EEE 133 Key concepts and Equations

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Differential Equations

A linear ordinary differential equation of constant coefficients follows the form

$$a_n \frac{\mathrm{d}^n x}{\mathrm{d}t^n} + a_{n-1} \frac{\mathrm{d}^{n-1} x}{\mathrm{d}t^{n-1}} + \ldots + a_1 \frac{\mathrm{d}x}{\mathrm{d}t} + a_0 x = f(t) \tag{1}$$

A homogenous differential equation is an equation where the independent variable appears to have the same power. Equation (1) is homogenous when $f(t) = 0$.
The solution to the linear ODE in (1) has the form

$$x(t) = x_h(t) + x_p(t) \tag{2}$$

Where $x_h(t)$ is a solution to the homogenous equation while $x_p(t)$ is a solution to the nonhomogenous equation.
A homogenous equation has solutions of the form

$$x_h(t) = \exp(mt) \tag{3}$$

for m is a root of the equation

$$m^n + \frac{a_{n-1}}{a_n} m^{n-1} + \ldots + \frac{a_1}{a_n} m + \frac{a_0}{a_n} = 0 \tag{4}$$

If a root m_i is repeated k times, the corresponding solutions are $\exp(m_i t),\ t\exp(m_i t),\ \ldots,\ t^{k-1}\exp(m_i t)$

Some common forms (with constant coefficients) and solutions

Equation	Characteristic Equation	Determinant	Solution
$a_1 \frac{\mathrm{d}x}{\mathrm{d}t} + a_0 x = 0$	$a_1 m + a_0 = 0$		$x = C \exp\left(-\frac{a_0}{a_1} t\right)$
$a_2 \frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + a_1 \frac{\mathrm{d}x}{\mathrm{d}t} + a_0 x = 0$	$a_2 m^2 + a_1 m + a_0 = 0$	$a_1^2 - 4a_2 a_0 > 0$	$x(t) = C_1 \exp(m_1 t) + C_2 \exp(m_2 t)$
		$a_1^2 - 4a_2 a_0 = 0$	$x(t) = (C_1 x + C_2) \exp(m_1 t)$
		$a_1^2 - 4a_2 a_0 < 0$	$x(t) = \exp(\alpha t) \left[C_1 \cos(\beta t) + C_2 \sin(\beta t)\right]$

In the last line, the characteristic equation has solutions $m_1 = \alpha + j\beta,\ m_2 = \alpha - j\beta$ where $j^2 = -1$

Laplace Transforms and Theorems

A Laplace transform of a function $f(t)$ defined for all $t \geq 0$ is the transformation

$$\mathcal{L}[f(t)] = F(s) = \int_0^\infty f(t)\exp(-st)\,\mathrm{d}t$$

An inverse Laplace transform of a function $F(s)$ is defined as:

$$f(t) = \mathcal{L}^{-1}[F(s)] = \frac{1}{2\pi j}\lim_{T\to\infty}\int_{\gamma-jT}^{\gamma+jT}\exp(st)F(s)\,\mathrm{d}s$$

Common Functional Tranforms

$f(t) = \mathcal{L}^{-1}[F(s)]$	$F(s) = \mathcal{L}[F(s)]$	$f(t) = \mathcal{L}^{-1}[F(s)]$	$F(s) = \mathcal{L}[F(s)]$
$\delta(t-a)$	$\exp(-as)$	$\frac{1}{\beta-\alpha}\left(\exp(-\alpha t)-\exp(-\beta t)\right)u(t)$	$\frac{1}{(s+\alpha)(s+\beta)}$
$u(t-a)$	$\frac{\exp(-as)}{s}$	$\sin(\omega t)u(t)$	$\frac{\omega}{s^2+\omega^2}$
$tu(t)$	$\frac{1}{s^2}$	$\cos(\omega t)u(t)$	$\frac{s}{s^2+\omega^2}$
$\frac{t^{n-1}}{(n-1)!}u(t)$	$\frac{1}{s^n}$	$\sin(\omega t+\theta)u(t)$	$\frac{s\sin(\theta)+\omega\cos(\theta)}{s^2+\omega^2}$
$\exp(-\alpha t)u(t)$	$\frac{1}{s+\alpha}$	$\cos(\omega t+\theta)u(t)$	$\frac{s\cos(\theta)-\omega\sin(\theta)}{s^2+\omega^2}$
$t\exp(-\alpha t)u(t)$	$\frac{1}{(s+\alpha)^2}$	$\exp(-\alpha t)\sin(\omega t)u(t)$	$\frac{\omega}{(s+\alpha)^2+\omega^2}$
$\frac{t^{n-1}}{(n-1)!}\exp(-\alpha t)u(t)$	$\frac{1}{(s+\alpha)^n}$	$\exp(-\alpha t)\cos(\omega t)u(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega^2}$

Common Operational Transforms

Operation	$f(t)$	$F(s)$
Multiplication by constant	$cf(t)$	$cF(s)$
Addition	$f_1(t)+f_2(t)-f_3(t)+\ldots$	$F_1(s)+F_2(s)-F_3(s)+\ldots$
First time derivative	$\frac{\mathrm{d}f(t)}{\mathrm{d}t}$	$sF(s)-f(0^-)$
Second time derivative	$\frac{\mathrm{d}^2f(t)}{\mathrm{d}t^2}$	$s^2F(s)-sf(0^-)-\frac{\mathrm{d}f(0^-)}{\mathrm{d}t}$
nth time derivative	$\frac{\mathrm{d}^nf(t)}{\mathrm{d}t^n}$	$s^nF(s)-\sum_{i=1}^{n}s^{n-i}\frac{\mathrm{d}^{i-1}f(0^-)}{\mathrm{d}t^{i-1}}$
Time integral	$\int_0^t f(x)\,\mathrm{d}x$	$\frac{F(s)}{s}$

Translation in time	$f(t-a)u(t-a),\ a>0$	$\exp(-as)F(s)$
Translation in frequency	$\exp(-at)f(t)$	$F(s+a)$
Scale change	$f(at),\ a>0$	$\frac{1}{a}F\left(\frac{s}{a}\right)$
First frequency derivative	$tf(t)$	$\frac{\mathrm{d}F(s)}{\mathrm{d}s}$
nth frequency derivative	$t^n f(t)$	$(-1)^n\frac{\mathrm{d}^n F(s)}{\mathrm{d}s^n}$
Frequency integral	$\frac{f(t)}{t}$	$\int_0^\infty F(u)\,\mathrm{d}u$

1 Passive components

Current and Voltage for Passive Components:

Component	Current	Voltage	Energy
Resistor R	$i=\frac{v}{R}$	$v=iR$	$w_R=\int iv\,\mathrm{d}t$
Capacitor C	$i(t)=C\frac{\mathrm{d}v}{\mathrm{d}t}$	$v(t)=v(t_0)+\frac{1}{C}\int_{t_0}^{t} i(t)\,\mathrm{d}t$	$w_C(t)=\frac{1}{2}Cv^2(t)$
Inductor L	$i(t)=i(t_0)+\frac{1}{L}\int_{t_0}^{t} v(t)\,\mathrm{d}t$	$v(t)=L\frac{\mathrm{d}i}{\mathrm{d}t}$	$w_L(t)=\frac{1}{2}Li^2(t)$

Combination of passive components:

Component	Series	Parallel
Resistor R	$R_{eq}=\sum_i R_i$	$\frac{1}{R_{eq}}=\sum_i\frac{1}{R_i}$
Capacitor C	$\frac{1}{C_{eq}}=\sum_i\frac{1}{C_i}$	$C_{eq}=\sum_i C_i$
Inductor L	$L_{eq}=\sum_i L_i$	$\frac{1}{L_{eq}}=\sum_i\frac{1}{L_i}$

1.1 Equilibrium Equations

1. **Loop Current formulation**:
    - number of unknown currents equal number of loops
    - KVL equation for each loop
2. **Node Voltage formulation**:
    - number of unknown voltage equal number of nodes except reference
    - KCL equation for each node

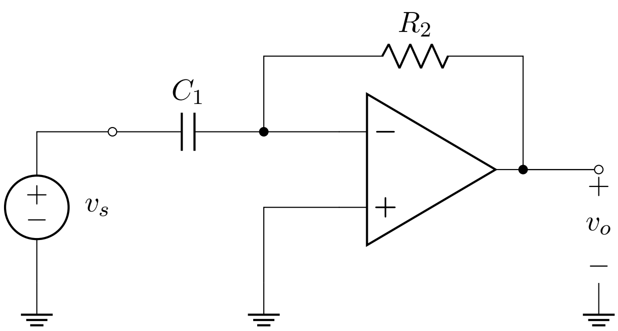


Figure 1: Op Amp Differentiator

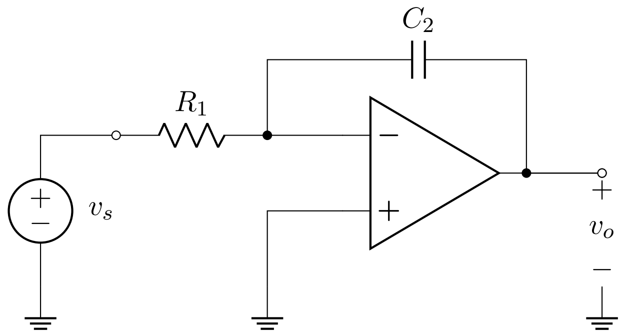


Figure 2: Op Amp Integrator

$$v_o(t) = -R_2 C_1 \frac{\mathrm{d}v_s}{\mathrm{d}t}$$

$$v_o(t) = -\frac{1}{R_1 C_2} \int_0^t v_s(t')\,\mathrm{d}t'$$

1.2 First Order Circuits

1. first order circuits are any circuit with a single energy storage element, an arbitrary number of sources and resistors
2. any current or voltage in such circuit is a solution to a first order differential equation

2 First order Unforced Response

Network	Current	Voltage	Time Constant	DC Steady State
Sourcefree RL	$i_L(t) = I_0 \exp\left(-\frac{R}{L}t\right)$	$v_L = -RI_0 \exp\left(-\frac{R}{L}t\right)$	$\tau = \frac{L}{R}$	$v_L = 0$
Sourcefree RC	$i_C(t) = -\frac{V_0}{R} \exp\left(-\frac{1}{RC}t\right)$	$v_c = V_0 \exp\left(-\frac{1}{RC}t\right)$	$\tau = RC$	$i_C = 0$

Typical time constant for RL is in ms, for RC in μs. For general RL and RC circuits, find the equivalent resistance as seen by the inductor/capacitor.

3 First order Forced Response

Consider a series RL circuit with voltage $v(t) = V_0 u(t)$. We have the following KVL equation of current

$$\begin{aligned} i(t) &= 0 && t < 0 \\ Ri + L\frac{\mathrm{d}i}{\mathrm{d}t} &= V_0 && t > 0 \end{aligned}$$

This differential equation gives a solution of

$$i(t) = \left[\underbrace{\frac{V_0}{R}}_{\text{forced response}} - \underbrace{\left(\frac{V_0}{R} - I_0\right) \exp\left(-\frac{R}{L}t\right)}_{\text{natural response}} \right] u(t) \tag{5}$$

1. Forced response

- dependent on forcing function
- steady state response $t \gg \tau$

2. Natural response

- similar to source-free circuit
- dependent on initial values and forcing function
- transient response

Generalization:

$$i_L(t) = \left[I_f - (I_f - I_i)\exp\left(-\frac{t}{\tau}\right)\right]u(t) \tag{6}$$

$$v_c(t) = \left[V_f - (V_f - V_i)\exp\left(-\frac{t}{\tau}\right)\right]u(t) \tag{7}$$

Note that equation (7) is for the voltage across the capacitor in an RC circuit.

3.1 Square Waves and Sequentially Switched Circuits

Now consider the series RL circuit with voltage $v(t) = V_0u(t) - V_0u(t-t_0)$ and $I_0 = 0$. We can apply superposition to find the current:

$$i(t) = i_1(t) + i_2(t)$$

$$i(t) = \underbrace{\left[\frac{V_0}{R}\left(1 - \exp\left(-\frac{R}{L}t\right)\right)\right]u(t)}_{\text{caused by } V_0u(t) \text{ alone}} - \underbrace{\left[\frac{V_0}{R}\left(1 - \exp\left(-\frac{R}{L}(t-t_0)\right)\right)\right]u(t-t_0)}_{\text{caused by } V_0u(t-t_0) \text{ alone}}$$

For sequentially switched circuits, we consider the pulse width (PW) and period (T) of the pulsing.

Condition	Output	Condition	Output
PW $\gg \tau$	time enough to fully charge	T - PW $\gg \tau$	time enough to fully discharge
PW $\ll \tau$	time NOT enough to fully charge	T - PW $\ll \tau$	time NOT enough to fully discharge

3.2 RC Oscillator

1. For a low pass RC circuit, the output (voltage at capacitor) is simply the input for low frequencies

   (a) at $\omega = 0$, $v_o = v_i$

   (b) at $\omega \to \infty$, $v_o \to 0$

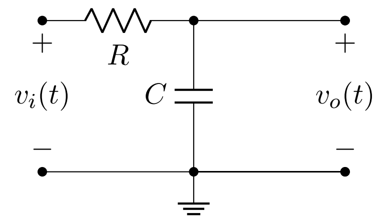
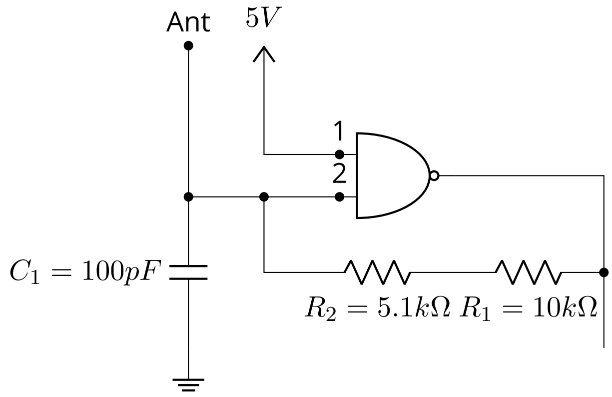


Figure 3: Low Pass Circuit



2. Schmitt Trigger RC

   (a) $V_p = 2.9V \approx 3.0V, V_n = 1.9V \approx 2.0V$

   (b) As the voltage on C_1 reaches V_p, the output will become voltage low. Then the voltage across the capacitor decays

   (c) When the voltage across C_1 decays to V_n, output will become voltage high. Then the voltage across the capacitor increases

   (d) The behavior oscillates.

   (e) the voltage across the charging capacitor is

$$v = V_{cc}\left[1 - \exp\left(-\frac{t}{\tau}\right)\right]$$

   (f) the time to charge to V_p is

$$t_1 = -\tau \ln\left(1 - \frac{V_p}{V_{cc}}\right)$$

   (g) the voltage across the discharging capacitor is

$$v = V_p \exp\left(-\frac{t}{\tau}\right)$$

   (h) the time to discharge to V_n is

$$t_2 = -\tau \ln\left(\frac{V_n}{V_p}\right)$$

   (i) the voltage across the charging capacitor from V_n to V_p is

$$v = V_{cc} - (V_{cc} - V_n)\exp\left(-\frac{t}{\tau}\right)$$

   (j) the time to charge to from V_n to V_p is

$$t_3 = -\tau \ln\left(\frac{V_{cc} - V_p}{V_{cc} - V_n}\right)$$

3. For a high pass RC circuit, the output (voltage at capacitor) is simply the input for high frequencies

   (a) at $\omega = 0$, $v_o = 0$

   (b) at $\omega \to \infty$, $v_o \to v_i$

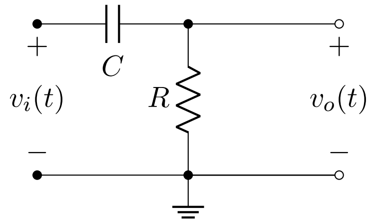


Figure 4: High Pass Circuit

4 Diode and Transistor Switching: Half Wave Rectifier

1. The half wave rectifier has the following circuit

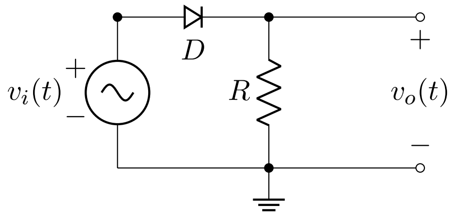


Figure 5: Half Wave Rectifier

(a) the diode is open only when $v_i(t) > v_{\text{on}}$, hence only the positive voltage are seen by $v_o(t)$

i. Average Value $\frac{V_{\text{m}}}{\pi}$

ii. $V_{\text{rms}} = \frac{V_{\text{m}}}{2}$

iii. $I_{\text{m}} = \frac{V_{\text{m}}}{R_{\text{L}}}$

iv. Ripple Factor $\frac{I_{\text{rms}}}{I_{\text{DC}}}$

v. Efficiency $e = \frac{\text{DC Output Power}}{\text{AC Output Power}}$

(b) a smoothing capacitor is added (capacitor filter) so that the output waveform does not have a 0 value:

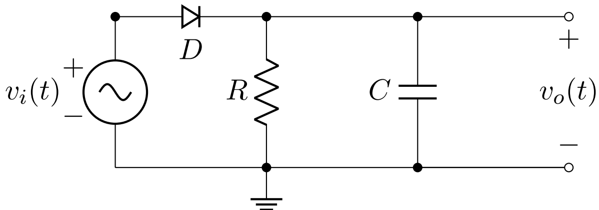


Figure 6: Half Wave Rectifier with Capacitor Filter

i. C is chosen such that $RC \gg T$ so the exponential seems linear

5 LC Oscillations

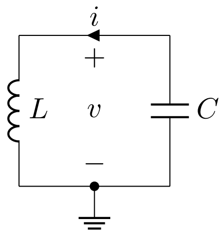


Figure 7: LC Oscillator

1. most of the time, either $v(0)$ or $i(0)$ are given.
2. by conservation of energy: $\frac{1}{2}Li^2 = \frac{1}{2}Cv^2$
3. the differential equations are $v = L\frac{\mathrm{d}i}{\mathrm{d}t}$ and $i = -C\frac{\mathrm{d}v}{\mathrm{d}t} \implies i = \frac{1}{LC}\frac{\mathrm{d}^2 i}{\mathrm{d}t^2}$
4. we have $\omega = \frac{1}{\sqrt{LC}}$
5. the solutions have the form

$$i = I_0 \cos\left(\frac{t}{\sqrt{LC}} + \phi\right)$$
$$v = -I_0\sqrt{\frac{L}{C}} \sin\left(\frac{t}{\sqrt{LC}} + \phi\right)$$

or

$$v = V_0 \cos\left(\frac{t}{\sqrt{LC}} + \phi\right)$$
$$i = -V_0\sqrt{\frac{C}{L}} \sin\left(\frac{t}{\sqrt{LC}} + \phi\right)$$

6 Source-free RLC Circuits

	Characteristic Equation	(α)	(ω_0)	Damping Factor (ζ)
Series $i(t)$	$s^2 + \left(\frac{R}{L}\right)s + \frac{1}{LC} = 0$	$\alpha = \frac{R}{2L}$	$\omega_0 = \frac{1}{\sqrt{LC}}$	$\zeta = \frac{R}{2}\sqrt{\frac{C}{L}}$
Parallel $v(t)$	$s^2 + \left(\frac{1}{RC}\right)s + \frac{1}{LC} = 0$	$\alpha = \frac{1}{2RC}$	$\omega_0 = \frac{1}{\sqrt{LC}}$	$\zeta = \frac{1}{2R}\sqrt{\frac{L}{C}}$

$$s_{1,\,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} \tag{8}$$

Case	Condition	Characteristic	Roots
Overdamped	$\zeta > 1$	Does not oscillate about the steady state value	Real, Distinct
Underdamped	$\zeta < 1$	Oscillation with decay envelope	Complex
Critacally damped	$\zeta = 1$	Decays fastest to steady state without oscillation	Real, equal

7 Resonance

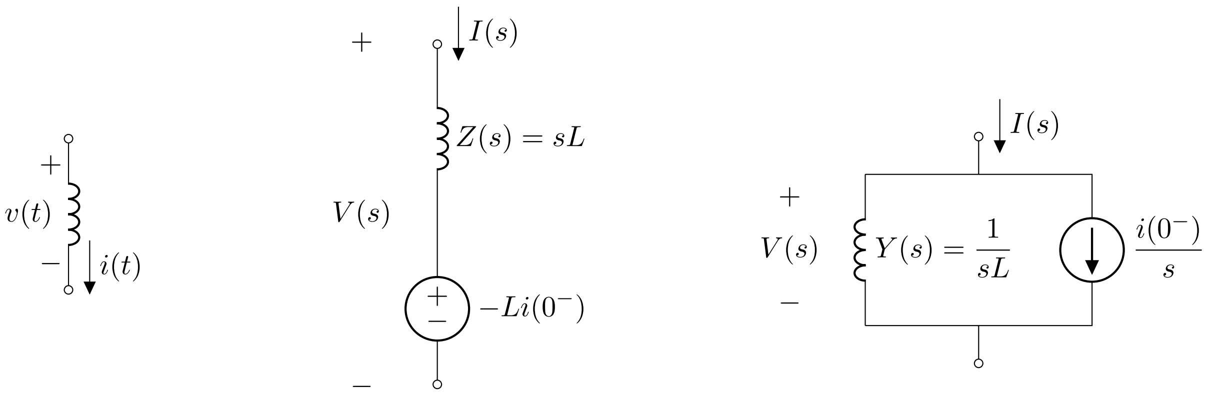
1. resonance - the fixed amplitude forcing function produces a response of maximum amplitude
2. Series Resonant Conditions:
   (a) $\omega_0 = \dfrac{1}{\sqrt{LC}}$
   (b) $Z_{in} = R$
   (c) voltage and current of source in phase
   (d) current magnitude is maximum
   (e) electric filter
   (f) the half-power-point frequency is where the power is half:
   $$\omega_{1,\,2} = \mp\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \tag{9}$$
   (g) bandwidth: $BW = \omega_2 - \omega_1 = \dfrac{R}{L}$
   (h) Quality factor: $Q = \dfrac{\omega_0}{BW} = \dfrac{1}{R}\sqrt{\dfrac{L}{C}}$
   (i) $\omega_0^2 = \omega_1\omega_2$
3. Parallel Resonant Conditions:
   (a) $\omega_0 = \dfrac{1}{\sqrt{LC}}$
   (b) $Y_{in} = \dfrac{1}{R}$
   (c) voltage and current of source in phase
   (d) voltage magnitude is maximum
   (e) electri c filter
   (f) the half-power-point frequency is where the power is half:
   $$\omega_{1,\,2} = \mp\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} \tag{10}$$
   (g) bandwidth: $BW = \omega_2 - \omega_1 = \dfrac{1}{RC}$
   (h) Quality factor: $Q = \dfrac{\omega_0}{BW} = R\sqrt{\dfrac{C}{L}}$
   (i) $\omega_0^2 = \omega_1\omega_2$
4. Physical model: R_1 series to L, C parallel to that series and to R_2
   (a) $\omega_0 = \sqrt{\dfrac{1}{LC} - \left(\dfrac{R_1}{L}\right)^2}$

8 Laplace Transform Applications

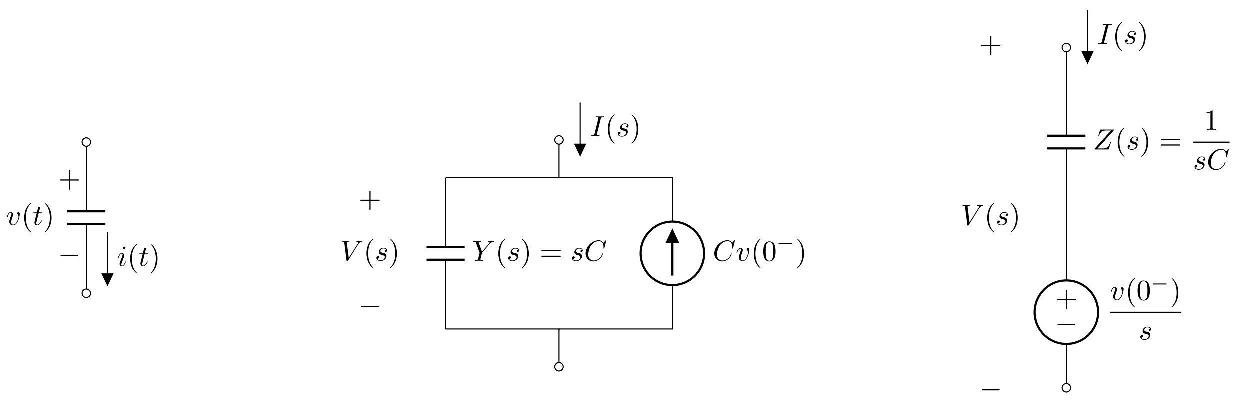
Transform of circuit elements:

1. Resistor:

2. Inductor:



3. Capacitor:



8.1 Method 1

1. Write the differential equations for the unknown function $x(t)$
2. Apply Laplace transform on the equation

3. Use algebraic manipulation for $X(s)$
4. Apply inverse Laplace transform to get $x(t)$

8.2 Method 2

1. Apply Laplace transform on each element
2. Use DC circuit analysis techniques to write the s-domain equations involving $X(s)$
3. Apply inverse Laplace transform to get $x(t)$

Examples

1. Resistors: $v = iR \iff V = IR$, where $V = \mathcal{L}\{v\},\ I = \mathcal{L}\{i\}$
2. Inductors: $v = L\dfrac{\mathrm{d}i}{\mathrm{d}t} \iff V = L\left[sI - i(0^-)\right] = sLI - LI_0$
3. Capacitors: $i = C\dfrac{\mathrm{d}v}{\mathrm{d}t} \iff I = C\left[sV - v(0^-)\right] = sCV - CV_0$

8.3 $v(t) = V_0 u(t)$

1. Find the equivalent circuit using the Laplace equivalent of the circuit
2. Solve for $V(s)$. May involve partial fraction decomposition. This is a nice guide
3. Use Inverse Laplace transform to get $v(t)$

Transfer function. The s-domain ratio of the output to input signal.

$$H(s) = \frac{Y(s)}{X(s)} \tag{11}$$

		$H(s)$ of series RL			$H(s)$ of parallel RC
Input is $V(t)$	Output is $i(t)$	$H(s) = \dfrac{1}{R+sL}$	Input is $I(t)$	Output is $v(t)$	$H(s) = \dfrac{R}{1+sRC}$
	Output is $v_L(t)$	$H(s) = \dfrac{sL}{R+sL}$		Output is $i_C(t)$	$H(s) = \dfrac{sRC}{1+sRC}$
	Output is $v_R(t)$	$H(s) = \dfrac{R}{R+sL}$		Output is $i_R(t)$	$H(s) = \dfrac{1}{1+sRC}$

9 Ramp and Sine Response

1. **Ramp Response:**
   (a) $\mathcal{L}\left[r\left(t\right)\right] = \dfrac{1}{s^2}$
2. **Sine Response:**
   (a) use the natural response equation as homogenous solution
   (b) use phasor analysis to find the forced response
   (c) add the two to find the coefficient of the natural response using initial values
   (d) the transient is 0 for some angle $\tan^{-1}\left(\dfrac{\omega L}{R}\right)$ or $\tan^{-1}\left(\dfrac{\omega}{RC}\right)$

10 Second Order System

1. the following transfer functions are used in RLC circuits:
2. Series RLC:

$$\frac{V_L(s)}{V_{in}(s)} = \frac{s^2}{s^2 + \left(\frac{R}{L}\right)s + \frac{1}{LC}} \tag{12}$$

$$\frac{V_R(s)}{V_{in}(s)} = \frac{\left(\frac{R}{L}\right)s}{s^2 + \left(\frac{R}{L}\right)s + \frac{1}{LC}} \tag{13}$$

$$\frac{V_C(s)}{V_{in}(s)} = \frac{\frac{1}{LC}}{s^2 + \left(\frac{R}{L}\right)s + \frac{1}{LC}} \tag{14}$$

3. Parallel RLC:

$$\frac{I_L(s)}{I_{in}(s)} = \frac{\frac{1}{LC}}{s^2 + \left(\frac{1}{RC}\right)s + \frac{1}{LC}} \tag{15}$$

$$\frac{I_R(s)}{I_{in}(s)} = \frac{\left(\frac{1}{RC}\right)s}{s^2 + \left(\frac{1}{RC}\right)s + \frac{1}{LC}} \tag{16}$$

$$\frac{I_C(s)}{I_{in}(s)} = \frac{s^2}{s^2 + \left(\frac{1}{RC}\right)s + \frac{1}{LC}} \tag{17}$$

4. to determine the damping, rewrite the denominator: $\dfrac{\frac{1}{\omega_0^2}}{\left(\frac{s}{\omega_0}\right)^2 + 2\zeta\left(\frac{s}{\omega_0}\right) + \frac{1}{\omega_0^2}}$

11 Transients in Power Systems

Load Energization

1. RL Circuit:
   (a) $P = pf * VI = I^2R$
   (b) $Q = I^2X_L$
   (c) $X_L = 2\pi fL$
2. LC (Capacitor Energization)
   (a) $Y_C = \dfrac{Q_C}{V^2} = 2\pi fC$
   (b) $\omega_i = \dfrac{1}{\sqrt{LC}}$. when $\omega_i \gg \omega_s$, we assume the source to be DC
   (c) $V_s = V\sqrt{2}$
   (d) $v_c(t) = [1 - \cos(\omega_i t)]\,V_s$
   (e) $I_{SC} = \dfrac{MVA_{SC}}{V}$
   (f) $X_L = \dfrac{V}{I_{SC}} = \dfrac{V^2}{MVA_{SC}} = 2\pi fL$
   (g) $Q_C = \dfrac{V^2}{X_C} = V^2 2\pi fC$

3. The Circuit Closing Transient (RL):
   (a) input is $V_m \sin(\omega t + \theta)$
   (b) $i(t) = \frac{V_m}{Z}\left[\sin\left(\omega t + \theta - \tan^{-1}\left(\frac{\omega L}{R}\right)\right) - \sin\left(\theta - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)\exp\left(-\frac{Rt}{L}\right)\right]$
   (c) $Z = \sqrt{R^2 + \omega^2 L^2}$
   (d) there is no transient when $\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$

12 Filter Transient Response and Design

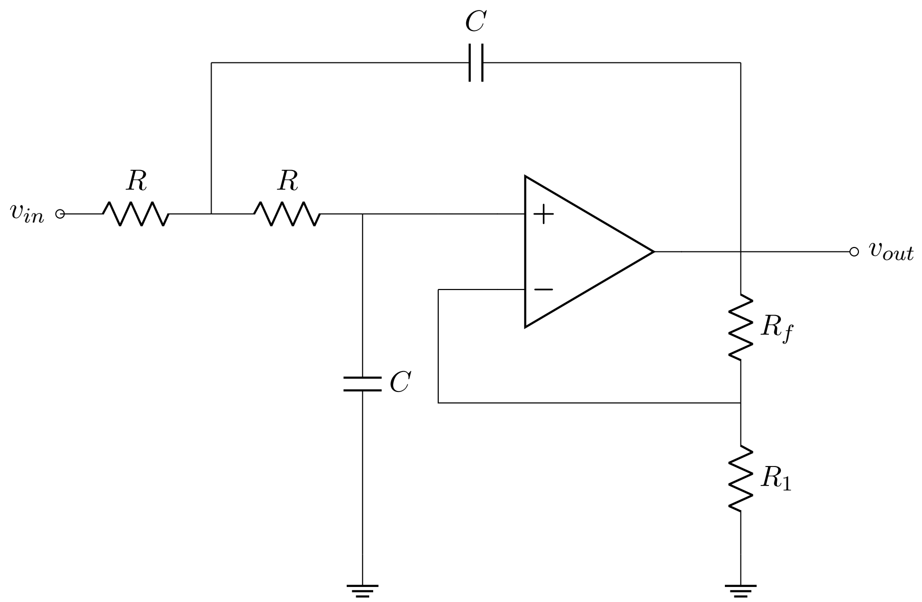


Figure 8: Sallen-Key Lowpass Filter Topology

1. The transfer function is:

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{s^2(RC)^2 + 2s\left(\frac{3-K}{2}\right) + 1} \tag{18}$$

   (a) $K = \frac{R_f + R_1}{R_1}$
   (b) $\omega_0 = \frac{1}{RC}$
   (c) $\alpha = \frac{3-K}{2RC}$
   (d) $\zeta = \frac{3-K}{2} = \frac{\alpha}{\omega_0}$

2. Analog Filters:
   (a) $R = \frac{k_1}{Cf_c}$
   (b) $\omega_0 = \frac{1}{RC} = \frac{f_c}{k_1}$
   (c) $R_f = R_1 k_2$

(d) $K = 1 + k_2$

(e) Chebychev filter: has the sharpest roll-off, but allows ripples in the passband

(f) Butterworth filter: 2nd sharpest roll-off *without* ripples in passband

(g) Bessel filter: has least ringing and overshoot in step response.

13 Switching-Mode Power Supply

1. SMPS: ransfers electric energy from asource to a load using circuits with power semiconductors that switch on and off at high frequency
   (a) AC - DC
   (b) DC - AC
   (c) AC - AC
   (d) DC - DC
      i. Step Up (Boost)
      ii. Step Down (Buck)
      iii. Step up/step down
      iv. multiple outputs
2. Pulse Width Modulation: uses a square wave with duty cycle modulated, resulting in change in average value of waveform
   (a) $\bar{y} = D y_{\text{max}} + (1 - D)\, y_{\text{min}}$
   (b) $D = \dfrac{t_{\text{on}}}{T}$
3. Switching converter: a constant DC supply is on only for the duty cycle time.
   (a) $V_o = V_s D$
4. Buck Converter: output voltage is lower but output current is higher

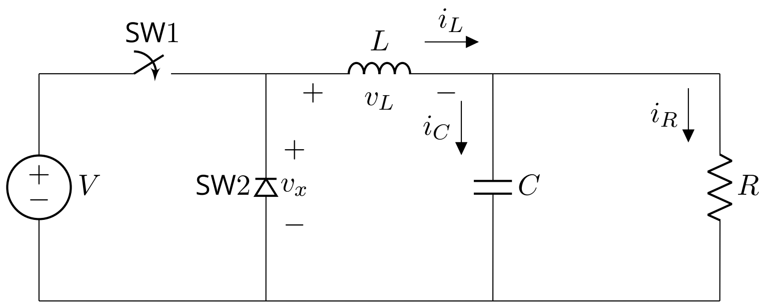


Figure 9: Buck Converter

   (a) assume the following:
   (b) steady state operation
   (c) periodic inductor current
   (d) average inductor voltage is 0
   (e) average capacitor current is 0

(f) $i_L = \begin{cases} \dfrac{V_S - V_o}{L} t + I_{\text{min}} & 0 \le t \le t_{\text{on}} \\ \dfrac{-V_o}{L}\left(t - t_{\text{on}}\right) + I_{\text{max}} & t_{\text{on}} \le t \le T \end{cases}$

(g) $I_o = \dfrac{V_o}{R} = I_{\text{min}} + \dfrac{1}{2}\left(\dfrac{V_s - V_o}{L}\right) t_{\text{on}}$

(h) $t_{\text{on}} = DT = \dfrac{D}{f}$

(i) $D = \dfrac{V_o}{V_s}$

(j) $L = \dfrac{1}{2}\dfrac{D}{f}\left(\dfrac{V_s - V_o}{\frac{V_o}{R} - I_{\text{min}}}\right)$

(k) boundary inductor (at $I_{\text{min}} = 0$): $L_b = \dfrac{D\left(V_s - V_o\right)R}{2fV_o} = \dfrac{R\left(1 - D\right)}{2f}$

i. $L < L_b$: DCM operation
ii. $L = L_b$: CDCM operation
iii. $L > L_b$: CCM operation