

Day 4: Classification

Summer STEM: Machine Learning

Department of Electrical and Computer Engineering
NYU Tandon School of Engineering
Brooklyn, New York

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Outline

- 1 Review
- 2 Non-linear Optimization
- 3 Logistic Regression
- 4 Lab: Diagnosing Breast Cancer
- 5 Multiclass Classification
- 6 Lab: Iris Dataset

Review

- Imagine you are preparing for the SATs and you come across a book full of practice questions you did not understand how to solve any of the problems. However, you memorized all of the answers.
- What do you think will happen if you try to solve practice questions in a different book.
- Why are you studying actual problem solving techniques instead of just memorizing solutions from practice questions?
- Assuming you have an eidetic memory will memorizing solutions from practice questions be a good strategy?

Review

- $J(w) = \frac{1}{N} \|Y - Xw\|^2 + \lambda \|w\|^2$
- $w = [10000, 20000, 30000, 10000]$ does this look good?

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Motivation

- Cannot rely on closed form solutions
 - Computation efficiency: operations like inverting a matrix is not efficient
 - For more complex problems such as neural networks, a closed-form solution is not always available
- Need an optimization technique to find an optimal solution
 - Machine learning practitioners use **gradient**-based methods

Gradient Descent Algorithm

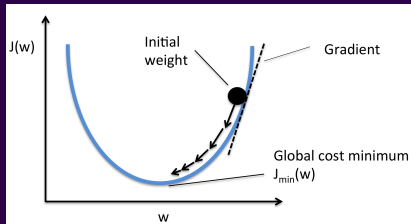
■ Update Rule

Repeat{

$$\mathbf{w}_{new} = \mathbf{w} - \alpha \nabla J(\mathbf{w})$$

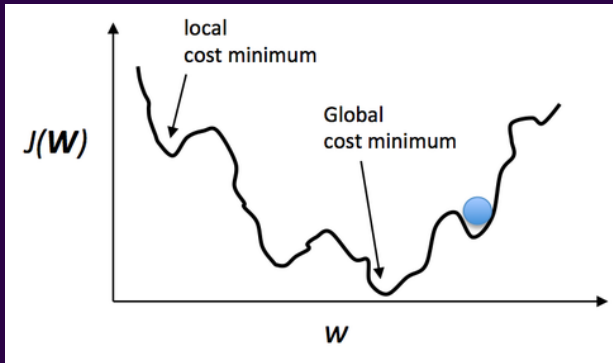
}

α is the learning rate

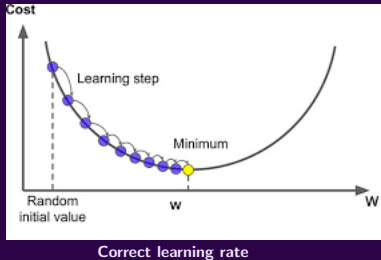
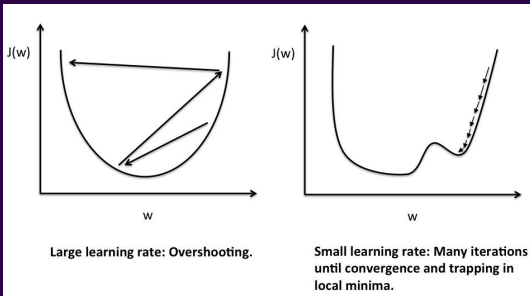


General Loss Function Contours

- Most loss function contours are not perfectly parabolic
- Our goal is to find a solution that is very close to global minimum by the right choice of hyper-parameters



Understanding Learning Rate



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Classification Vs. Regression

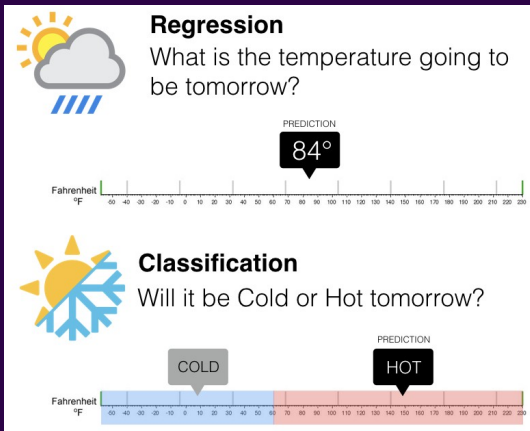
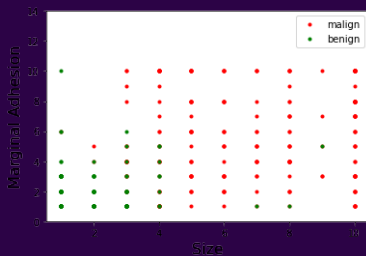
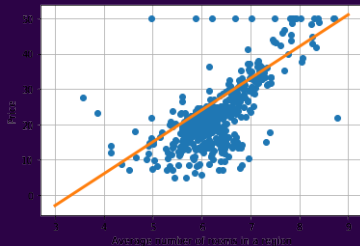


Figure: <https://www.pinterest.com/pin/672232681855858622/?lp=true>

Classification Vs. Regression



(a) Breast cancer dataset

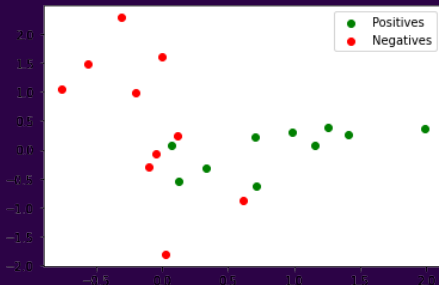


(b) Boston Housing dataset

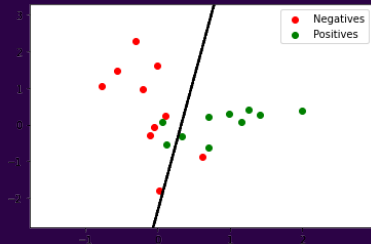
Classification

Given the dataset (x_i, y_i) for $i = 1, 2, \dots, N$, find a function $f(x)$ (model) so that it can predict the label \hat{y} for some input x , even if it is not in the dataset, i.e. $\hat{y} = f(x)$.

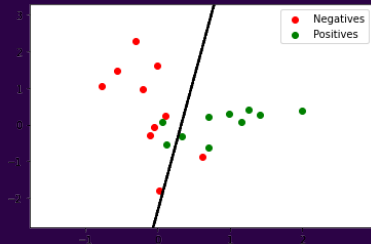
- Positive : $y = 1$
- Negative : $y = 0$



Decision Boundary



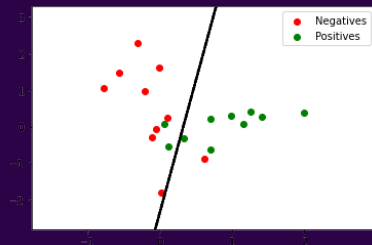
Decision Boundary



- Evaluation metric :

$$\text{Accuracy} = \frac{\text{Number of correct prediction}}{\text{Total number of prediction}}$$

- What is the accuracy in this example ?



■ Evaluation metric :

$$\text{Accuracy} = \frac{\text{Number of correct prediction}}{\text{Total number of prediction}} = \frac{17}{20} = 0.85 = 85\%$$

Need for a new model

- What would happen if we used the linear regression model :

$$\hat{y} = w_0 + w_1x$$

Need for a new model

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- y is 0 or 1
- \hat{y} will take any value between $-\infty$ and ∞

Need for a new model

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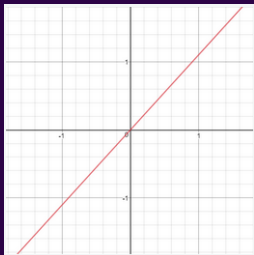
$$\hat{y} = w_0 + w_1 x$$

- y is 0 or 1
- \hat{y} will take any value between $-\infty$ and ∞
- It will be hard to find w_0 and w_1 that make the prediction \hat{y} match the label y .

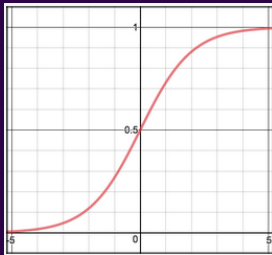
Sigmoid Function

- By applying the sigmoid function, we enforce $0 \leq \hat{y} \leq 1$

$$\hat{y} = \text{sigmoid}(w_0 + w_1x) = \frac{1}{1 + e^{-(w_0 + w_1x)}}$$



(a) Linear model



(b) Sigmoid model

A new loss function

- Binary cross entropy loss :

$$\text{Loss} = \frac{1}{N} \sum_{i=1}^N \left[-y_i \log(\hat{y}_i) - (1 - y_i) \log(1 - \hat{y}_i) \right]$$

pause

- What happens if $y_i = 0$:

$$\left[-y_i \log(\hat{y}_i) - (1 - y_i) \log(1 - \hat{y}_i) \right] = ?$$

A new loss function

- Binary cross entropy loss :

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- If $y_i = 0$:

$$\left[-y_i \log(\hat{y}_i) - (1 - y_i) \log(1 - \hat{y}_i) \right] = -\log(1 - \hat{y}_i)$$

A new loss function

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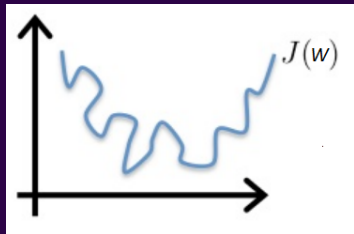
$$\left[-y_i \log(\hat{y}_i) - (1 - y_i) \log(1 - \hat{y}_i) \right] = -\log(1 - \hat{y}_i)$$

- If $y_i = 1$:

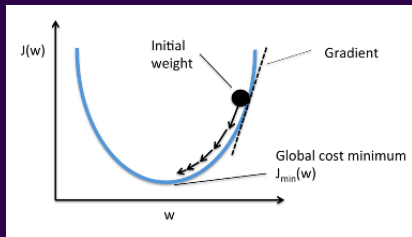
$$\left[-y_i \log(\hat{y}_i) - (1 - y_i) \log(1 - \hat{y}_i) \right] = -\log(\hat{y}_i)$$

MSE vs Binary cross entropy loss

- MSE of a logistic function has many local minima.
- The Binary cross entropy loss has only one minimum.



(a) MSE



(b) Binary cross entropy loss

Classifier

$$\hat{y} = \text{sigmoid}(w_0 + w_1 x) = \frac{1}{1 + e^{-(w_0 + w_1 x)}}$$

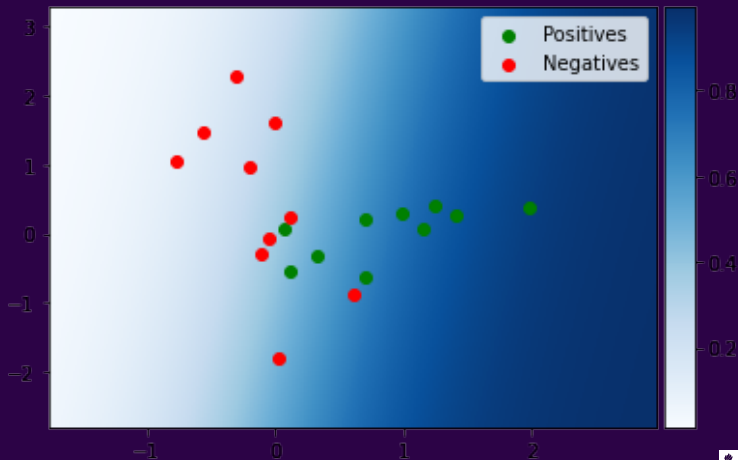
- How to deal with uncertainty ?
 - Thanks to the sigmoid, $\hat{y} = f(x)$ is between 0 and 1.

Classifier

$$\hat{y} = \text{sigmoid}(w_0 + w_1 x) = \frac{1}{1 + e^{-(w_0 + w_1 x)}}$$

- How to deal with uncertainty ?
 - Thanks to the sigmoid, $\hat{y} = f(x)$ is between 0 and 1.
- If \hat{y} is close to 0, the data is probably negative
- If \hat{y} is close to 1, the data is probably positive
- If \hat{y} is around 0.5, we are not sure.

Classifier



Decision Boundary

- Once, we have a classifier outputting a score $0 < \hat{y} < 1$, we need to create a decision rule.

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- Let $0 < t < 1$ be a Threshold :
 - If $\hat{y} > t$, \hat{y} is classified as positive.
 - If $\hat{y} < t$, \hat{y} is classified as negative.

Decision Boundary

- Once, we have a classifier outputting a score $0 < \hat{y} < 1$, we need to create a decision rule.
- Let $0 < t < 1$ be a Threshold :
 - If $\hat{y} > t$, \hat{y} is classified as positive.
 - If $\hat{y} < t$, \hat{y} is classified as negative.
- How to choose t ?

Impact of the threshold

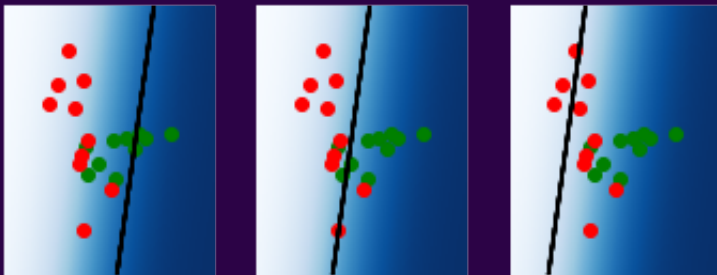


Figure: $t = 0.2, 0.5, 0.8$

Performance metrics for a classifier

- Accuracy of a classifier: percentage of correct classification
- Why accuracy alone is not a good measure for assessing the model ?

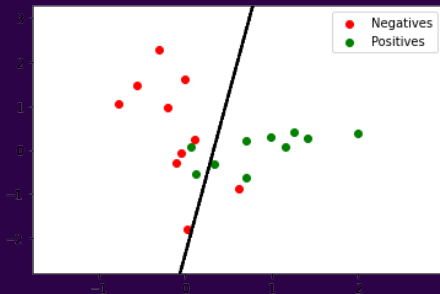
Performance metrics for a classifier

- Accuracy of a classifier: percentage of correct classification
- Why accuracy alone is not a good measure for assessing the model ?
 - Example: A rare disease occurs 1 in ten thousand people
 - A test that classifies everyone as free of the disease can achieve 99.999% accuracy when tested with people drawn randomly from the entire population

Types of Errors in Classification

- Correct predictions:
 - True Positive (TP) : Predict $\hat{y} = 1$ when $y = 1$
 - True Negative (TN) : Predict $\hat{y} = 0$ when $y = 0$
- Two types of errors:
 - False Positive/ False Alarm (FP): $\hat{y} = 1$ when $y = 0$
 - False Negative/ Missed Detection (FN): $\hat{y} = 0$ when $y = 1$

Example



- How many True Positive (TP) are there ?
- How many True Negative (TN) are there ?
- How many False Positive (FP) are there ?
- How many False Negative (FN) are there ?

Other metrics

- Sensitivity/Recall/TPR (How many positives are detected among all positive?)

$$\frac{TP}{TP + FN}$$

- Precision (How many detected positives are actually positive?)

$$\frac{TP}{TP + FP}$$

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Lab: Diagnosing Breast Cancer

- We're going to use the breast cancer dataset to predict whether the patients' scans show a malignant tumour or a benign tumour.
- Let's try to find the best linear classifier using logistic regression.

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Multiclass Classification

- Previous model: $f(\mathbf{x}) = \sigma(\phi(\mathbf{x})w)$
- Representing Multiple Classes:
 - One-hot / 1-of-K vectors, ex : 4 Class
 - Class 1 : $\mathbf{y} = [1, 0, 0, 0]$
 - Class 2 : $\mathbf{y} = [0, 1, 0, 0]$
 - Class 3 : $\mathbf{y} = [0, 0, 1, 0]$
 - Class 4 : $\mathbf{y} = [0, 0, 0, 1]$

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 - Class 3 : $\mathbf{y} = [0, 0, 1, 0]$
 - Class 4 : $\mathbf{y} = [0, 0, 0, 1]$
- Multiple outputs: $f(\mathbf{x}) = \text{softmax}(\phi(\mathbf{x})W)$
- Shape of $\phi(\mathbf{x})W$: $(N, K) = (N, D) \times (D, K)$
- $\text{softmax}(\mathbf{z})_k = \frac{e^{z_k}}{\sum_j e^{z_j}}$

Multiclass Classification

- Multiple outputs: $f(\mathbf{x}) = \text{softmax}(\mathbf{z})$ with $\mathbf{z} = \phi(\mathbf{x})W$

- $\text{softmax}(\mathbf{z})_k = \frac{e^{z_k}}{\sum_j e^{z_j}}$

- Softmax example: If $\mathbf{z} = \begin{bmatrix} -1 \\ 2 \\ 1 \\ -4 \end{bmatrix}$ then,

$$\text{softmax}(\mathbf{z}) = \begin{bmatrix} \frac{e^{-1}}{e^{-1}+e^2+e^1+e^{-4}} \\ \frac{e^2}{e^{-1}+e^2+e^1+e^{-4}} \\ \frac{e^1}{e^{-1}+e^2+e^1+e^{-4}} \\ \frac{e^{-4}}{e^{-1}+e^2+e^1+e^{-4}} \end{bmatrix} \approx \begin{bmatrix} 0.035 \\ 0.704 \\ 0.259 \\ 0.002 \end{bmatrix}$$

Cross-entropy

- Multiple outputs: $\hat{\mathbf{y}}_i = \text{softmax}(\phi(\mathbf{x}_i)W)$
- Cross-Entropy: $J(W) = - \sum_{i=1}^N \sum_{k=1}^K \mathbf{y}_{ik} \log(\hat{\mathbf{y}}_{ik})$
- Example : $K = 4$

$$\text{If, } \mathbf{y}_i = [0, 0, 1, 0] \text{ then, } \sum_{k=1}^K \mathbf{y}_{ik} \log(\hat{\mathbf{y}}_{ik}) = \log(\hat{\mathbf{y}}_{i3})$$

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Lab: Iris Dataset

- Open `demo_iris.ipynb`