Cards Game Project

Consider the following game:

A box contains 20 standard decks of shuffled playing cards. A player picks 3 cards at random, without replacement. The player will only win a prize if either of the following scenarios occur:

Scenario	**Prize Payout**
All 3 cards drawn are the number 7 and are the same suit.	1000
The cards drawn are the numbers 6,7 and 8 respectively, of the same suit.	500
All 3 cards drawn are the number 7 but at least one card is a different suit.	250
The cards drawn are the numbers 6,7 and 8 respectively, and at least one card is a different suit.	100
All 3 cards drawn are of the same suit, and the values add up to 21. The cards cannot be three 7's of the same suit or 6,7, 8 of the same suit.	50
The sum of all 3 cards drawn is 20.	20

Each card has its own value:

Card	Value
Ace	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	10
Jack	11
Queen	12
King	13

Project milestones

1. Determine each scenario probability and calculate the average prize that the player will win.
2. RTP value.
3. Create a script to run the MC simulation.

M1:

A box contains 20 standard decks of shuffled playing cards.

That means 1040 total cards, and we have 20 of each: 20 A♠, 20 K♣, etc.

This also means that if we do not care about the suit, we have 80 cards of each: 80 A, 80 K, …

A player will pick three random cards from the box and will win a prize under a set of scenarios.

For the proposed Scenarios, I'll present the probability for each case:

Scenario 1: All 3 cards drawn are the number 7 and are the same suit.

$$p = \frac{4 \times 20C3}{1040C3} = \frac{4560}{186936880} \cong 0.00002439$$

Scenario 2: The cards drawn are the numbers 6,7 and 8 respectively, of the same suit.

$$p = \frac{20 \times 20 \times 20 \times 4}{1040C3} = \frac{32000}{186936880} \cong 0.00017118$$

Scenario 3: All 3 cards drawn are the number 7 but at least one card is a different suit.

Only two same suit sevens: 4x20C2x60C1 = 45600

Three unsuited sevens: 20C1x20C1x20C1x4C3 = 32000

$$p = \frac{45600 + 32000}{1040C3} = \frac{77600}{186936880} = 0.00041511$$

Alternatively: we can combine all sevens and remove the combinations with same suit: 80C3 – 4x20C3 = 77600

Scenario 4: The cards drawn are the numbers 6,7 and 8 respectively, and at least one card is a different suit.

I'll combine all 6, 7 and 8's and remove the combinations with same suit

$$p = \frac{80 \times 80 \times 80 - 4 \times 20 \times 20 \times 20}{1040C3} = \frac{480000}{1040C3} \cong 0.00256771$$

Scenario 5: All 3 cards drawn are of the same suit, and the values add up to 21. The cards cannot be three 7's of the same suit or 6,7, 8 of the same suit.

For this case we have lots of possible sets (study made on excel file to provide if needed)

With different cards:
{A,7,K}, {2,6,K}, {3,5,K}, {A,8,Q}, {2,7,Q}, {3,6,Q}, {4,5,Q}, {A,9,J}, {2,8,J}, {3,7,J}, {4,6,J}, {2,9,10}, {3,8,10}, {4,7,10}, {5,6,10}, {4,8,9}, {5,7,9}
For each of these 17 cases we have 4x20x20x20 = 32000 possibilities

With two equal cards (and the other different)
{4,4,K}, {5,5,J}, {A,10,10}, {3,9,9}, {6,6,9},{5,8,8}
For each of these 6 cases we have 4x20x20C2 = 15200 possibilities

$$p = \frac{6 \times 15200 + 17 \times 32000}{1040C3} = \frac{635200}{186936880} = 0.0033979$$

Scenario 6: The sum of all 3 cards drawn is 20.

Like Scenario 5.

With different cards:
{A,6,K}, {2,5,K}, {3,4,K}, {A,7,Q}, {2,6,Q}, {3,5,Q}, {A,8,J}, {2,8,J}, {2,7,J}, {3,6,J}, {4,5,J}, {A,9,10}, {2,8,10}, {3,7,10}, {4,6,10}, {3,8,9}, {4,7,9}, {5,6,9},{5,7,8},
For each of these 18 cases we have 80x80x80 = 512000 possibilities

With two equal cards (and the other different)
{4,4,Q}, {5,5,10}, {2,9,9}, {6,6,8},{4,8,8}, {6,7,7}
For each of these 6 cases we have 80x80C2 = 252800 possibilities

$$p = \frac{6 \times 252800 + 18 \times 512000}{1040C3} = \frac{10732800}{186936880} \cong 0.057414$$

The average prize value is determined by the formula: $\sum prize \times prizeProbability$

prize	1000	500	250	100	50	20	
relative freq	0,00002439	0,00017118	0,00041511	0,002567711	0,003397938	0,05741403	sum:
average prize	0,02439	0,08559	0,1037775	0,2567711	0,1698969	1,1482806	1,7887061

The average prize is 1,788 credits

M2:

A casino hosting this game will be profitable if the bet value exceeds the average prize. This means that if the bet = €1 the RTP will be 179% and the casino will lose.

The RTP can be calculated by the formula:

$$RTP = \frac{TotalWin}{TotalBets} \times 100 = \frac{TotalGames \times AverageWin}{TotalGames \times bet} \times 100 = \frac{AverageWin}{bet} \times 100$$

So, for instance, if bet = €2 the RTP will be 89,4% and the casino will be profitable

M3:

In the code project the Main class contains the actual code to run the Monte Carlo simulation.

So for the implementation, I've created a small java project with the classes Card, DeckOfCards, DeckBox and Main, of course. The first three classes were created to represent a card, a deck of cards and the Box with the 20 decks. In the Main class I've set a loop with a large number of iterations, and on each iteration I've defined conditions in order to capture when each of the scenarios occur.

I've got results that are consistent with the theoretical findings:

Number of games: 100000000
Bet value: 2
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Prizes frequency:

Prize	hits	probability
prize 20	→ 5742744	→p = 0,057427
prize 50	→ 340250	→p = 0,003403
prize 100	→255758	→p = 0,002558
prize 250	→41354	→p = 0,000414
prize 500	→16848	→p = 0,000168
prize 1000	→2422	→p = 0,000024

total wins = 178627680
total bets = 200000000
total games = 100000000
RTP = 89,31