EEE 133 Key concepts and Equations

AJ Mesa Jr.

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1 Diode Models and Circuits

1.1 History of Electronics

1. 1904: John Fleming invented vacuum tubes. First time electron flow is controlled in non-conducting medium
   (a) if the cathode is heated, some electrons can escape
   (b) if a voltage is applied ($+$) on anode ($-$) on cathode, there will be current
2. 1906: the grid was added to control the current between the anode and cathode
   (a) vacuum tubes allowed the development of amplifiers, transmitters, receivers, signal processor
3. 1946: the ENIAC computer was completed. it could execute 5000 additions per second
4. 1947: invention of transistor by Shockley, Bardeen, Brattain at the Bell Labs
   (a) they were point-contact transistors
5. 1948: Schockley invented the BJT. the BJT is monolithic (containted in a single semiconductor crystal)
6. 1958: the first Integrated Circuits (IC) was invented by Jack Kilby of Texas Instruments (non-monolithic)
7. 1959: the first monolithic IC was developed by Robert Noyce at Fairchild Semiconduction
   (a) uses silicon instead of germanium
   (b) 2 interconnected BJT transistor
   (c) SiO_2 insulator, Al interconnection
   (d) planar IC, 0.06 in diameter
8. Bipolar ICs with BJT devices domidated until early 80s
   (a) S/M/L Scale Integration : 10, 100, 1000 transistors
9. 1959: MOS ICs started in the 60s after the MOS transistor was develpoed at Bell Labs
   (a) easier to fabricate than bipolar
   (b) uses less power
   (c) can fit more transistor in the same silicon area
   (d) slower than bipolar IC
   (e) less robust than bipolar IC

10. MOS ICs
    (a) early 70s: MOS technology improved in speed and reliability
    (b) first microprocessor: Intel 4004 (1971)
    (c) emergence of VLSI (>10000 trasistors)
    (d) microprocessors evolved into microcontrollers (MCUs) ad system on a chip (SOCs)
11. Moore's Law: the transistor density would double every two years

1.2 Piecewise Linear Diode Models

1. 1st Approximation: (when $v_s \gg V_T$)
    (a) Open when $v_D < 0$
    (b) Short when $v_D \geq 0$
2. 2nd Approximation (when above condition not satisfied):
    (a) Open when $v_D < V_T$
    (b) Voltage of V_T when $v_D \geq V_T$
3. 3rd Approximation (2nd with resistor R in series)
    (a) Open when $v_D < V_T$
    (b) Voltage of V_T in series with resistance R when $v_D \geq V_T$
4. To determine diode state:
    (a) Assume it is conducting
    (b) Check direction of current
    (c) If current is from anode to cathode, the assumption is correct and equivalent circuit is valid
    (d) Otherwise, diode should be open
5. Another method:
    (a) Replace diode with open circuit
    (b) Determine the voltage across the diode terminals ($+$) on anode, ($-$) on cathode
    (c) If v_D < the needed threshold, assumption is correct
    (d) Otherwise, diode should be replaced with the appropriate model

1.3 Practical Diode Circuits

1. Half Wave Rectifier

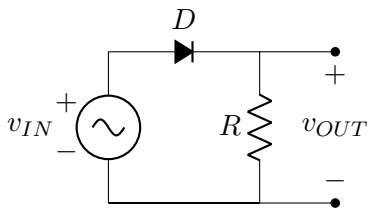


Figure 1: Half Wave Rectifier

   (a) When diode is IDEAL:

$$v_{OUT} = \begin{cases} 0, & v_{IN} < 0 \\ v_{IN} & v_{IN} \geq 0 \end{cases}$$

   (b) When diode has constant voltage model:

$$v_{OUT} = \begin{cases} 0, & v_{IN} < V_T \\ v_{IN} - V_T & v_{IN} \geq V_T \end{cases}$$

2. Full Wave Rectifier ($v_1 = v_2$)

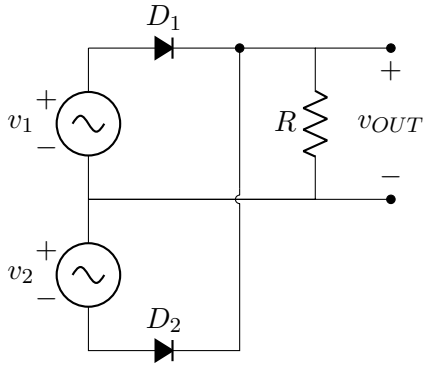


Figure 2: Full Wave Rectifier

   (a) When diode is IDEAL:
   $v_{OUT} = |v_{IN}|$

   (b) When diode has constant voltage model:

$$v_{OUT} = \begin{cases} v_1 - V_T, & v_1 > V_T \\ -v_2 - V_T, & v_2 < -V_T \\ 0 & \text{otherwise} \end{cases}$$

   (c) This requires a transformer with center-tapped secondary ($n_1 : n_2 = 1 : 2$)

3. Full Wave Bridge Rectifier

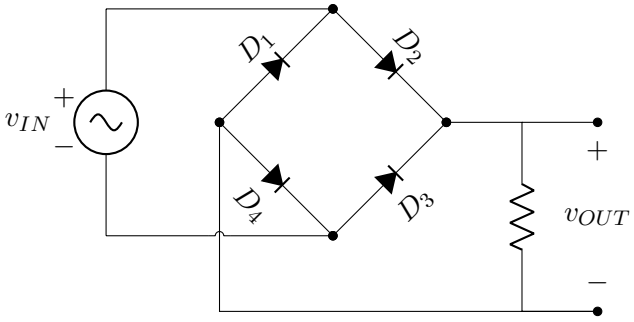


Figure 3: Full Wave Bridge Rectifier

(a) When diode is IDEAL:
$v_{OUT} = |v_{IN}|$

(b) When diode has constant voltage model:

$$v_{OUT} = \begin{cases} v_1 - 2V_T, & v_1 > 2V_T \\ -v_2 - 2V_T, & v_2 < -2V_T \\ 0 & \text{otherwise} \end{cases}$$

4. Positive Clipper

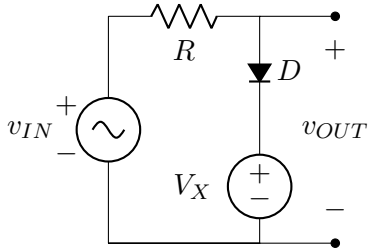


Figure 4: Positive Clipper

(a) $$v_{OUT} = \begin{cases} V_X + V_T, & v_{IN} - V_X > V_T \\ v_{IN}, & v_{IN} - V_X < V_T \end{cases}$$

5. Negative Clipper

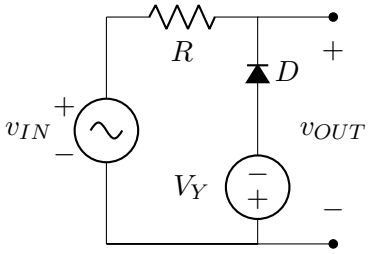


Figure 5: Negative Clipper

(a) $$v_{OUT} = \begin{cases} -(V_Y + V_T), & -V_Y - v_{IN} > V_T \\ v_{IN}, & -V_Y - v_{IN} < V_T \end{cases}$$

6. The positive and negative clippers can be combined

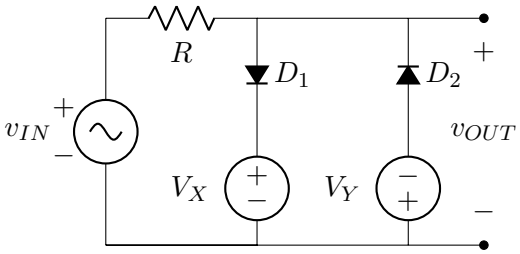


Figure 6: Positive and Negative Clipper

(a) $$v_{OUT} = \begin{cases} V_X + V_T, & v_{IN} - V_X > V_T \\ -(V_Y + V_T), & -V_Y - v_{IN} > V_T \\ v_{IN}, & \text{otherwise} \end{cases}$$

7. Peak Detector

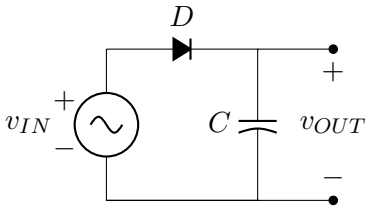


Figure 7: Peak Detector

(a) Let the time t_1 be the first time the input reaches its maximum value V_P and t_0 be the time to reach V_T

(b) When diode is IDEAL:

$$v_{OUT} = \begin{cases} v_{IN}, & t < t_1 \\ V_P, & t \geq t_1 \end{cases}$$

(c) When diode has constant voltage model:

$$v_{OUT} = \begin{cases} 0, & 0 \le t < t_0 \\ v_{IN} - V_T, & t_0 \le t < t_1 \\ V_P - V_T & t_1 \le t \end{cases}$$

8. Peak Detector with Load Resistor (Ideal Diode)

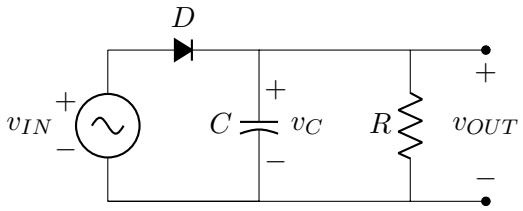


Figure 8: Peak Detector with load resistor

(a) Let the time t_1 be the first time the input reaches its maximum value V_P and the period be T. When the capacitor voltage decays, let time t_2 be when $v_C = v_{IN}$ again

(b) $$v_{OUT} = \begin{cases} v_{IN}, & t < t_1 \\ V_P \exp\left(-\dfrac{t - t_1}{RC}\right), & t_1 \le t < t_2 \\ v_{IN}, & t_2 \le t < t_1 + T \\ V_P \exp\left(-\dfrac{t - (t_1 + T)}{RC}\right), & t_1 + T \le t < t_2 + T \\ \vdots \end{cases}$$

9. Negative Clamper (ideal diode)

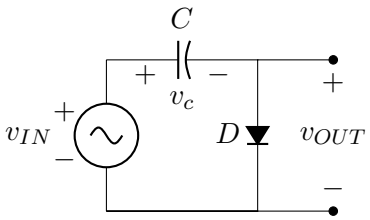


Figure 9: Negative Clamper

(a) Let the time t_1 be the first time the input reaches its maximum value V_P

(b) This is similar to the peak detector, only that $v_{OUT} = v_{IN} - v_C$ where v_C follows the characteristic of the peak detector

(c) $$v_C = \begin{cases} v_{IN}, & t < t_1 \\ V_P, & t_1 \le t \end{cases}$$

(d) $v_{OUT} = \begin{cases} 0, & t < t_1 \\ v_{IN} - V_P, & t \geq t_1 \end{cases}$

10. Positive Clamper (ideal diode)

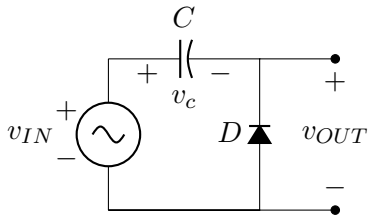


Figure 10: Positive Clamper

(a) Let the period be T

(b) $v_C = \begin{cases} 0, & 0 \leq t < T/2 \\ v_{IN}, & T/2 \leq t < 3T/4 \\ -V_P, & 3T/4 \leq t \end{cases}$

(c) $v_{OUT} = \begin{cases} v_{IN}, & 0 \leq t < T/2 \\ 0, & T/2 \leq t < 3T/4 \\ v_{IN} + V_P, & 3T/4 \leq t \end{cases}$

1.4 Exponential Diode Model

1. Shockley's Diode Equation

$$i_D = I_S \left[\exp\left(\frac{v_D}{\eta V_T} \right) - 1 \right] \tag{1}$$

   (a) v_D: diode voltage with positive at anode
   (b) i_D: diode current from anode to cathode
   (c) I_S: reverse saturation current due to minority carriers. $10^{-15}\text{A} < I_s < 10^{-9}\text{A}$
   (d) η: ideality factor, $1 < \eta < 2$; close to 1 for well fabricated diodes
   (e) V_T: thermal voltage, $V_T = \dfrac{kT_K}{e} \approx \dfrac{T_K}{11600}$

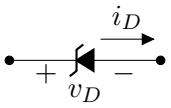
2. for typical operating voltages (forward biased): $\exp\left(\dfrac{v_D}{\eta V_T} \right) \gg 1 \implies i_D \approx I_S \exp\left(\dfrac{v_D}{\eta V_T} \right)$

3. for typical reverse bias voltages, $\exp\left(\dfrac{v_D}{\eta V_T} \right) \ll 1 \implies i_D \approx -I_S$

1.5 Small Signal Model

1. $v_D = V_D + v_d$
2. $i_D = I_D + i_d$. I_D is calculated using large signal analysis
3. if the amplitude of v_d is very small, the diode characteristic equation is approximately a line with conductance $g_d = \dfrac{\partial i_D}{\partial v_D} = \dfrac{I_S}{\eta V_T}\exp\left(\dfrac{v_D}{\eta V_T}\right) \approx \dfrac{I_D}{\eta V_T} \implies r_d = \dfrac{\eta V_T}{I_D}$

1.6 Zener Diode



1. has 3 modes of operations:

Operation	Condition	Equivalent
Forward-biased	$v_D \leq V_T$	$v_D = -V_T$
Non-conducting	$-V_T < v_D < V_Z$	open circuit
Zener region	$v_D \geq V_Z$	$v_D = V_Z$

2 PN Juncion and BJT Operation

2.1 Semiconductor Fundamentals

1. The Silicon (14) Atom:
   (a) has 4 valence electrons
   (b) the semiconductor lattice has bounded valence electrons at 0K
   (c) energy bands: electrons can occupy discrete energy levels
   (d) there is a **band gap** between the valence band and conduction band
   (e) at higher temperatures, an electron can gain enough energy to break the bonds
   (f) when an electron leaves a bond, it leaves a positively charged **hole**
2. Groups by electrical properties:
   (a) insulators: huge band gap
   (b) conductor: conduction band and valence overlap
   (c) semiconductor: small band gap that can be easily reached
3. Semiconductors:
   (a) narrow band gap
   (b) has 2 groups:

   i. intrinsic: just enough valence electrons to complete the matrix. either has group IV semiconductors (Si and Ge) or III-V compound (GaAs)
   ii. extrinsic: produced by doping intrinsic semiconductors
      A. N-type: dopants have excess electrons. energy level just below the conduction band
      B. P-type: dopants have less electrons. energy level just above the valence

4. Fermi level: hypothetical energy level where there is 50% probability that the level is occuppied by an electron at any given time
   (a) intrinsic semiconductor: midway between E_C and E_V
   (b) extrinsic semiconductor:
      i. N-type: closer to E_C
      ii. P-type: closer to E_V

2.2 Carrier Actions in Semiconductors

1. Charge Carriers: the electrons and holes. units in coulombs (C)
2. Intrinsic semiconductor: $n = p$ equal concentration of electrons and holes
3. N-type semiconductors:
   (a) $n > p$
   (b) majority charge carriers: electrons
   (c) minority charge carriers: holes
4. P-type semiconductors:
   (a) $p > n$
   (b) majority charge carriers: holes
   (c) minority charge carriers: electrons
5. Current: rate of movement of charge past a point or region. units in ampere (A). $+$ is the direction of the hole/opposite the electron
6. Mobility: ability of charge carrier to move. $\mu_e > \mu_h$
7. Factors that affect mobility:
   (a) Scattering - the random motion of charges. One mechanism is the *lattice scattering* when electrons bump with the vibrating lattice or other electrons and change directions
   (b) another scattering mechanism is *impurity scattering* when dopant ions deflect charge carriers
   (c) Temperature vs Scattering vs Mobility. At low temperatures, impurity scattering dominates (and with low mobility). at high temperatures, lattice scattering dominates (and low mobility). there is a temperature in the middle with maximum mobility. (like a parabolic log-log graph)
   (d) Doping vs Scattering vs Mobility. more doping = more impurity scattering
   (e) scattering has a net current of 0
8. There are 3 primary carrier actions:
   (a) Generation and Recombination: generation = electron gains enough energy to break a covalent bond and leaves a hole. recombination = moving electron moves close to a hole and experience attractive force

(b) Diffusion: electrons move from higher concentration/temperature to lower concentration/temperature. $+$ diffusion is parallel to positive charges. becomes 0 when there is uniform distribution of charge carriers

(c) Drift: caused by applied external voltage. modeled by a constant drift velocity $v_d = \mu E$ (directly related to the electric field with the mobility as constant)

i. $I_n = -Aqnv_{\text{n-drift}}$

ii. $I_p = Aqpv_{\text{p-drift}}$

2.2.1 PN Junction

1. PN Junction: formed when in a single silicon crystal, one region is doped with donor dopants (N-type) and the other with acceptor dopants (P-type). PN Junction is the boundary.
2. Depletion Region: happens at the boundary when the electrons diffuse and recombine with the holes at the P-type region. This results to a potential difference (and thus an electric field) across the depletion region (points from N to P)
   (a) drift current of minority carriers are drifted by this electric field
3. There are 3 junction operations:
   (a) Open-circuit (equilibrium condition):
      i. the diffusion current I_D is limited by the built in voltage V_0 and equal to the drift current I_S

3 BJTs as Amplifiers

4 MOSFETs as Amplifiers

5 Small Signal Analysis of Transistor Amplifiers

6 Amplifier Frequency Response