Homework 4
STA 4321/5325, Fall 2019, MWF 8:30am
Professor: Aaron J. Molstad
Due date: Wednesday, October 9th, 2019

All work must be shown for complete credit. W.M.S. denotes the course textbook (*Mathematical Statistics with Applications*).

1. Let X be a random variable which denotes the number of failures in a sequence of independent Bernoulli trials, each with success probability p, before observing the rth success. That is, X follows the negative binomial distribution with success probability p and number of successes r. Prove that
$$\mathrm{E}(X) = \frac{r(1-p)}{p}.$$
Note that the negative binomial distribution described on Wikipedia is defined differently than the one in our course, and thus gives a different expected value.

2. (Similar to W.M.S, 3.97) A geological survey indicates that an exploratory oil well should strike oil with probability 0.1.
   (a) What is the probability that the first strike of oil comes on the third well drilled?
   (b) What is the probability that the third strike of oil comes on the seventh well drilled?
   (c) What assumptions did you make in answering (a) and (b)?
   (d) Find the expected value (mean) and variance of the number of wells that must be drilled in the company wants to set up four producing wells.

3. Let Y be a random variable following the Poisson distribution with expected value λ.
   (a) Prove that $\mathrm{E}\left[Y(Y-1)\right] = \lambda^2$ using the probability mass function of Y and the formula for expected value (i.e., you cannot use the variance of Y in your proof).
   (b) Using your solution to (a), and the fact that $\mathrm{E}(Y) = \lambda$ (which we will now treat as known and need not be proven), prove $\mathrm{Var}(Y) = \lambda$.

4. Suppose that on a given weekend the number of accidents at a certain intersection has the Poisson distribution with mean 0.7. What is the probability that there will be at least three accidents at the intersection during the weekend?

5. Given that we have already tossed a balanced coin ten times and obtained zero heads, what is the probability that we must toss it at least two more times to obtain the first head? What property did you use in obtaining your answer?

6. Which distribution do the following random variables follow? State their parameters explicitly, (e.g, if the random variable follows the binomial distribution, state both the values of n and p.) If the random variable does not follow any "named" distribution from class, briefly describe how you might construct a probability mass function for it.

   (a) A thumbtack will land with the point directly up 30% of the time. It is repeatedly flipped until it lands point up twice. Let U be the number of times to thumbtack is flipped.

   Let U be the number of times the thumbtack is flipped. Then, we know that $U + 2 = W$ where W is the number of times that the thumback is flipped and does not land on the point. That is, W the number of failures before the second success (i.e., W follows the negative binomial distribution with $r = 2$ and $p = 0.30$). Hence,

   $$P(U = k) = P(W = k - 2) \quad \text{for } k = 2, 3, 4, \ldots,$$

   where W is a negative binomial random variable with $r = 2$ and $p = 0.30$.

   (b) A specific textbook has a 10% chance of falling apart after a semester of use. If a class of 30 all has this textbook, let V be the number of textbooks that have fallen apart by the end of the semester.

   (c) An archer hits her target with probability 0.6. Let W be the number of arrows she shoots in order to hit the target 5 times.

   (d) A machine that makes pencils makes correctly made pencils with probability 0.99. The boss asks to see what a defective pencil looks like, and waits until a defective pencil is made. Let X be the number of pencils that the boss watches made.

   (e) (continued from (d)...) If 10 pencils come in a box, let Y be the number of correctly made pencils in a given box.

   (f) I have 10 pencils in my drawer, but only two are sharpened. I randomly choose pencils from my drawer one at a time until I find a sharp one. I don't bother to return the unsharpened pencils to my drawer before choosing again. Let Z be the number of pencils I choose from my drawer.

7. (Similar to W.M.S, 3.104) Thirty identical looking packets of white powder are such that 20 contain cocaine and 10 do not. Four packets are randomly selected, and the contents were tested and found to contain cocaine. Two additional packets were selected from the remainder and sold by undercover police officers to a single buyer. What is the probability that the 6 packets randomly selected are such that the first four all contain cocaine and the two sold to the buyer do not?