EEE 131 Key concepts and Equations

AJ Mesa Jr.

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1 Diode Models and Circuits

1.1 History of Electronics

1. 1904: John Fleming invented vacuum tubes. First time electron flow is controlled in non-conducting medium
   (a) if the cathode is heated, some electrons can escape
   (b) if a voltage is applied $(+)$ on anode $(-)$ on cathode, there will be current
2. 1906: the grid was added to control the current between the anode and cathode
   (a) vacuum tubes allowed the development of amplifiers, transmitters, receivers, signal processor
3. 1946: the ENIAC computer was completed. it could execute 5000 additions per second
4. 1947: invention of transistor by Shockley, Bardeen, Brattain at the Bell Labs
   (a) they were point-contact transistors
5. 1948: Schockley invented the BJT. the BJT is monolithic (containted in a single semiconductor crystal)
6. 1958: the first Integrated Circuits (IC) was invented by Jack Kilby of Texas Instruments (non-monolithic)
7. 1959: the first monolithic IC was developed by Robert Noyce at Fairchild Semiconduction
   (a) uses silicon instead of germanium
   (b) 2 interconnected BJT transistor
   (c) SiO_2 insulator, Al interconnection
   (d) planar IC, 0.06 in diameter
8. Bipolar ICs with BJT devices domidated until early 80s
   (a) S/M/L Scale Integration : 10, 100, 1000 transistors
9. 1959: MOS ICs started in the 60s after the MOS transistor was develpoed at Bell Labs
   (a) easier to fabricate than bipolar
   (b) uses less power
   (c) can fit more transistor in the same silicon area
   (d) slower than bipolar IC
   (e) less robust than bipolar IC

10. MOS ICs
    (a) early 70s: MOS technology improved in speed and reliability
    (b) first microprocessor: Intel 4004 (1971)
    (c) emergence of VLSI (>10000 trasistors)
    (d) microprocessors evolved into microcontrollers (MCUs) ad system on a chip (SOCs)
11. Moore's Law: the transistor density would double every two years

1.2 Piecewise Linear Diode Models

1. 1st Approximation: (when $v_s \gg V_T$)
    (a) Open when $v_D < 0$
    (b) Short when $v_D \geq 0$
2. 2nd Approximation (when above condition not satisfied):
    (a) Open when $v_D < V_T$
    (b) Voltage of V_T when $v_D \geq V_T$
3. 3rd Approximation (2nd with resistor R in series)
    (a) Open when $v_D < V_T$
    (b) Voltage of V_T in series with resistance R when $v_D \geq V_T$
4. To determine diode state:
    (a) Assume it is conducting
    (b) Check direction of current
    (c) If current is from anode to cathode, the assumption is correct and equivalent circuit is valid
    (d) Otherwise, diode should be open
5. Another method:
    (a) Replace diode with open circuit
    (b) Determine the voltage across the diode terminals $(+)$ on anode, $(-)$ on cathode
    (c) If v_D < the needed threshold, assumption is correct
    (d) Otherwise, diode should be replaced with the appropriate model

1.3 Practical Diode Circuits

1. Half Wave Rectifier

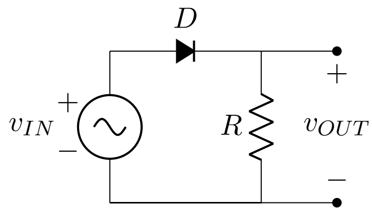


Figure 1: Half Wave Rectifier

   (a) When diode is IDEAL:

$$v_{OUT} = \begin{cases} 0, & v_{IN} < 0 \\ v_{IN} & v_{IN} \geq 0 \end{cases}$$

   (b) When diode has constant voltage model:

$$v_{OUT} = \begin{cases} 0, & v_{IN} < V_T \\ v_{IN} - V_T & v_{IN} \geq V_T \end{cases}$$

2. Full Wave Rectifier ($v_1 = v_2$)

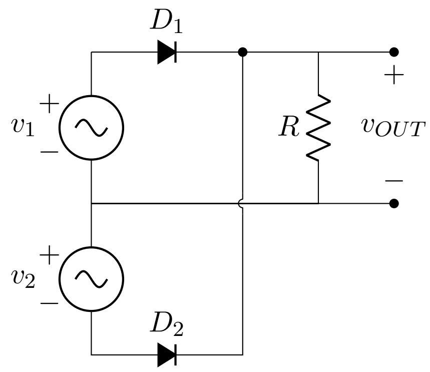


Figure 2: Full Wave Rectifier

   (a) When diode is IDEAL:
   $v_{OUT} = |v_{IN}|$

   (b) When diode has constant voltage model:

$$v_{OUT} = \begin{cases} v_1 - V_T, & v_1 > V_T \\ -v_2 - V_T, & v_2 < -V_T \\ 0 & \text{otherwise} \end{cases}$$

   (c) This requires a transformer with center-tapped secondary ($n_1 : n_2 = 1 : 2$)

3. Full Wave Bridge Rectifier

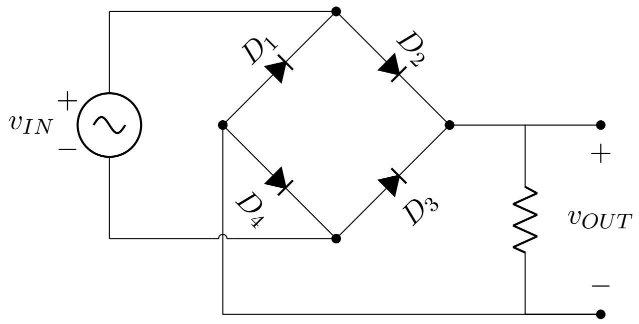


Figure 3: Full Wave Bridge Rectifier

(a) When diode is IDEAL:
$v_{OUT} = |v_{IN}|$

(b) When diode has constant voltage model:

$$v_{OUT} = \begin{cases} v_1 - 2V_T, & v_1 > 2V_T \\ -v_2 - 2V_T, & v_2 < -2V_T \\ 0 & \text{otherwise} \end{cases}$$

4. Positive Clipper

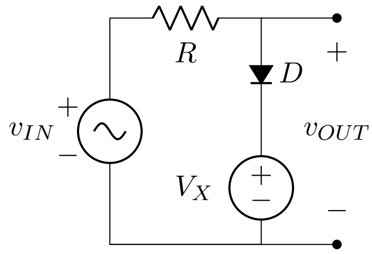


Figure 4: Positive Clipper

(a) $$v_{OUT} = \begin{cases} V_X + V_T, & v_{IN} - V_X > V_T \\ v_{IN}, & v_{IN} - V_X < V_T \end{cases}$$

5. Negative Clipper

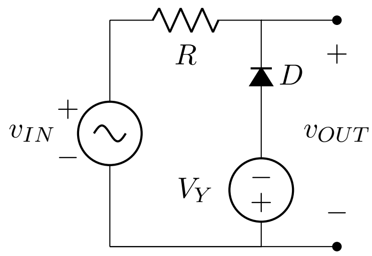


Figure 5: Negative Clipper

(a) $$v_{OUT} = \begin{cases} -\left(V_Y + V_T\right), & -V_Y - v_{IN} > V_T \\ v_{IN}, & -V_Y - v_{IN} < V_T \end{cases}$$

6. The positive and negative clippers can be combined

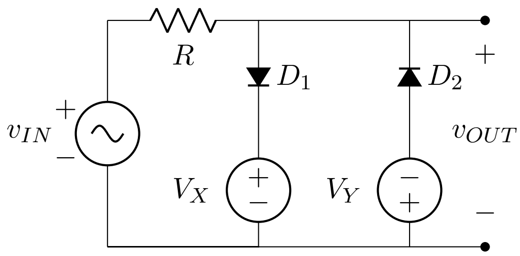


Figure 6: Positive and Negative Clipper

(a) $$v_{OUT} = \begin{cases} V_X + V_T, & v_{IN} - V_X > V_T \\ -\left(V_Y + V_T\right), & -V_Y - v_{IN} > V_T \\ v_{IN}, & \text{otherwise} \end{cases}$$

7. Peak Detector

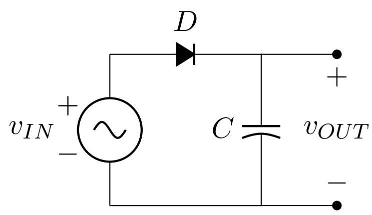


Figure 7: Peak Detector

(a) Let the time t_1 be the first time the input reaches its maximum value V_P and t_0 be the time to reach V_T

(b) When diode is IDEAL:

$$v_{OUT} = \begin{cases} v_{IN}, & t < t_1 \\ V_P, & t \geq t_1 \end{cases}$$

(c) When diode has constant voltage model:

$$v_{OUT} = \begin{cases} 0, & 0 \le t < t_0 \\ v_{IN} - V_T, & t_0 \le t < t_1 \\ V_P - V_T & t_1 \le t \end{cases}$$

8. Peak Detector with Load Resistor (Ideal Diode)

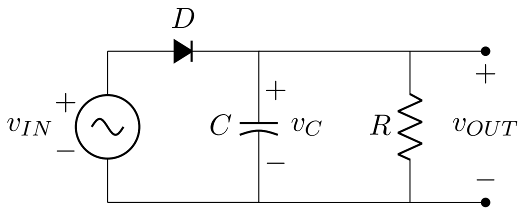


Figure 8: Peak Detector with load resistor

(a) Let the time t_1 be the first time the input reaches its maximum value V_P and the period be T. When the capacitor voltage decays, let time t_2 be when $v_C = v_{IN}$ again

(b) $$v_{OUT} = \begin{cases} v_{IN}, & t < t_1 \\ V_P \exp\left(-\dfrac{t - t_1}{RC}\right), & t_1 \le t < t_2 \\ v_{IN}, & t_2 \le t < t_1 + T \\ V_P \exp\left(-\dfrac{t - (t_1 + T)}{RC}\right), & t_1 + T \le t < t_2 + T \\ \vdots \end{cases}$$

9. Negative Clamper (ideal diode)

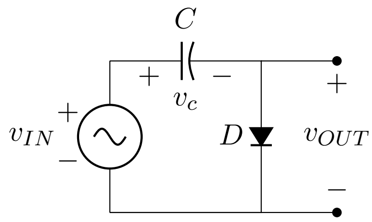


Figure 9: Negative Clamper

(a) Let the time t_1 be the first time the input reaches its maximum value V_P

(b) This is similar to the peak detector, only that $v_{OUT} = v_{IN} - v_C$ where v_C follows the characteristic of the peak detector

(c) $$v_C = \begin{cases} v_{IN}, & t < t_1 \\ V_P, & t_1 \le t \end{cases}$$

(d) $v_{OUT} = \begin{cases} 0, & t < t_1 \\ v_{IN} - V_P, & t \geq t_1 \end{cases}$

10. Positive Clamper (ideal diode)

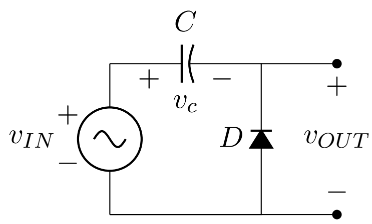


Figure 10: Positive Clamper

(a) Let the period be T

(b) $v_C = \begin{cases} 0, & 0 \leq t < T/2 \\ v_{IN}, & T/2 \leq t < 3T/4 \\ -V_P, & 3T/4 \leq t \end{cases}$

(c) $v_{OUT} = \begin{cases} v_{IN}, & 0 \leq t < T/2 \\ 0, & T/2 \leq t < 3T/4 \\ v_{IN} + V_P, & 3T/4 \leq t \end{cases}$

1.4 Exponential Diode Model

1. Shockley's Diode Equation

$$i_D = I_S \left[\exp\left(\frac{v_D}{\eta V_T} \right) - 1 \right] \tag{1}$$

   (a) v_D: diode voltage with positive at anode
   (b) i_D: diode current from anode to cathode
   (c) I_S: reverse saturation current due to minority carriers. 10^{-15}A $< I_s < 10^{-9}$A
   (d) η: ideality factor, $1 < \eta < 2$; close to 1 for well fabricated diodes
   (e) V_T: thermal voltage, $V_T = \dfrac{kT_K}{e} \approx \dfrac{T_K}{11600}$

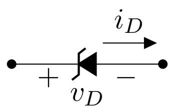
2. for typical operating voltages (forward biased): $\exp\left(\dfrac{v_D}{\eta V_T}\right) \gg 1 \implies i_D \approx I_S \exp\left(\dfrac{v_D}{\eta V_T}\right)$

3. for typical reverse bias voltages, $\exp\left(\dfrac{v_D}{\eta V_T}\right) \ll 1 \implies i_D \approx -I_S$

1.5 Small Signal Model

1. $v_D = V_D + v_d$
2. $i_D = I_D + i_d$. I_D is calculated using large signal analysis
3. if the amplitude of v_d is very small, the diode characteristic equation is approximately a line with conductance $g_d = \dfrac{\partial i_D}{\partial v_D} = \dfrac{I_S}{\eta V_T} \exp\left(\dfrac{v_D}{\eta V_T}\right) \approx \dfrac{I_D}{\eta V_T} \implies r_d = \dfrac{\eta V_T}{I_D}$

1.6 Zener Diode



1. has 3 modes of operations:

Operation	Condition	Equivalent
Forward-biased	$v_D \leq V_T$	$v_D = -V_T$
Non-conducting	$-V_T < v_D < V_Z$	open circuit
Zener region	$v_D \geq V_Z$	$v_D = V_Z$

2 PN Juncion and BJT Operation

2.1 Semiconductor Fundamentals

1. The Silicon (14) Atom:
   (a) has 4 valence electrons
   (b) the semiconductor lattice has bounded valence electrons at 0K
   (c) energy bands: electrons can occupy discrete energy levels
   (d) there is a **band gap** between the valence band and conduction band
   (e) at higher temperatures, an electron can gain enough energy to break the bonds
   (f) when an electron leaves a bond, it leaves a positively charged **hole**
2. Groups by electrical properties:
   (a) insulators: huge band gap
   (b) conductor: conduction band and valence overlap
   (c) semiconductor: small band gap that can be easily reached
3. Semiconductors:
   (a) narrow band gap
   (b) has 2 groups:
      i. intrinsic: just enough valence electrons to complete the matrix. either has group IV semiconductors (Si and Ge) or III-V compound (GaAs)
      ii. extrinsic: produced by doping intrinsic semiconductors
         A. N-type: dopants have excess electrons. energy level just below the conduction band
         B. P-type: dopants have less electrons. energy level just above the valence
4. Fermi level: hypothetical energy level where there is 50% probability that the level is occuppied by an electron at any given time
   (a) intrinsic semiconductor: midway between E_C and E_V
   (b) extrinsic semiconductor:
      i. N-type: closer to E_C
      ii. P-type: closer to E_V

2.2 Carrier Actions in Semiconductors

1. Charge Carriers: the electrons and holes. units in coulombs (C)
2. Intrinsic semiconductor: $n = p$ equal concentration of electrons and holes
3. N-type semiconductors:
   (a) $n > p$
   (b) majority charge carriers: electrons
   (c) minority charge carriers: holes
4. P-type semiconductors:
   (a) $p > n$

(b) majority charge carriers: holes
(c) minority charge carriers: electrons

5. Current: rate of movement of charge past a point or region. units in ampere (A). $+$ is the direction of the hole/opposite the electron
6. Mobility: ability of charge carrier to move. $\mu_e > \mu_h$
7. Factors that affect mobility:
   (a) Scattering - the random motion of charges. One mechanism is the *lattice scattering* when electrons bump with the vibrating lattice or other electrons and change directions
   (b) another scattering mechanism is *impurity scattering* when dopant ions deflect charge carriers
   (c) Temperature vs Scattering vs Mobility. At low temperatures, impurity scattering dominates (and with low mobility). at high temperatures, lattice scattering dominates (and low mobility). there is a temperature in the middle with maximum mobility. (like a parabolic log-log graph)
   (d) Doping vs Scattering vs Mobility. more doping = more impurity scattering
   (e) scattering has a net current of 0
8. There are 3 primary carrier actions:
   (a) Generation and Recombination: generation = electron gains enough energy to break a covalent bond and leaves a hole. recombination = moving electron moves close to a hole and experience attractive force
   (b) Diffusion: electrons move from higher concentration/temperature to lower concentration/temperature. $+$ diffusion is parallel to positive charges. becomes 0 when there is uniform distribution of charge carriers
   (c) Drift: caused by applied external voltage. modeled by a constant drift velocity $v_d = \mu E$ (directly related to the electric field with the mobility as constant)
      i. $I_n = -Aqnv_{\text{n-drift}}$
      ii. $I_p = Aqpv_{\text{p-drift}}$

2.2.1 PN Junction

1. PN Junction: formed when in a single silicon crystal, one region is doped with donor dopants (N-type) and the other with acceptor dopants (P-type). PN Junction is the boundary.
2. Depletion Region: happens at the boundary when the electrons diffuse and recombine with the holes at the P-type region. This results to a potential difference (and thus an electric field) across the depletion region (points from N to P)
   (a) drift current of minority carriers are drifted by this electric field
3. There are 3 junction operations:
   (a) Open-circuit (equilibrium condition):
      i. the diffusion current I_D is limited by the built in voltage V_0 and equal to the drift current I_S
   (b) Forward Bias ($+$ on the P-type)
      i. the $+$ potential pushes the holes in the P-type and the depletion region in the P-side decreases

    ii. the $-$ potential pushes the electrons in the N-type and the depletion region in this side also decreases
    iii. the diffusion current I_D dominates, but the temperature dependent I_S remain the same
    iv. the net current $I = I_D - I_S$ in the direction from P to N
    v. when $V_F \geq V_o$, the depletion region disappears and $I \approx I_D$

(c) Reverse Bias ($+$ on the N-type)

    i. the $+$ potential attracts the electrons in the N-type, the depletion region becomes larger
    ii. the $-$ potential attracts the holes in the P-type, the depletion region becomes larger
    iii. the voltage V_o increases so I_D decreases
    iv. net current $I = I_D - I_S$ from N to P
    v. if I_D becomes very small, $I \approx -I_S$

4. The energy band diagram will align the Fermi levels, thus bending the E_C and E_V with the maximum deviation V_0

    (a) at forward bias, V_0 decreases so the bending energy will also decrease and the Fermi level will not be aligned anymore (E_{FN} is higher)
    (b) at reverse bias, V_0 increases so the bending energy will also inrease and the Fermi level will not be aligned anymore (E_{FN} is lower)

5. The PN Junction implements a Diode. anode at P, cathode at N

    (a) the Shockley Equation summarizes the 3 conditions for the PN Junction operations
    (b) Diode resistivity:
        i. at higher temperature, more electrons are freed and resistivity decreases
        ii. higher doping density lowers the resistivity

3 BJTs as Amplifiers

3.1 DC Characteristics and Operating Modes

Mode of Operation	EB Junction	CB Junction
Cut-off	Reverse-biased	Reverse-biased
Active	Forward-biased	Reverse-biased
Saturation	Forward-biased	Forward-biased
Reverse Active	Forward-biased	Reverse-biased

1. Cutoff Mode Relationships:
   (a) $I_B = I_C = I_E = 0$
2. Active Mode Relationships:
   (a) $I_B = \frac{I_S}{\beta}\left[\exp\left(\frac{eV_{BE}}{kT}\right) - 1\right]\left[1 + \frac{V_{CE}}{V_A}\right]$
   (b) $I_C = \beta I_B = I_S\left[\exp\left(\frac{eV_{BE}}{kT}\right) - 1\right]\left[1 + \frac{V_{CE}}{V_A}\right]$
   (c) $I_E = \frac{1}{\alpha} I_C = (\beta + 1)\, I_B$
   (d) $\alpha = \frac{\beta}{\beta + 1} \iff \beta = \frac{\alpha}{1 - \alpha}$
   (e) In most cases, $V_A \gg V_{CE}$ so the factor is approximately 1
   (f) Model:

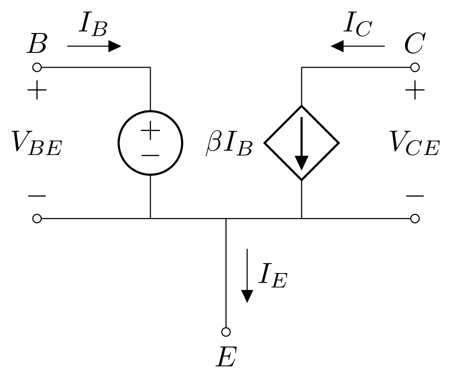


Figure 11: NPN Transistor Active Mode

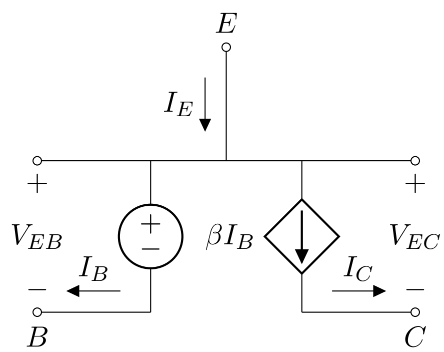


Figure 12: PNP Transistor Active Mode

3. Saturation Mode Relationships
   (a) $I_{C,\,sat} = \beta_{forced} I_B$
   (b) $V_{BE} = 0.7\ V$, $V_{EB} = 0.7\ V$
   (c) $V_{CE} = 0.3\ V$, $V_{EC} = 0.3\ V$ edge of saturation
   (d) deep saturation is normally $0.2\ V$

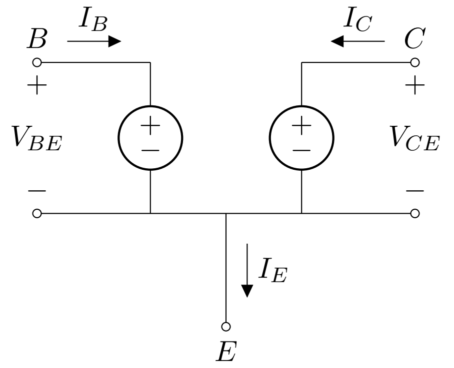


Figure 13: NPN Transistor Saturation Mode

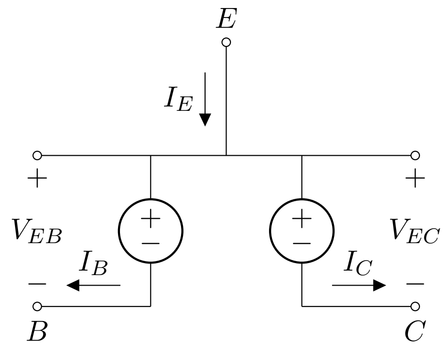


Figure 14: PNP Transistor Saturation

3.2 BJT Biasing Circuits

1. Q point: fixed characteristics of the transistor
   (a) active region for amplifier
   (b) cut-off/saturation for switches
2. Biasing: application of DC voltages to establish the operating point
3. Transistor Parameter variations on Temperature:
   (a) leakage current: $I_{Co}(25°C) = 1\,nA$. doubles every $6°$ rise
   (b) $V_{BE}(25°C) = 0.7\,V$. drops about $2.2\,mV/°C$
   (c) β doubles with an increase of $80°C$

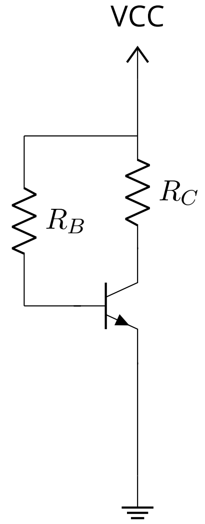


Figure 15: Fixed Bias Circuit

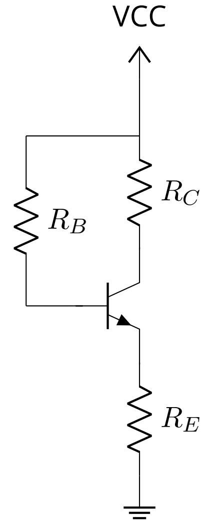


Figure 16: Emitter Stabilized

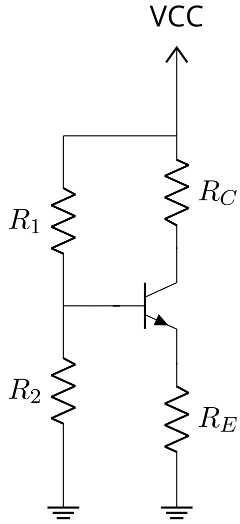


Figure 17: Voltage Divier Bias

3.3 BJT Amplifiers

1. General steps in circuit analysis:
   (a) In the DC part, treat the capacitors as open
   (b) The DC analysis calculates the Q point values
   (c) In the AC part, treat the capacitors and DC voltage sources as shorts
2. Common Emitter Amplifier:
   (a) Inverting amplifier with very huge gains

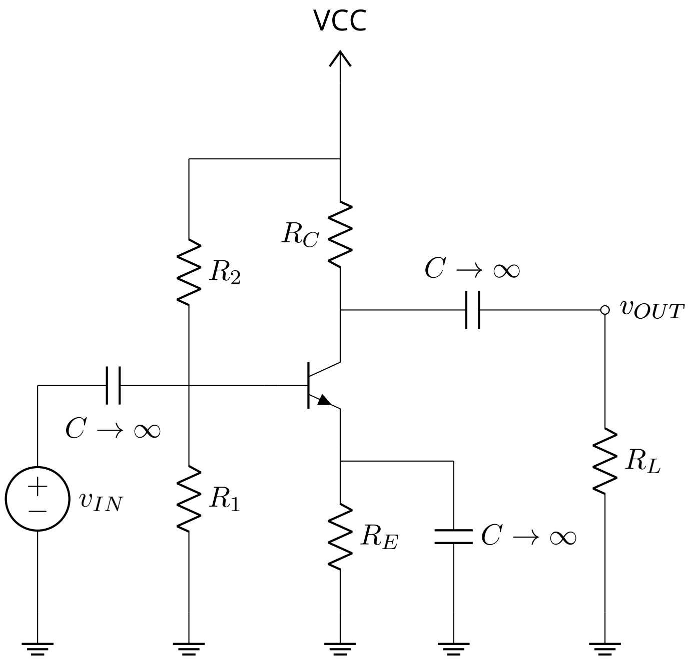
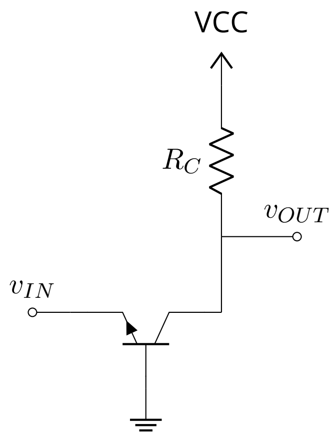
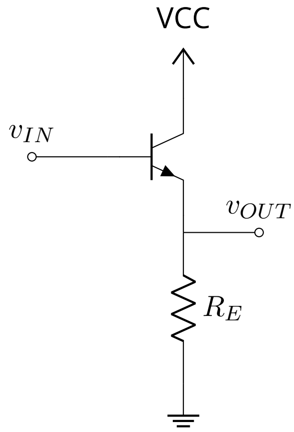


Figure 18: CE Amplifier

3. Common Base Amplifier:
   (a) has a sudden rise of gain in the vicinity of the V_{BE} threshold



4. Common Collector Amplifier:
   (a) has unity gain
   (b) is a voltage follower



4 MOSFETs as Amplifiers

4.1 MOSFET Fundamentals

1. FET: an electric field is applied normal to the surface (gate) of a semiconductor that modulates the conductance of the semiconductor
   (a) NMOS: uses electrons as charge carriers
   (b) PMOS: uses holes as charge carriers
2. The source and drain terminals are heavily doped (n for NMOS, p for PMOS)
   (a) Enhancement Mode: default off
   (b) Depletion Mode: default on
3. Threshold voltage: minimum V_{GS} for the MOSFET to conduct
4. Saturation Voltage: minimum V_{DS} that causes the channel of the transistor to pinch-off in the drain side due to widening depletion region in the drain bulk junction
5. MOSFET Regions of Operation:

Region of Operation	Conditions (NMOS)	Current I_D
Cut-off	$V_{GS} < V_{Th,\,n}$	$I_D = 0$
Linear	$V_{GS} \geq V_{Th,\,n}$ $V_{DS} < V_{GS} - V_{Th,\,n}$	$I_D = k_n \left(V_{GS} - V_{Th,\,n} - \frac{V_{DS}}{2} \right) V_{DS}$
Saturation	$V_{GS} \geq V_{Th,\,n}$ $V_{DS} \geq V_{GS} - V_{Th,\,n}$	$I_D = \frac{1}{2} k_n \left(V_{GS} - V_{Th,\,n} \right)^2$
Region of Operation	Conditions (PMOS)	Current I_D
Cut-off	$V_{GS} < V_{Th,\,n}$	$I_D = 0$
Linear	$V_{SG} \geq \lvert V_{Th,\,p} \rvert$ $V_{SD} < V_{SG} - V_{Th,\,p}$	$I_D = -k_p \left(V_{SG} - V_{Th,\,p} - \frac{V_{SD}}{2} \right) V_{SD}$
Saturation	$V_{SG} \geq V_{Th,\,p}$ $V_{SD} \geq V_{SG} - V_{Th,\,p}$	$I_D = -\frac{1}{2} k_p \left(V_{SG} - V_{Th,\,p} \right)^2$

4.2 MOSFET DC Characterization

1. Steps in solving MOSFET circuits:
   (a) Identify the terminals
   (b) Solve for voltage terminals
   (c) Use the current equation to ge I_D. assume saturation if region of operation is unknown

4.3 MOSFET Amplifier

1. Common Source Amplifier:
   (a) inverting amplifier
   (b) can have large gains

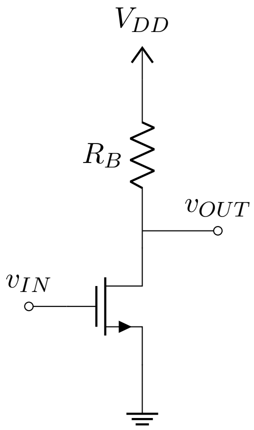


Figure 19: NMOS CS Amplifier

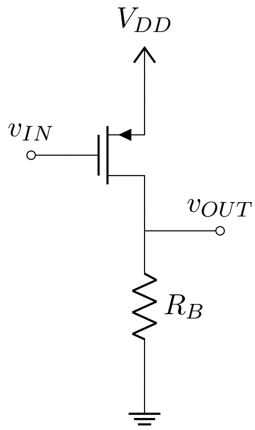


Figure 20: PMOS CS Amplifier

2. Common Drain Amplifier:
   (a) has a unity gain
   (b) non-inverting amplifier

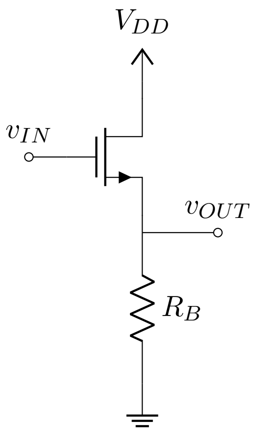


Figure 21: NMOS CD Amplifier

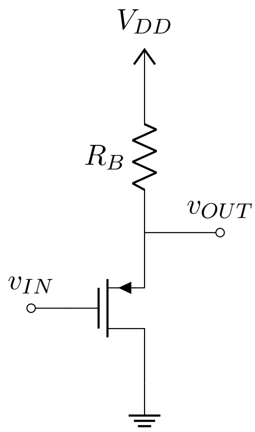


Figure 22: PMOS CD Amplifier

3. Common Gate Amplifier:
   (a) non-inverting amplifier
   (b) can have gain more than 1

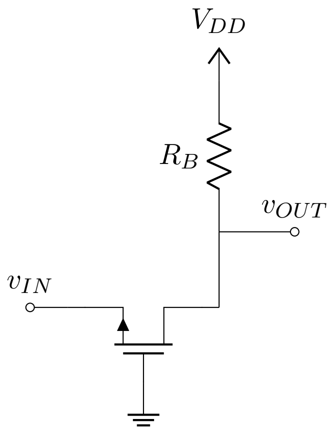


Figure 23: NMOS CG Amplifier

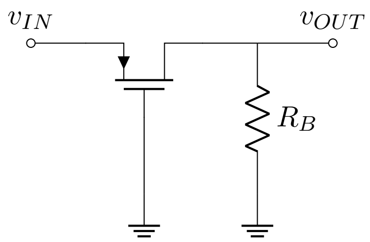


Figure 24: PMOS CG Amplifier

5 Small Signal Analysis of Transistor Amplifiers

5.1 Two Port Networks

1. one port serves as input, the other as output
2. Z parameters: impedance parameters. taken for open circuit of both ports

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{1,\,1} & Z_{1,\,2} \\ Z_{2,\,1} & Z_{2,\,2} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

3. Y parameters: admittance parameters. taken for short circuit of both ports

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{1,\,1} & Y_{1,\,2} \\ Y_{2,\,1} & Y_{2,\,2} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

4. h parameters: hybrid parameters. first column is short circuit of Port 2, second column is open circuit for Port 1

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{1,\,1} & h_{1,\,2} \\ h_{2,\,1} & h_{2,\,2} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

5. g parameters: inverse hybrid parameters. tfirst column is open circuit of Port 2, second column is short circuit for Port 1

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{1,\,1} & g_{1,\,2} \\ g_{2,\,1} & g_{2,\,2} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

6. Unilateral Hybrid π Network: only has 3 parameters
   (a) $R_i,\ R_o,\ A_v$ for Thevenin Equivalent
   (b) $R_i,\ R_o,\ G_m$ for Norton Equivalent

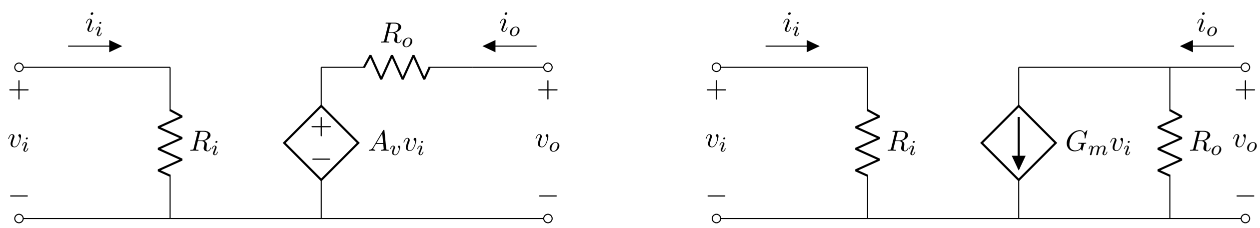


Figure 25: Thevenin Equivalent

Figure 26: Norton Equivalent

Figure 27: Unilateral Hybrid π Network

5.2 Small Signal Model

Parameter	BJT	MOSFET
g_m Transconductance	$g_m = \dfrac{I_C}{V_T}$	$g_m = \dfrac{2I_D}{V_{GS} - V_{Th,\,n}} = \sqrt{2K_n I_D (1 + \lambda V_{DS})} \approx \sqrt{2K_n I_D}$
r_π Input Resistance	$r_\pi = \dfrac{\beta}{g_m} = \dfrac{\beta V_T}{I_C}$	$r_\pi = \infty$
r_o Output Resistance	$r_o = \dfrac{V_A + V_{CE}}{I_C} \approx \dfrac{V_A}{I_C}$	$\dfrac{1}{I_D}\left(\dfrac{1}{\lambda} + V_{DS}\right) \approx \dfrac{1}{\lambda I_D}$

1. the approximations above are from:
   (a) $V_A \gg V_{CE}$
   (b) $\lambda \ll \dfrac{1}{V_{DS}}$

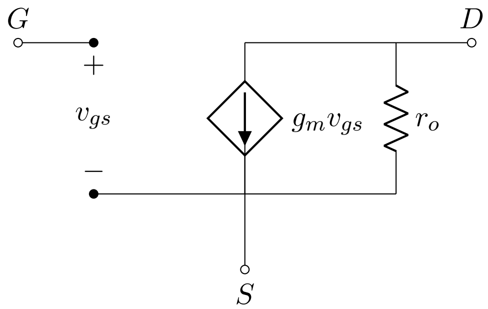


Figure 28: MOSFET Small Signal Model

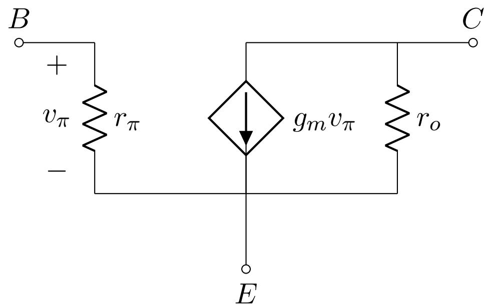


Figure 29: BJT Small Signal Model

5.3 Small Signal Analysis

1. Amplifier Parameters:

Voltage Gain	$A_v = \dfrac{v_o}{v_i}$
Open-loop Gain	$A_{vo} = \left.\dfrac{v_o}{v_i}\right\vert_{R_L \to \infty}$
Input Resistance	$R_i = \left.\dfrac{v_i}{i_i}\right\vert_{i_{out}=0}$
Output Resistance of Amplifier	$R_o = \left.\dfrac{v_o}{i_o}\right\vert_{v_{in}=0}$
Output Resistance of the circuit	$R_{out} = \left.\dfrac{v_o}{i_o}\right\vert_{v_{sig}=0}$
Equivalent Transistor Gain	$G_m = \left.\dfrac{i_o}{v_i}\right\vert_{v_o=0}$
Overall Gain	$A = \dfrac{R_i}{R_i + R_{sig}} A_v$

2. Design considerations:
   (a) $R_i \gg R_{sig}$
   (b) $R_o \ll R_L$
3. Solving Amplifier Circuits
   (a) Find Bias and Signal Circuits
   (b) Get the bias (Q point) values
      i. capacitors are open
      ii. large signal models
   (c) Get the signal paramaters
      i. capacitors are shorts
      ii. independent sources are suppressed
      iii. replace with small signal model
   (d) Terminal Resistance:
      i. Consider that a voltage v_x and current i_x is supplied through the terminal. the terminal resistance is $\frac{v_x}{i_x}$

6 Amplifier Frequency Response

6.1 Introduction to Frequency Response

1. Transfer function:

$$A_v(\omega) = \frac{v_o(\omega)}{v_i(\omega)}$$
$$|A_v| = \sqrt{\mathfrak{Re}(A_v)^2 + \mathfrak{Im}(A_v)^2}$$
$$\angle A_v = \tan^{-1}\left(\frac{\mathfrak{Im}(A_v)}{\mathfrak{Re}(A_v)}\right)$$

2. In phasor analysis: $|v_o| \exp(j\angle v_o) = |A_v| |v_i| \exp(j(\angle A_v + \angle v_o))$
   (a) the -3 dB happens at ω_0
   (b) power $P_v(\omega_0) = \frac{1}{2}$
3. Bode Plots: approximate plots for the magnitude and phase of the transfer function

Term	Magnitude	Phase
Constant	$20\log_{10}\lvert K\rvert$	$K > 0 : 0^\circ$ $K < 0 : \pm 180^\circ$
Real Pole	$\omega < \omega_0 : 0$ dB/dec slope $\omega \geq \omega_0 : -20$ dB/dec slope	$0.1\omega_0 : 0^\circ$ $10\omega_0 : -90^\circ$
Real Zero	$\omega < \omega_0 : 0$ dB/dec slope $\omega \geq \omega_0 : +20$ dB/dec slope	$0.1\omega_0 : 0^\circ$ $10\omega_0 : +90^\circ$
Pole at Origin	-20 dB/dec slope at $(1, 0)$	-90°
Zero at Origin	$+20$ dB/dec slope at $(1, 0)$	$+90^\circ$

   (a) a pole/zero repeated n times multiplies the slopes and $\mp 90^\circ$ by n
   (b) if $\omega_0 < 0$, change the signs of the phase (pole/zero) but not the magnitude

6.2 Transistor Capacitances

1. BJT Parasitic Capacitances

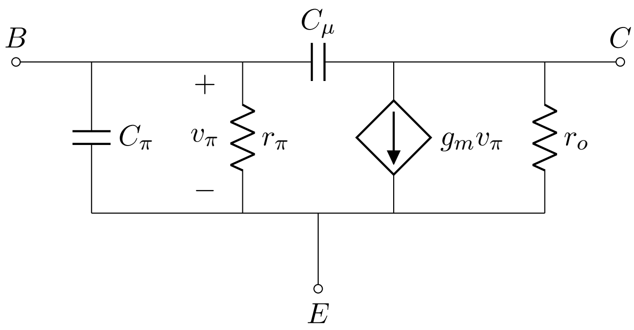


Figure 30: BJT Small Signal Model with Parasitic Capacitances

   (a) $C_\pi = C_b + C_{je}$
   (b) BCJ Capacitance ($5 - 10$ fF): $C_\mu = \dfrac{C_{\mu 0}}{\sqrt{1 + \frac{V_{CB}}{V_{j,\ CB}}}}$
      i. Miller Effect when Common Emitter
   (c) Base-Charging Capacitance (100s fF): $C_b = \tau_F g_m = \tau_F \dfrac{I_C}{V_T}$
      i. forward base transit time τ_F: average time per carrier spent crossing the base
   (d) BEJ Capacitance (10s of fF): $C_{je} = \dfrac{C_{je0}}{\sqrt{1 + \frac{V_{BE}}{V_{j,\ BE}}}} \approx 2C_{je0}$

2. MOSFET Parasitic Capacitances:

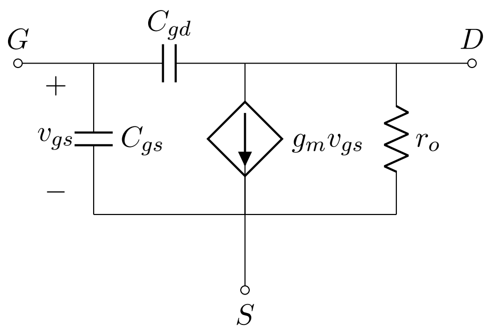


Figure 31: MOSFET Small Signal Model

   (a) GS Capacitance (Parallel Plate + Gate Overlap): C_{gs}
      i. largest parasitic in MOSFET
   (b) GD Capacitace (Gate Overlap only)
      i. Miller Effect when Common Source
   (c) Source/Drain Junction: normally reverse biased and omitted in the small signal model

6.3 Transition Frequency

1. Transition frequency : frequency where $\frac{i_o}{i_i} = 1$ (short circuit output gain). if $i_o < i_i$, the transistor is not working as an amplifier that that frequency
2. at $\omega = 0$, the BJT current gain is β
3. the BJT current transfer function: $\dfrac{i_o(s)}{i_i(s)} = \dfrac{g_m - sC_\mu}{1/r_\pi + s(C_\pi + C_\mu)} = g_m r_\pi \dfrac{1 - s\left(\frac{C_\mu}{g_m}\right)}{1 + s(r_\pi(C_\mu + C_\pi))}$
   (a) $\omega_z = -\dfrac{g_m}{C_\mu}$
   (b) $\omega_p = \dfrac{1}{r_\pi(C_\mu + C_\pi)}$
4. the BJT transition frequency is $f_T = \dfrac{\omega_T}{2\pi} = \dfrac{1}{2\pi}\sqrt{\dfrac{g_m^2 - \left(\frac{1}{r_\pi}\right)^2}{(C_\pi + C_\mu)^2 - C_\mu^2}} \approx \dfrac{1}{2\pi}\dfrac{g_m}{C_\pi - C_\mu}$
5. at $\omega = 0$, the MOSFTET current gain is $+\infty$
6. the MOSFET current transfer function: $\dfrac{i_o}{i_i} = \dfrac{g_m - sC_{gd}}{s(C_{gd} + C_{gs})} = \dfrac{1 - s\left(\frac{C_{gd}}{g_m}\right)}{s\left(\frac{C_{gd}+C_{gs}}{g_m}\right)}$
   (a) $\omega_z = -\dfrac{g_m}{C_{gd}}$
   (b) $\omega_p = \dfrac{g_m}{C_{gd} + C_{gs}}$
7. the MOSFET transition frequency is $f_T = \dfrac{\omega_T}{2\pi} = \dfrac{1}{2\pi}\dfrac{g_m}{\sqrt{(C_{gd} + C_{gs})^2 - C_{gd}^2}} \approx \dfrac{1}{2\pi}\dfrac{g_m}{C_{gd} + C_{gs}}$

6.4 Frequency Response of CE Amplifier

1. $A_v = \left.\dfrac{v_o}{v_i}\right|_{i_o=0}$
2. $G_m = \left.\dfrac{i_o}{v_i}\right|_{v_o=0}$
3. $Z_i = \left.\dfrac{v_i}{i_i}\right|_{v_o=0}$
4. $Z_o = \left.\dfrac{v_o}{i_o}\right|_{v_i=0}$
5. Single Pole CE Amplifier:
   (a) $A_v = (-g_m R_C)\dfrac{1}{1 + s(R_C C_L)}$
   (b) $Z_i = r_\pi$
   (c) $Z_o = \dfrac{R_C}{1 + s(R_C C_L)}$

6. Considering the input resistance and input capacitance

   (a) $A_v = \left(-g_m R_C \dfrac{r_\pi}{r_\pi + R_S}\right) \dfrac{1}{\left(1 + s\left(\frac{R_s r_\pi C_\pi}{R_s + r_\pi}\right)\right)}$

   (b) $Z_i = (R_s + r_\pi) \dfrac{1 + s\left(\frac{R_s r_\pi C_\pi}{R_s + r_\pi}\right)}{1 + s\,(r_\pi C_\pi)}$

   (c) $Z_o = (R_C) \dfrac{1}{1 + s\,(R_C C_L)}$

7. Considering the input resistance and Miller capacitance

   (a) $A_v \approx \left(-g_m R_C \dfrac{r_\pi}{r_\pi + R_S}\right) \dfrac{1 - s\left(\frac{C_\mu}{g_m}\right)}{[1 + s\,(R\,[C_L + C_\mu])]\left[1 + s\left(\frac{R_s r_\pi C_\mu (1 + g_m R_C)}{R_s + r_\pi}\right)\right]}$

6.5 Gain Bandwidth Product

1. Single Pole Amplifier: $A(\omega) = \dfrac{A_o}{1 + j\frac{\omega}{\omega_p}}$

   (a) $\omega_u = \omega_p \sqrt{A_o^2 - 1} \approx A_o \omega_p$

   (b) for all $\omega_x \in [\omega_p,\ \omega_u]\,,\ A_x \omega_x = A_o \omega_p = GBP$

2. Two Pole Amplifer: $A(\omega) = \dfrac{A_o}{\left[1 + j\frac{\omega}{\omega_{p1}}\right]\left[1 + j\frac{\omega}{\omega_{p2}}\right]}$

   (a) case 1: $\omega_u > \omega_{p2} : \omega_u = A_o \omega_{p1}$

   (b) case 2: $\omega_u < \omega_{p2} : \omega_u = \sqrt{A_o \omega_{p1} \omega_{p2}}$