

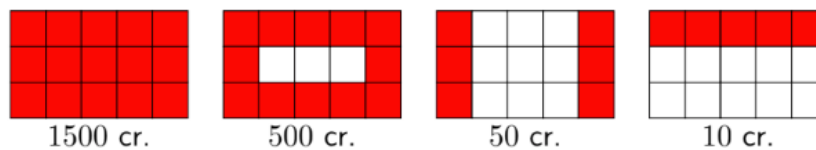
Bingo Project

In a Video Bingo game, players are randomly drawn 30 balls from a set of balls numbered from 1 to 60. Each player has one (or more) cards with 15 numbers from this universe (as in the image to the side). The players goal is to fill in all the numbers on his card (doing Bingo).

BINGO				
1	14	26	39	50
3	19	28	45	55
7	20	30	47	58

The player can also obtain other prizes.

An example would be the prize patterns below. To do this, the player simply fill in all the cells marked in red, with no influence on the remaining card cells marked on white . However, if the pattern of a prize with a higher value entirely contains the pattern of another prize, only the highest prize will be paid.



Example:

If the player has obtained the prize of 50 cr., and later obtains the prize of 500 cr., he will only receive 500 credits. If the player wins the prizes of 50 cr. and 10 cr., he will receive 60 credits, since their patterns do not overlap.

Project milestones:

- 1) Determine each prize probability.
- 2) Calculate the average prize that the player will win and determine the RTP, if bet = 1cr.
- 3) Create a script to run the MC simulation.
- 4) The business needs game's RTP to be 95%. To deliver that, the player may receive an extra free ball, on each play. What percentage of plays should the player have access to this extra ball, to achieve the needed RTP?

M1:

P(1500cr)

$$p = \frac{15C15 \times 45C15}{60C30} = \frac{3}{1028785} \cong 2.916 \times 10^{-6}$$

P(500cr)

$$p = \frac{48C18 - 45C15}{60C30} \cong 0.00005889$$

P(60cr)

$$p = \frac{51C21 - 48C18}{60C30} \cong 0.000905992$$

P(50cr)

$$p = \frac{54C24 - 51C21}{60C30} \cong 0.01089255$$

P(10cr)

$$p = \frac{55C25 - 51C21}{60C30} \cong 0.02512497$$

M2:

Thus we have the prize probability model:

xi	1500	500	60	50	10
p(X=xi)	0,000002916	0,000058893	0,00090599	0,01089255	0,02512497
xi*freq	0,004374	0,0294465	0,05435952	0,5446275	0,25124973

The average prize value is determined by the formula: $\sum prize \times prizeProbability$

So the average prize is 0.884 cr.

The RTP can be calculated by the formula:

$$RTP = \frac{TotalWin}{TotalBets} \times 100 = \frac{TotalGames \times AverageWin}{TotalGames \times bet} \times 100 = \frac{AverageWin}{bet} \times 100$$

So if the bet = 1 cr, then RTP = 88,4%

M3:

The script to run the Monte Carlo simulation is on Git Hub:

<https://github.com/ajmferreira/bingo>

Running the script we get:

```
=====BINGO=====
=====
Prize      →   hits   →   probability
=====
cr_1500    →    26    →   0,0000026
cr_500     →   641    →   0,000064
cr_60      →  8986    →   0,000899
cr_50      → 108774   →   0,010877
cr_10      → 251635   →   0,025164
=====
total games → 10000000
total wins  → 8853710
rtp → 88,5%
```

M4:

The model for the extraction of 31 balls:

xi	1500	500	60	50	10
p(X=xi)	0,000005649	9,51967E-05	0,00126287	0,01334312	0,02975
xi*freq	0,0084735	0,04759834	0,0757722	0,66715575	0,2975

This would give us $RTP = 1,09649979 \times 100\% = 110\%$

If the probability of getting a free ball is x, we have:

$$0,88(1-x) + 1,10x = 0,95$$

$$x = 31,8\%$$

To get $RTP = 95\%$, the player should have an extra ball with probability close to 31.8%