Solutions to these problems will be posted on Monday, September 23rd, 2019.

1. Suppose that a random variable X has a discrete distribution with the following probability mass function:
$$p_X(x) = \begin{cases} cx & \text{for } x = 1, 2, 3, 4, 5 \\ 0 & \text{otherwise} \end{cases}$$
Answer the following:

   (a) Determine the value of c so that p_X is a valid probability mass function.
   (b) With the valid probability mass function p_X (i.e., with the correct c used in the definition of p_X), compute:
      i. $E(X)$
      ii. $V(X)$
      iii. A lower bound for $P(|X - E(X)| < 2.2)$ (using Chebyshev's inequality)

2. We have five balls which are numbered 1, 2, 3, 4, and 5. Suppose that we randomly select one ball, put it back, and then select another ball, and record their numbers. Compute the probability mass function, the expected value, and the variance for

   (a) the largest of the two balls' numbers, Y
   (b) the sum of the two balls' numbers, X

3. We have five balls which are numbered 1, 2, 3, 4, and 5. Suppose that we randomly select two balls simultaneously and record their numbers. Compute the probability mass function, the expected value, and the variance for

   (a) the largest of the two balls' numbers, Y
   (b) the sum of the two balls' numbers, X

4. An engineer is studying a left-turn lane that is long enough to hold seven cars. Let X be the number of cars in the lane at the end of a randomly chosen red light. The engineer believes that the probability that $X = x$ is proportional to
$$(x + 1)(8 - x) \quad \text{for } x = 0, 1, 2, \ldots, 7 \text{ (all the values in } \mathfrak{X})$$

   (a) Find the probability distribution function of X.
   (b) Compute the probability that X will be at least 5.

5. A multiple-choice exam has 25 questions, each with four possible answers, only one of which is correct. Suppose a student answers each question with an independent random guess. What's the probability that he answers at least ten questions correctly?