Homework 6
STA 4321/5325, Fall 2019, MWF 8:30am
Professor: Aaron J. Molstad
Due date: Wednesday, October 23rd, 2019

All work must be shown for complete credit. W.M.S. denotes the course textbook (*Mathematical Statistics with Applications*). Problem #8 is worth 2pts of extra credit.

1. Let Y be a continuous random variable with mean 11 and variance 9. Using Tchebyshev's inequality, find
   (a) a lower bound for $P(6 < Y < 16)$;
   (b) the value of C such that $P(|Y - 11| \geq C) \leq 0.09$.

2. (W.M.S., 4.48) If a point is randomly located on an interval (a, b) and if Y denotes the location of the point, then Y is assumed to have a uniform distribution over (a, b). A plant efficiency expert randomly selects a location along a 500-foot assembly line from whcih to observe the work habits of the workers on the line. What is the probability that the point she selects is:
   (a) within 25 feet of the end of the line?
   (b) within 25 feet of the beginning of the line?
   (c) closer to the beginning of the line than to the end of the line?

3. (Similar to W.M.S, 4.43) A circle of radius r has area $A = \pi r^2$. If a random circle has radius that is uniformly distributed on $[0, \theta]$:
   (a) What are the mean and variance of the area of the circle?
   (b) Suppose $\theta = 1$. What is the probability distribution function of A? That is, what is $F_A(a) = P(A \leq a)$?

4. Suppose X is a random variable having the Uniform$[0, 6]$ distribution. Compute the following conditional probabilities:
   (a) $P(X \in [0, 1] \mid X < 3)$
   (b) $P(X \in [0, 4] \mid X < 3)$
   (c) $P(X \in [0, 4] \mid X \geq 3)$

5. Prove that if Y is a random variable following the Gamma distribution with parameters $\alpha > 0$ and $\beta > 0$
$$\mathrm{E}(Y^3) = \frac{\beta^3 \Gamma(3 + \alpha)}{\Gamma(\alpha)}.$$

6. Let $Z = \log(Y)$ where Z is a random variable following the standard normal distribution. Compute $\mathrm{E}(Y)$.

7. Let Z be a standard normal random variable. Then, using statistical software, we know $P(Z \leq 1) = 0.841$ and $P(Z \leq 2) = 0.977$. Using this information, answer the following: Suppose that the measured voltage in a certain electric circuit has the normal distribution with mean 120 and standard deviation 2. If three independent measurements of the voltage are made, what is the probability that all three measurements will lie between 116 and 118?

8. (Optional) Suppose X is a random variable whose distribution depends on parameters $\alpha > 0$ and $\beta > 0$ with density

$$f_X(x) = \begin{cases} \frac{\beta\alpha^\beta}{x^{\beta+1}} & \alpha < x < \infty \\ 0 & x \leq \alpha \end{cases}$$

   (a) Verify that f_X is a valid probability density function (Hint: be mindful of the $\mathrm{Range}(X)$).

   (b) Derive the expected value and variance of X. Does the variance exist (i.e., is it finite) for all possible values of α and β?