

# EEE 131 Key concepts and Equations

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## 1 Diode Models and Circuits

### 1.1 History of Electronics

1. 1904: John Fleming invented vacuum tubes. First time electron flow is controlled in non-conducting medium
  - (a) if the cathode is heated, some electrons can escape
  - (b) if a voltage is applied (+) on anode (–) on cathode, there will be current
2. 1906: the grid was added to control the current between the anode and cathode
  - (a) vacuum tubes allowed the development of amplifiers, transmitters, receivers, signal processor
3. 1946: the ENIAC computer was completed. it could execute 5000 additions per second
4. 1947: invention of transistor by Shockley, Bardeen, Brattain at the Bell Labs
  - (a) they were point-contact transistors
5. 1948: Schockley invented the BJT. the BJT is monolithic (containted in a single semiconductor crystal)
6. 1958: the first Integrated Circuits (IC) was invented by Jack Kilby of Texas Instruments (non-monolithic)
7. 1959: the first monolithic IC was developed by Robert Noyce at Fairchild Semiconduction
  - (a) uses silicon instead of germanium
  - (b) 2 interconnected BJT transistor
  - (c)  $SiO_2$  insulator,  $Al$  interconnection
  - (d) planar IC, 0.06 in diameter
8. Bipolar ICs with BJT devices domidated until early 80s
  - (a) S/M/L Scale Integration : 10, 100, 1000 transistors
9. 1959: MOS ICs started in the 60s after the MOS transistor was developoed at Bell Labs
  - (a) easier to fabricate than bipolar
  - (b) uses less power
  - (c) can fit more transistor in the same silicon area
  - (d) slower than bipolar IC
  - (e) less robust than bipolar IC

10. MOS ICs

- (a) early 70s: MOS technology improved in speed and reliability
- (b) first microprocessor: Intel 4004 (1971)
- (c) emergence of VLSI (>10000 transistors)
- (d) microprocessors evolved into microcontrollers (MCUs) and system on a chip (SOCs)

11. Moore's Law: the transistor density would double every two years

## 1.2 Piecewise Linear Diode Models

1. 1st Approximation: (when  $v_s \gg V_T$ )

- (a) Open when  $v_D < 0$
- (b) Short when  $v_D \geq 0$

2. 2nd Approximation (when above condition not satisfied):

- (a) Open when  $v_D < V_T$
- (b) Voltage of  $V_T$  when  $v_D \geq V_T$

3. 3rd Approximation (2nd with resistor  $R$  in series)

- (a) Open when  $v_D < V_T$
- (b) Voltage of  $V_T$  in series with resistance  $R$  when  $v_D \geq V_T$

4. To determine diode state:

- (a) Assume it is conducting
- (b) Check direction of current
- (c) If current is from anode to cathode, the assumption is correct and equivalent circuit is valid
- (d) Otherwise, diode should be open

5. Another method:

- (a) Replace diode with open circuit
- (b) Determine the voltage across the diode terminals (+) on anode, (−) on cathode
- (c) If  $v_D < \text{the needed threshold}$ , assumption is correct
- (d) Otherwise, diode should be replaced with the appropriate model

### 1.3 Practical Diode Circuits

#### 1. Half Wave Rectifier

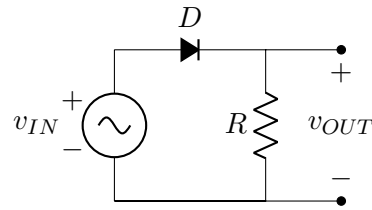


Figure 1: Half Wave Rectifier

(a) When diode is IDEAL:

$$v_{OUT} = \begin{cases} 0, & v_{IN} < 0 \\ v_{IN} & v_{IN} \geq 0 \end{cases}$$

(b) When diode has constant voltage model:

$$v_{OUT} = \begin{cases} 0, & v_{IN} < V_T \\ v_{IN} - V_T & v_{IN} \geq V_T \end{cases}$$

#### 2. Full Wave Rectifier ( $v_1 = v_2$ )

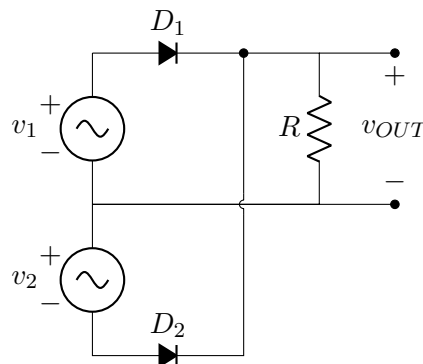


Figure 2: Full Wave Rectifier

(a) When diode is IDEAL:

$$v_{OUT} = |v_{IN}|$$

(b) When diode has constant voltage model:

$$v_{OUT} = \begin{cases} v_1 - V_T, & v_1 > V_T \\ -v_2 - V_T, & v_2 < -V_T \\ 0 & \text{otherwise} \end{cases}$$

(c) This requires a transformer with center-tapped secondary ( $n_1 : n_2 = 1 : 2$ )

### 3. Full Wave Bridge Rectifier

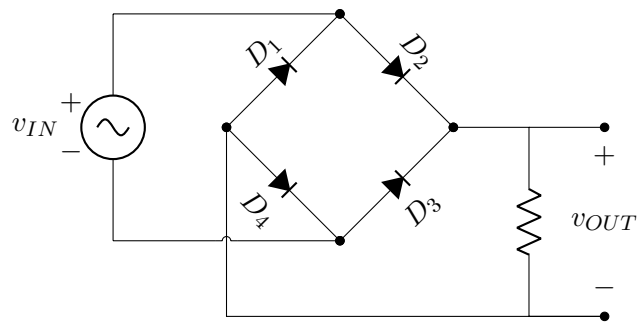


Figure 3: Full Wave Bridge Rectifier

(a) When diode is IDEAL:

$$v_{OUT} = |v_{IN}|$$

(b) When diode has constant voltage model:

$$v_{OUT} = \begin{cases} v_1 - 2V_T, & v_1 > 2V_T \\ -v_2 - 2V_T, & v_2 < -2V_T \\ 0 & \text{otherwise} \end{cases}$$

### 4. Positive Clipper

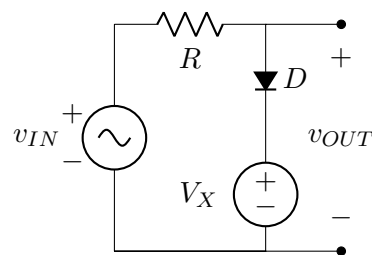


Figure 4: Positive Clipper

$$(a) \ v_{OUT} = \begin{cases} V_X + V_T, & v_{IN} - V_X > V_T \\ v_{IN}, & v_{IN} - V_X < V_T \end{cases}$$

### 5. Negative Clipper

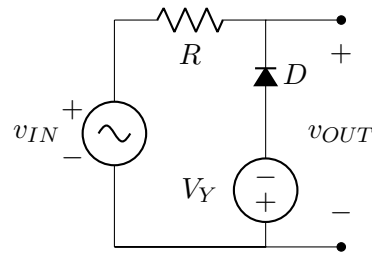


Figure 5: Negative Clipper

$$(a) v_{OUT} = \begin{cases} -(V_Y + V_T), & -V_Y - v_{IN} > V_T \\ v_{IN}, & -V_Y - v_{IN} < V_T \end{cases}$$

### 6. The positive and negative clippers can be combined

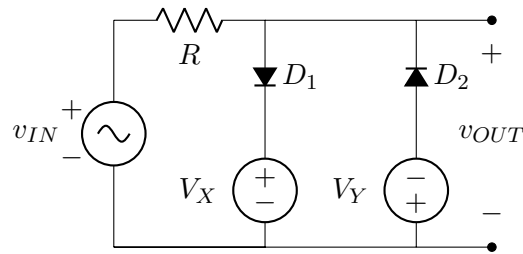


Figure 6: Positive and Negative Clipper

$$(a) v_{OUT} = \begin{cases} V_X + V_T, & v_{IN} - V_X > V_T \\ -(V_Y + V_T), & -V_Y - v_{IN} > V_T \\ v_{IN}, & \text{otherwise} \end{cases}$$

### 7. Peak Detector

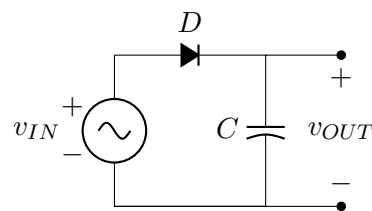


Figure 7: Peak Detector

(a) Let the time  $t_1$  be the first time the input reaches its maximum value  $V_P$  and  $t_0$  be the time to reach  $V_T$

(b) When diode is IDEAL:

$$v_{OUT} = \begin{cases} v_{IN}, & t < t_1 \\ V_P, & t \geq t_1 \end{cases}$$

(c) When diode has constant voltage model:

$$v_{OUT} = \begin{cases} 0, & 0 \leq t < t_0 \\ v_{IN} - V_T, & t_0 \leq t < t_1 \\ V_P - V_T & t_1 \leq t \end{cases}$$

### 8. Peak Detector with Load Resistor (Ideal Diode)

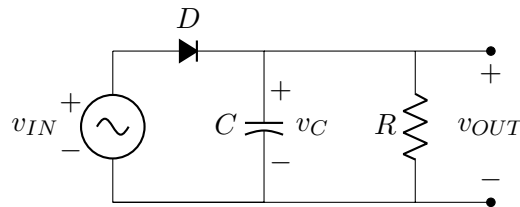


Figure 8: Peak Detector with load resistor

(a) Let the time  $t_1$  be the first time the input reaches its maximum value  $V_P$  and the period be  $T$ . When the capacitor voltage decays, let time  $t_2$  be when  $v_C = v_{IN}$  again

$$(b) v_{OUT} = \begin{cases} v_{IN}, & t < t_1 \\ V_P \exp\left(-\frac{t - t_1}{RC}\right), & t_1 \leq t < t_2 \\ v_{IN}, & t_2 \leq t < t_1 + T \\ V_P \exp\left(-\frac{t - (t_1 + T)}{RC}\right), & t_1 + T \leq t < t_2 + T \\ \vdots & \end{cases}$$

### 9. Negative Clamper (ideal diode)

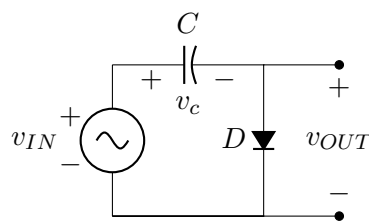


Figure 9: Negative Clamper

(a) Let the time  $t_1$  be the first time the input reaches its maximum value  $V_P$

(b) This is similar to the peak detector, only that  $v_{OUT} = v_{IN} - v_C$  where  $v_C$  follows the characteristic of the peak detector

$$(c) v_C = \begin{cases} v_{IN}, & t < t_1 \\ V_P, & t_1 \leq t \end{cases}$$

$$(d) v_{OUT} = \begin{cases} 0, & t < t_1 \\ v_{IN} - V_P, & t \geq t_1 \end{cases}$$

#### 10. Positive Clamper (ideal diode)

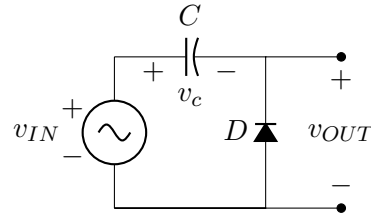


Figure 10: Positive Clamper

(a) Let the period be  $T$

$$(b) v_C = \begin{cases} 0, & 0 \leq t < T/2 \\ v_{IN}, & T/2 \leq t < 3T/4 \\ -V_P, & 3T/4 \leq t \end{cases}$$

$$(c) v_{OUT} = \begin{cases} v_{IN}, & 0 \leq t < T/2 \\ 0, & T/2 \leq t < 3T/4 \\ v_{IN} + V_P, & 3T/4 \leq t \end{cases}$$

### 1.4 Exponential Diode Model

#### 1. Shockley's Diode Equation

$$i_D = I_S \left[ \exp \left( \frac{v_D}{\eta V_T} \right) - 1 \right] \quad (1)$$

(a)  $v_D$ : diode voltage with positive at anode

(b)  $i_D$ : diode current from anode to cathode

(c)  $I_S$ : reverse saturation current due to minority carriers.  $10^{-15} \text{A} < I_S < 10^{-9} \text{A}$

(d)  $\eta$ : ideality factor,  $1 < \eta < 2$ ; close to 1 for well fabricated diodes

(e)  $V_T$ : thermal voltage,  $V_T = \frac{kT_K}{e} \approx \frac{T_K}{11600}$

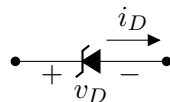
2. for typical operating voltages (forward biased):  $\exp \left( \frac{v_D}{\eta V_T} \right) \gg 1 \Rightarrow i_D \approx I_S \exp \left( \frac{v_D}{\eta V_T} \right)$

3. for typical reverse bias voltages,  $\exp \left( \frac{v_D}{\eta V_T} \right) \ll 1 \Rightarrow i_D \approx -I_S$

## 1.5 Small Signal Model

1.  $v_D = V_D + v_d$
2.  $i_D = I_D + i_d$ .  $I_D$  is calculated using large signal analysis
3. if the amplitude of  $v_d$  is very small, the diode characteristic equation is approximately a line with conductance  $g_d = \frac{\partial i_D}{\partial v_D} = \frac{I_S}{\eta V_T} \exp\left(\frac{v_D}{\eta V_T}\right) \approx \frac{I_D}{\eta V_T} \implies r_d = \frac{\eta V_T}{I_D}$

## 1.6 Zener Diode



1. has 3 modes of operations:

Operation	Condition	Equivalent
Forward-biased	$v_D \leq V_T$	$v_D = -V_T$
Non-conducting	$-V_T < v_D < V_Z$	open circuit
Zener region	$v_D \geq V_Z$	$v_D = V_Z$



## 2 PN Junction and BJT Operation

### 2.1 Semiconductor Fundamentals

1. The Silicon (14) Atom:
  - (a) has 4 valence electrons
  - (b) the semiconductor lattice has bounded valence electrons at 0K
  - (c) energy bands: electrons can occupy discrete energy levels
  - (d) there is a **band gap** between the valence band and conduction band
  - (e) at higher temperatures, an electron can gain enough energy to break the bonds
  - (f) when an electron leaves a bond, it leaves a positively charged **hole**
2. Groups by electrical properties:
  - (a) insulators: huge band gap
  - (b) conductor: conduction band and valence overlap
  - (c) semiconductor: small band gap that can be easily reached
3. Semiconductors:
  - (a) narrow band gap
  - (b) has 2 groups:
    - i. intrinsic: just enough valence electrons to complete the matrix. either has group IV semiconductors (Si and Ge) or III-V compound (GaAs)
    - ii. extrinsic: produced by doping intrinsic semiconductors
      - A. N-type: dopants have excess electrons. energy level just below the conduction band
      - B. P-type: dopants have less electrons. energy level just above the valence
4. Fermi level: hypothetical energy level where there is 50% probability that the level is occupied by an electron at any given time
  - (a) intrinsic semiconductor: midway between  $E_C$  and  $E_V$
  - (b) extrinsic semiconductor:
    - i. N-type: closer to  $E_C$
    - ii. P-type: closer to  $E_V$

### 2.2 Carrier Actions in Semiconductors

1. Charge Carriers: the electrons and holes. units in coulombs (C)
2. Intrinsic semiconductor:  $n = p$  equal concentration of electrons and holes
3. N-type semiconductors:
  - (a)  $n > p$
  - (b) majority charge carriers: electrons
  - (c) minority charge carriers: holes
4. P-type semiconductors:
  - (a)  $p > n$

- (b) majority charge carriers: holes
- (c) minority charge carriers: electrons
- 5. Current: rate of movement of charge past a point or region. units in ampere (A). + is the direction of the hole/opposite the electron
- 6. Mobility: ability of charge carrier to move.  $\mu_e > \mu_h$
- 7. Factors that affect mobility:
  - (a) Scattering - the random motion of charges. One mechanism is the *lattice scattering* when electrons bump with the vibrating lattice or other electrons and change directions
  - (b) another scattering mechanism is *impurity scattering* when dopant ions deflect charge carriers
  - (c) Temperature vs Scattering vs Mobility. At low temperatures, impurity scattering dominates (and with low mobility). at high temperatures, lattice scattering dominates (and low mobility). there is a temperature in the middle with maximum mobility. (like a parabolic log-log graph)
  - (d) Doping vs Scattering vs Mobility. more doping = more impurity scattering
  - (e) scattering has a net current of 0
- 8. There are 3 primary carrier actions:
  - (a) Generation and Recombination: generation = electron gains enough energy to break a covalent bond and leaves a hole. recombination = moving electron moves close to a hole and experience attractive force
  - (b) Diffusion: electrons move from higher concentration/temperature to lower concentration/temperature. + diffusion is parallel to positive charges. becomes 0 when there is uniform distribution of charge carriers
  - (c) Drift: caused by applied external voltage. modeled by a constant drift velocity  $v_d = \mu E$  (directly related to the electric field with the mobility as constant)
    - i.  $I_n = -Aqnv_{n\text{-drift}}$
    - ii.  $I_p = Aqp v_{p\text{-drift}}$

### 2.2.1 PN Junction

- 1. PN Junction: formed when in a single silicon crystal, one region is doped with donor dopants (N-type) and the other with acceptor dopants (P-type). PN Junction is the boundary.
- 2. Depletion Region: happens at the boundary when the electrons diffuse and recombine with the holes at the P-type region. This results to a potential difference (and thus an electric field) across the depletion region (points from N to P)
  - (a) drift current of minority carriers are drifted by this electric field
- 3. There are 3 junction operations:
  - (a) Open-circuit (equilibrium condition):
    - i. the diffusion current  $I_D$  is limited by the built in voltage  $V_0$  and equal to the drift current  $I_S$
  - (b) Forward Bias (+ on the P-type)
    - i. the + potential pushes the holes in the P-type and the depletion region in the P-side decreases
    - ii. the - potential pushes the electrons in the N-type and the depletion region in this side also decreases
    - iii. the diffusion current  $I_D$  dominates, but the temperature dependent  $I_S$  remain the same

- iv. the net current  $I = I_D - I_S$  in the direction from P to N
- v. when  $V_F \geq V_o$ , the depletion region disappears and  $I \approx I_D$
- (c) Reverse Bias (+ on the N-type)
  - i. the + potential attracts the electrons in the N-type, the depletion region becomes larger
  - ii. the – potential attracts the holes in the P-type, the depletion region becomes larger
  - iii. the voltage  $V_o$  increases so  $I_D$  decreases
  - iv. net current  $I = I_D - I_S$  from N to P
  - v. if  $I_D$  becomes very small,  $I \approx -I_S$
- 4. The energy band diagram will align the Fermi levels, thus bending the  $E_C$  and  $E_V$  with the maximum deviation  $V_0$ 
  - (a) at forward bias,  $V_0$  decreases so the bending energy will also decrease and the Fermi level will not be aligned anymore ( $E_{FN}$  is higher)
  - (b) at reverse bias,  $V_0$  increases so the bending energy will also increase and the Fermi level will not be aligned anymore ( $E_{FN}$  is lower)
- 5. The PN Junction implements a Diode. anode at P, cathode at N
  - (a) the Shockley Equation summarizes the 3 conditions for the PN Junction operations
  - (b) Diode resistivity:
    - i. at higher temperature, more electrons are freed and resistivity decreases
    - ii. higher doping density lowers the resistivity

### 3 BJTs as Amplifiers

#### 3.1 DC Characteristics and Operating Modes

Mode of Operation	EB Junction	CB Junction
Cut-off	Reverse-biased	Reverse-biased
Active	Forward-biased	Reverse-biased
Saturation	Forward-biased	Forward-biased
Reverse Active	Forward-biased	Reverse-biased

##### 1. Cutoff Mode Relationships:

$$(a) I_B = I_C = I_E = 0$$

##### 2. Active Mode Relationships:

$$(a) I_B = \frac{I_S}{\beta} \left[ \exp \left( \frac{eV_{BE}}{kT} \right) - 1 \right] \left[ 1 + \frac{V_{CE}}{V_A} \right]$$

$$(b) I_C = \beta I_B = I_S \left[ \exp \left( \frac{eV_{BE}}{kT} \right) - 1 \right] \left[ 1 + \frac{V_{CE}}{V_A} \right]$$

$$(c) I_E = \frac{1}{\alpha} I_C = (\beta + 1) I_B$$

$$(d) \alpha = \frac{\beta}{\beta + 1} \iff \beta = \frac{\alpha}{1 - \alpha}$$

(e) In most cases,  $V_A \gg V_{CE}$  so the factor is approximately 1

(f) Model:

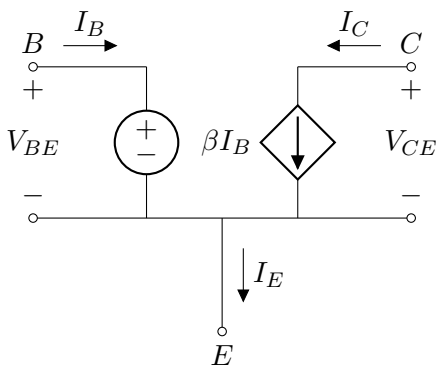


Figure 11: NPN Transistor Active Mode

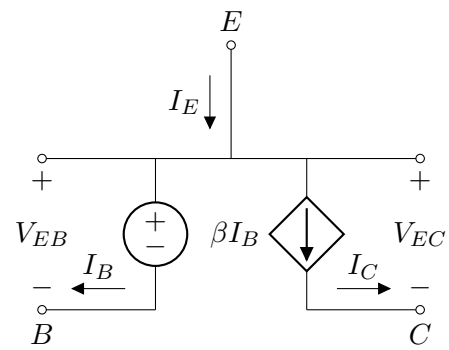


Figure 12: PNP Transistor Active Mode

##### 3. Saturation Mode Relationships

$$(a) I_{C, sat} = \beta_{forced} I_B$$

$$(b) V_{BE} = 0.7 \text{ V}, V_{EB} = 0.7 \text{ V}$$

$$(c) V_{CE} = 0.3 \text{ V}, V_{EC} = 0.3 \text{ V} \text{ edge of saturation}$$

$$(d) \text{ deep saturation is normally } 0.2 \text{ V}$$

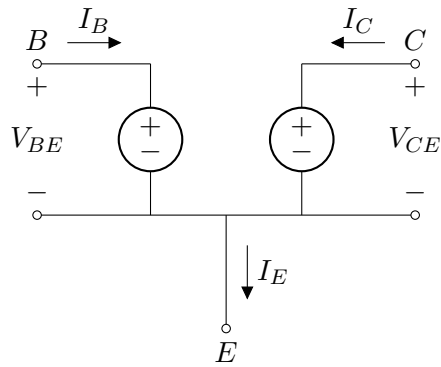


Figure 13: NPN Transistor Saturation Mode

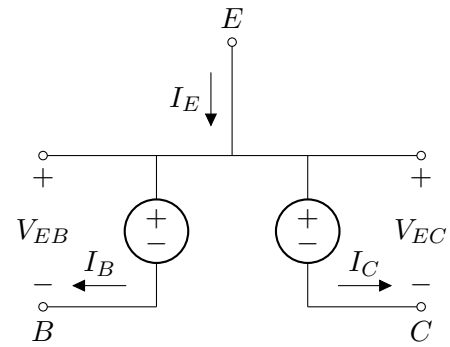


Figure 14: PNP Transistor Saturation

### 3.2 BJT Biasing Circuits

1. Q point: fixed characteristics of the transistor
  - (a) active region for amplifier
  - (b) cut-off/saturation for switches
2. Biasing: application of DC voltages to establish the operating point
3. Transistor Parameter variations on Temperature:
  - (a) leakage current:  $I_{C_o}(25^\circ C) = 1 \text{ nA}$ . doubles every  $6^\circ$  rise
  - (b)  $V_{BE}(25^\circ C) = 0.7 \text{ V}$ . drops about  $2.2 \text{ mV}/^\circ C$
  - (c)  $\beta$  doubles with an increase of  $80^\circ C$

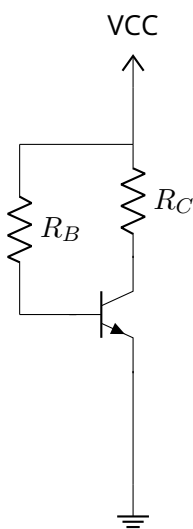


Figure 15: Fixed Bias Circuit

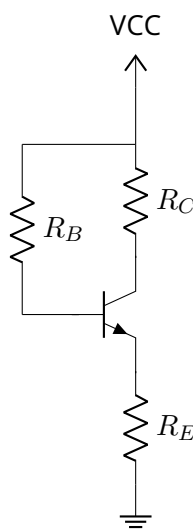


Figure 16: Emitter Stabilized

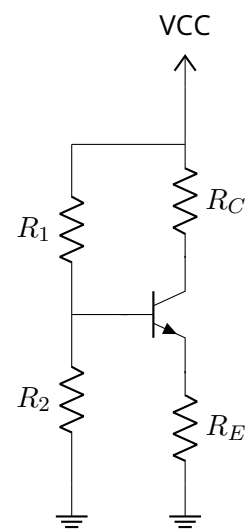


Figure 17: Voltage Divider Bias

### 3.3 BJT Amplifiers

1. General steps in circuit analysis:

- (a) In the DC part, treat the capacitors as open
- (b) The DC analysis calculates the Q point values
- (c) In the AC part, treat the capacitors and DC voltage sources as shorts

2. Common Emitter Amplifier:

- (a) Inverting amplifier with very huge gains

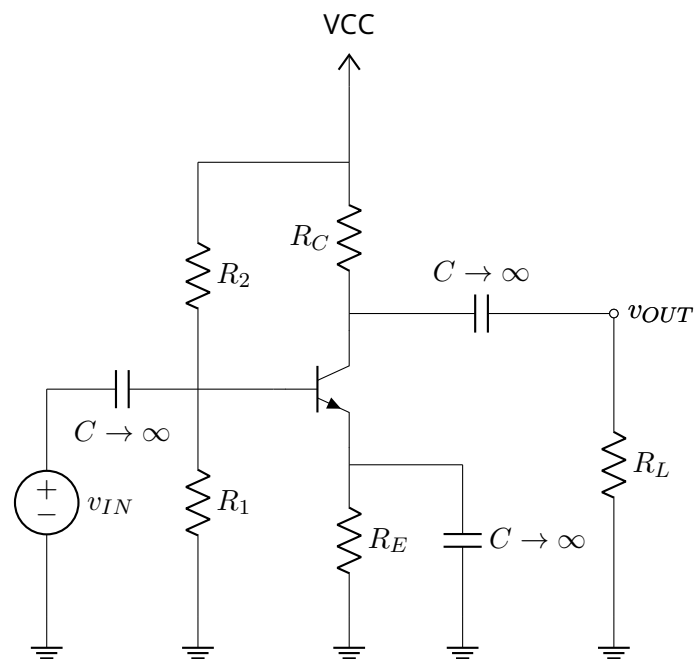
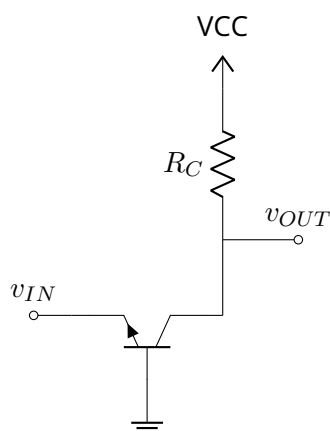


Figure 18: CE Amplifier

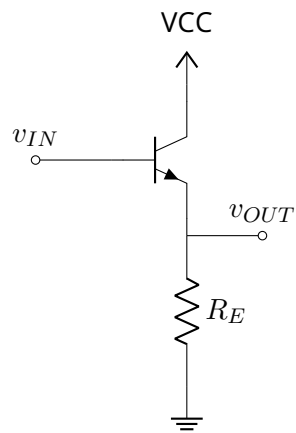
3. Common Base Amplifier:

- (a) has a sudden rise of gain in the vicinity of the  $V_{BE}$  threshold



4. Common Collector Amplifier:

- (a) has unity gain
- (b) is a voltage follower



## 4 MOSFETs as Amplifiers

### 4.1 MOSFET Fundamentals

1. FET: an electric field is applied normal to the surface (gate) of a semiconductor that modulates the conductance of the semiconductor
  - (a) NMOS: uses electrons as charge carriers
  - (b) PMOS: uses holes as charge carriers
2. The source and drain terminals are heavily doped (n for NMOS, p for PMOS)
  - (a) Enhancement Mode: default off
  - (b) Depletion Mode: default on
3. Threshold voltage: minimum  $V_{GS}$  for the MOSFET to conduct
4. Saturation Voltage: minimum  $V_{DS}$  that causes the channel of the transistor to pinch-off in the drain side due to widening depletion region in the drain bulk junction
5. MOSFET Regions of Operation:

Region of Operation	Conditions (NMOS)	Current $I_D$
Cut-off	$V_{GS} < V_{Th, n}$	$I_D = 0$
Linear	$V_{GS} \geq V_{Th, n}$ $V_{DS} < V_{GS} - V_{Th, n}$	$I_D = k_n \left( V_{GS} - V_{Th, n} - \frac{V_{DS}}{2} \right) V_{DS}$
Saturation	$V_{GS} \geq V_{Th, n}$ $V_{DS} \geq V_{GS} - V_{Th, n}$	$I_D = \frac{1}{2} k_n (V_{GS} - V_{Th, n})^2$
Region of Operation	Conditions (PMOS)	Current $I_D$
Cut-off	$V_{GS} < V_{Th, n}$	$I_D = 0$
Linear	$V_{SG} \geq  V_{Th, p} $ $V_{SD} < V_{SG} - V_{Th, p}$	$I_D = -k_p \left( V_{SG} - V_{Th, p} - \frac{V_{SD}}{2} \right) V_{SD}$
Saturation	$V_{SG} \geq V_{Th, p}$ $V_{SD} \geq V_{SG} - V_{Th, p}$	$I_D = -\frac{1}{2} k_p (V_{SG} - V_{Th, p})^2$

### 4.2 MOSFET DC Characterization

1. Steps in solving MOSFET circuits:
  - (a) Identify the terminals
  - (b) Solve for voltage terminals
  - (c) Use the current equation to get  $I_D$ . assume saturation if region of operation is unknown



### 4.3 MOSFET Amplifier

#### 1. Common Source Amplifier:

- (a) inverting amplifier
- (b) can have large gains

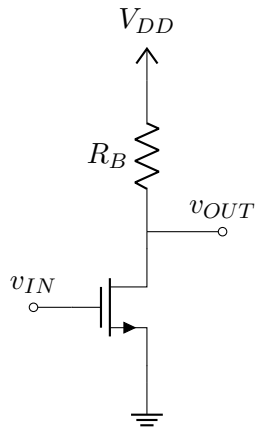


Figure 19: NMOS CS Amplifier

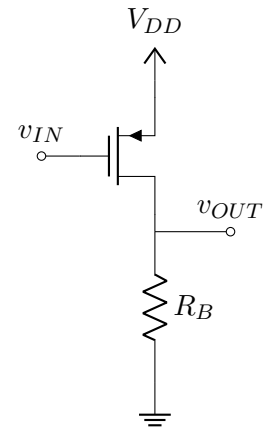


Figure 20: PMOS CS Amplifier

#### 2. Common Drain Amplifier:

- (a) has a unity gain
- (b) non-inverting amplifier

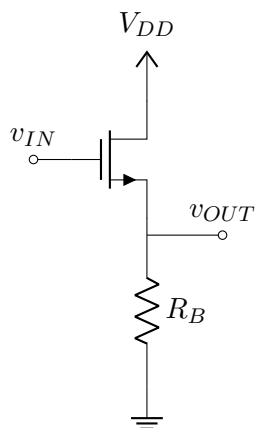


Figure 21: NMOS CD Amplifier

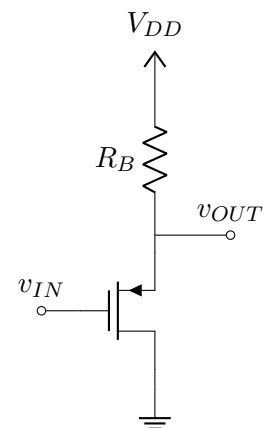


Figure 22: PMOS CD Amplifier

### 3. Common Gate Amplifier:

- (a) non-inverting amplifier
- (b) can have gain more than 1

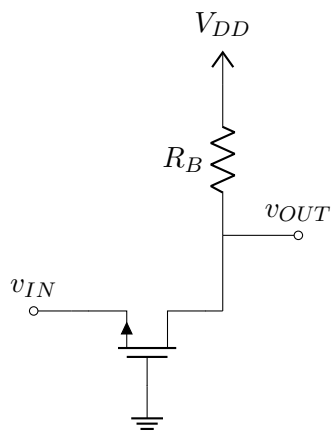


Figure 23: NMOS CG Amplifier

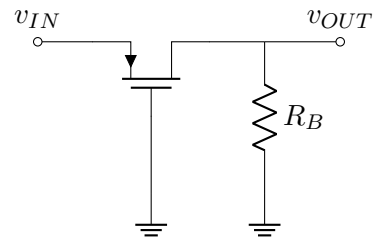


Figure 24: PMOS CG Amplifier

## 5 Small Signal Analysis of Transistor Amplifiers

### 5.1 Two Port Networks

1. one port serves as input, the other as output
2.  $Z$  parameters: impedance parameters. taken for open circuit of both ports

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{1,1} & Z_{1,2} \\ Z_{2,1} & Z_{2,2} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

3.  $Y$  parameters: admittance parameters. taken for short circuit of both ports

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{1,1} & Y_{1,2} \\ Y_{2,1} & Y_{2,2} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

4.  $h$  parameters: hybrid parameters. first column is short circuit of Port 2, second column is open circuit for Port 1

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

5.  $g$  parameters: inverse hybrid parameters. first column is open circuit of Port 2, second column is short circuit for Port 1

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{1,1} & g_{1,2} \\ g_{2,1} & g_{2,2} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

6. Unilateral Hybrid  $\pi$  Network: only has 3 parameters

(a)  $R_i$ ,  $R_o$ ,  $A_v$  for Thevenin Equivalent

(b)  $R_i$ ,  $R_o$ ,  $G_m$  for Norton Equivalent

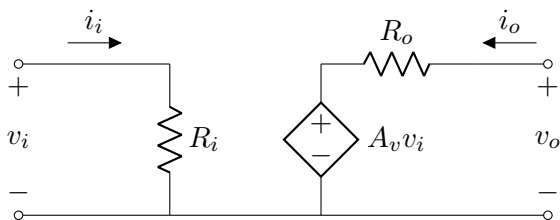


Figure 25: Thevenin Equivalent

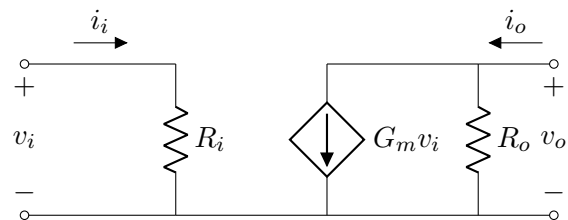


Figure 26: Norton Equivalent

Figure 27: Unilateral Hybrid  $\pi$  Network

## 5.2 Small Signal Model

Parameter	BJT	MOSFET
$g_m$ Transconductance	$g_m = \frac{I_C}{V_T}$	$g_m = \frac{2I_D}{V_{GS} - V_{Th,n}} = \sqrt{2K_n I_D (1 + \lambda V_{DS})} \approx \sqrt{2K_n I_D}$
$r_\pi$ Input Resistance	$r_\pi = \frac{\beta}{g_m} = \frac{\beta V_T}{I_C}$	$r_\pi = \infty$
$r_o$ Output Resistance	$r_o = \frac{V_A + V_{CE}}{I_C} \approx \frac{V_A}{I_C}$	$\frac{1}{I_D} \left( \frac{1}{\lambda} + V_{DS} \right) \approx \frac{1}{\lambda I_D}$

1. the approximations above are from:

(a)  $V_A \gg V_{CE}$

(b)  $\lambda \ll \frac{1}{V_{DS}}$

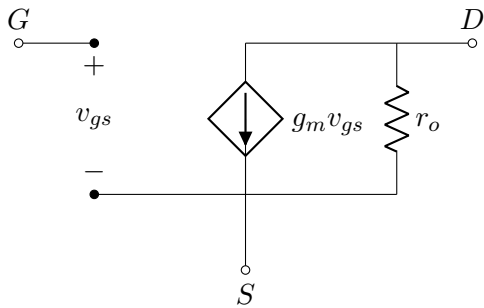


Figure 28: MOSFET Small Signal Model

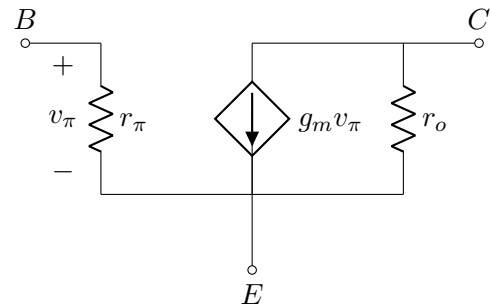


Figure 29: BJT Small Signal Model

## 5.3 Small Signal Analysis

1. Amplifier Parameters:

Voltage Gain	$A_v = \frac{v_o}{v_i}$
Open-loop Gain	$A_{vo} = \frac{v_o}{v_i} \Big _{R_L \rightarrow \infty}$
Input Resistance	$R_i = \frac{v_i}{i_i}$
Output Resistance of Amplifier	$R_o = \frac{v_o}{i_o} \Big _{v_i \rightarrow 0}$
Output Resistance of the circuit	$R_{out} = \frac{v_o}{i_o} \Big _{v_{sig} \rightarrow 0}$
Overall Gain	$A = \frac{R_i}{R_i + R_{sig}} A_v$

2. Design considerations:

(a)  $R_i \gg R_{sig}$

(b)  $R_o \ll R_L$

### 3. Solving Amplifier Circuits

- (a) Find Bias and Signal Circuits
- (b) Get the bias (Q point) values
  - i. capacitors are open
  - ii. large signal models
- (c) Get the signal parameters
  - i. capacitors are shorts
  - ii. independent sources are suppressed
  - iii. replace with small signal model
- (d) Terminal Resistance:
  - i. Consider that a voltage  $v_x$  and current  $i_x$  is supplied through the terminal. the terminal resistance is  $\frac{v_x}{i_x}$

## 6 Amplifier Frequency Response