Barycentric Interpolation

How do you interpolate values defined at vertices across the entire triangle?

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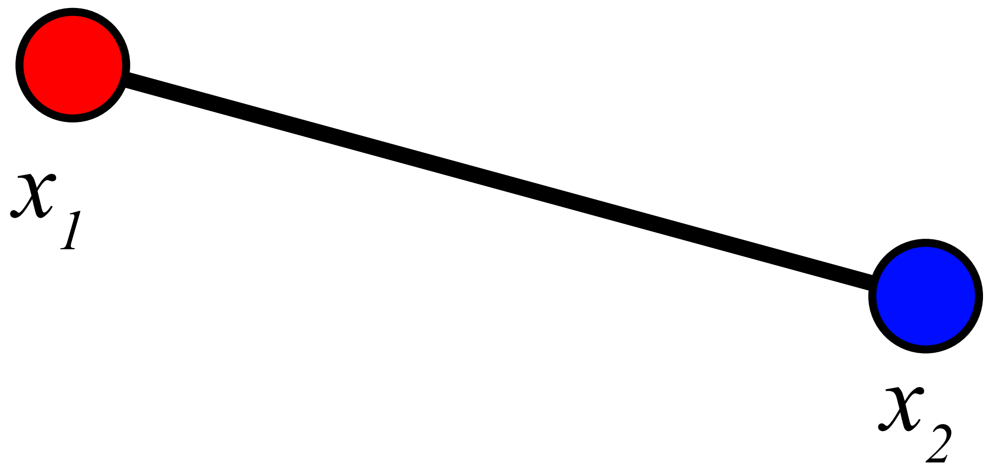
Solve a simpler problem first:



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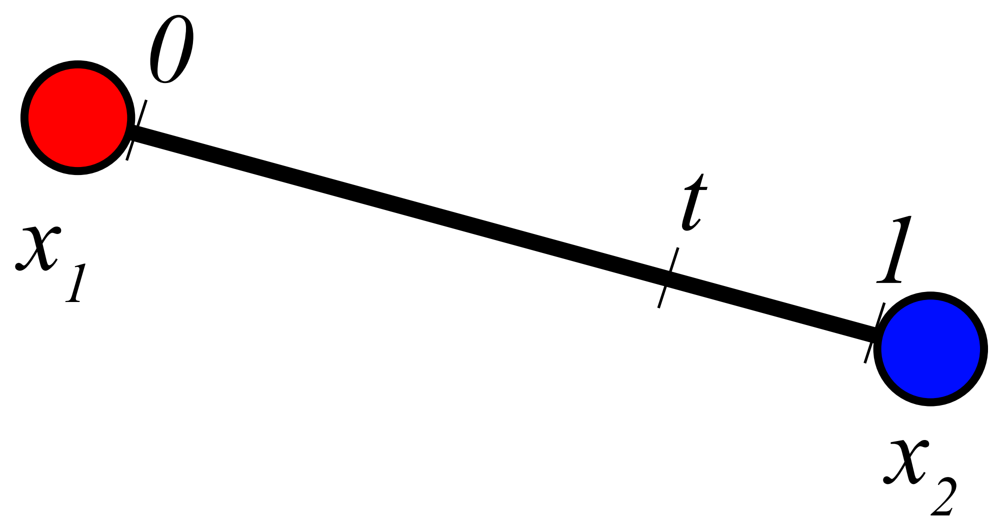
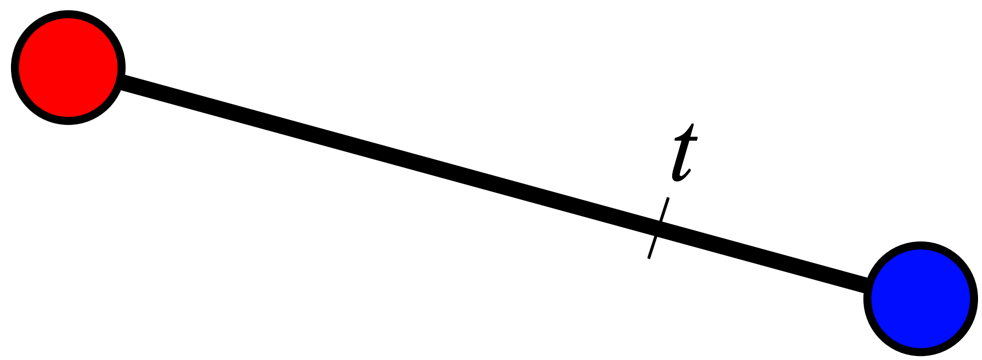


Want to define a value for every $t \in [0,1]$:

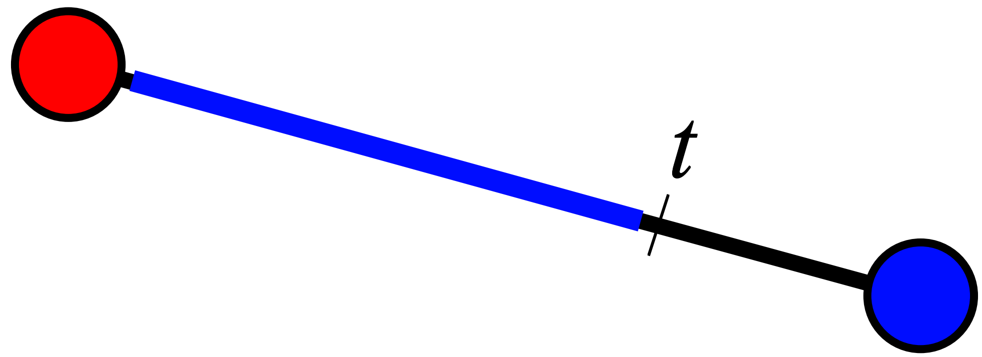
Barycentric Interpolation

How do we come up with this equation?

Look at the picture!

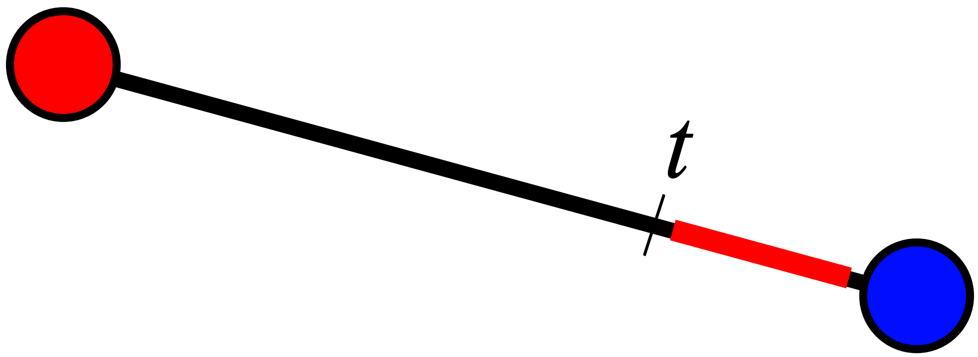


Barycentric Interpolation



The further t is from the red point, the more blue we want.

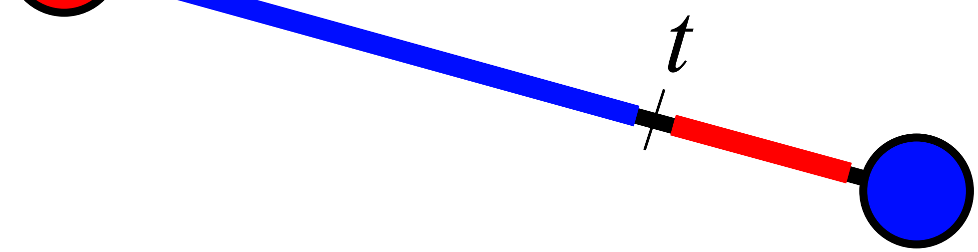
Barycentric Interpolation



The further t is from the red point, the more blue we want.

The further t is from the blue point, the more red we want.

Barycentric Interpolation



The further t is from the red point, the more blue we want.

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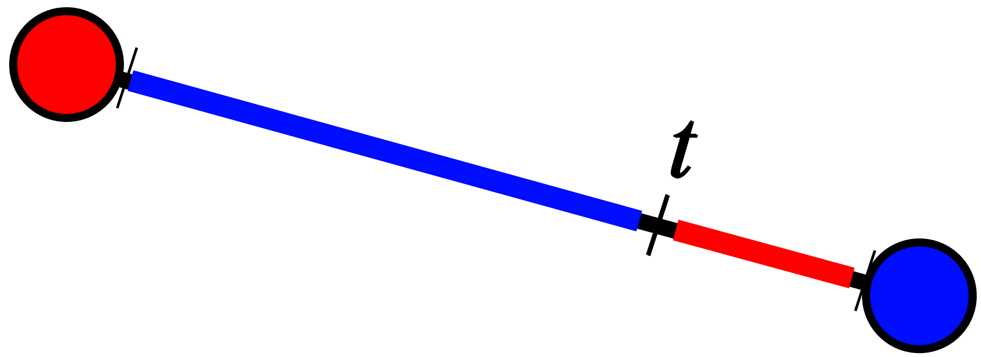
Percent blue = (length of blue segment)/(total length)

Barycentric Interpolation



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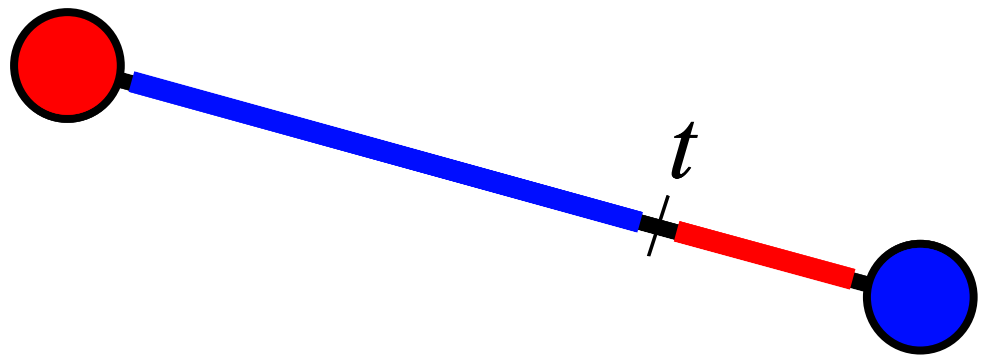
Barycentric Interpolation



The further t is from the red point, the more blue we want.

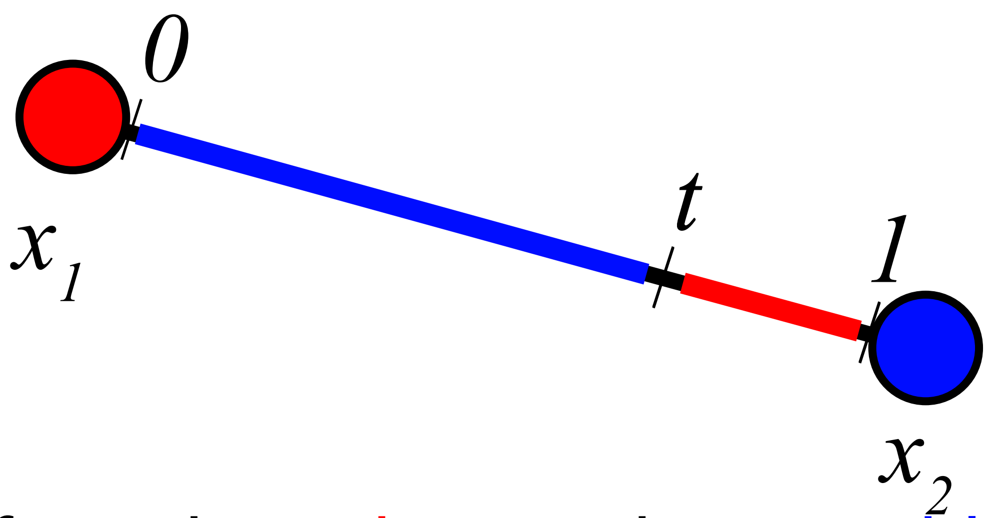
The further t is from the blue point, the more red we want.

Percent blue = (length of blue segment)/(total length)

Percent red = (length of red segment)/(total length)

Value at t = (% blue)(value at blue) + (% red)(value at red)

Barycentric Interpolation



The further t is from the red point, the more blue we want.

The further t is from the blue point, the more red we want.

Percent blue = t

Percent red = $1-t$

Value at $t = tx_1 + (1-t)x_2$

Barycentric Interpolation

Now what about triangles?

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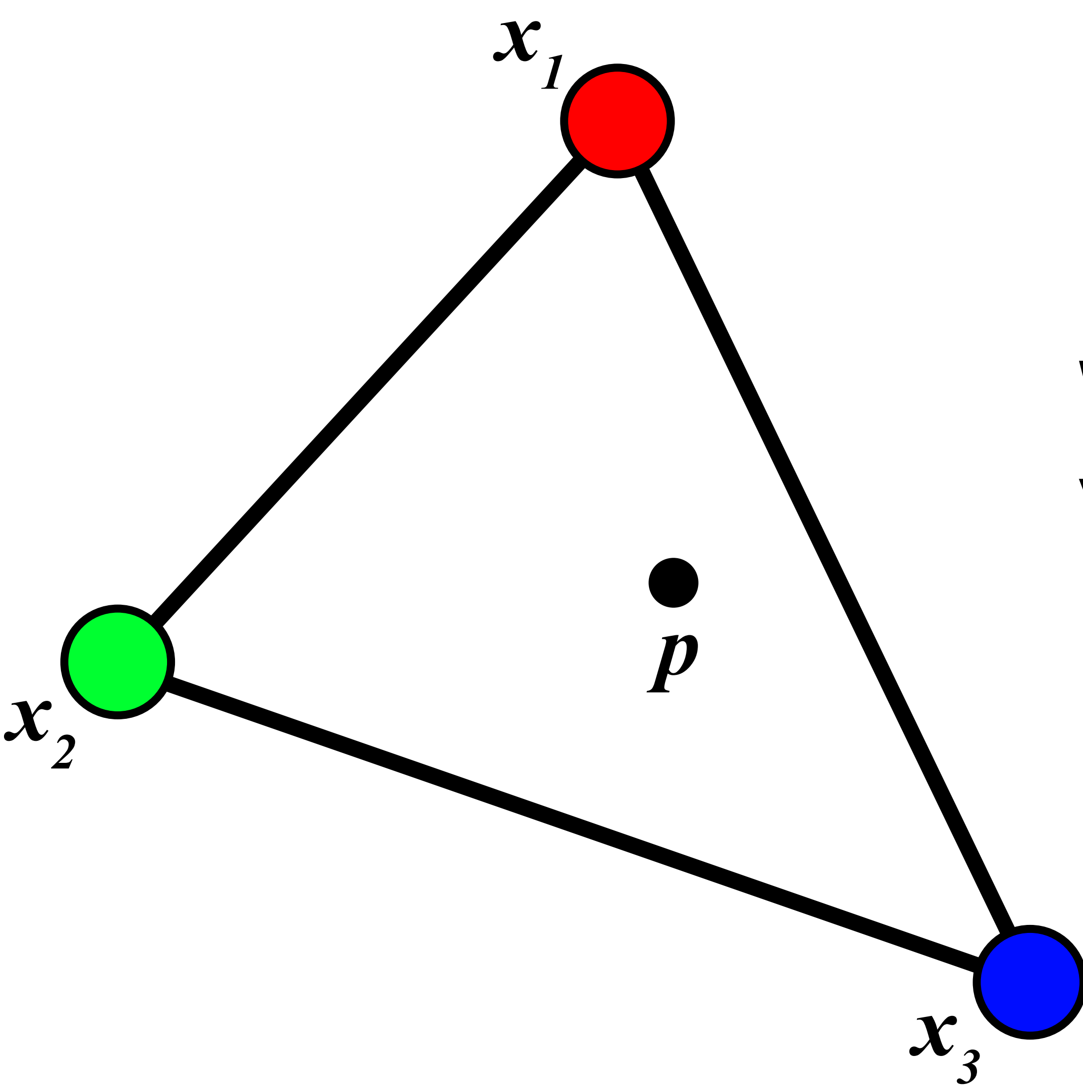
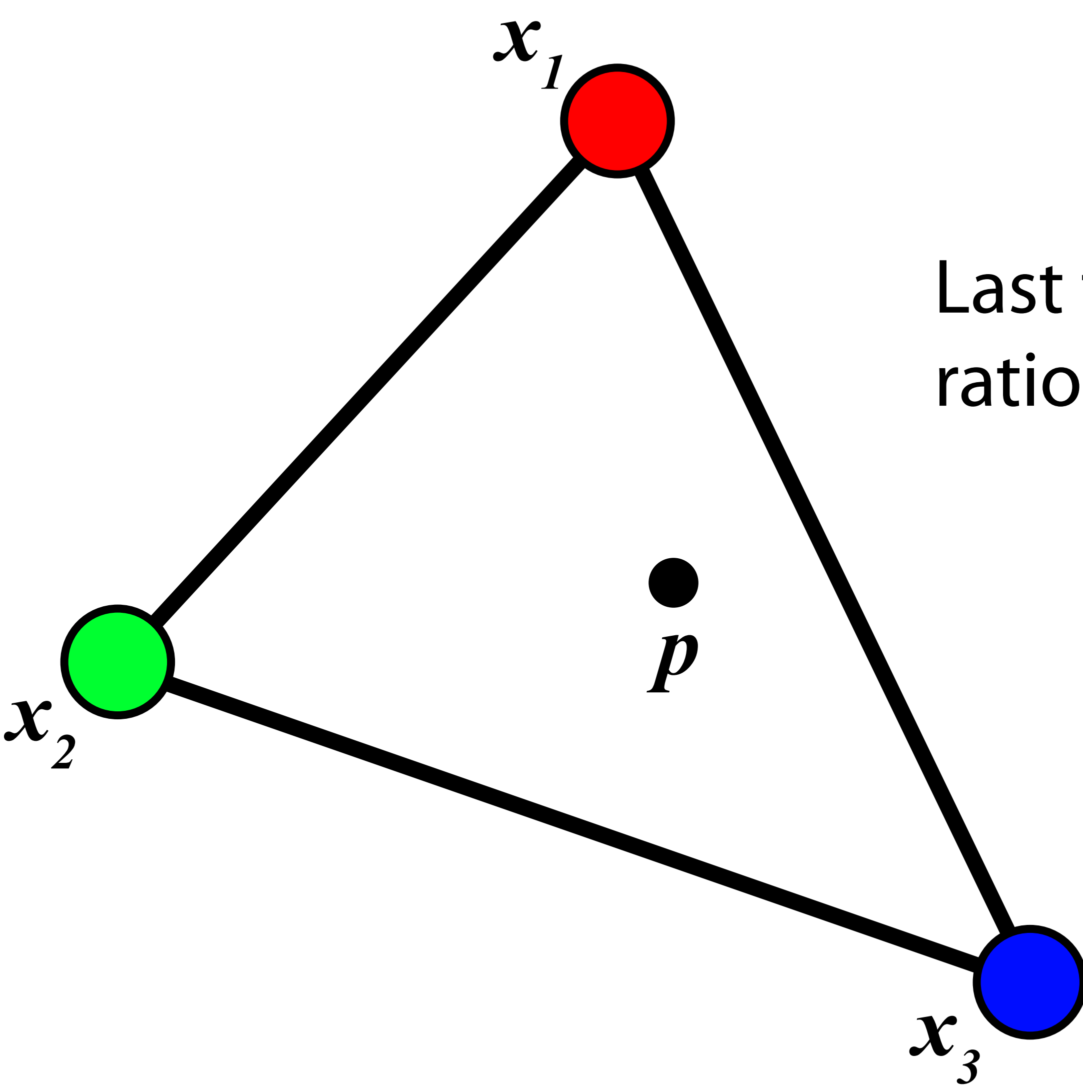
Just consider the geometry:

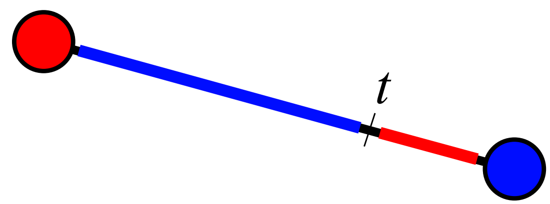


What's the interpolated value at the point p?

Barycentric Interpolation



Last time (in 1D) we used ratios of lengths.



Barycentric Interpolation

Now what about triangles?

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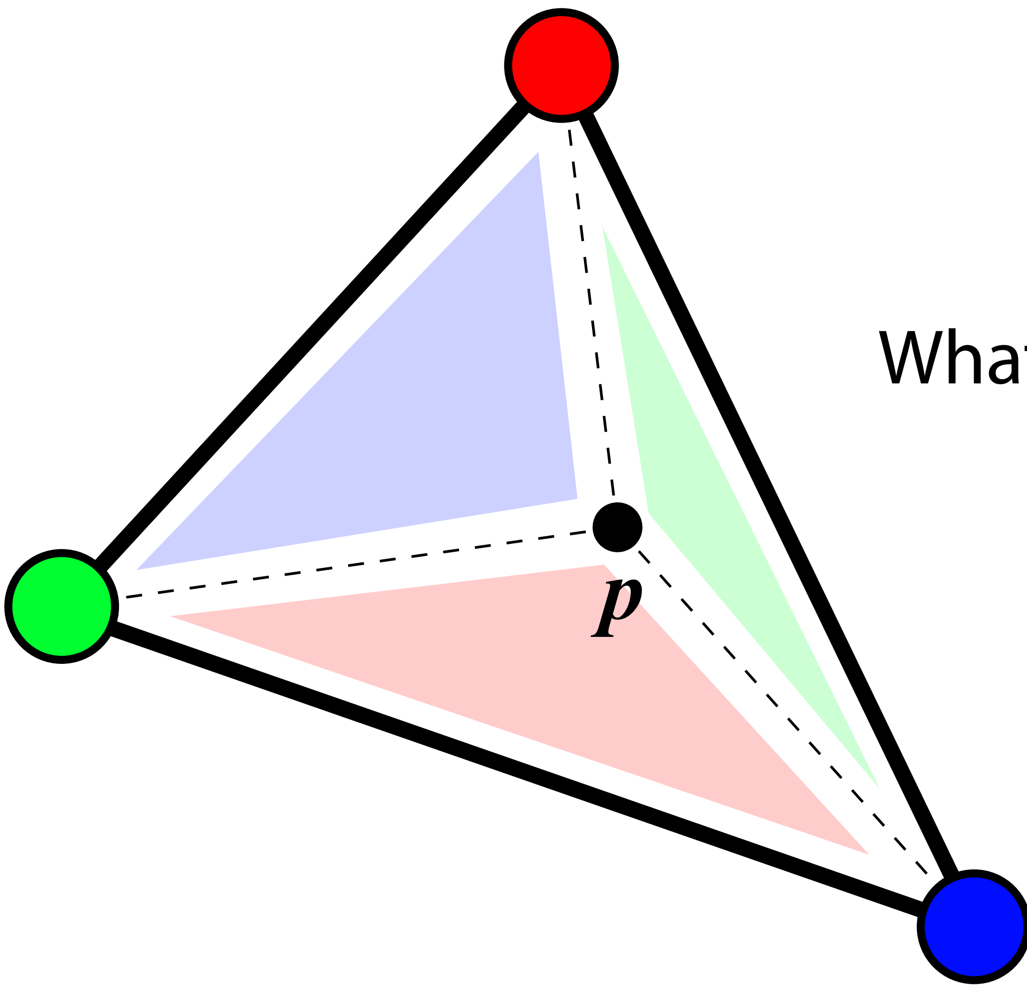
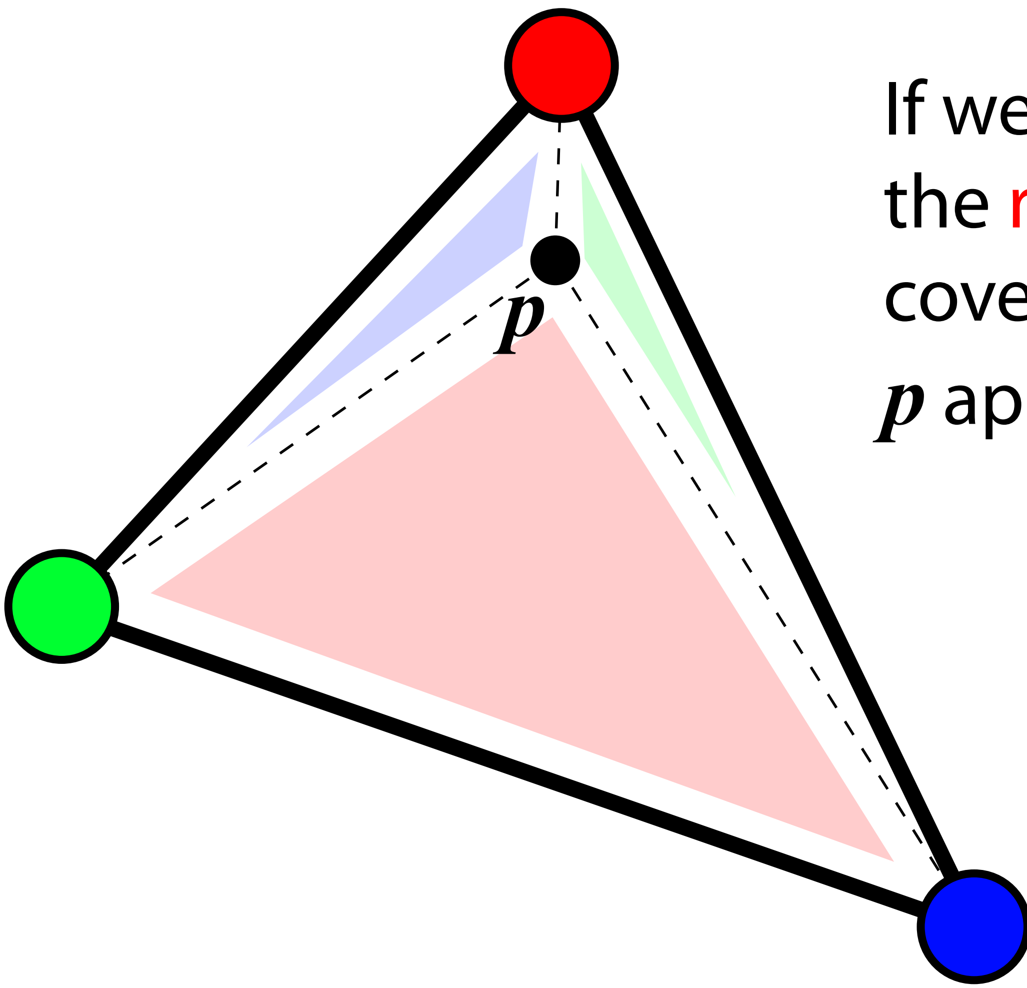


What about ratios of areas (2D)?

Barycentric Interpolation

Now what about triangles?

Just consider the geometry:



If we color the areas carefully, the red area (for example) covers more of the triangle as $\boldsymbol{p}$ approaches the red point.

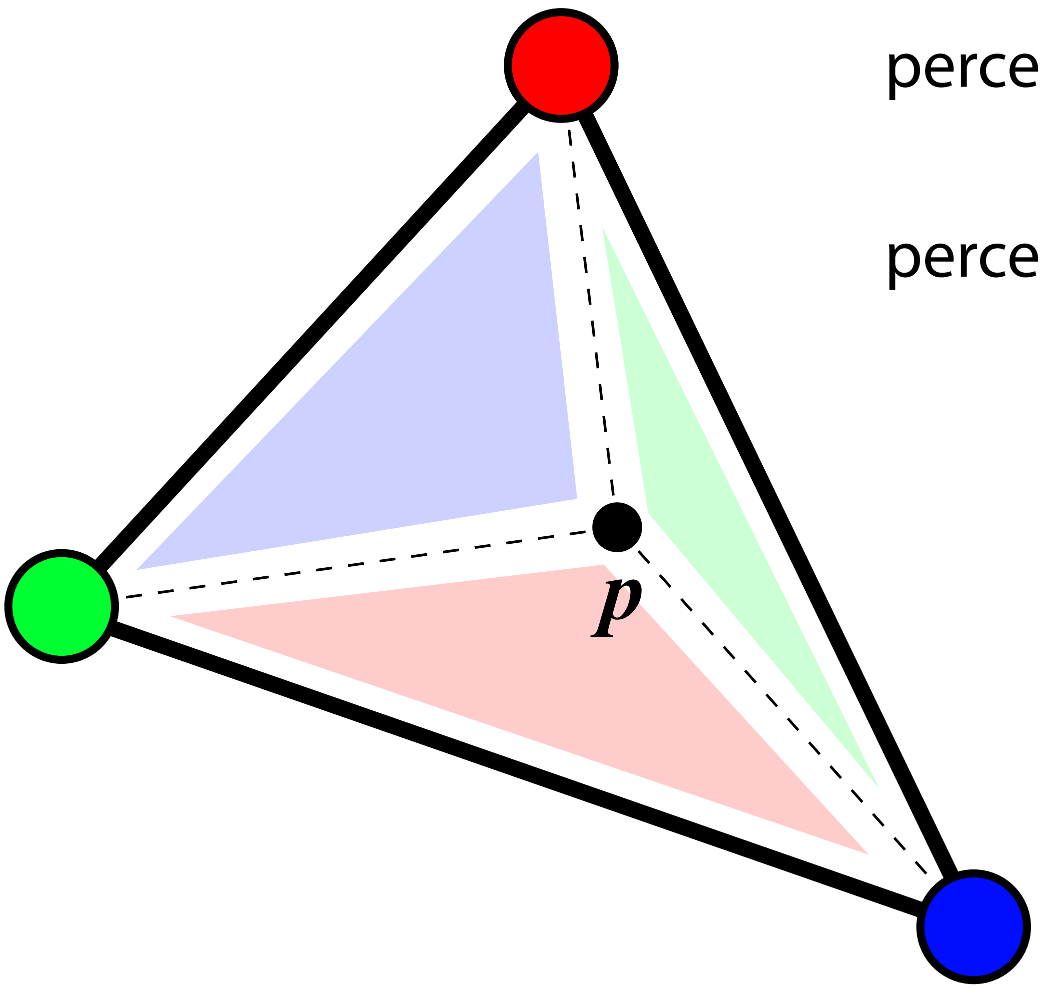
Barycentric Interpolation

Just like before:

$$\text{percent red} = \frac{\text{area of red triangle}}{\text{total area}}$$

$$\text{percent green} = \frac{\text{area of green triangle}}{\text{total area}}$$

$$\text{percent blue} = \frac{\text{area of blue triangle}}{\text{total area}}$$



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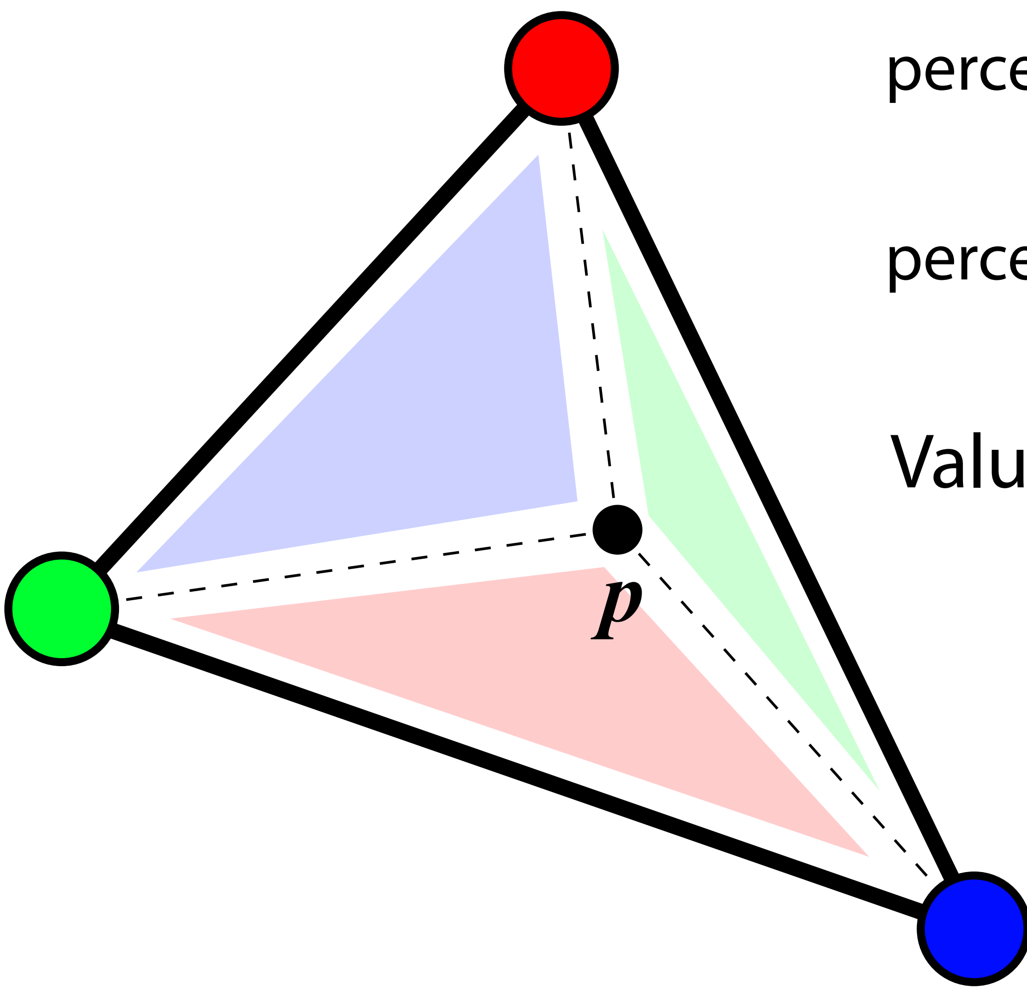


Just like before:

percent red = $\frac{\text{area of red triangle}}{\text{total area}}$

percent green = $\frac{\text{area of green triangle}}{\text{total area}}$

percent blue = $\frac{\text{area of blue triangle}}{\text{total area}}$

Value at ***p***:

(% red)(value at red) +
(% green)(value at green) +
(% blue)(value at blue)

Barycentric Interpolation

$$\Sigma_i \lambda_i = 1$$

Why? Look at the picture!

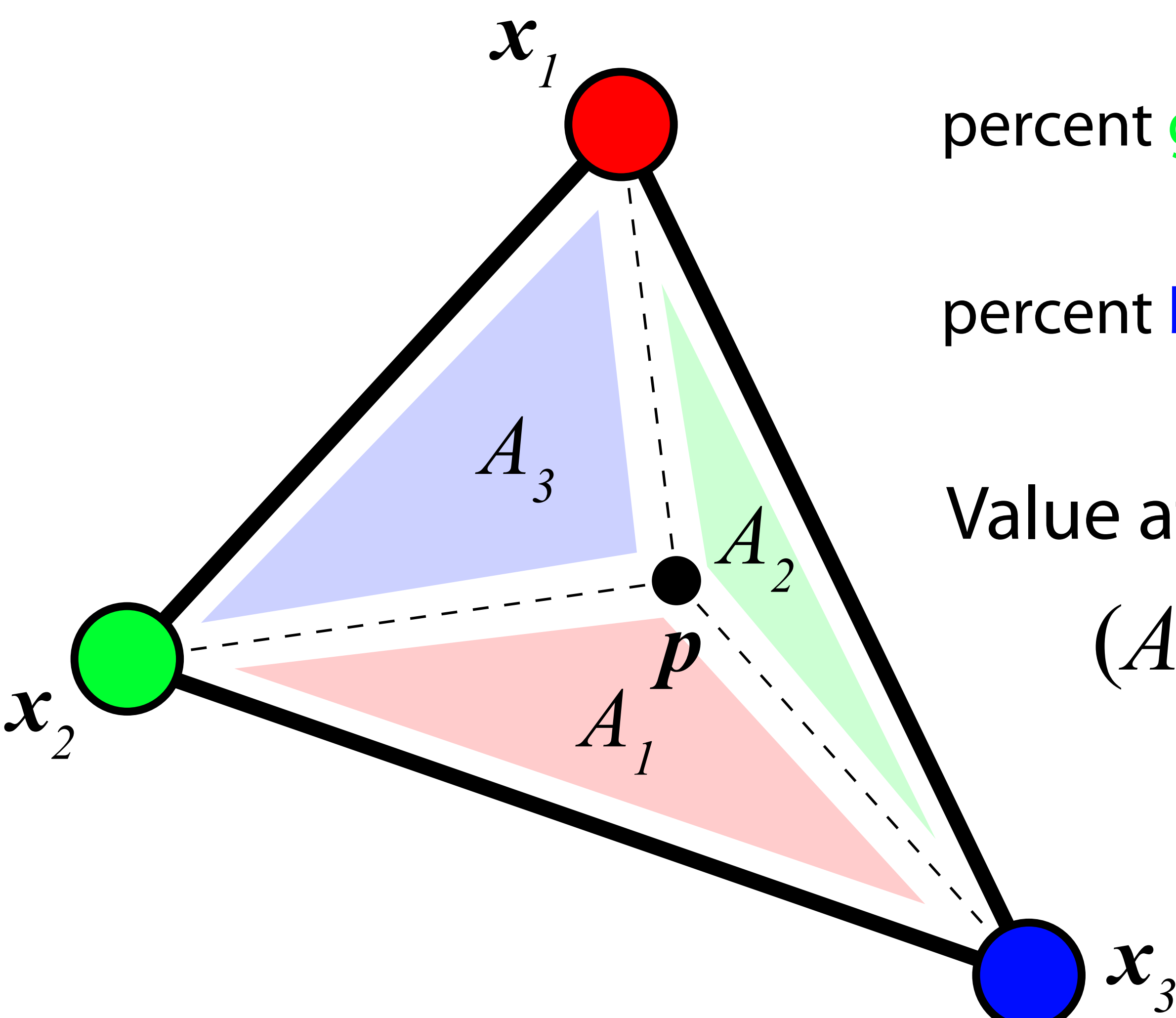
Just like before:

$$\text{percent red} = \frac{A_1}{A} = \lambda_1$$

$$\text{percent green} = \frac{A_2}{A} = \lambda_2$$

$$\text{percent blue} = \frac{A_3}{A} = \lambda_3$$

"barycentric coordinates"



Value at $\boldsymbol{p}$:

$$(A_1\boldsymbol{x}_1 + A_2\boldsymbol{x}_2 + A_3\boldsymbol{x}_3)/A$$

"barycentric interpolation"
a.k.a. "convex combination"
a.k.a. "affine linear extension"

Now convert this to a bunch of ugly symbols if you want... just don't think about it that way!