

**Exam #1 practice problems**

STA 4321/5325, Fall 2019, MWF 8:30am

Professor: Aaron J. Molstad

Solutions to these problems will be posted on Monday, September 23rd, 2019.

1. Suppose that a random variable  $X$  has a discrete distribution with the following probability mass function:

$$p_X(x) = \begin{cases} cx & \text{for } x = 1, 2, 3, 4, 5 \\ 0 & \text{otherwise} \end{cases}$$

Answer the following:

- (a) Determine the value of  $c$  so that  $p_X$  is a valid probability mass function.
  - (b) With the valid probability mass function  $p_X$  (i.e., with the correct  $c$  used in the definition of  $p_X$ ), compute:
    - i.  $E(X)$
    - ii.  $V(X)$
    - iii. A lower bound for  $P(|X - E(X)| < 2.2)$  (using Chebyshev's inequality)
2. We have five balls which are numbered 1, 2, 3, 4, and 5. Suppose that we randomly select two of these ball from the five, and record their numbers. Compute the probability distribution, the expected value, and the variance for
- (a) the largest of the two balls' numbers,  $X$
  - (b) the sum of the two balls' numbers,  $Y$
3. An engineer is studying a left-turn lane that is long enough to hold seven cars. Let  $X$  be the number of cars in the lane at the end of a randomly chosen red light. The engineer believes that the probability that  $X = x$  is proportional to

$$(x - 1)(8 - x) \quad \text{for } x = 0, 1, 2, \dots, 7 \text{ (all the values in } \mathcal{X})$$

- (a) Find the probability distribution function of  $X$ .
  - (b) Compute the probability that  $X$  will be at least 5.
4. A multiple-choice exam has 25 questions, each with four possible answers, only one of which is correct. Suppose a student answers each question with an independent random guess. What's the probability that he answers at least ten questions correctly?