University of North Carolina at Chapel Hill

SPRING, 2017

COMPUTER SCIENCE

Optimal Estimation in Image Analysis: Homework 1

(Time allowed: due on Tuesday, February 21st, 2017)

Feel free to discuss these problems with other students in class, but turn in your own work. Turn in all the source code you write as well as detailed derivations for the problems.

1. Otsu Thresolding

Otsu thresholding as covered in class finds an optimal threshold for image segmentation by assuming that the image can be partitioned by thresholding into two sets, the foreground and the background, where the image intensities can be modeled by Gaussian distributions with means μ_1 and μ_2 (for foreground and background respectively) and given standard deviation σ for both foreground and background. All pixels in an image are expected to be independent. Pixels within partitions are identically distributed. I.e.,

$$p(I|BG) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(I-\mu_1)^2}{2\sigma^2}} \quad \text{and} \quad p(I|FG) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(I-\mu_2)^2}{2\sigma^2}}, \tag{1}$$

where μ_1 and μ_2 are the means over the partitions. As independence is assumed (for all problems in this homework) we can write the joint likelihood in the background and foreground partitions as

$$p(\{I_i\}|BG) = \prod_{i \in BG} p(I_i|BG) \quad \text{and} \quad p(\{I_i\}|FG) = \prod_{i \in FG} p(I_i|FG). \tag{2}$$

Optimization problems

1) Derive the energy to be minimized for the standard Otsu-thresholding approach (as discussed in class), assuming that σ is given and identical in the foreground and the background partitions.
2) Derive the energy to be minimized as in 1), but assume that both means and standard deviations need to be estimated.
3) Add a prior on the sizes of the partitions to 1). I.e., assume that the size of the foreground partition is Gaussian distributed with a given mean μ_{A-FG} and given standard deviation σ_A and the one for the background partition with a given mean μ_{A-BG} and the same given standard deviation σ_A. Derive the corresponding energy to be minimized. Assume for simplicity that these size distributions are independent, i.e., neglect the fact that the areas of the partitions have to sum up to the overall image area.
4) Add a prior (as in 3)) to the Otsu thresholding formulation in 2) and state the energy to be minimized.

For what follows we call the method of 1) OTSU-M, of 2) OTSU-MS, of 3) OTSU-M-A, and of 4) OTSU-MS-A.

Experiments To explore the behavior of the different methods we have four different test scenarios as follows:

Scenario 1: Noisy image with an equal-sized background and foreground area. Noise in these areas is Gaussian with unknown means, but the same standard deviation.

Scenario 2: Noisy image with an equal-sized background and foreground area. Noise in these areas is Gaussian with unknown means and standard deviations.

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Scenario 3: Noisy image where the backround area is only 5% of the total area. Noise in the foreground and background areas is Gaussian with unknown means, but the same standard deviation.

Scenario 4: Noisy image where the backround area is only 5% of the total area. Noise in the foreground and background areas is Gaussian with unknown means and standard deviation.

For all the four scenarios test the performance of the four different Otsu thresholding methods derived above (OTSU-M, OTSU-MS, OTSU-M-A, and OTSU-MS-A) by brute-force optimization: i.e., discretize the possible thresholds to the integers of the maximum and minimum itensity values in an image and search for the threshold θ which minimizes the respective energies. Here, the background partition is the set of pixels for which $I_i \leq \theta$ and the foreground partition is the set of pixels for which $I_i > \theta$. Assume $\sigma = 25$ for OTSU-M and OTSU-M-A (the standard deviation will need to be estimated for OTSU-MS and OTSU-MS-A). For OTSU-M-A and OTSU-MS-A assume $\sigma_A = 100$ and set the means for the forground and background areas μ_{A-FG} and μ_{A-BG} to the true value for the different scenarios. Means are expressed in numbers of pixels. E.g., $\mu_{A-FG} = 0.5\#pixels$ for scenario 1.

To present the results given the ground-truth segmentation, S compute Dice overlap scores with respect to the estimated segmentation $\hat{S}$ for a given threshold as

$$DC(S, \hat{S}) = \frac{2|S \bigcap \hat{S}|}{|S| + |\hat{S}|}, \tag{3}$$

where $|S|$ denotes the cardinality of set S, i.e., the number of elements in this set, or equivalently the number of pixels in paritition S. Use the provided matlab code, getImages.m, to generate noisy images for the different scenarios. Run 25 repetitions for each experiment (i.e., call getImages 25 times to generate 25 random images for a given scenario). Report the results for each scenario given the four methods using a boxplot summarizing the 25 tries. I.e., you will get 4 images total (one for each scenario) containing 4 boxplots (one for each method) containing the results for the 25 tries. Briefly discuss the behavior you observe.

Figure 1 shows example images for the four scenarios.

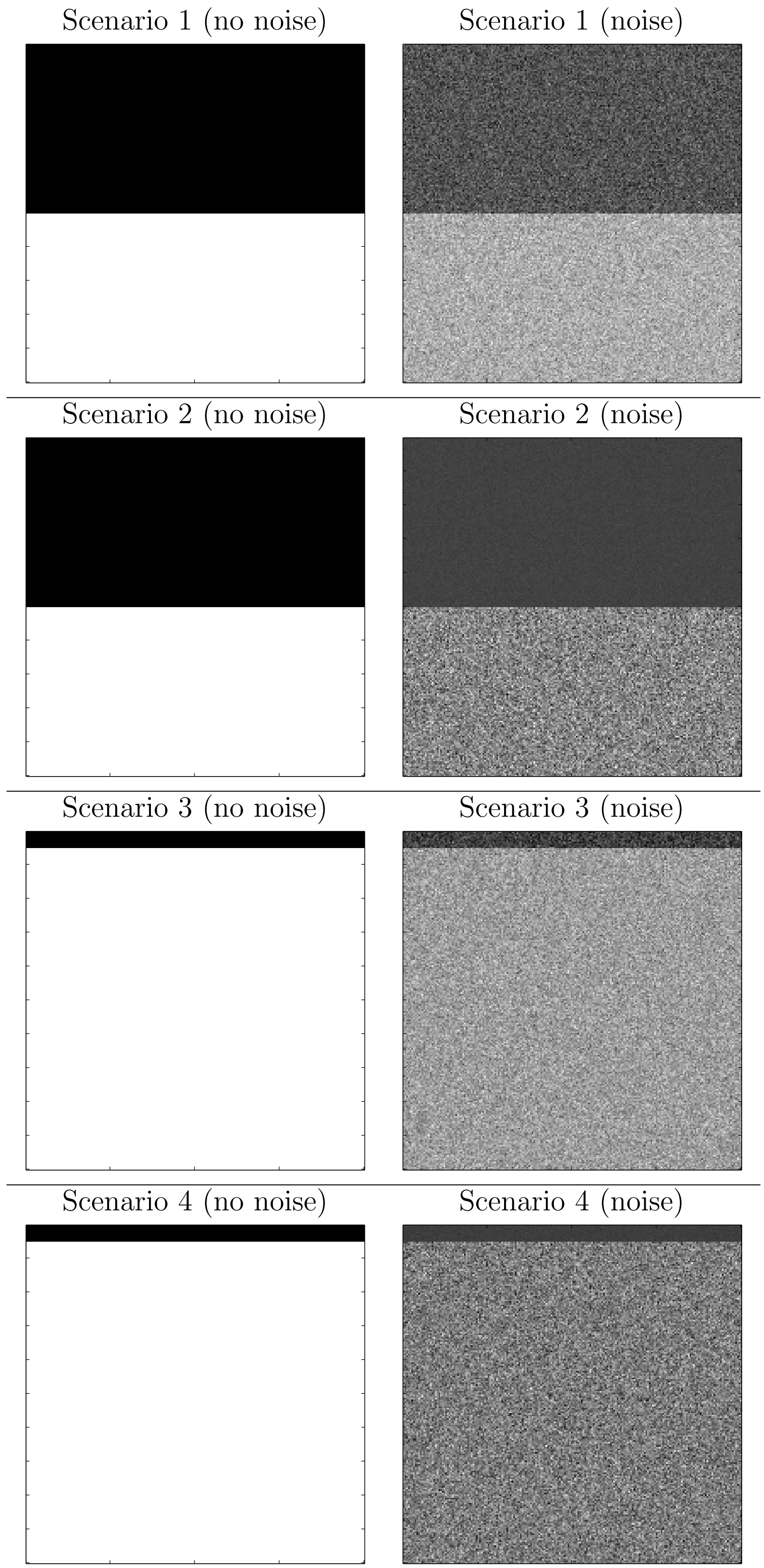

Figure 1: Scenarios 1-4.