell'L}(x')p_{HL})\beta_j(H) + (\sigma_{\ell'H}(x')p_{LH} + \sigma_{\ell'L}(x')p_{LL})\beta_j(L)](p_{Hj}r_H + p_{Lj}r_L)}. \end{aligned} \quad (4)$$

In addition to updating its belief about the true demand state, a firm constructs beliefs about the possible types (i.e., signals) of its competitor given its own signal and its competitor's action, i.e., it calculates $\tilde{\delta}_{\ell j}(j'|x') \equiv P(S_{\ell'} = j'|S_\ell = j, X_{\ell'} = x')$, where

$$\begin{aligned} \tilde{\delta}_{\ell j}(j'|x') &= \frac{P(S_{\ell'} = j', S_\ell = j, X_{\ell'} = x')}{P(X_{\ell'} = x', S_\ell = j)} \\ &= \frac{P(X_{\ell'} = x'|S_{\ell'} = j', S_\ell = j) P(S_{\ell'} = j', S_\ell = j)}{P(X_{\ell'} = x', S_\ell = j)} \end{aligned}$$

$$= \frac{P(X_{\ell'} = x' | S_{\ell'} = j') P(S_{\ell'} = j' | S_{\ell} = j) P(S_{\ell} = j)}{P(X_{\ell'} = x', S_{\ell} = j)}. \quad (5)$$

We already showed how to compute $P(S_{\ell'} = j' | S_{\ell} = j)$ in equation (2). The only remaining term in equation (5) is $P(X_{\ell'} = x', S_{\ell} = j)$, which can be computed using repeated application of Bayes rule as shown below.

$$\begin{aligned} P(X_{\ell'} = x', S_{\ell} = j) &= P(X_{\ell'} = x', S_{\ell'} = H, S_{\ell} = j) + P(X_{\ell'} = x', S_{\ell'} = L, S_{\ell} = j) \\ &= P(X_{\ell'} = x' | S_{\ell'} = H, S_{\ell} = j) P(S_{\ell'} = H | S_{\ell} = j) P(S_{\ell} = j) \\ &\quad + P(X_{\ell'} = x' | S_{\ell'} = L, S_{\ell} = j) P(S_{\ell'} = L | S_{\ell} = j) P(S_{\ell} = j) \\ &= [\sigma_{\ell' H}(x') \delta_j(H) + \sigma_{\ell' L}(x') \delta_j(L)] (p_{Hj} r_H + p_{Lj} r_L). \end{aligned}$$

Combining the last expression with equation (2), we get

$$\tilde{\delta}_{\ell j}(j' | x') = \frac{\sigma_{\ell' j'}(x') \delta_j(j')}{(\sigma_{\ell' H}(x') \delta_j(H) + \sigma_{\ell' L}(x') \delta_j(L))}. \quad (6)$$

So far, we have derived the expressions for the belief updates done at the beginning of the first and second stages. For convenience, we collect the key notations introduced so far along with a brief description of each symbol in Table 1.

Symbol	Conditional probability	Description
$\beta_j(i)$	$P(\alpha = \alpha_i S_{\ell} = j)$	probability that the true demand state is α_i given that firm ℓ 's signal is j
$\delta_{j'}(j)$	$P(S_{\ell} = j S_{\ell'} = j')$	probability that firm ℓ 's signal is j given that firm ℓ' 's signal is j'
$\mu_{\ell j}(i \phi)$	$P(\alpha = \alpha_i S_{\ell} = j, \phi)$	probability that the true demand state is α_i given that firm ℓ 's signal is j and firm ℓ 's information about firm ℓ' is ϕ
$\tilde{\delta}_{\ell j}(j' x')$	$P(S_{\ell'} = j' S_{\ell} = j, X_{\ell'} = x')$	probability that firm ℓ' 's signal is j' given that firm ℓ 's signal is j and firm ℓ' 's action is x'

Table 1 Table of symbols

Having expressed the firm's beliefs corresponding to each of the possible information states in Φ , we next explain how firms use these belief updates in determining their production quantities at the beginning of the second stage and their information sharing strategies in the first stage.

2.3. Profit Expressions in the Ex-Post Setting

Under strategic information sharing, firms make decisions on a case-by-case basis, i.e., having observed their signal, they decide whether it would be beneficial to share it with their competitor. Hence we need to compare the expected profits under no information sharing and information

sharing for each signal separately. We use backwards induction to calculate the expected profits under the two strategies (sharing and not sharing information).

Let us start with the case in which both firms agree to share their signals after observing them. This means that firms have access to each other's signals and the second-stage profit in this case is given by

$$\pi_\ell(j, j') = [\alpha_H \mu_{\ell j}(H|Sj') + \alpha_L \mu_{\ell j}(L|Sj') - (q_\ell(j, j') + q_{\ell'}(j', j))] q_\ell(j, j'), \quad j, j' \in \{H, L\}. \quad (7)$$

When firms do not agree to share their private information after observing it, at the beginning of the second stage each firm knows its own signal and the other firm's action. However, the second-stage expected profit calculations cannot be carried out based on this information alone. Each firm has to take into account the fact that its own decision to share or conceal information is factored into the quantity decision of its competitor. Taking this into consideration, we can write the second-stage expected profit for a firm under no information sharing as

$$\pi_\ell^x(j|\phi) = \left[\alpha_H \mu_{\ell j}(H|\phi) + \alpha_L \mu_{\ell j}(L|\phi) - \left(q_\ell^x(j|\phi) + \sum_{j' \in \{H, L\}} \tilde{\delta}_{\ell j}(j'|x') q_{\ell'}^{x'}(j'|\phi') \right) \right] q_\ell^x(j|\phi), \quad j, j' \in \{H, L\}. \quad (8)$$

In the last expression, x is the action (S or NS) of firm ℓ in the first stage and ϕ is the information set of firm ℓ ; x' and ϕ' are firm ℓ' 's action and information set, respectively. Here, $\phi, \phi' \in \{S\emptyset, NS\}$. (We point out that ϕ and ϕ' cannot be $S\emptyset$ simultaneously because that would imply that both firms want to share information and information sharing would take place.) In expression (8), $q_\ell^x(j|\phi)$ is the equilibrium production quantity given the information available at the beginning of the second stage and $\pi_\ell^x(j|\phi)$ represents the corresponding second-stage profit of firm ℓ when it has received signal j and has information ϕ about its competitor.

We are mainly interested in the existence of an “information sharing” equilibrium, i.e., an equilibrium where both firms want to share their private information. Under strategic information sharing, a firm wants to share its information only if doing so results in at least as much profit as concealing it. More importantly, this condition should hold for both H and L signals. (Note that sharing one signal type and concealing the other one is outcome-equivalent to sharing both.) Assuming that its competitor wants to share information for both signal types, firm ℓ , having seen signal j , can calculate its expected profit under information sharing as follows:

$$\Pi_{\ell, j}(S) = \sum_{j' \in \{H, L\}} \pi_\ell(j, j') \delta_{\ell j}(j'), \quad \ell \in \{1, 2\}, \quad j \in \{H, L\}. \quad (9)$$

If a firm decides against sharing information, then the information available to that firm at the beginning of the second stage would be its own signal and the fact that the other firm was willing

to share information. Hence, the expected profit for the firm under no information sharing would be

$$\Pi_{\ell,j}(NS) = \pi_{\ell}^{NS}(j|S\emptyset), \ell \in \{1, 2\}, j \in \{H, L\}. \quad (10)$$

Using (9) and (10) and the definition of BNE, we see that information sharing is an equilibrium strategy for a firm if

$$\Pi_{\ell,j}(S) - \Pi_{\ell,j}(NS) \geq 0, \ell \in \{1, 2\}, j \in \{H, L\}. \quad (11)$$

We briefly remark on the off-the-equilibrium-path beliefs when firms decide ex-post whether to share information. Recall that the belief updates $P(\alpha = i | S_{\ell} = j, X_{\ell'} = x')$ and $P(S_{\ell'} = j' | S_{\ell} = j, X_{\ell'} = x')$ (done at the beginning of the second stage) take into account the competitor's choice to share/conceal information. In case of a pooling equilibrium (where a firm's action is the same for both signals), when a firm observes a non-equilibrium action from its competitor, the above-mentioned belief updates cannot be calculated correctly since the denominators in expressions (4) and (6) become zero. To overcome this issue, we assume that the competitor's non-equilibrium action is ignored when updating these beliefs, i.e., $P(\alpha = i | S_{\ell} = j, X_{\ell'} = x') = P(\alpha = i | S_{\ell} = j)$ and $P(S_{\ell'} = j' | S_{\ell} = j, X_{\ell'} = x') = P(S_{\ell'} = j' | S_{\ell} = j)$. Here x' is the competitor's off-the-equilibrium action. Ignoring a non-equilibrium action when making Bayesian updates is consistent with the literature (see Caminal and Vives 1996, Hart and Tirole 1990).

3. The Pros and Cons of Sharing Signals

Before characterizing the equilibrium behavior in Section 4, we explore the tradeoff a firm faces when deciding whether or not to share its demand signal with its competitor. This discussion will prove useful in understanding the results to come.

We attribute the impact of information sharing to two effects, which we name the *signal accuracy* effect and the *quantity* effect. Briefly, the signal accuracy effect refers to the improvement in the accuracy of a firm's demand forecast when it observes two demand signals instead of one. This will generally create an incentive for the firm to share its signal, so as to acquire its competitor's signal in return. By quantity effect, we refer to the fact that firms' production quantities become perfectly correlated when they share signals and therefore have access to the same information. By symmetry, when firms share signals, each firm captures half the industry profits. Depending on its situation prior to sharing, this can be good or bad for the firm.

In the two subsections to follow, we discuss these two effects in more detail and characterize for which problem parameters they are strongest and weakest.

3.1. The Signal Accuracy Effect

In general, increased accuracy has a positive effect on firms' expected profits because it enables them to better match production with demand (Kirby 1988). We consider the impact of signal accuracy both from an ex-ante and ex-post standpoint — this provides a deeper understanding of the signal accuracy effect as well as facilitates a study of ex-ante information sharing in Section 6. To capture the effect of signal accuracy from an ex-ante standpoint, we adopt the following measure used by Malueg and Tsutsui (1998): $E_{SA} = (Var(e_{ns}) - Var(e_s)) / Var(e_{ns})$, where $e_{ns} = E[\alpha|S_1] - \alpha$ and $e_s = E[\alpha|S_1, S_2] - \alpha$. E_{SA} measures the relative reduction in the variance of the error in the demand estimate when a firm uses two demand signals instead of one. Figure 2 shows how E_{SA} varies with signal accuracy (p) for different prior beliefs about the true demand state (r_H). For any value of r_H , the relative reduction in the error variance as a result of information sharing is increasing in p . Combining two very accurate signals eliminates most of the uncertainty surrounding the true demand state and hence the signal accuracy effect is strongest when the firms receive very accurate signals.

E_{SA} exhibits a different behavior with respect to r_H . E_{SA} is symmetric with respect to $r_H = 1/2$, hence we focus on prior beliefs that are skewed towards the high-demand state, i.e., $r_H \geq 1/2$. Notice in Figure 2 that for relatively low signal accuracies (e.g., p lower than 0.75), the largest reduction in the error variance is attained when both demand states are equally likely and the error variance reduction due to information sharing decreases with r_H . The opposite is true when the signal accuracy is high. The relative reduction in error variance due to information sharing is large if the priors are highly skewed (i.e., r_H is close to 0 or 1), and the smallest reduction is observed when both demand states are equally likely. Based on these observations, we can conclude that the signal accuracy effect is likely to have a significant (positive) impact on the information sharing strategy when the prior beliefs of the firms are highly skewed and their signals are fairly accurate.

To understand the signal accuracy effect from an ex-post standpoint, consider Figures 3(a) and 3(b), where $r_H = 0.1$ and $r_H = 0.9$ respectively. In both figures, we plot the updated probability of high demand as a function of signal accuracy (p) under no information sharing ("Signal-L" and "Signal-H": where a firm knows only its own signal) and information sharing ("Signal-(H,L)" and "Signal-(H,H)": where a firm knows its own signal as well as its competitor's). When $r_H = 0.1$ and a firm has already observed a high-demand signal, there is a large discrepancy between the updated beliefs under no information sharing and information sharing (Note the distances between the H, (H,L), and (H,H) curves.) whereas when $r_H = 0.9$, the discrepancy is relatively small. Thus, information accuracy gains from sharing information are large when the firm's prior belief and the signal it observes are contradictory. Having a second demand signal is most valuable in this situation. We would expect this effect to be accentuated when the demand uncertainty (α_H relative to α_L) is large.

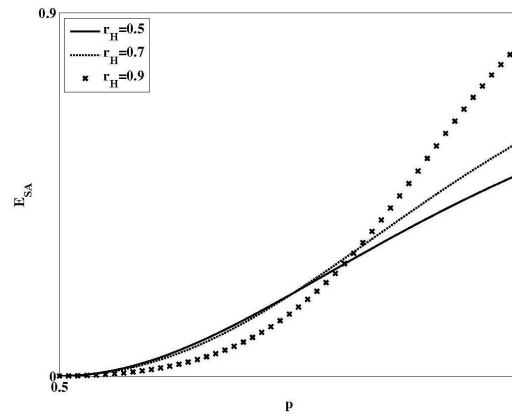


Figure 2 Relative reduction in error variance as a function of signal accuracy (p) for various prior probabilities (r_H) of the high demand state.

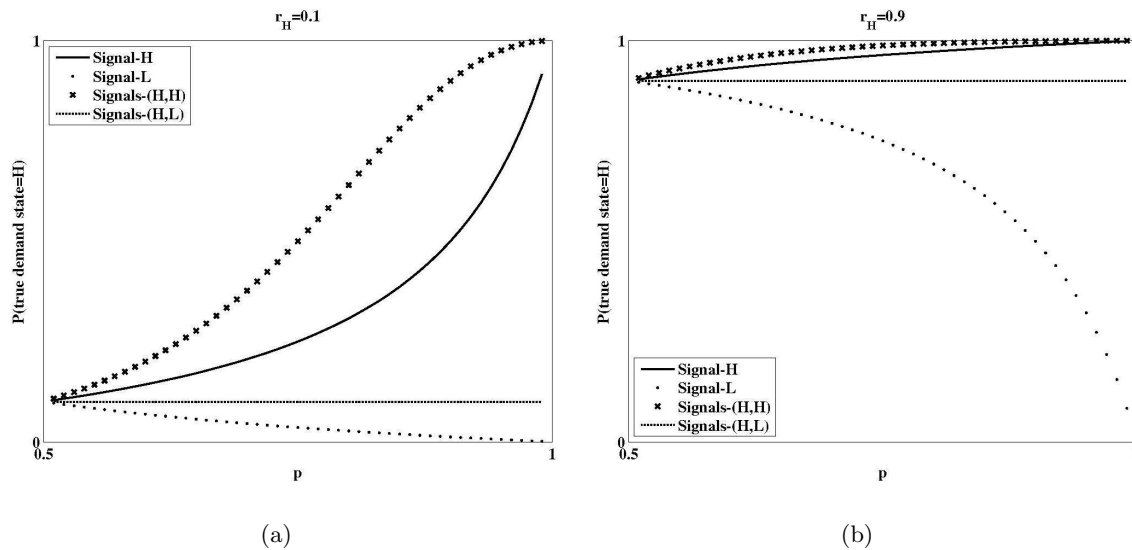


Figure 3 Updated beliefs for various signals.

3.2. The Quantity Effect

Assessing the impact of the quantity effect on information sharing is less straightforward. When a firm shares a low-demand signal with its competitor, it results in “under-production” by the competitor and a larger share of the industry profit (compared to the no-information sharing case). On the other hand, sharing a high-demand signal results in “over-production” by the competitor and, as a result, a smaller share of the total industry profit (compared to the no-information-sharing case). As Gal-Or (1985) points out, when firms observe noisy signals of equal accuracy, the amount of “over-production” need not coincide with the amount of “under-production”. Therefore, choosing the right information sharing strategy is a non-trivial task and the decision depends on

the magnitude of the benefit obtained from making more precise updates and the potential loss as a result of correlation of production strategies.

Figures 3(a) and 3(b) yield some intuition about the production quantities under different prior beliefs. First, consider the situation where a firm observes a high-demand signal and its competitor observes a low-demand signal. If firms do not share information, the firm with the high-demand signal overestimates the probability of high demand (compared to the updated belief under information sharing) whereas its competitor underestimates it. When $r_H = 0.1$, the (relative) overestimation is larger than the underestimation by the competitor, thereby resulting in excess production and lower prices. The overestimation by the firm grows with its confidence in the estimate and thus is more pronounced at high signal accuracies. Notice in Figure 3(b) that the opposite is true when $r_H = 0.9$ —underestimation by the competitor with a low-demand signal is greater than overestimation by the firm with the high-demand signal. In summary, the quantity effect encourages information sharing when $r_H = 0.1$ and discourages it when $r_H = 0.9$, even more so when α_H is large and therefore getting demand wrong has severe implications.

4. Results in the Ex-Post Setting

To understand the equilibrium behavior under strategic information sharing and identify conditions for information sharing, we need to analyze expression (11). Expressions (1)-(4) and (6)-(8) provide a glimpse of the complexity involved in analytically characterizing the equilibrium behavior. The profit expressions are ratios of high-order polynomials in the model parameters; hence a complete analytical characterization of the equilibrium behavior is not possible. Our approach is to first present a numerical study to provide an overview of the equilibrium behavior and then support our observations and insights with the strongest possible analytical results. In Section 6, we provide an overview of results for the ex-ante version of the problem, which extend results of Malueg and Tsutsui (1998). We also compare the equilibrium behavior in the two versions to understand the impact of decision timing on the firms' incentives to share information. Additional details on the ex-ante case are provided in Appendix B.

In what follows, we first make several observations about the equilibrium information sharing strategy using numerical studies, and then establish related analytical results. In our numerical examples, we vary three parameters, namely signal accuracy (p), prior probability of high demand (r_H) and the intercept ratio α_H/α_L . In addition, for ease of exposition, we normalize α_L to 1 and use the value of α_H as a measure of the level of demand uncertainty.

Under ex-post or strategic information sharing, firms decide whether or not to share their private demand signals after observing them. Theoretically, there are four possible equilibrium strategies in the ex-post setting: pooling information sharing equilibrium (i.e., sharing both high- and low-demand signals), pooling no-information-sharing equilibrium (sharing neither signal), a separating

equilibrium where firms share their low-demand signal and conceal the high-demand signal, and a separating equilibrium where firms conceal their low-demand signal and share the high-demand signal. In the following proposition, proved in Appendix A, we rule out the existence of the two separating equilibria.

PROPOSITION 1. *There does not exist a separating equilibrium under ex-post information sharing.*

The pooling no-information-sharing equilibrium exists trivially for all values of p , r_H and α_H . The reasoning is as follows: given that a firm's competitor is willing to share neither the high- nor the low-demand signal, the firm's possible actions, "share signal" and "conceal signal" are outcome-equivalent; in both cases, firms do not exchange signals and choose production quantities using their own signals. Recall our assumption (introduced at the end of Section 3.2) that the competitor ignores the non-equilibrium action (share signal) of the firm and uses only its own signal to decide its production quantity. Thus, given that its competitor is not sharing, the firm has no incentive to deviate from a no-information-sharing strategy.

Hence, the only equilibrium that remains to be studied is the pooling information sharing equilibrium. We seek to identify conditions that would induce information sharing both when firms receive a low-demand as well as when they receive a high-demand signal. We begin by considering the information sharing strategy of a firm that receives a low-demand signal. For a firm with a low-demand signal, the signal accuracy and quantity effects are aligned. By sharing a low-demand signal, a firm signals to its competitor that the true demand state might be low (i.e., lower than originally expected) and the competitor responds by reducing the production quantity because $\gamma_{Lj'}(H) \leq \beta_{j'}(H)$, $j' \in \{H, L\}$. Because under-production by the competitor is always good for a firm, we would expect firms to always share a low-demand signal. We confirm this intuition rigorously in the following proposition.

PROPOSITION 2. *When a firm observes a low-demand signal and its competitor is willing to share information, sharing the low-demand signal is more profitable than concealing it.*

Proposition 2 implies that a pooling information sharing equilibrium exists if and only if it is beneficial for firms to share a high-demand signal. In the remainder of this section, we test for the existence of such an equilibrium by examining the strategy of a firm whose competitor is assumed to be following a pooling information-sharing strategy (i.e., offering to share regardless of the signal it receives). Recall that in a pooling no-information-sharing equilibrium, the deviation profit from not sharing information is equal to the equilibrium profit. Therefore, a pooling information sharing equilibrium, if it exists, at least weakly Pareto-dominates the pooling no-information-sharing equilibrium.

Figure 4, generated numerically, plots where in the parameter space information sharing is an equilibrium for a high-demand signal, and therefore where an information sharing equilibrium exists. We see that information sharing is an equilibrium if the signal accuracy of the firm (p) is sufficiently high, the prior probability of high demand (r_H) is fairly low, and the demand intercept α_H is large enough. As we saw in Section 3, this is precisely the region of the parameter space where the signal accuracy effect is largest and the quantity effect is most conducive to sharing. The fact that there exist regions where firms are better off sharing their high-demand signal is particularly important from a competition standpoint, indicating that information exchange between competitors can increase efficiency and need not necessarily harm competition. Consumers are better off due to the information exchange because, by sharing a high-demand signal, a firm prevents under-production by the competitor and higher market prices.

Because pooling information sharing and pooling no-information-sharing are the only possible equilibria in pure strategies, in the regions where sharing a high-demand signal is not an equilibrium action in Figure 4, *the unique equilibrium strategy is to share neither high- nor low-demand signals*. In all our numerical tests, we observed that information sharing is not an equilibrium at relatively small values (roughly less than or equal to 2) of α_H for all p and r_H . We also notice in Figure 4 that the size of the region where information sharing is an equilibrium grows as we increase α_H .

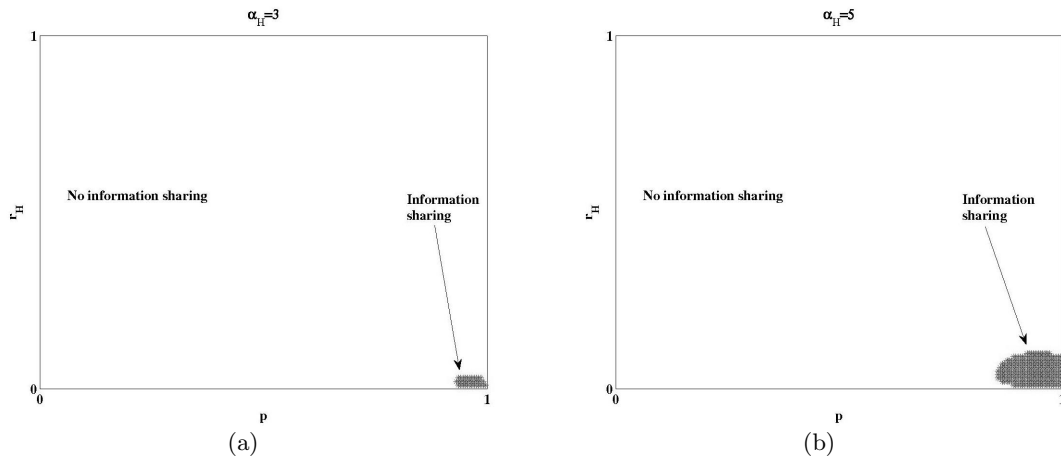


Figure 4 Equilibrium in the ex-post setting.

Due to the complexity of the profit expressions, we are not able to prove that information sharing is an equilibrium for all points within the shaded region in Figures 4(a) and 4(b). However, we formally establish the existence of the information sharing equilibrium for a restricted set of parameters, i.e., for points close enough to the diagonal $p = 1 - r_H$ and where $p > p^*$, where $p^* = 0.8315$ is a threshold value.

PROPOSITION 3. *Let $B_{\epsilon_p}(p, 1 - p)$ denote an ϵ -ball around the point $(p, 1 - p)$. When α_H is sufficiently large and the competitor is willing to share information, for every $p \in (p^*, 1)$, there exists an $\epsilon_p > 0$ such that sharing a high-demand signal is more profitable for a firm than concealing it for all $(\hat{p}, \hat{r}_H) \in B_{\epsilon_p}(p, 1 - p)$.*

The following corollary, stated without proof, immediately follows from Propositions 2 and 3.

COROLLARY 1. *When α_H is sufficiently large, for every $p \in (p^*, 1)$, there exists an $\epsilon_p > 0$ such that information sharing is a pooling equilibrium for all $(\hat{p}, \hat{r}_H) \in B_{\epsilon_p}(p, 1 - p)$.*

Conversely, it is informative to examine the regions where information sharing is not an equilibrium action. In Figure 4, we see that firms are better off not sharing their demand forecasts if they originally believed demand would be high with sufficiently high probability, i.e., when $r_H > 1/2$. As we found in Section 3.2, this is exactly the case where the quantity effect provides a strong disincentive to share information. For a firm with a high-demand signal, its share of the industry profit reduces under information sharing, from greater than or equal to $1/2$ to $1/2$. When the demand signal “confirms” prior beliefs, as is the case when $r_H > 1/2$, the increase in total expected industry profits (via the signal accuracy effect) is not sufficient to compensate for the reduction in the firm’s share of the industry profit (via the quantity effect). The next proposition shows analytically that information sharing will not happen when α_H is large and $r_H \geq 1/2$, albeit for a restricted set of parameters.

PROPOSITION 4. *Let $B_{\epsilon_p}(p, p)$ denote an ϵ -ball around the point (p, p) . When α_H is sufficiently large and the competitor is willing to share information, for every $p \in (1/2, 1)$, there exists $\epsilon_p > 0$ such that concealing a high-demand signal is more profitable for a firm than sharing it for all $(\hat{p}, \hat{r}_H) \in B_{\epsilon_p}(p, p)$.*

Proposition 4 implies that the unique equilibrium is pooling no-information-sharing for parameter values close enough to the leading diagonal ($p = r_H$) when α_H is large enough and $r_H > 1/2$. Note that the requirement that α_H be sufficiently large is purely to facilitate the proof of the proposition; all our numerical results suggest that, for points close to the diagonal, firms are better off concealing a high-demand signal for practically all values of α_H when $r_H > 1/2$.

In summary, when demand uncertainty is large and the demand signals are sufficiently accurate, it is beneficial for a firm whose prior belief is very skewed towards low demand to reveal a high-demand signal to its competitor regardless of what signal the competitor receives. Hence, in this region, a pooling information sharing equilibrium exists. Outside this region, the only equilibrium is pooling no-information-sharing.

5. Non-Mutual or Non-Verifiable Information Sharing

In this section, we analyze what happens when we relax the mutual and truthful information sharing assumptions under ex-post information sharing. In this relaxed setting, each firm, having observed its signal, can choose to announce one of the following to its competitor: $\{H, L, NS\}$. To completely characterize the equilibrium behavior, we need to consider several possibilities. We analyze them one by one.

Throughout this section, we will take firm 2's action as given and derive the best-response strategy from firm 1's perspective. First, let us consider pooling strategies, where firm 2's action is independent of the signal received. There exist three such pooling strategies: 1) not share any signal; 2) always announce a low-demand signal; 3) always announce a high-demand signal. Under all three pooling strategies, firm 1 does not learn anything about firm 2's signal and hence they are outcome-equivalent from firm 2's perspective. For ease of explanation, we will focus on the case where firm 2 shares neither the high- nor the low-demand signal. In response to this, firm 1 could possibly employ the following strategies when it observes a high-demand or a low-demand signal respectively: $\{(H,L), (L,H), (H,H), (L,L), (NS,L), (NS,H), (H,NS), (L,NS), (NS,NS)\}$. For example, the notation (H,L) indicates a strategy of announcing H when firm 1 observes a high-demand signal and announcing L when it observes a low-demand signal. Let us begin with the strategy (H,L) for firm 1. Under this strategy, firm 1's profit when it observes a high-demand signal (and announces H) is $q_1(H)^2$ where

$$q_1(H) = \frac{\alpha_H \beta_H(H) + \alpha_L \beta_H(L) - \delta_H(H) q_2(H, H) - \delta_H(L) q_2(L, H)}{2}.$$

Instead, if firm 1 announces that it observed signal L , then its profit would be $\tilde{q}_1(H)^2$ where,

$$\tilde{q}_1(H) = \frac{\alpha_H \beta_H(H) + \alpha_L \beta_H(L) - \delta_H(H) q_2(H, L) - \delta_H(L) q_2(L, L)}{2}.$$

Clearly, $\tilde{q}_1(H) \geq q_1(H)$ and hence, (H,L) is not a best response from firm 1's perspective. The key idea here is that firm 1 is better off pretending to be the low type all the time by employing the same action that it uses for a low-demand signal. Carrying out a similar analysis for the other eight possible response strategies reveals that there are three, outcome-equivalent, best-response strategies: $\{(H,H), (L,L), (NS,NS)\}$, i.e., the best response to 'no information sharing' by a competitor is to not provide any discernible information to the competing firm. The analysis is identical for the other two pooling strategies.

Two key results emerge from the study of pooling strategies: (1) There does not exist an ex-post equilibrium where one firm employs a pooling strategy and the other firm uses a separating strategy. (2) There exist six equilibrium strategies where both firms employ a pooling strategy and they are all outcome-equivalent.

We now move on to explore the possibility of firm 2 employing separating strategies, i.e., different actions for the high- and low-demand signals. We should point out that any such separating strategy (even when firm 2 is not sharing information truthfully) amounts to truth-telling from firm 2's perspective since signals are revealed by the share/conceal actions alone. Thus, although we only discuss the separating equilibrium where firm 2 reveals the true demand signal, all the insights from this analysis hold equally well for the other separating equilibria.

Straightaway, we can rule out the three possible equilibria where firm 2 shares information truthfully and firm 1 employs a pooling strategy. This is because, as we showed earlier in this section, a firm (in this case firm 2) should respond to "no information sharing" by a competitor (firm 1) by employing a pooling strategy. Thus, it only remains to be seen if there exists an equilibrium where both firms employ a separating strategy. To check this, suppose that firm 1 also shares information truthfully (any other separating strategy used by firm 1 is outcome-equivalent). Under this strategy, firm 1's profit when it observes a high-demand signal (and announces H) is given by $q_1(H, H)^2 P(S_1 = H|S_2 = H) + q_1(H, L)^2 P(S_1 = L|S_2 = H)$, while its profit if it announced that it observed a low-demand signal would be $\tilde{q}_1(H, H)^2 P(S_1 = H|S_2 = H) + \tilde{q}_1(H, L)^2 P(S_1 = L|S_2 = H)$. In the proof of Proposition 1, we show that $\tilde{q}_1(H, H) \geq q_1(H, H)$ and $\tilde{q}_1(H, L) \geq q_1(H, L)$. Hence, firm 1's best response is not to share information truthfully.

Overall, our analysis leads us to conclude that when the mutual and truthful information sharing assumptions are relaxed, no information sharing occurs in equilibrium. However, the story does not end there. Interestingly, having the option to lie and share information in a non-mutual way hurts both firms. To see why, notice that under no information sharing, a firm's profit is given by expression (10). However, had both firms shared information truthfully (which is not an equilibrium), each firm's profit would be given by expression (9) and at least for certain values of p and r_H , both firms would have been better-off sharing information truthfully as can be seen in Figure 4. This leads to a Prisoner's Dilemma type of situation as illustrated in Table 2.

		Firm 2	
		Share Truthfully	Not Share
Firm 1	Share Truthfully	(A,A)	(B,C)
	Not Share	(C,B)	(D,D)

Table 2 Illustration of the Prisoner's Dilemma type situation

In Table 2, the ordering of A, B, C, and D is $C > A > D > B$. Therefore, it is clear that, at least for certain parameter values, it is in the best interest of both firms to establish a mechanism to exchange information truthfully in order to move from the Pareto-dominated (Not share, Not share) equilibrium towards truthful and mutual information sharing. A portion of the increase in

profits obtained from such a transition could be used to offset the costs associated with setting up and running the information sharing mechanism. In fact, it has been shown that efficient outcomes can be achieved in Prisoner's Dilemma type situations by designing appropriate transfer payment schemes that reward desired behavior (e.g., Qin 2005 and Varian 1994). For symmetric games like ours, where the transfer payments are the same for both firms, this implies that information sharing can be sustained as an equilibrium even without any monetary transactions actually taking place. The transfer payments simply prevent the firms from giving into the temptation of achieving higher profits by misreporting demand signals.

6. Ex-Ante Information Sharing

6.1. Results

Under ex-ante information sharing, firms have to commit to an information sharing strategy before observing their private demand signals. When a firm decides whether to share or withhold private information in the ex-ante case, it compares the expected profit if it were to share its private signal with the expected profit if it were to withhold the signal. We analyze whether information sharing would be beneficial for a firm assuming that its competitor wants to share information. If a firm decides to share its private information, its expected profit is a weighted average of the second-stage profits given by (7). We denote the ex-ante expected profit under information sharing by $\Pi_\ell(S)$, where

$$\Pi_\ell(S) = \sum_{j \in \{H, L\}} \sum_{j' \in \{H, L\}} \pi_\ell(j, j') P(S_\ell = j, S_{\ell'} = j'), \quad \ell, \ell' \in \{1, 2\}. \quad (12)$$

If a firm decides not to share information, its expected profit $\Pi_\ell(NS)$ is a weighted average of the second-stage expected profits $\pi_\ell(j)$, i.e.,

$$\Pi_\ell(NS) = \sum_{j \in \{H, L\}} \pi_\ell(j) P(S_\ell = j), \quad \ell \in \{1, 2\}, \quad (13)$$

where

$$\pi_\ell(j) = \left[\alpha_H \beta_{\ell j}(H) + \alpha_L \beta_{\ell j}(L) - \left(q_\ell(j) + \sum_{j'=H, L} q_{\ell'}(j') \delta_{\ell j}(j') \right) \right] q_\ell(j), \quad j \in \{H, L\}. \quad (14)$$

For information sharing to be an equilibrium when firms have to commit to it, it must be at least as profitable as committing to not share information. This means that information sharing is an equilibrium if

$$\Pi_\ell(S) - \Pi_\ell(NS) \geq 0, \quad \ell \in \{1, 2\}. \quad (15)$$

From expression (15), it is clear that a unique equilibrium exists for every parameter set except when $\Pi_\ell(S) = \Pi_\ell(NS)$, $\ell = 1, 2$. Figure 5, which is similar to Figure 1 in Malueg and Tsutsui (1998),

illustrates the equilibrium for different parameter values. The following observations made from Figure 5 are supported by theoretical results (Propositions 5-7) presented and proved in Appendix B .

1. There exist regions where committing to information sharing is the equilibrium. Malueg and Tsutsui (1998)’s Proposition 2, stated without proof, claims the existence of an information sharing equilibrium ex-ante for $r_H > 0.81$ and for $p > 0.75$. Proposition 5 of our paper includes a result with proof for the existence of an information sharing equilibrium for parameter values close to the leading diagonal ($p = r_H$) in Figure 5 when p is larger than a stated threshold.
2. The equilibrium strategy is symmetric with respect to the line $r_H=0.5$. We show this result analytically in Proposition 6 and will discuss it later in this section.
3. The region where information sharing is the equilibrium is invariant in size with respect to α_H . We prove this result rigorously in Proposition 7.

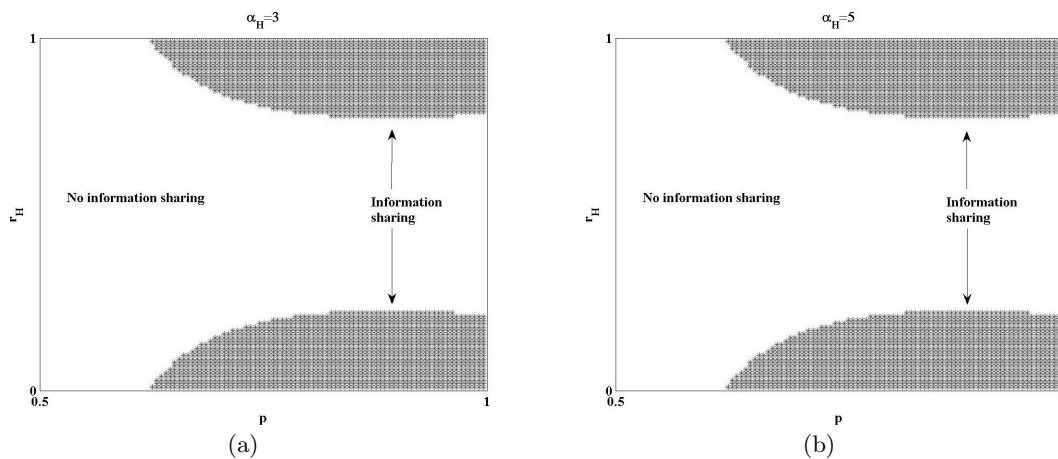


Figure 5 Equilibrium in the ex-ante setting.

Our first observation, made also by Malueg and Tsutsui (1998), is that an information sharing equilibrium exists. This result opposes the classic “no information sharing” result, established by Clarke (1983), Li (1985), and Gal-Or (1985). Kirby (1988) shows that the “no information sharing” result is sensitive to the functional form of the cost function. Figure 5 demonstrates that the equilibrium is also sensitive to how demand and information about it are modeled. Why does the normal distribution framework (as in Clarke 1983) preclude information sharing but the binary framework allows it? Malueg and Tsutsui (1998) explain this by pointing out that the normal distribution framework imposes an upper bound on the improvement in forecast accuracy due to information sharing, where $E_{SA} \leq 1/2$. Therefore, the upside due to the signal accuracy effect is limited. As seen in Figure 2, this restriction does not hold in our model. The reason is that a

binary framework allows extreme events to have a significant probability of occurrence, which is not possible in case of a normal distribution.

We briefly remark about the invariance property. We conjecture that this property may be specific to our setup and may not hold true for other competition/information sharing models. With that caveat, the invariance property offers an interesting insight into the role of market uncertainty in decision making. The level of uncertainty in the market does not determine the equilibrium strategy itself but impacts the gains/losses from information sharing. In fact, from the proof of Proposition 7, we see that the gains (losses) from information sharing increase in α_H monotonically, i.e., strategic mistakes on the part of firms can prove to be costly when the two demand states are further apart, and choosing the right information sharing strategy becomes more important.

6.2. Ex-Ante vs. Ex-Post Information Sharing

Before we compare the equilibria under ex-ante and ex-post information sharing, we try to reconcile the existence of an ex-ante information sharing equilibrium with the intuition we developed in Section 3 and with the ex-post information sharing results. To understand the drivers of the existence (of an ex-ante information sharing equilibrium) result, we recall Propositions 2 and 4 (and the fact that both propositions assume that the competitor is willing to share information). Proposition 2 established that sharing information is always advantageous for a firm observing a low-demand signal. On the contrary, for parameter values close to $p = r_H$, we established through Proposition 4 that a firm is better off concealing its high-demand signal. When the firm decides whether to commit to information sharing ex ante, it essentially has to consider the probabilities of ending up in these two situations: receiving a low-demand signal and receiving a high-demand signal. The intuition we developed in Section 3 suggests that a high prior probability of low demand (i.e., small r_H) and the belief that demand signals are accurate (i.e., high p) would be most conducive to sharing information. Both Figure 5 and Proposition 5 in Appendix B confirm this intuition.

Comparing Figures 4 and 5, we immediately notice that the region where sharing information is an equilibrium ex-post is a subset of the information sharing region ex-ante. This is not surprising *per se* because a firm might find a blanket agreement on information sharing profitable based on expectations but suboptimal once it realizes it has to share a high-demand signal. However, this demonstrates that the information sharing agreements made ex-ante may not survive ex-post. Given that these agreements may not be enforceable, a study of the incentives for ex-post information sharing arguably is more relevant than its ex-ante counterpart.

Another feature that distinguishes ex-ante from ex-post information sharing is the symmetry we observed in the ex-ante case. An implication of this symmetry is that a firm that is fairly

certain it will receive a high-demand signal would prefer to conceal the signal ex post but commit to share it ex ante. To understand this counterintuitive result, recall our discussion regarding the signal accuracy effect in Section 3.1: information accuracy gains from sharing information are large when a firm's prior belief and the signal it receives are contradictory. This contradiction can arise ex-post only if the firm has a highly skewed prior belief about the state of demand. In addition, the information accuracy gains are the same if a firm believes that demand will be high with probability 0.9 and receives a low-demand signal, and if it believes that demand will be low with probability 0.9 and receives a high-demand signal. When a firm's belief is skewed towards the high-demand state and it receives a low-demand signal, it gains significantly from information sharing. This more than compensates for the losses incurred by sharing a high-demand signal and in expectation, information sharing is more beneficial than not sharing information.

7. Summary and Conclusions

Although it is reasonable to expect (and has been argued) that information sharing between competitors might lead to substantial efficiency gains, the majority of the existing literature posits that competitors are better off concealing market demand information from each other. By using a model that can serve as a building block to study intentional and unintentional information sharing in more complex supply chains, we demonstrate that this conclusion is not true in a setting with binary demand uncertainty and binary demand signals. We prove the existence of an information sharing equilibrium both when firms decide about sharing information prior to observing their private demand signals (ex ante) and when they make that decision after they have observed them (ex post). Furthermore, we show that the incentives to share or conceal information change depending on the timing of the information sharing decision. In some situations, firms want to commit to sharing information but prefer concealing their signals ex post.

When firms commit ex ante to either share or conceal any information they receive later on, we analytically show and extend findings in Malueg and Tsutsui (1998). However, the main contribution of this paper is to demonstrate the existence of a pooling information-sharing equilibrium in the ex-post case. Specifically, we find that strategic information sharing between competitors is particularly valuable when either of the following two situations arise: 1) A firm receives private data that demand will be lower than expected; 2) A firm that is confident in its ability to accurately estimate demand receives private data that demand will be higher than expected, even though its priors were strongly suggesting the opposite. In the first case, the firm wants to share its information to prevent overproduction by its competitor (and thus low prices). In the second case, the firm wants to resolve the uncertainty resulting from two conflicting sources of information—prior beliefs and a demand signal.

We also analyze the possibility that firms exchange non-verifiable information. In line with evidence suggesting this is a more theoretical than practical concern, we show that firms are better off establishing mechanisms that can verify the information being exchanged. Overall, we offer compelling evidence that truthful and mutual information sharing between competitors within the same supply chain or across supply chains extends beyond simply maintaining high prices and, as one could argue, mitigating competition. For example, we illustrate instances where information sharing leads to a higher level of aggregate production and thus lower prices (relative to the case where no information sharing takes place). Therefore, information sharing between competitors is not as harmful or unlikely as it has been suspected. We hope that this paper will spur a deeper understanding of information flows in more complex supply chain relationships.

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Appendix A: Proofs

For convenience and brevity, we introduce the following symbols.

$$A = p^2 r_H, B = (1-p)^2(1-r_H), C = p(1-p)r_H, D = p(1-p)(1-r_H), E = (1-p)^2 r_H, F = p^2(1-r_H), G = pr_H, H = (1-p)(1-r_H), I = (1-p)r_H, J = p(1-r_H)$$

$$a = \beta_H(H), c = \beta_L(H), e = \beta_H(L), g = \beta_L(L), i = \delta_H(H), j = \delta_H(L), m = \delta_L(H), n = \delta_L(L)$$

$$K_1 = 2a + an - cj, K_2 = 2e + en - gj, K_3 = 2c + ci - am, K_4 = 2g + gi - em, K_5 = 2i + 2n + in - mj + 4.$$

$$T_1 = \frac{1}{9} \left(\frac{A^2}{A+B} + \frac{2C^2}{C+D} + \frac{E^2}{E+F} \right), T_2 = \frac{1}{9} \left(\frac{B^2}{A+B} + \frac{2D^2}{C+D} + \frac{F^2}{E+F} \right), T_3 = \frac{1}{9} \left(\frac{AB}{A+B} + \frac{2CD}{C+D} + \frac{EF}{E+F} \right)$$

$$T_4 = \frac{K_1^2}{K_5^2} (G+H) + \frac{K_2^2}{K_5^2} (I+J), T_5 = \frac{K_2^2}{K_5^2} (G+H) + \frac{K_4^2}{K_5^2} (I+J), T_6 = \frac{K_1 K_2}{K_5^2} (G+H) + \frac{K_3 K_4}{K_5^2} (I+J)$$

$$T_7 = \frac{1}{9} \left(\frac{A^2}{A+B} + \frac{C^2}{C+D} \right) \left(\frac{1}{G+H} \right) - \frac{K_1^2}{K_5^2}, T_8 = \frac{1}{9} \left(\frac{B^2}{A+B} + \frac{D^2}{C+D} \right) \left(\frac{1}{G+H} \right) - \frac{K_2^2}{K_5^2}, T_9 = \frac{1}{9} \left(\frac{AB}{A+B} + \frac{CD}{C+D} \right) \left(\frac{1}{G+H} \right) - \frac{K_1 K_2}{K_5^2}$$

$$T_{10} = \frac{1}{9} \left(\frac{C^2}{C+D} + \frac{E^2}{E+F} \right) \left(\frac{1}{I+J} \right) - \frac{K_3^2}{K_5^2}, T_{11} = \frac{1}{9} \left(\frac{D^2}{C+D} + \frac{F^2}{E+F} \right) \left(\frac{1}{I+J} \right) - \frac{K_4^2}{K_5^2}, T_{12} = \frac{1}{9} \left(\frac{CD}{C+D} + \frac{EF}{E+F} \right) \left(\frac{1}{I+J} \right) - \frac{K_3 K_4}{K_5^2}.$$

Proof of Proposition 1

To prove the proposition, we need to demonstrate the non-existence of the two possible separating equilibria: (1) Share a low-demand signal and conceal a high-demand signal and (2) Conceal a low-demand signal and share a high-demand signal. We prove them one by one.

Without loss of generality, let us take the perspective of firm 1 in our analysis, assuming that firm 2 follows the first equilibrium considered, i.e., it shares a low-demand signal and conceals a high-demand signal. We will check if this strategy is also an equilibrium for firm 1, assuming firm 1 sees a high-demand signal. To begin with, consider the situation where firm 2 has observed a low-demand signal. Following the equilibrium strategy, firm 2 would be willing to share its demand signal. In this situation, firm 1's actions "not share" and "share" information are outcome equivalent. This is easy to see because, from firm 2's perspective, firm 1's "not share" action implies that firm 1 has observed a high-demand signal. On the other hand, if firm 1 announces that it is willing to share its demand signal, then firms would end up exchanging their demand signals. In both cases, firm 2's production quantity would be the same and so will firm 1's reaction. Thus, to test the equilibrium strategy, we only need to consider the situation where firm 2 has observed a high-demand signal. In this situation, firm 2 would interpret firm 1's willingness to share demand information as an indication of the firm observing a low-demand signal. Then, firm 1's profit under the equilibrium strategy "not share" information is

$$[\alpha_H \mu_{HH}(H) + \alpha_L \mu_{HH}(L) - (q_1(H, H) + q_2(H, H))] q_1(H, H), \quad (16)$$

and firm 1's profit if it announces that it is willing to share its signal is given by

$$[\alpha_H \mu_{HH}(H) + \alpha_L \mu_{HH}(L) - (\tilde{q}_1(H, H) + q_2(H, L))] \tilde{q}_1(H, H). \quad (17)$$

Here, $\tilde{q}_1(H, H)$ is the production quantity of firm 1 when it deviates from the equilibrium strategy. In this case, firm 1 makes its quantity decision knowing that $S_1 = S_2 = H$ but firm 2, misled by firm 1's deviation, makes its quantity decision believing that $S_1 = L, S_2 = H$. It is easy to see that (17) \geq (16) if $\tilde{q}_1(H, H) \geq q_1(H, H)$ and $\tilde{q}_1(H, H) + q_2(H, L) \leq q_1(H, H) + q_2(H, H)$. Since the firms are symmetric, $q_1(H, H) = q_2(H, H)$. The first-order conditions on the Cournot-stage profits imply that

$$\tilde{q}_1(H, H) = \frac{\alpha_H \mu_{HH}(H) + \alpha_L \mu_{HH}(L) - q_2(H, L)}{2} = \frac{3q_1(H, H) - q_2(H, L)}{2} \geq q_1(H, H).$$

The last inequality holds because $q_1(H, H) \geq q_2(H, L)$. Also, notice that $2\tilde{q}_1(H, H) + q_2(H, L) = 3q_1(H, H)$, which directly implies $\tilde{q}_1(H, H) + q_2(H, L) \leq 2q_1(H, H)$. Hence, (17) \geq (16), proving the non-existence of the first separating equilibrium.

The proof is similar in spirit for the other separating equilibrium. Again, assume that firm 2 follows the equilibrium strategy — conceal a low-demand signal and share a high-demand signal. Consider the situation where firm 1 has observed a high-demand signal and firm 2's demand signal is low. Firm 2, following the equilibrium strategy, would conceal its low-demand signal. If firm 1 follows the equilibrium strategy “share a high-demand signal”, then its profit is

$$[\alpha_H \mu_{HL}(H) + \alpha_L \mu_{HL}(L) - (q_1(H, L) + q_2(H, L))] q_1(H, L), \quad (18)$$

and firm 1's profit if it deviates from the equilibrium strategy is

$$[\alpha_H \mu_{HL}(H) + \alpha_L \mu_{HL}(L) - (\tilde{q}_1(H, L) + q_2(L, L))] \tilde{q}_1(H, L). \quad (19)$$

From the first-order conditions on the Cournot-stage profit expressions, we have

$$\tilde{q}_1(H, L) = \frac{\alpha_H \mu_{HL}(H) + \alpha_L \mu_{HL}(L) - q_2(L, L)}{2} = \frac{3q_1(H, L) - q_2(L, L)}{2} \geq q_1(H, L).$$

Also, $\tilde{q}_1(H, L) + q_2(L, L) \leq q_1(H, L) + q_2(H, L)$. Hence, (19) \geq (18) holds. When firm 2 observes a high-demand signal, firm 1's profit under the equilibrium strategy (share a high-demand signal) is given by (16) and its profit under deviation from the equilibrium strategy is given by (17). We already showed that (17) \geq (16). Therefore, when the competitor conceals a low-demand signal and conceals a high-demand signal, a firm is actually better off concealing its high-demand signal. This rules out the existence of the second separating equilibrium.

Proof of Proposition 2

Since firms are symmetric, we prove the proposition from firm 1's perspective. We have

$$\Pi_{1,L}(S) = q_1(L, H)^2 \delta_L(H) + q_1(L, L)^2 \delta_L(L)$$

and

$$\Pi_{1,L}(NS) = q_1(L)^2.$$

From the definition of $q_1(L)$, we see that

$$\begin{aligned} \alpha_H \beta_L(H) + \alpha_L \beta_L(L) &= 2q_1(L) + \delta_L(H)q_2(H) + \delta_L(L)q_2(L) \\ &\geq 2q_1(L) + \delta_L(H)q_2(L) + \delta_L(L)q_2(L) \\ &= 3q_1(L). \end{aligned} \quad (20)$$

The above inequality holds if $q_2(H) \geq q_2(L)$. We show that this is indeed the case. Notice that

$$2q_2(L) = \alpha_H \beta_L(H) + \alpha_L \beta_L(L) - \delta_L(H)q_1(H) - \delta_L(L)q_1(L)$$

and

$$2q_2(H) = \alpha_H \beta_H(H) + \alpha_L \beta_H(L) - \delta_H(H)q_1(H) - \delta_H(L)q_1(L).$$

Since $\alpha_H\beta_H(H) + \alpha_L\beta_H(L) \geq \alpha_H\beta_L(H) + \alpha_L\beta_L(L)$, we have

$$2q_2(H) + \delta_H(H)q_1(H) + \delta_H(L)q_1(L) \geq 2q_2(L) + \delta_L(H)q_1(H) + \delta_L(L)q_1(L).$$

By symmetry, $q_1(j) = q_2(j)$, $j \in \{H, L\}$ and hence $q_2(H) \geq q_2(L)$. Also,

$$\begin{aligned} \alpha_H\beta_L(H) + \alpha_L\beta_L(L) &= [\alpha_H\gamma_{LL}(H) + \alpha_L\gamma_{LL}(L)]\delta_L(L) + [\alpha_H\gamma_{LH}(H) + \alpha_L\gamma_{LH}(L)]\delta_L(H) \\ &= 3q_1(L, H)\delta_L(H) + 3q_1(L, L)\delta_L(L). \end{aligned} \quad (21)$$

Combining equations (20) and (21), we get $q_1(L, H)\delta_L(H) + q_1(L, L)\delta_L(L) \geq q_1(L)$. Finally,

$$\begin{aligned} q_1(L, H)^2\delta_L(H) + q_1(L, L)^2\delta_L(L) &\geq (q_1(L, H)\delta_L(H) + q_1(L, L)\delta_L(L))^2 \\ &\geq q_1(L)^2 \end{aligned}$$

if $(q_1(L, H) - q_1(L, L))^2 \geq 0$, which trivially holds. Hence the proposition.

Proof of Proposition 3.

From expressions (9) and (10), we have

$$\Pi_{\ell H}(S) = q_{\ell}(H, H)^2\delta_H(H) + q_{\ell}(H, L)^2\delta_H(L)$$

and

$$\Pi_{\ell H}(NS) = q_{\ell}(H)^2$$

In terms of T_7, T_8, \dots, T_{12} , $\Pi_{\ell H}(S) - \Pi_{\ell H}(NS) = T_7\alpha_H^2 + T_8\alpha_L^2 + 2T_9\alpha_H\alpha_L$. Replacing r_H by $1 - p$, we get

$$T_7 = \left[\frac{2p - 1}{18(12p^2 - 12p + 5)^2} \right] [144p^5 - 360p^4 + 408p^3 - 264p^2 + 96p - 17].$$

When $1/2 < p < 1$, the term in the first square bracket is always positive. The polynomial in the second bracket is verified to be non-negative for $p > 0.8315$. Thus, for $p^* = 0.8315 < p < 1$, $\Pi_{\ell H}(S) - \Pi_{\ell H}(NS)$ is convex in p . Hence, when $p = 1 - r_H$, $\Pi_{\ell H}(S) - \Pi_{\ell H}(NS) \geq 0$ for large enough α_H making “sharing information” the equilibrium strategy.

For $(\hat{p}, \hat{r}_H) \in B_{\epsilon_p}(p, 1 - p)$ when $p \in (p^*, 1)$, we use the continuity of T_7 in p and r_H . T_7 is the sum of rational functions in p and r_H , all of which are well-defined for $p, r_H \in (0, 1)$. This implies continuity of T_7 in p and r_H and therefore, $T_7 \geq 0$ for any $(\hat{p}, \hat{r}_H) \in B_{\epsilon_p}(p, 1 - p)$ when $p \in (p^*, 1)$. Hence the proposition holds.

Proof of Proposition 4

We mentioned in the proof of Proposition 3 that $\Pi_{\ell H}(S) - \Pi_{\ell H}(NS) = T_7\alpha_H^2 + T_8\alpha_L^2 + 2T_9\alpha_H\alpha_L$. When $p = r_H$,

$$T_7 = \left[\frac{-p^2}{9(12p^2 - 12p + 5)^2} \right] \left[\frac{288p^8 - 1440p^7 + 3048p^6 - 3624p^5 + 2660p^4 - 1218p^3 + 327p^2 - 42p + 1}{6p^4 - 12p^3 + 11p^2 - 5p + 1} \right]$$

which is negative for $1/2 < p < 1$. Hence, when $\alpha_L = 1$, $\Pi_{\ell H}(S) - \Pi_{\ell H}(NS)$ is concave in α_H meaning that for large enough α_H , “not sharing information” is the equilibrium strategy when $p = r_H$.

For points close to the diagonal, i.e., $(\hat{p}, \hat{r}_H) \in B_{\epsilon_p}(p, p)$, the proposition holds because of the continuity of T_7 in p and r_H . The reasoning is the same as provided in the proof of Proposition 3.

Appendix B: Further Results on the Ex-ante Case

This section provides analytical support for our observations made in Section 6 about Figure 5. This material is intended to supplement the discussion of Section 6.

In the following proposition, we prove the existence of an information sharing equilibrium for parameter values close to the leading diagonal ($p = r_H$) in Figure 5 when $p > p'$, where p' is a threshold value. We do the same for parameter values close to the other diagonal ($p = 1 - r_H$) in Figure 5.

PROPOSITION 5. (a) Let $B_{\epsilon_p}(p, p)$ denote a ϵ -ball around the point (p, p) . For every $p \in (p', 1)$, there exists $\epsilon_p > 0$ such that committing to share information is an equilibrium for any $(\hat{p}, \hat{r}_H) \in B_{\epsilon_p}(p, p)$.

(b) Let $B_{\epsilon_p}(p, 1 - p)$ denote a ϵ -ball around the point $(p, 1 - p)$. For every $p \in (p', 1)$, there exists $\epsilon_p > 0$ such that committing to share information is an equilibrium for any $(\hat{p}, \hat{r}_H) \in B_{\epsilon_p}(p, 1 - p)$.

Proof. From (12), we see that

$$\begin{aligned} \Pi_\ell(S) = & \pi_\ell(H, H)P(S_\ell = H, S_{\ell'} = H) + \pi_\ell(H, L)P(S_\ell = H, S_{\ell'} = L) \\ & + \pi_\ell(L, H)P(S_\ell = L, S_{\ell'} = H) + \pi_\ell(L, L)P(S_\ell = L, S_{\ell'} = L). \end{aligned}$$

In the above expression, $\pi_\ell(H, H) = q_\ell(H, H)^2$, $\pi_\ell(H, L) = q_\ell(H, L)^2$, $\pi_\ell(L, H) = q_\ell(L, H)^2$, $\pi_\ell(L, L) = q_\ell(L, L)^2$. The first-order conditions on the Cournot profit expressions given by (7) imply that

$$q_\ell(j, j') = \frac{\alpha_H \gamma_{jj'}(H) + \alpha_L \gamma_{jj'}(L)}{3}, \quad j, j' \in \{H, L\}.$$

In terms of symbols A, B, \dots, J , we have

$$\begin{aligned} q_\ell(H, H) &= \frac{A\alpha_H + B\alpha_L}{3(A + B)} \\ q_\ell(H, L) = q_\ell(L, H) &= \frac{C\alpha_H + D\alpha_L}{3(C + D)} \\ q_\ell(L, L) &= \frac{E\alpha_H + F\alpha_L}{3(E + F)} \end{aligned}$$

and $P(S_\ell = H, S_{\ell'} = H) = A + B$, $P(S_\ell = H, S_{\ell'} = L) = P(S_\ell = L, S_{\ell'} = H) = C + D$, $P(S_\ell = L, S_{\ell'} = L) = E + F$. Then it follows directly that

$$\Pi_\ell(S) = T_1\alpha_H^2 + T_2\alpha_L^2 + 2T_3\alpha_H\alpha_L.$$

Similarly, (13) implies that

$$\Pi_\ell(NS) = \pi_\ell(H)P(S_1 = H) + \pi_\ell(L)P(S_1 = L).$$

Again, $\pi_\ell(H) = q_\ell(H)^2$ and $\pi_\ell(L) = q_\ell(L)^2$. The first-order conditions on the Cournot profit expressions given by (14) yield

$$\begin{aligned} q_\ell(H) &= \frac{\alpha_H\beta_H(H) + \alpha_L\beta_H(L) - \delta_H(L)q_{\ell'}(L)}{2 + \delta_H(H)} \quad \text{and} \\ q_\ell(L) &= \frac{\alpha_H\beta_L(H) + \alpha_L\beta_L(L) - \delta_L(H)q_{\ell'}(H)}{2 + \delta_L(L)}. \end{aligned}$$

In terms of the symbols defined, $q_\ell(H) = (K_1\alpha_H + K_2\alpha_L)/K_5$, $q_\ell(L) = (K_3\alpha_H + K_4\alpha_L)/K_5$, $P(S_\ell = H) = G + H$ and $P(S_\ell = L) = I + J$. Thus, we have

$$\Pi_\ell(NS) = T_4\alpha_H^2 + T_5\alpha_L^2 + 2T_6\alpha_H\alpha_L.$$

For information sharing to be an equilibrium strategy we need $\Pi_\ell(S) - \Pi_\ell(NS) \geq 0$, i.e., $(T_1 - T_4)\alpha_H^2 + (T_2 - T_5)\alpha_L^2 + 2(T_3 - T_6)\alpha_H\alpha_L \geq 0$. Straightforward algebra shows that $(T_1 - T_4) + (T_2 - T_5) = 2(T_6 - T_3)$ and the following equalities also hold: $(T_1 - T_4) = (T_2 - T_5) = -(T_3 - T_6)$. Thus, the condition for information sharing to be an equilibrium strategy is simply $(T_1 - T_4)(\alpha_H^2 + \alpha_L^2 - 2\alpha_H\alpha_L) \geq 0$. Hence, the sign of $\Pi_\ell(S) - \Pi_\ell(NS)$ is determined by the sign of $T_1 - T_4$. Substituting $r_H = p$ and solving for $T_1 - T_4 = 0$, we obtain the following real roots of the equation: $p = 0, 1/2, 1, 1/2 - \sqrt{3}/6$ and $1/2 + \sqrt{3}/6$. The derivatives of $T_1 - T_4$ with respect to p are 0.0311, 0, -0.0311, -0.0095 and 0.0095 at the five roots respectively. The derivative, 0.0095 at $p = 1/2 + \sqrt{3}/6$ implies that the equilibrium strategy switches from “not sharing” to “sharing” at $p = 1/2 + \sqrt{3}/6$ and “sharing” remains the equilibrium strategy when $p = r_H > p' = 1/2 + \sqrt{3}/6$.

When $(\hat{p}, \hat{r}_H) \in B_{\epsilon_p}(p, p)$ and $p \in (p', 1)$, the proposition holds because $T_1 - T_4$, being a sum of rational functions, is continuous in p and r_H and therefore remains positive within the ϵ_p -ball.

The proof is similar in spirit for $(\hat{p}, \hat{r}_H) \in B_{\epsilon_p}(p, 1 - p)$ and $p \in (p', 1)$. This is because when we substitute $r_H = 1 - p$ and solve for $T_1 - T_4 = 0$, we obtain the same five roots and the derivative values at these five roots are also the same.

The following proposition establishes the symmetry of Figure 5 about the line $r_H = 0.5$.

PROPOSITION 6. $\Pi_\ell(S) - \Pi_\ell(NS)$ is symmetric with respect to $r_H = 0.5$.

Proof. Let $q_\ell(S)$ be the “expected” production quantity of firm ℓ under information sharing and $q_\ell(NS)$ be the “expected” production quantity under no information sharing. Then,

$$q_\ell(S) = \sum_{j \in \{H, L\}} \sum_{j' \in \{H, L\}} q_\ell(j, j') P(S_\ell = j, S_{\ell'} = j')$$

and

$$q_\ell(NS) = \sum_{j \in \{H, L\}} q_\ell(j) P(S_\ell = j).$$

It is straightforward to show that

$$q_\ell(S) = q_\ell(NS) = \frac{\alpha_H r_H + \alpha_L (1 - r_H)}{3}.$$

The expected profits under information sharing and no information sharing can be expressed in terms of these expected production quantities. Specifically,

$$\Pi_\ell(S) = q_\ell(S)^2 + f_1(p, r_H)$$

and

$$\Pi_\ell(NS) = q_\ell(NS)^2 + f_2(p, r_H),$$

where

$$f_1(p, r_H) = \frac{r_H^2(2p-1)^2(\alpha_H - \alpha_L)^2(1-r_H)^2(2p^2-2p+1)}{9(p^4-2p^3-4p^2r_H^2+4p^2r_H+p^2+4pr_H^2-4pr_H-r_H^2+r_H)}$$

and

$$f_2(p, r_H) = \frac{r_H^2(2p-1)^2(\alpha_H - \alpha_L)^2(1-r_H)^2(4p^2r_H-4p^2r_H^2+p-p^2+4pr_H^2-4pr_H+r_H-r_H^2)}{(2p-2p^2-12p^2r_H^2+12p^2r_H+p^2+12pr_H^2-12pr_H-3r_H^2+3r_H)}.$$

It is easily verified that functions f_1 and f_2 are symmetric in r_H . Hence,

$$\begin{aligned} \Pi_\ell(S)|_{r_H} - \Pi_\ell(S)|_{1-r_H} &= q_\ell(S)^2|_{r_H} - q_\ell(S)^2|_{1-r_H} \text{ and} \\ \Pi_\ell(NS)|_{r_H} - \Pi_\ell(NS)|_{1-r_H} &= q_\ell(NS)^2|_{r_H} - q_\ell(NS)^2|_{1-r_H}. \end{aligned}$$

We already mentioned that $q_\ell(S) = q_\ell(NS)$. Hence the symmetry of $\Pi_\ell(S) - \Pi_\ell(NS)$ in r_H .

In essence, Proposition 6 says that firms' incentives to share information (or not) depend on how certain firms are about the future demand state; whether they are certain that demand will be low or high is irrelevant.

Proposition 7 establishes the invariance of the information sharing strategy with respect to α_H .

PROPOSITION 7. *When firms decide ex-ante whether or not to share information, the equilibrium strategy is independent of α_H .*

Proof. This result follows almost directly from the proof of Proposition 5. Note that there we showed that $\Pi_\ell(S) - \Pi_\ell(NS) = (T_1 - T_4)(\alpha_H^2 + \alpha_L^2 - 2\alpha_H\alpha_L)$. The equilibrium strategy is to share if $T_1 - T_4$ is non-negative and not share otherwise. Notice that $T_1 - T_4$ is a function of only p and r_H . Hence the result.

In the proof of Proposition 7, we show that the invariance property holds because of the specific structure of $\Pi_\ell(S) - \Pi_\ell(NS)$, i.e., the difference in profits between sharing and not sharing private information. In our model, the difference in profits, given by (15), is separable in α_H and (p, r_H) . In addition, its sign depends on (p, r_H) but not on α_H . This separability property may be specific to our setup and may not hold true for other competition/information sharing models.