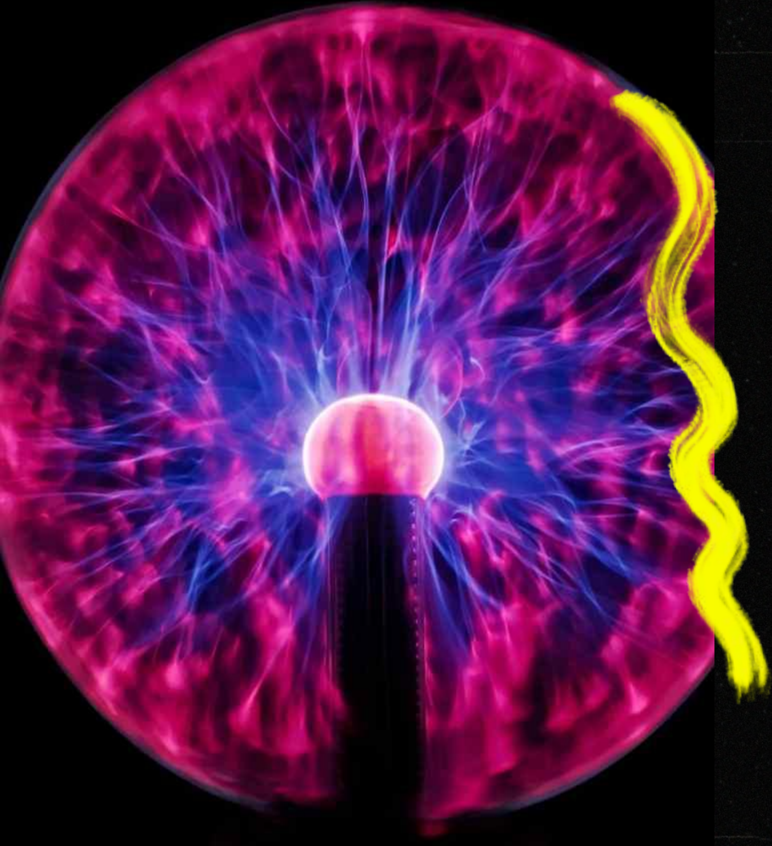

Introducción básica a la Computación Cuántica ICC01

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Introducción física

Haremos una breve introducción a la física necesaria para iniciarse en la computación cuántica

Hay mucho más, pero esto es lo más básico para empezar:

- Sistema cuántico
- Operaciones cuánticas
- Qubit y Puertas cuánticas
- Circuitos cuánticos

Estado físico

Vídeo explicativo de todo
https://youtu.be/6QAm1a7ZIXw?si=Wdm7YopgTZir0Tdc

Un elemento

$$\vec{v} = v_{\uparrow}\hat{\uparrow} + v_{\downarrow}\hat{\downarrow}$$

Dos elementos

$$v = v_{\uparrow\uparrow}\hat{\uparrow}\hat{\uparrow} + v_{\uparrow\downarrow}\hat{\uparrow}\hat{\downarrow} + v_{\downarrow\uparrow}\hat{\downarrow}\hat{\uparrow} + v_{\downarrow\downarrow}\hat{\downarrow}\hat{\downarrow}$$

Bit clásico

$$\vec{b} = b_0\hat{b_0} + b_1\hat{b_1}$$

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Qubit

Notación de Dirac

Notación original	Notación de Dirac
$\vec{v}$	$\|v\rangle$
$\vec{v}^\dagger$	$\langle v\| = (\|v\rangle)^\dagger$
$\vec{v}\cdot\vec{w}$	$\langle v\|w\rangle$
$A\vec{v}$	$A\|v\rangle$
$\vec{v}^\dagger A$	$\langle v\|A = (A^\dagger\|v\rangle)^\dagger$
$\vec{v}^\dagger A\vec{w}$	$\langle v\|A\|w\rangle$
$\vec{v}^\dagger A\vec{v}$	$\langle A\rangle_v$
$\vec{v}\otimes\vec{w}$	$\|v\rangle\otimes\|w\rangle = \|v,w\rangle = \|vw\rangle$

$$|v\rangle = \sum_i v_i |b_i\rangle$$

$$A = \sum_{ij} A_{ij} |b_i\rangle \langle b_j|$$

$$A|v\rangle = \sum_{ijk} A_{ij} v_k |b_i\rangle \langle b_j\rangle b_k = \sum_{ij} A_{ij} v_j |b_i\rangle$$

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Qubit

Estado de un qubit

$$|\psi\rangle = \psi_0 |0\rangle + \psi_1 |1\rangle$$

Condición de normalización

$$|\psi_0|^2 + |\psi_1|^2 = 1$$

$$|\psi\rangle = \cos(\theta)|0\rangle + e^{i\varphi}\sin(\theta)|1\rangle, \qquad \theta, \varphi \in \mathbb{R}$$

Cambio de base

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \qquad |i\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle).$$

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \qquad |-i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle).$$

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Qubit

Estado de dos qubits

$$|\psi\rangle = \psi_{00}\,|0\rangle\otimes|0\rangle + \psi_{01}\,|0\rangle\otimes|1\rangle + \psi_{10}\,|1\rangle\otimes|0\rangle + \psi_{11}\,|1\rangle\otimes|1\rangle = \sum_{ij}\psi_{ij}\,|i\rangle\otimes|j\rangle = \sum_{ij}\psi_{ij}\,|ij\rangle$$

Estado de 3 qubits

$$|\psi\rangle = \sum_{ijk}\psi_{ijk}\,|ijk\rangle$$

Estado de N qubits

$$|\psi\rangle = \sum_{i_0,i_1,\ldots,i_{N-1}\in\{0,1\}^N}\psi_{i_0,i_1,\ldots,i_{N-1}}\,|i_0,i_1,\ldots,i_{N-1}\rangle$$

$$|\psi\rangle = \sum_{i=0}^{2^N-1}\psi_i\,|i\rangle$$

$$\sum_{i_0,i_1,\ldots,i_{N-1}\in\{0,1\}^N}|\psi_{i_0,i_1,\ldots,i_{N-1}}|^2$$

$$\sum_{i=0}^{2^N-1}|\psi_i|^2$$

Qubit

$$|0\rangle^{\otimes N} = |0\rangle \otimes |0\rangle \otimes \cdots \otimes |0\rangle$$

$$|+\rangle^{\otimes N} = |+\rangle \otimes |+\rangle \otimes \cdots \otimes |+\rangle$$

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

$$|GHZ\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

$$|W\rangle = \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle)$$

$$|W\rangle = \frac{1}{\sqrt{N}}(|00\ldots1\rangle + \cdots + |01\ldots0\rangle + |10\ldots0\rangle)$$

Puertas cuánticas

$$|\psi\rangle = \sum_{ijkl} \psi_{ijkl} |i\rangle \otimes |j\rangle \otimes |k\rangle \otimes |l\rangle$$

Puerta single-qubit

$$U(\theta, \phi, \lambda) = \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) & -e^{i\lambda} \sin\left(\frac{\theta}{2}\right) \\ e^{i\phi} \sin\left(\frac{\theta}{2}\right) & e^{i(\phi+\lambda)} \cos\left(\frac{\theta}{2}\right) \end{pmatrix}$$

Nuestro estado

$$V = \mathbb{I} \otimes \mathbb{I} \otimes U(\theta, \phi, \lambda) \otimes \mathbb{I}$$

Aplicar la puerta U en el tercer qubit

$$V|\psi\rangle = \sum_{ijkl} \psi_{ijkl} (\mathbb{I} \otimes \mathbb{I} \otimes U(\theta, \phi, \lambda) \otimes \mathbb{I}) |i\rangle \otimes |j\rangle \otimes |k\rangle \otimes |l\rangle$$

$$V|\psi\rangle = \sum_{ijkl} \psi_{ijkl} \mathbb{I}|i\rangle \otimes \mathbb{I}|j\rangle \otimes U(\theta, \phi, \lambda)|k\rangle \otimes \mathbb{I}|l\rangle = \sum_{ijkl} \psi_{ijkl} |i\rangle \otimes |j\rangle \otimes U(\theta, \phi, \lambda)|k\rangle \otimes |l\rangle$$

$$V|\psi\rangle = \sum_{ijkl} \sum_{nm} U_{nm} \psi_{ijkl} |i\rangle \otimes |j\rangle \otimes |n\rangle \langle m|k\rangle \otimes |l\rangle = \sum_{ijnl} \sum_{k} U_{nk} \psi_{ijkl} |ijnl\rangle$$

$$V = \mathbb{I} \otimes \mathbb{I} \otimes \cdots \otimes U \otimes \cdots \otimes \mathbb{I}$$

$$V|\psi\rangle = \sum_{i_0, i_1, \ldots, i_N} \sum_{k} U_{k, i_j} \psi_{i_0, i_1, \ldots, i_N} |i_0, i_1, \ldots, i_{j-1}, k, i_{j+1}, \ldots, i_N\rangle$$

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Puertas cuánticas

Puerta multiqubit

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{aligned} |00\rangle &= |0\rangle \to |00\rangle = |0\rangle \\ |01\rangle &= |1\rangle \to |01\rangle = |1\rangle \\ |10\rangle &= |2\rangle \to |11\rangle = |3\rangle \\ |11\rangle &= |3\rangle \to |10\rangle = |2\rangle \end{aligned}$$

$$V = \mathbb{I} \otimes U \otimes \mathbb{I}$$

$$V|\psi\rangle = \sum_{ijkl} \psi_{ijkl} \mathbb{I}|i\rangle \otimes U(|j\rangle \otimes |k\rangle) \otimes \mathbb{I}|l\rangle$$

$$V|\psi\rangle = \sum_{ijklmn} U_{2m+n,2j+k} \psi_{ijkl} |i,m,n,l\rangle$$

$$U_{2m+n,2j+k} = |m,n\rangle \langle j,k|$$

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Puertas cuánticas

$$|\psi\rangle = \sum_{i_0,i_1,\ldots,i_j,i_{j+1},\ldots,i_{j+n-1},\ldots,i_N} \psi_{i_0,i_1,\ldots,i_j,i_{j+1},\ldots,i_{j+n-1},\ldots,i_N} |i_0,i_1,\ldots,i_j,i_{j+1},\ldots,i_{j+n-1},\ldots,i_N\rangle$$

$$|\psi\rangle = \sum_{i_0,i_1,\ldots,m,\ldots,i_N} \psi_{i_0,i_1,\ldots,m,\ldots,i_N} |i_0,i_1,\ldots,m,\ldots,i_N\rangle$$

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$$V = \mathbb{I} \otimes \mathbb{I} \otimes \cdots \otimes U \otimes \cdots \otimes \mathbb{I}$$

$$V|\psi\rangle = \sum_{i_0,i_1,\ldots,m,\ldots,i_N} \sum_k U_{k,m} \psi_{i_0,i_1,\ldots,m,\ldots,i_N} |i_0,i_1,\ldots,k,\ldots,i_N\rangle$$

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & U_{0,0} & U_{0,1} \\ 0 & 0 & U_{1,0} & U_{1,1} \end{pmatrix}$$

Puerta controlada

Puertas cuánticas

Ejemplos

$$\sigma_X = \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_Y = \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_Z = \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Sigmas de Pauli

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Hadamard

$$S = \sqrt{Z} = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

$$T = \sqrt{S} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$

$$P(\lambda) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\lambda} \end{pmatrix}$$

Fase

$$U(\theta, \phi, \lambda) = \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) & -e^{i\lambda}\sin\left(\frac{\theta}{2}\right) \\ e^{i\phi}\sin\left(\frac{\theta}{2}\right) & e^{i(\phi+\lambda)}\cos\left(\frac{\theta}{2}\right) \end{pmatrix}$$

General single-qubit

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Puertas cuánticas

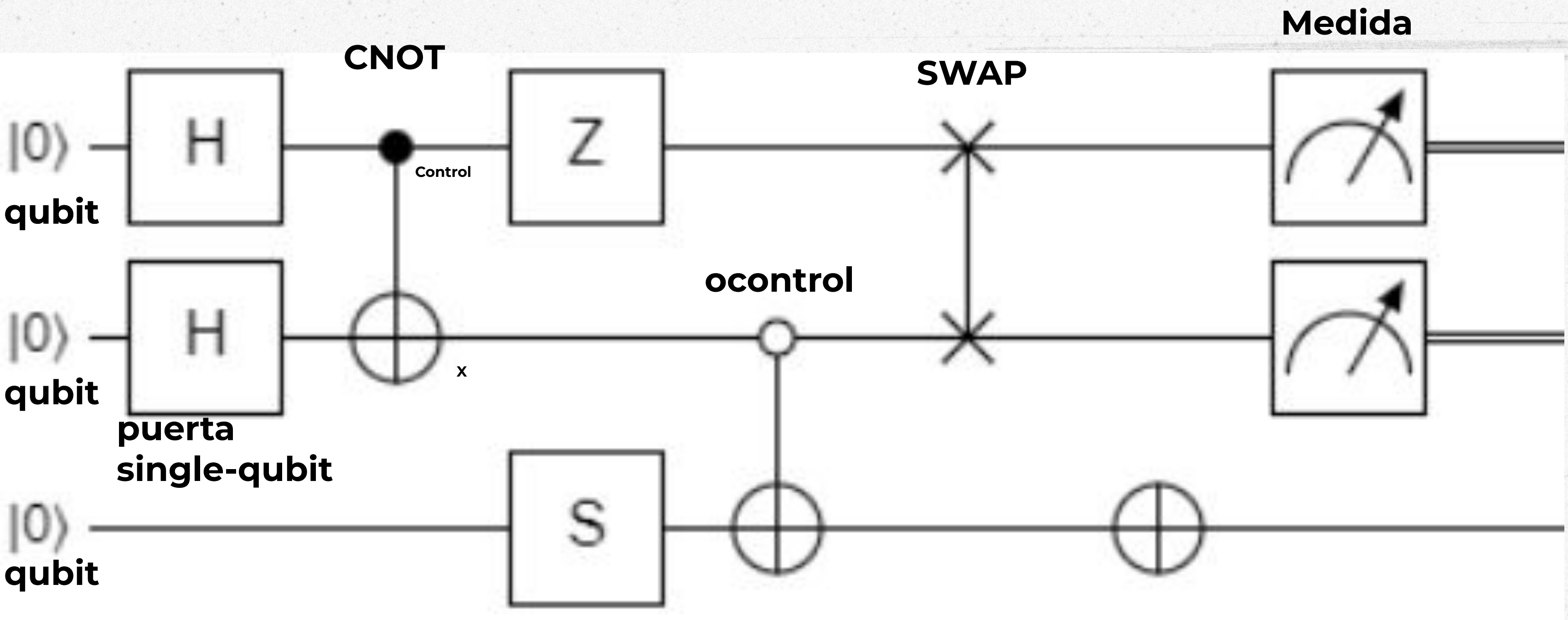
$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & U_{0,0} & U_{0,1} \\ 0 & 0 & U_{1,0} & U_{1,1} \end{pmatrix}$$

$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$CZ = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$SWAP = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Circuitos cuánticos



https://algassert.com/quirk#circuit={%22cols%22:[]}

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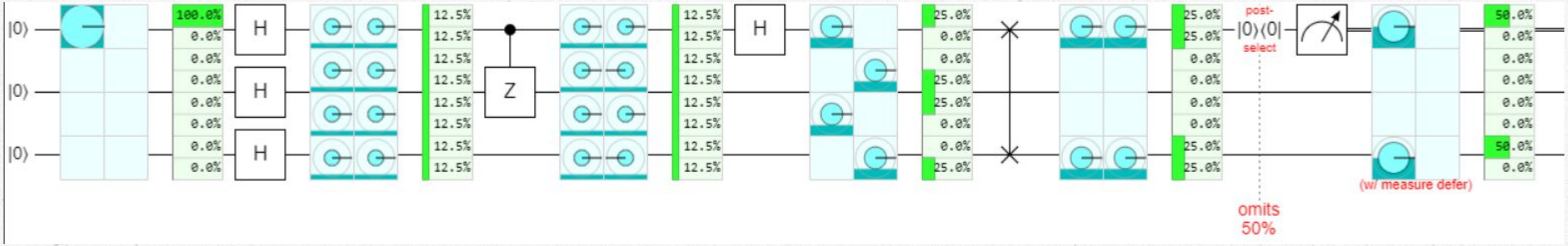
Circuitos cuánticos

Ejemplo



https://algassert.com/quirk#circuit={%22cols%22:[[%22Amps3%22],[],[%22Chance3%22],[%22H%22,%22H%22,%22H%22],[%22Amps3%22],[],[%22Chance3%22],[%22%E2%80%A2%22,%22Z%22],[%22Amps3%22],[],[%22Chance3%22],[%22H%22],[%22Amps3%22],[],[%22Chance3%22],[%22Swap%22,1,%22Swap%22],[%22Amps3%22],[],[%22Chance3%22],[%22|0%E2%9F%A9%E2%9F%A80|%22],[%22Measure%22],[%22Amps3%22],[],[%22Chance3%22]]}

¡Gracias!

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