Slot Machine project

A Slot Machine game is made up of several reels, each of which contains different graphic elements arranged on its side. In its most classic form, a Slots game rewards the player when all the reels stop in a position such that the same symbol is repeated in the central position as seen in the figure to the side.

Nowadays, this classic model has been expanded. The playing area has more than one visible cylinder position, with the most widespread model having a playing area with 5 columns and 3 rows. The award is made according to pre-defined lines. If the player manages to obtain a certain number of consecutive symbols, in the order defined by one or more pay lines, he obtains a prize.

Imagine a game with the following conditions:

• Game area with 5 columns and 3 rows.
• Each of the reels has a total of 20 symbols.
• There are 7 different symbols distributed on each reel:

- A, B, C, D, E → Normal Symbols.
- W → Wild. Special symbol that, when present, combines with any of the normal symbols.
- S → Scatter. Special symbol. These symbols do not have to respect the line patterns, nor do they have to be sequential, for there to be awards. This means that a pre-defined number of symbols simply appear in any position on the playing area to win.

• Prize table (defines the number of symbols required to award a prize and its value):

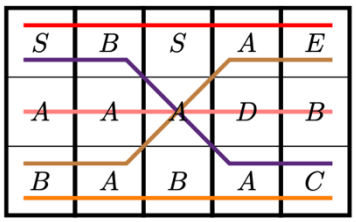
Symbol	x3 Symbols	x4 Symbols	x5 Symbols
A	5	20	50
B	5	20	50
C	10	50	100
D	15	75	150
E	25	150	400
S	5	15	25

- The W symbol does not award directly. Must be in sequence with normal symbols.
- The player wins when he gets a sequence with 3, 4 or 5 identical symbols on the same line, starting in the first position from left to the right, excluding the Scatter situation.

The symbol distribution by the five reels are as follows:

B	A	B	E	C
D	B	E	D	D
A	D	S	A	A
A	E	D	D	A
E	C	E	C	A
S	E	S	C	S
A	C	A	A	A
B	D	B	A	B
C	S	A	A	D
C	D	B	B	C
C	B	C	D	B
B	A	D	C	D
E	S	C	B	D
D	C	S	W	E
B	D	D	B	E
D	A	B	C	B
C	W	W	S	C
D	C	C	D	A
A	B	A	A	C
A	A	W	S	B

Consider that the game pays on 5 defined lines (figure below) and that, on each play, the player spends 2 cr. per line.

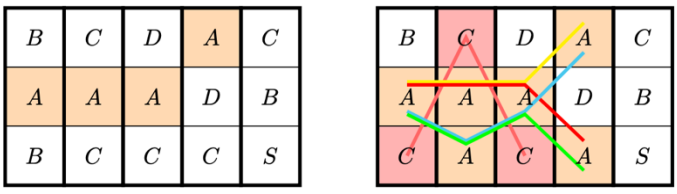


Project milestones:

1) Determine the frequency of plays with the award of a Scatter prize.
2) Determine the RTP value.
3) Run a Monte Carlo Simulation for the game and extract its parameters.
4) When the player receives a Scatter prize, they have access to FreeSpins, with 3 free spins. What is the RTP in FreeSpins mode?
   Remark:
   FreeSpins is a special game mode in which the player is entitled to a pre-defined number of free spins. If the player receives the Scatter prize again within the FreeSpins mode, the number of free spins is increased.

5) ***Ways***
   Consider a game with the same configuration as the previous exercise. However, instead of paying only 5 lines, the game now pays in **243Ways**. This means that there are no specific paylines, the game pays on any consecutive combination on the reels.

   Ex: In the case below, in the figure on the left, only a prize of x4 A is paid. In the figure on the right, a prize of x3 C and four prizes of x4 A are paid.



   Furthermore, the Wild symbol has an 80% chance of turning into an Expanding Wild. Expanding Wild means that, when present, it will replace all positions on the column in which it appears with Wild.
   Assuming the player pays 250 cr. for each play, **run a Monte Carlo simulation** to determine the RTP of the defined game, considering only the normal game (without FreeSpins).

M1:

Each Reel has some number of Scatter symbols. Since they are separated along the reel by two or more normal symbols, each S symbol has three positions on the column where we have a single hit.

#S per Reel:

1	2	3	2	1

A			B	
A			C	
E			S	
S			D	
A			A	
B			S	
C			E	

Nr. of possible hits for each reel per column:

3	6	9	6	3

Nr. of possible misses for each reel per column:

17	14	11	14	17

Since the total number of possible situations for random reels stop is 20^5 and there are a lot of situations to face, I'll count the hits for each one of them, and by the end, the probability calculation value will be determined by hits/20^5.

So using S to represent a scatter symbol in the cell line and $\bar{S}$ to represent the opposite, we have the following number of hits in the playing area:

Prize: 25cr

$$SSSSS \rightarrow 3 \times 6 \times 9 \times 6 \times 3 = 2916 \rightarrow \$72900$$

Prize: 15cr

$$\bar{S}SSSS \rightarrow 17 \times 6 \times 9 \times 6 \times 3 = 16524$$
$$S\bar{S}SSS \rightarrow 3 \times 14 \times 9 \times 6 \times 3 = 6804$$
$$SS\bar{S}SS \rightarrow 3 \times 6 \times 11 \times 6 \times 3 = 3564$$
$$SSS\bar{S}S \rightarrow 3 \times 6 \times 9 \times 14 \times 3 = 6804$$
$$SSSS\bar{S} \rightarrow 3 \times 6 \times 9 \times 6 \times 17 = 16524$$
$$\Sigma = 50220 \rightarrow \$753300$$

Prize: 5cr

$$\bar{S}\bar{S}SSS \rightarrow 17 \times 14 \times 9 \times 6 \times 3 = 38556$$
$$\bar{S}S\bar{S}SS \rightarrow 17 \times 6 \times 11 \times 6 \times 3 = 20196$$
$$\bar{S}SS\bar{S}S \rightarrow 17 \times 6 \times 9 \times 14 \times 3 = 38556$$
$$\bar{S}SSS\bar{S} \rightarrow 17 \times 6 \times 9 \times 6 \times 17 = 93636$$
$$S\bar{S}\bar{S}SS \rightarrow 3 \times 14 \times 11 \times 6 \times 3 = 8316$$
$$S\bar{S}S\bar{S}S \rightarrow 3 \times 14 \times 9 \times 14 \times 3 = 15876$$
$$S\bar{S}SS\bar{S} \rightarrow 3 \times 14 \times 9 \times 6 \times 17 = 38556$$
$$SS\bar{S}\bar{S}S \rightarrow 3 \times 6 \times 11 \times 14 \times 3 = 8316$$
$$SS\bar{S}S\bar{S} \rightarrow 3 \times 6 \times 11 \times 6 \times 17 = 20196$$
$$SSS\bar{S}\bar{S} \rightarrow 3 \times 6 \times 9 \times 14 \times 17 = 38556$$
$$\Sigma = 320760 \rightarrow \$1603800$$

So the total number of hits is 2916 + 50220 + 320760 = 373869 and we get the probability of getting a scatter prize (of any kind) is:

$$p = \frac{373896}{20^5} \times 100\% \cong 11.684$$

M2:

In order to calculate the RTP we need to have the probability model for all the prizes (not only for the Scatter).

So we need to evaluate for the other symbols (A, B, C, D and E).
The frequency that they occur on the reels is as follows:

Symbol	Reel#1	Reel#2	Reel#3	Reel#4	Reel#5	Total
A	5	4	3	5	5	22
B	4	3	4	3	4	18
C	4	4	3	4	4	19
D	4	4	3	4	4	19
E	2	2	2	1	2	9
S	1	2	3	2	1	9
W	0	1	2	1	0	4
Total	20	20	20	20	20	

And adding the W (wild symbol) occurrence to each of the other symbols, we have:

$$A + W \rightarrow 5,\ 5,\ 5,\ 6,\ 5$$
$$\overline{A + W} \rightarrow 15,\ 15,\ 15,\ 14,\ 15$$
$$B + W \rightarrow 4,\ 4,\ 6,\ 4,\ 4$$
$$\overline{B + W} \rightarrow 16,\ 16,\ 14,\ 16,\ 16$$
$$C + W \rightarrow 4,\ 5,\ 5,\ 5,\ 4$$
$$\overline{C + W} \rightarrow 16,\ 15,\ 15,\ 15,\ 16$$
$$D + W \rightarrow 4,\ 5,\ 5,\ 5,\ 4$$
$$\overline{D + W} \rightarrow 16,\ 15,\ 15,\ 15,\ 16$$
$$E + W \rightarrow 2,\ 3,\ 4,\ 2,\ 2$$
$$\overline{E + W} \rightarrow 18,\ 17,\ 16,\ 18,\ 18$$

Symbol A prizes (with wild):

$$AAAAA \rightarrow 5 \times 5 \times 5 \times 6 \times 5 = 3750 \rightarrow \$187500$$
$$AAAA\bar{A} \rightarrow 5 \times 5 \times 5 \times 6 \times 15 = 11250 \rightarrow \$225000$$
$$AAA\bar{A}X \rightarrow 5 \times 5 \times 5 \times 14 \times 20 = 35000 \rightarrow \$175000$$

Symbol B prizes (with wild):

$$BBBBB \rightarrow 4 \times 4 \times 6 \times 4 \times 4 = 1536 \rightarrow \$76800$$
$$BBBB\bar{B} \rightarrow 4 \times 4 \times 6 \times 4 \times 16 = 6144 \rightarrow \$122880$$
$$BBB\bar{B}X \rightarrow 4 \times 4 \times 6 \times 16 \times 20 = 30720 \rightarrow \$153600$$

Symbol C prizes (with wild):

$$CCCCC \rightarrow 4 \times 5 \times 5 \times 5 \times 4 = 2000 \rightarrow \$200000$$
$$CCCC\bar{C} \rightarrow 4 \times 5 \times 5 \times 5 \times 16 = 8000 \rightarrow \$400000$$
$$CCC\bar{C}X \rightarrow 4 \times 5 \times 5 \times 15 \times 20 = 30000 \rightarrow \$175000$$

Symbol D prizes (with wild):

$$DDDDD \rightarrow 4 \times 5 \times 5 \times 5 \times 4 = 2000 \rightarrow \$187500$$
$$DDDD\bar{D} \rightarrow 4 \times 5 \times 5 \times 5 \times 16 = 8000 \rightarrow \$225000$$
$$DDD\bar{D}X \rightarrow 4 \times 5 \times 5 \times 15 \times 20 = 30000 \rightarrow \$300000$$

Symbol E prizes (with wild):

$$EEEEE \rightarrow 2 \times 3 \times 4 \times 2 \times 2 = 96 \rightarrow \$38400$$
$$EEEE\bar{E} \rightarrow 2 \times 3 \times 4 \times 2 \times 18 = 8640 \rightarrow \$129600$$
$$EEE\bar{E}X \rightarrow 2 \times 3 \times 4 \times 18 \times 20 = 864 \rightarrow \$21600$$

(of course, the X above situations represents any symbol whatsoever.)

Using an Excel sheet:

prize	400	150	100	75	50	25	20	15	10	5	
S						2916		50220		320760	
A					3750		11250			35000	
B					1536		6144			30720	
C			2000		8000				30000		
D		2000		8000				30000			
E	96	864				8640					
# Total	96	2864	2000	8000	13286	8640	17394	30000	30000	65720	
$ Total	38400	429600	200000	600000	664300	216000	347880	450000	300000	328600	
prob %	0,00003	0,000895	0,000625	0,0025	0,00415188	0,0027	0,00543563	0,009375	0,009375	0,0205375	
rtp (line)	0,6	6,7125	3,125	9,375	10,3796875	3,375	5,435625	7,03125	4,6875	5,134375	55,8559375
rtp (scatter)						0,2278125		2,3540625		5,011875	5,011875
											60,8678125

We have both, the RTP for the (five) lines and the scatter, obtaining an RTP = 60,87%

M3:

In the code project the Main class contains the script to run the Monte Carlo simulation. The code may be found in the public repository:
https://github.com/ajmferreira/slots

We need to parametrize the project (with the wild symbol but no free spins)
This is the kind of result we get:

Initial (payed) spins 100000000
SCATTER PRIZEs won: 11681945
==========
Prizes frequency:
premio 5 → 19159392 →0,0958
premio 10 →4686196 →0,0469
premio 15 →6261724 →0,0939
premio 20 →2409940 →0,0482
premio 25 →1441534 →0,0360
premio 50 →1982848 →0,0991
premio 75 →1250225 →0,0938
premio 100 →312888 →0,0313
premio 150 →446927 →0,0670
premio 400 →14910 →0,0060
total win = 618023055
total bets = 1000000000
total spins = 100000000
RTP = 61,8023

M4:

Turning on the FreeSpins option on the script, we get similar results like this one:

Initial (payed) spins 100000000
SCATTER PRIZEs won: 17993230
==========
Prize frequency:
premio 5 → 29503136 → 0,1475
premio 10 → 7216649 → 0,0722
premio 15 → 9637800 → 0,1446
premio 20 → 3710773 → 0,0742
premio 25 → 2221315 → 0,0555
premio 50 → 3050912 → 0,1525
premio 75 → 1924971 → 0,1444
premio 100 → 480178 → 0,0480
premio 150 → 688040 → 0,1032
premio 400 → 23044 → 0,0092
total win = 951357330
total bets = 1000000000
total spins = 153979690
RTP = 95,1357

We may observe that even with the free spins, the scatter prize distribution remains very close to the previous result:
17993230/153979690=0,11685

And with the free spins the RTP value is upgraded close to 95%
But this is the value that we are getting from the script.

The theoretical RTP is calculated as follows:

$$rtp = \frac{average\ win}{total\ bets}$$

With the free spins, the total bets will not change, but the average win changes:

$$rtp = \frac{avWin + avWin \times 3 \times f_s}{totalBets}$$

Where f_s is the frequency which the scatter occurs.
We may assume that the average win is the sum of the prize values multiplied by the probability of each prize to occur. We can assume as well that the average win is the same for a free spin or for a regular spin.

Then we get:

$$rtp(withFreeSpins) = rtp + 3 \times f_s \times rtp$$

In the formula, the rtp is multiplied by 3 (the 3 spins offered to the player) and f_s = 11.7

But a scatter prize may occur when a free spin is in place, and in that situation, the player wins three more spins (we may call them second degree free spins). Using the same reasoning, the player may get third degree free spins, and fourth degree, and so on.

So, the expected free spin value can be determined by:

$$\begin{aligned} E(freeSpin) &= 3 \times 0{,}117 + (3 \times 0{,}117)^2 + (3 \times 0{,}117)^3 + \cdots \\ &= \sum_{i=1} 0.371^i \quad (this\ is\ a\ convergente\ geometric\ series) \\ &= \frac{0.351}{0.649} \cong 0.541 \end{aligned}$$

So,

$$rtp(withFreeSpins) = rtp + 0.541 \times rtp \cong 93.85\%$$

Which is close to the value obtained in the Monte Carlo simulation (95%).

M5. Slots – Ways

In the script, we need to turn off the free spin mode

As per specification, 80% of the times that we got a W (wild) symbol, this will expand to the other cells in the same column.

In the next lines, we may observe some random games generated in the MC simulation.

game: 26
15 2 4 13 5 ← reels random stop position, generating the window area below
D D E W S
C E S B A
D C A C B

==changes=to==

D D E W S
C E S W A
D C A W B

Dismissing any eventual Scatter prize, the total win for this table is: 0

game: 27
13 17 11 2 10
D C D A B
B B C D D
D A S C D

Dismissing any eventual Scatter prize, the total win for this table is: 0

game: 46
9 7 0 17 13
prizes 3S = 1 with value 5
C D B D E
C S E A E
B D S S B

Dismissing any eventual Scatter prize, the total win for this table is: 0

game: 72
11 16 17 0 11
B W C E D
E C A D D
D B W A E

==changes=to==

B W W E D
E W W D D
D W W A E

prizes 3B = 9 with value 5
prizes 5E = 9 with value 400
prizes 5D = 18 with value 150
Dismissing any eventual Scatter prize, the total win for this table is: 6345

game: 96
4 2 18 13 11
E D A W D
S E W B D
A C B C E

==changes=to==

E D A W D (the W on the first line expanded, but not the one in the second line).
S E W W D
A C B W E

prizes 5E = 3 with value 400
Dismissing any eventual Scatter prize, the total win for this table is: 1200

game: 97
6 13 1 19 11
A C E S D
B D S E D
C A D D E
Dismissing any eventual Scatter prize, the total win for this table is: 0

game: 98
14 2 18 13 1
B D A W D
D E W B A
C C B C A

==changes=to==

B D W W D (in this case both W expanded)
D E W W A
C C W W A

prizes 5D = 9 with value 150
prizes 4C = 9 with value 50
Dismissing any eventual Scatter prize, the total win for this table is: 1800

game: 99
2 19 1 6 13
A A E A E
A A S A E
E B D A B
Dismissing any eventual Scatter prize, the total win for this table is: 0

Generating a total of 1000M games we get results like:

Initial (payed) spins 1000000000
SCATTER PRIZEs won: 116831852
==========
Prizes frequency:
prize 5 → 619273390 → 0,0124
prize 10 → 272740744 → 0,0109
prize 15 → 219163355 → 0,0131
prize 20 → 520195399 → 0,0416
prize 25 → 151596821 → 0,0152
prize 50 → 1056099992 → 0,2112
prize 75 → 335803788 → 0,1007
prize 100 → 330888807 → 0,1324
prize 150 → 495874404 → 0,2975
prize 400 → 34899746 → 0,0558
total win = 222725276620
total bets = 250000000000
total spins = 1000000000
RTP = 89,0901

And the game RTP is close to 89%

Of course, if this result is too low and doesn't meet some game regulations, this value may be "fixed" changing the number of symbols in the reels, or the value of the prizes, or changing the 'expanding wild' frequency, or adding free spins to the game, etc. (and these changes were studied in the project to achieve a 96% RTP.)