## **EEE 133 Key concepts and Equations**

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#### 1 Diode Models and Circuits

### 1.1 History of Electronics

- 1. 1904: John Fleming invented vacuum tubes. First time electron flow is controlled in non-conducting medium
  - (a) if the cathode is heated, some electrons can escape
  - (b) if a voltage is applied (+) on anode (-) on cathode, there will be current
- 2. 1906: the grid was added to control the current between the anode and cathode
  - (a) vacuum tubes allowed the development of amplifiers, transmitters, receivers, signal processor
- 3. 1946: the ENIAC computer was completed. it could execute 5000 additions per second
- 4. 1947: invention of transistor by Shockley, Bardeen, Brattain at the Bell Labs
  - (a) they were point-contact transistors
- 5. 1948: Schockley invented the BJT. the BJT is monolithic (containted in a single semiconductor crystal)
- 6. 1958: the first Integrated Circuits (IC) was invented by Jack Kilby of Texas Instruments (non-monolithic)
- 7. 1959: the first monolithic IC was developed by Robert Noyce at Fairchild Semiconduction
  - (a) uses silicon instead of germanium
  - (b) 2 interconnected BJT transistor
  - (c)  $SiO_2$  insulator, Al interconnection
  - (d) planar IC, 0.06 in diameter
- 8. Bipolar ICs with BJT devices domidated until early 80s
  - (a) S/M/L Scale Integration: 10, 100, 1000 transistors
- 9. 1959: MOS ICs started in the 60s after the MOS transistor was develooed at Bell Labs
  - (a) easier to fabricate than bipolar
  - (b) uses less power
  - (c) can fit more transistor in the same silicon area
  - (d) slower than bipolar IC
  - (e) less robust than bipolar IC

- 10. MOS ICs
  - (a) early 70s: MOS technology improved in speed and reliability
  - (b) first microprocessor: Intel 4004 (1971)
  - (c) emergence of VLSI (>10000 trasistors)
  - (d) microprocessors evolved into microcontrollers (MCUs) ad system on a chip (SOCs)
- 11. Moore's Law: the transistor density would double every two years

#### 1.2 Piecewise Linear Diode Models

- 1. 1st Approximation: (when  $v_s \gg V_T$ )
  - (a) Open when  $v_D < 0$
  - (b) Short when  $v_D \ge 0$
- 2. 2nd Approximation (when above condition not satisfied):
  - (a) Open when  $v_D < V_T$
  - (b) Voltage of  $V_T$  when  $v_D \geq V_T$
- 3. 3rd Approximation (2nd with resistor R in series)
  - (a) Open when  $v_D < V_T$
  - (b) Voltage of  $V_T$  in series with resistance R when  $v_D \ge V_T$
- 4. To determine diode state:
  - (a) Assume it is conducting
  - (b) Check direction of current
  - (c) If current is from anode to cathode, the assumption is correct and equivalent circuit is valid
  - (d) Otherwise, diode should be open
- 5. Another method:
  - (a) Replace diode with open circuit
  - (b) Determine the voltage across the diode terminals (+) on anode, (-) on cathode
  - (c) If  $v_D$  < the needed threshold, assumption is correct
  - (d) Otherwise, diode should be replaced with the appropriate model

#### 1.3 Practical Diode Circuits

1. Half Wave Rectifier

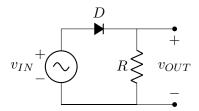


Figure 1: Half Wave Rectifier

(a) When diode is IDEAL:

$$v_{OUT} = \begin{cases} 0, & v_{IN} < 0 \\ v_{IN} & v_{IN} \ge 0 \end{cases}$$

(b) When diode has constant voltage model:

$$v_{OUT} = \begin{cases} 0, & v_{IN} < V_T \\ v_{IN} - V_T & v_{IN} \ge V_T \end{cases}$$

2. Full Wave Rectifier ( $v_1 = v_2$ )

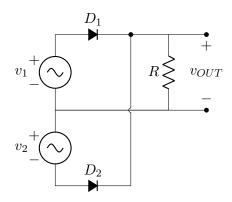


Figure 2: Full Wave Rectifier

(a) When diode is IDEAL:

$$v_{OUT} = |v_{IN}|$$

(b) When diode has constant voltage model

$$v_{OUT} = \left\{ egin{array}{ll} v_1 - V_T, & v_1 > V_T \\ -v_2 - V_T, & v_2 < -V_T \\ 0 & ext{otherwise} \end{array} 
ight.$$

(c) This requires a transformer with center-tapped secondary ( $n_1:n_2=1:2$ )

### 3. Full Wave Bridge Rectifier

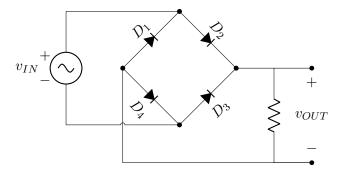


Figure 3: Full Wave Bridge Rectifier

(a) When diode is IDEAL:

$$v_{OUT} = |v_{IN}|$$

(b) When diode has constant voltage model:

$$v_{OUT} = \left\{ egin{array}{ll} v_1 - 2V_T, & v_1 > 2V_T \\ -v_2 - 2V_T, & v_2 < -2V_T \\ 0 & ext{otherwise} \end{array} 
ight.$$

#### 4. Positive Clipper

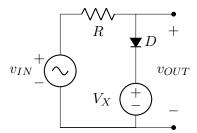


Figure 4: Positive Clipper

(a) 
$$v_{OUT} = \left\{ egin{array}{ll} V_X + V_T, & v_{IN} - V_X > V_T \\ v_{IN}, & v_{IN} - V_X < V_T \end{array} \right.$$

#### 5. Negative Clipper

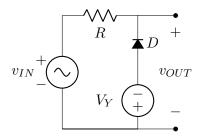


Figure 5: Negative Clipper

(a) 
$$v_{OUT} = \begin{cases} -\left(V_Y + V_T\right), & -V_Y - v_{IN} > V_T \\ v_{IN}, & -V_Y - v_{IN} < V_T \end{cases}$$

6. The positive and negative clippers can be combined

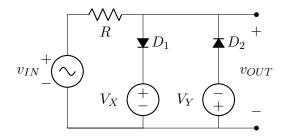


Figure 6: Positive and Negative Clipper

(a) 
$$v_{OUT}=\left\{egin{array}{ll} V_X+V_T, & v_{IN}-V_X>V_T \\ & -\left(V_Y+V_T\right), & -V_Y-v_{IN}>V_T \\ & v_{IN}, & ext{otherwise} \end{array}
ight.$$

#### 7. Peak Detector

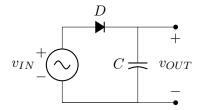


Figure 7: Peak Detector

- (a) Let the time  $t_1$  be the first time the input reaches its maximum value  $V_P$  and  $t_0$  be the time to reach  $V_T$
- (b) When diode is IDEAL:

$$v_{OUT} = \begin{cases} v_{IN}, & t < t_1 \\ V_P, & t \ge t_1 \end{cases}$$

(c) When diode has constant voltage model:

$$v_{OUT} = \begin{cases} 0, & 0 \le t < t_0 \\ v_{IN} - V_T, & t_0 \le t < t_1 \end{cases}$$
$$V_P - V_T \quad t_1 \le t$$

8. Peak Detector with Load Resistor (Ideal Diode)

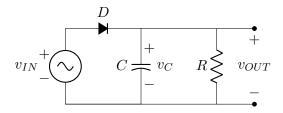


Figure 8: Peak Detector with load resistor

(a) Let the time  $t_1$  be the first time the input reaches its maximum value  $V_P$  and the period be T. When the capacitor voltage decays, let time  $t_2$  be when  $v_C = v_{IN}$  again

$$\text{(b) } v_{OUT} = \left\{ \begin{array}{ll} v_{IN}, & t < t_1 \\ V_P \exp\left(-\frac{t-t_1}{RC}\right), & t_1 \leq t < t_2 \\ \\ v_{IN}, & t_2 \leq t < t_1 + T \\ \\ V_P \exp\left(-\frac{t-(t_1+T)}{RC}\right), & t_1 + T \leq t < t_2 + T \\ \\ \vdots & \vdots \end{array} \right.$$

9. Negative Clamper (ideal diode)

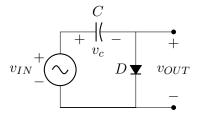


Figure 9: Negative Clamper

- (a) Let the time  $t_1$  be the first time the input reaches its maximum value  $V_P$
- (b) This is similar to the peak detector, only that  $v_{OUT}=v_{IN}-v_{C}$  where  $v_{C}$  follows the characteristic of the peak detector

(c) 
$$v_C = \left\{ egin{array}{ll} v_{IN}, & t < t_1 \\ V_P, & t_1 \leq t \end{array} \right.$$

(d) 
$$v_{OUT} = \left\{ egin{array}{ll} 0, & t < t_1 \\ \\ v_{IN} - V_P, & t \geq t_1 \end{array} \right.$$

10. Positive Clamper (ideal diode)

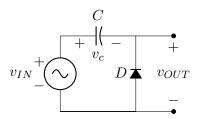


Figure 10: Positive Clamper

(a) Let the period be T

(b) 
$$v_C = \begin{cases} 0, & 0 \le t < T/2 \\ v_{IN}, & T/2 \le t < 3T/4 \\ -V_P, & 3T/4 \le t \end{cases}$$

(c) 
$$v_{OUT} = \begin{cases} v_{IN}, & 0 \le t < T/2 \\ 0, & T/2 \le t < 3T/4 \\ v_{IN} + V_P, & 3T/4 \le t \end{cases}$$

#### 1.4 Exponential Diode Model

1. Shockley's Diode Equation

$$i_D = I_S \left[ \exp\left(\frac{v_D}{\eta V_T}\right) - 1 \right] \tag{1}$$

- (a)  $v_D$ : diode voltage with positive at anode
- (b)  $i_D$ : diode current from anode to cathode
- (c)  $I_S$ : reverse saturation current due to minority carriers.  $10^{-15} {\rm A} < I_s < 10^{-9} {\rm A}$
- (d)  $\eta$ : ideality factor,  $1 < \eta < 2$ ; close to 1 for well fabricated diodes
- (e)  $V_T$ : thermal voltage,  $V_T = \frac{kT_K}{e} \approx \frac{T_K}{11600}$
- 2. for typical operating voltages (forward biased):  $\exp\left(\frac{v_D}{\eta V_T}\right)\gg 1 \implies i_D\approx I_S\exp\left(\frac{v_D}{\eta V_T}\right)$
- 3. for typical reverse bias voltages,  $\exp\left(\frac{v_D}{\eta V_T}\right) \ll 1 \implies i_D \approx -I_S$

### 1.5 Small Signal Model

1. 
$$v_D = V_D + v_d$$

2. 
$$i_D = I_D + i_d$$
.  $I_D$  is calculated using large signal analysis

3. if the amplitude of  $v_d$  is very small, the diode characteristic equation is approximately a line with conductance  $g_d = \frac{\partial i_D}{\partial v_D} = \frac{I_S}{\eta V_T} \exp\left(\frac{v_D}{\eta V_T}\right) \approx \frac{I_D}{\eta V_T} \implies r_d = \frac{\eta V_T}{I_D}$ 

#### 1.6 Zener Diode



1. has 3 modes of operations:

Operation	Condition	Equivalent
Forward-biased	$v_D \le V_T$	$v_D = -V_T$
Non-conducting	$-V_T < v_D < V_Z$	open circuit
Zener region	$v_D \ge V_Z$	$v_D = V_Z$

### 2 PN Juncion and BJT Operation

#### 2.1 Semiconductor Fundamentals

- 1. The Silicon (14) Atom:
  - (a) has 4 valence electrons
  - (b) the semiconductor lattice has bounded valence electrons at 0K
  - (c) energy bands: electrons can occupy discrete energy levels
  - (d) there is a **band gap** between the valence band and conduction band
  - (e) at higher temperatures, an electron can gain enough energy to break the bonds
  - (f) when an electron leaves a bond, it leaves a positively charged hole
- 2. Groups by electrical properties:
  - (a) insulators: huge band gap
  - (b) conductor: conduction band and valence overlap
  - (c) semiconductor: small band gap that can be easily reached
- 3. Semiconductors:
  - (a) narrow band gap
  - (b) has 2 groups:
    - i. intrinsic: just enough valence electrons to complete the matrix. either has group IV semiconductors (Si and Ge) or III-V compound (GaAs)
    - ii. extrinsic: produced by doping intrinsic semiconductors
      - A. N-type: dopants have excess electrons. energy level just below the conduction band
      - B. P-type: dopants have less electrons. energy level just above the valence
- 4. Fermi level: hypothetical energy level where there is 50% probability that the level is occuppied by an electron at any given time
  - (a) intrinsic semiconductor: midway between  $E_C$  and  $E_V$
  - (b) extrinsic semiconductor:
    - i. N-type: closer to  $E_C$  ii. P-type: closer to  $E_V$

#### 2.2 Carrier Actions in Semiconductors

- 1. Charge Carriers: the electrons and holes. units in coulombs (C)
- 2. Intrinsic semiconductor: n = p equal concentration of electrons and holes
- 3. N-type semiconductors:
  - (a) n > p
  - (b) majority charge carriers: electrons
  - (c) minority charge carriers: holes
- 4. P-type semiconductors:
  - (a) p > n

- (b) majority charge carriers: holes
- (c) minority charge carriers: electrons
- 5. Current: rate of movement of charge past a point or region. units in ampere (A). + is the direction of the hole/opposite the electron
- 6. Mobility: ability of charge carrier to move.  $\mu_e > \mu_h$
- 7. Factors that affect mobility:
  - (a) Scattering the random motion of charges. One mechanism is the *lattice scattering* when electrons bump with the vibrating lattice or other electrons and change directions
  - (b) another scattering mechanism is impurity scattering when dopant ions deflect charge carriers
  - (c) Temperature vs Scattering vs Mobility. At low temperatures, impurity scattering dominates (and with low mobility). at high temperatures, lattice scattering dominates (and low mobility). there is a temperature in the middle with maximum mobility. (like a parabolic log-log graph)
  - (d) Doping vs Scattering vs Mobility. more doping = more impurity scattering
  - (e) scattering has a net current of 0
- 8. There are 3 primary carrier actions:
  - (a) Generation and Recombination: generation = electron gains enough energy to break a covalent bond and leaves a hole. recombination = moving electron moves close to a hole and experience attractive force
  - (b) Diffusion: electrons move from higher concentration/temperature to lower concentration/temperature. + diffusion is parallel to positive charges. becomes 0 when there is uniform distribution of charge carriers
  - (c) Drift: caused by applied external voltage. modeled by a constant drift velocity  $v_d = \mu E$  (directly related to the electric field with the mobility as constant)
    - i.  $I_n = -Aqnv_{\text{n-drift}}$
    - ii.  $I_p = Aqpv_{p-drift}$

#### 2.2.1 PN Junction

- 1. PN Junction: formed when in a single silicon crystal, one region is doped with donor dopants (N-type) and the other with acceptor dopants (P-type). PN Junction is the boundary.
- 2. Depletion Region: happens at the boundary when the electrons diffuse and recombine with the holes at the P-type region. This results to a potential difference (and thus an electric field) across the depletion region (points from N to P)
  - (a) drift current of minority carriers are drifted by this electric field
- 3. There are 3 junction operations:
  - (a) Open-circuit (equilibrium condition):
    - i. the diffusion current  $I_D$  is limited by the built in voltage  $V_0$  and equal to the drift current  $I_S$
  - (b) Forward Bias (+ on the P-type)
    - i. the + potential pushes the holes in the P-type and the depletion region in the P-side decreases
    - ii. the potential pushes the electrons in the N-type and the depletion region in this side also decreases
    - iii. the diffusion current  $I_D$  dominates, but the temperature dependent  $I_S$  remain the same

- iv. the net current  $I=I_D-I_S$  in the direction from P to N
- v. when  $V_F \geq V_o$ , the depletion region disappears and  $I \approx I_D$
- (c) Reverse Bias (+ on the N-type)
  - i. the + potential attracts the electrons in the N-type, the depletion region becomes larger
  - ii. the potential attracts the holes in the P-type, the depletion region becomes larger
  - iii. the voltage  $V_o$  increases so  $I_D$  decreases
  - iv. net current  $I = I_D I_S$  from N to P
  - v. if  $I_D$  becomes very small,  $I \approx -I_S$
- 4. The energy band diagram will align the Fermi levels, thus bending the  $E_C$  and  $E_V$  with the maximum deviation  $V_0$ 
  - (a) at forward bias,  $V_0$  decreases so the bending energy will also decrease and the Fermi level will not be aligned anymore ( $E_{FN}$  is higher)
  - (b) at reverse bias,  $V_0$  increases so the bending energy will also increase and the Fermi level will not be aligned anymore ( $E_{FN}$  is lower)
- 5. The PN Junction implements a Diode. anode at P, cathode at N
  - (a) the Shockley Equation summarizes the 3 conditions for the PN Junction operations
  - (b) Diode resistivity:
    - i. at higher temperature, more electrons are freed and resistivity decreases
    - ii. higher doping density lowers the resistivity

### 3 BJTs as Amplifiers

### 3.1 DC Characteristics and Operating Modes

Mode of Operation	EB Junction	CB Junction
Cut-off	Reverse-biased	Reverse-biased
Active	Forward-biased	Reverse-biased
Saturation	Forward-biased	Forward-biased
Reverse Active	Forward-biased	Reverse-biased

#### 1. Cutoff Mode Relationships:

(a) 
$$I_B = I_C = I_E = 0$$

#### 2. Active Mode Relationships:

(a) 
$$I_B = rac{I_S}{eta} \left[ \exp\left(rac{eV_{BE}}{kT}
ight) - 1 
ight] \left[ 1 + rac{V_{CE}}{V_A} 
ight]$$

(b) 
$$I_C = \beta I_B = I_S \left[ \exp\left(\frac{eV_{BE}}{kT}\right) - 1 \right] \left[ 1 + \frac{V_{CE}}{V_A} \right]$$

(c) 
$$I_E=rac{1}{lpha}I_C=\left(eta+1
ight)I_B$$

(d) 
$$\alpha = \frac{\beta}{\beta + 1} \Longleftrightarrow \beta = \frac{\alpha}{1 - \alpha}$$

- (e) In most cases,  $V_A\gg V_{CE}$  so the factor is approximately 1
- (f) Model:

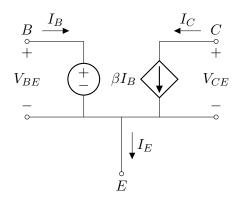


Figure 11: NPN Transistor Active Mode

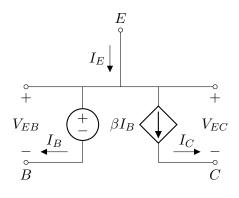
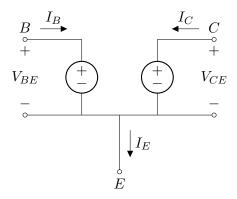


Figure 12: PNP Transistor Active Mode

### 3. Saturation Mode Relationships

- (a)  $I_{C, sat} = \beta_{forced} I_B$
- (b)  $V_{BE} = 0.7 V$ ,  $V_{EB} = 0.7 V$
- (c)  $V_{CE}=0.3\ V$ ,  $V_{EC}=0.3\ V$  edge of saturation
- (d) deep saturation is normally 0.2~V





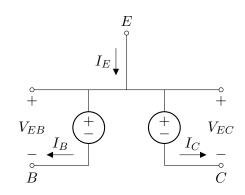


Figure 14: PNP Transistor Saturation

#### **BJT Biasing Circuits** 3.2

- 1. Q point: fixed characteristics of the transistor
  - (a) active region for amplifier
  - (b) cut-off/saturation for switches
- 2. Biasing: application of DC voltages to establish the operating point
- 3. Transistor Parameter variations on Temperature:
  - (a) leakage current:  $I_{Co}(25^{\circ}C) = 1 \ nA$ . doubles every  $6^{\circ}$  rise
  - (b)  $V_{BE}(25^{\circ}C) = 0.7 V$ . drops about  $2.2 mV/^{\circ}C$
  - (c)  $\beta$  doubles with an increase of  $80^{\circ}C$

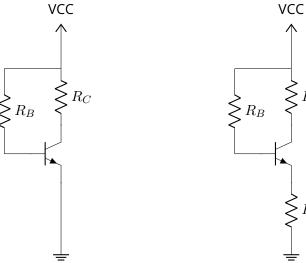


Figure 15: Fixed Bias Circuit

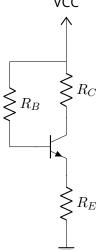


Figure 16: Emitter Stabilized

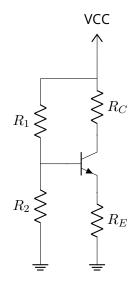


Figure 17: Voltage Divier Bias

# 3.3 BJT Amplifiers

- 1. General steps in circuit analysis:
  - (a) In the DC part, treat the capacitors as open
  - (b) The DC analysis calculates the Q point values
  - (c) In the AC part, treat the capacitors and DC voltage sources as shorts
- 2. Common Emitter Amplifier:
  - (a) Inverting amplifier with very huge gains

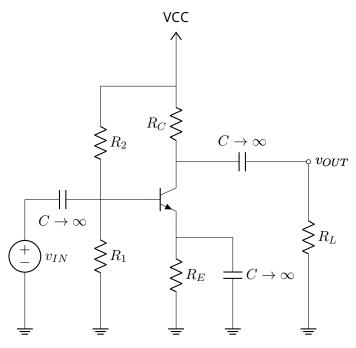
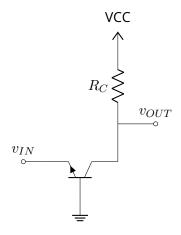
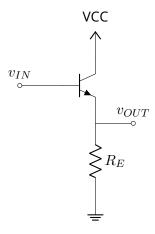


Figure 18: CE Amplifier

- 3. Common Base Amplifier:
  - (a) has a sudden rise of gain in the vicinity of the  $\mathcal{V}_{BE}$  threshold



- 4. Common Collector Amplifier:
  - (a) has unity gain
  - (b) is a voltage follower



## 4 MOSFETs as Amplifiers

#### 4.1 MOSFET Fundamentals

- 1. FET: an electric field is applied normal to the surface (gate) of a semiconductor that modulates the conductance of the semiconductor
  - (a) NMOS: uses electrons as charge carriers
  - (b) PMOS: uses holes as charge carriers
- 2. The source and drain terminals are heavily doped (n for NMOS, p for PMOS)
  - (a) Enhancement Mode: default off
  - (b) Depletion Mode: default on
- 3. Threshold voltage: minimum  $V_{GS}$  for the MOSFET to conduct
- 4. Saturation Voltage: minimum  $V_{DS}$  that causes the channel of the transistor to pinch-off in the drain side due to widening depletion region in the drain bulk junction
- 5. MOSFET Regions of Operation:

Region of Operation	Conditions (NMOS)	Current $I_D$	
Cut-off	$V_{GS} < V_{Th, n}$	$I_D = 0$	
Linear	$V_{GS} \ge V_{Th, n}$	$I_D = k_n \left( V_{GS} - V_{Th, n} - \frac{V_{DS}}{2} \right) V_{DS}$	
	$V_{DS} < V_{GS} - V_{Th, n}$		
Saturation	$V_{GS} \ge V_{Th, n}$	$I_D = \frac{1}{2} k_n \left( V_{GS} - V_{Th, n} \right)^2$	
Saturation	$V_{DS} \ge V_{GS} - V_{Th, n}$		
Region of Operation	Conditions (PMOS)	Current $I_D$	
Cut-off	$V_{GS} < V_{Th, n}$	$I_D = 0$	
Linear	$V_{SG} \ge  V_{Th, p} $	$I_D = -k_p \left( V_{SG} - V_{Th, p} - \frac{V_{SD}}{2} \right) V_{SD}$	
	$V_{SD} < V_{SG} - V_{Th, p}$	$ID = -\kappa_p \left( v_{SG} - v_{Th, p} - \frac{1}{2} \right) v_{SD}$	
Saturation	$V_{SG} \ge V_{Th, p}$	$I_D = -\frac{1}{2} k_p \left( V_{SG} - V_{Th, p} \right)^2$	
	$V_{SD} \ge V_{SG} - V_{Th, p}$	$ID = -\frac{1}{2} \kappa_p \left( v_{SG} - v_{Th, p} \right)$	

#### 4.2 MOSFET DC Characterization

- 1. Steps in solving MOSFET circuits:
  - (a) Identify the terminals
  - (b) Solve for voltage terminals
  - (c) Use the current equation to ge  $I_D$ . assume saturation if region of operation is unknown

### 4.3 MOSFET Amplifier

- 1. Common Source Amplifier:
  - (a) inverting amplifier
  - (b) can have large gains

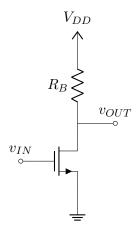


Figure 19: NMOS CS Amplifier

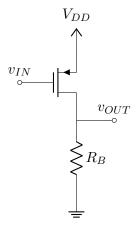


Figure 20: PMOS CS Amplifier

- 2. Common Drain Amplifier:
  - (a) has a unity gain
  - (b) non-inverting amplifier

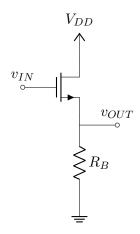


Figure 21: NMOS CD Amplifier

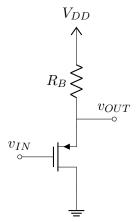


Figure 22: PMOS CD Amplifier

### 3. Common Gate Amplifier:

- (a) non-inevrting amplifier
- (b) can have gain more than 1

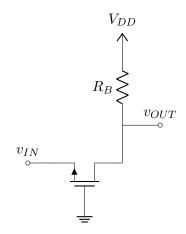


Figure 23: NMOS CG Amplifier

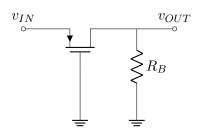


Figure 24: PMOS CG Amplifier

## 5 Small Signal Analysis of Transistor Amplifiers

#### 5.1 Two Port Networks

- 1. one port serves as input, the other as output
- 2.  $\it Z$  parameters: impedance parameters. taken for open circuit of both ports

$$egin{bmatrix} V_1 \ V_2 \end{bmatrix} = egin{bmatrix} Z_{1, \ 1} & Z_{1, \ 2} \ Z_{2, \ 1} & Z_{2, \ 2} \end{bmatrix} egin{bmatrix} I_1 \ I_2 \end{bmatrix}$$

3. Y parameters: admittance parameters. taken for short circuit of both ports

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{1, 1} & Y_{1, 2} \\ Y_{2, 1} & Y_{2, 2} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

4. h parameters: hybrid parameters. first column is short circuit of Port 2, second column is open circuit for Port 1

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{1, 1} & h_{1, 2} \\ h_{2, 1} & h_{2, 2} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

5. g parameters: inverse hybrid parameters. tfirst column is open circuit of Port 2, second column is short circuit for Port 1

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{1, 1} & g_{1, 2} \\ g_{2, 1} & g_{2, 2} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

- 6. Unilateral Hybrid  $\pi$  Network: only has 3 parameters
  - (a)  $R_i$ ,  $R_o$ ,  $A_v$  for Thevenin Equivalent
  - (b)  $R_i$ ,  $R_o$ ,  $G_m$  for Norton Equivalent

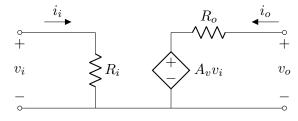


Figure 25: Thevenin Equivalent

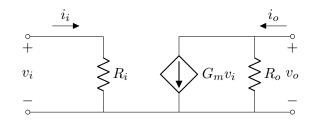


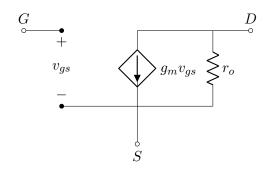
Figure 26: Norton Equivalent

Figure 27: Unilateral Hybrid  $\pi$  Network

## 5.2 Small Signal Model

Parameter	ВЈТ	MOSFET
$g_m$ Transconductance	$g_m = \frac{I_C}{V_T}$	$g_m = \frac{2I_D}{V_{GS} - V_{Th, n}} = \sqrt{2K_n I_D (1 + \lambda V_{DS})} \approx \sqrt{2K_n I_D}$
$r_{\pi}$ Input Resistance	$r_{\pi} = \frac{\beta}{g_m} = \frac{\beta V_T}{I_C}$	$r_{\pi} = \infty$
$r_o$ Output Resistance	$r_o = \frac{V_A + V_{CE}}{I_C} \approx \frac{V_A}{I_C}$	$\frac{1}{I_D} \left( \frac{1}{\lambda} + V_{DS} \right) pprox \frac{1}{\lambda I_D}$

- 1. the approximations above are from:
  - (a)  $V_A\gg V_{CE}$
  - (b)  $\lambda \ll \frac{1}{V_{DS}}$



 $\begin{array}{c|c}
 & C \\
 & + \\
 & v_{\pi} \\
 & - \\
\end{array}$   $\begin{array}{c|c}
 & C \\
 & g_m v_{\pi} \\
 & F_o
\end{array}$ 

Figure 28: MOSFET Small Signal Model

Figure 29: BJT Small Signal Model

### 5.3 Small Signal Analysis

### 1. Amplifier Parameters:

Voltage Gain	$A_v = \frac{v_o}{v_i}$
Open-loop Gain	$A_{vo} = \left. \frac{v_o}{v_i} \right _{R_L \to \infty}$
Input Resistance	$R_i = \frac{v_i}{i_i}$
Output Resistance of Amplifier	$R_o = \frac{v_o}{i_o} \bigg _{v_i \to 0}$
Output Resistance of the circuit	$R_{out} = \frac{v_o}{i_o} \bigg _{v_{sig} \to 0}$
Overall Gain	$A = \frac{R_i}{R_i + R_{sig}} A_v$

### 2. Design considerations:

- (a)  $R_i \gg R_{sig}$
- (b)  $R_o \ll R_L$

- 3. Solving Amplifier Circuits
  - (a) Find Bias and Signal Circuits
  - (b) Get the bias (Q point) values
    - i. capacitors are open
    - ii. large signal models
  - (c) Get the signal paramaters
    - i. capacitors are shorts
    - ii. independent sources are suppressed
    - iii. replace with small signal model
  - (d) Terminal Resistance:
    - i. Consider that a voltage  $v_x$  and current  $i_x$  is supplied through the terminal. the terminal resistance is  $\frac{v_x}{i_x}$

## **6 Amplifier Frequency Response**